

# Joint probabilistic forecasting of wind speed and temperature using Bayesian model averaging

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## Abstract

Ensembles of forecasts are typically employed to account for the forecast uncertainties inherent in predictions of future weather states. However, biases and dispersion errors often present in forecast ensembles require statistical post-processing. Univariate post-processing models such as Bayesian model averaging (BMA) have been successfully applied for various weather quantities. Nonetheless, BMA and many other standard post-processing procedures are designed for a single weather variable, thus ignoring possible dependencies among weather quantities. In line with recently upcoming research to develop multivariate post-processing procedures, e.g., BMA for bivariate wind vectors, or flexible procedures applicable for multiple weather quantities of different types, a bivariate BMA model for joint calibration of wind speed and temperature forecasts is proposed based on the bivariate truncated normal distribution. It extends the univariate truncated normal BMA model designed for post-processing ensemble forecast of wind speed by adding a normally distributed temperature component with a covariance structure representing the dependency among the two weather quantities.

The method is applied to wind speed and temperature forecasts of the eight-member University of Washington mesoscale ensemble and of the eleven-member ALADIN-HUNEPS ensemble of the Hungarian Meteorological Service and its predictive performance is compared to that of the independent BMA calibration of these weather quantities and the general Gaussian copula method. The results indicate improved calibration of probabilistic and accuracy of point forecasts in comparison to the raw ensemble and the independent BMA approach and the overall performance of this bivariate model is able to keep up with that of the Gaussian copula method.

*Key words:* Bayesian model averaging, Gaussian copula, energy score, ensemble calibration, Euclidean error, truncated normal distribution.

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# 1 Introduction

The main objective of weather forecasting is to give a reliable prediction of future atmospheric states on the basis of observational data, prior forecasts valid for the initial time of the forecasts and mathematical models describing the dynamical and physical behaviour of the atmosphere. These models numerically solve the set of the hydro-thermodynamic non-linear partial differential equations of the atmosphere and its coupled systems. A disadvantage of these numerical weather prediction models is that since the atmosphere has a chaotic character the solutions depend on the initial conditions and also on other uncertainties related to the numerical weather prediction process. In practice it means that the results of such models are never fully accurate and the forecast uncertainties should be also taken into account in the forecast preparation. One can reduce the uncertainties by running the model with different initial conditions and produce an ensemble of forecasts. Using a forecast ensemble one can estimate the probability distribution of future weather variables which allows probabilistic weather forecasting (Gneiting and Raftery, 2005), where not only the future atmospheric states are predicted, but also the related uncertainty information such as variance, probabilities of various events, etc. The ensemble prediction method was proposed by Leith (1974) and since its first operational implementation (Buizza *et al.*, 1993; Toth and Kalnay, 1997) it became a widely used technique all over the world (see, e.g., Eckel and Mass, 2005; Leutbecher and Palmer, 2008; Gebhardt *et al.*, 2011; Horányi *et al.*, 2011). However, although, e.g., the ensemble mean on average yields better forecasts of a meteorological quantity than any of the individual ensemble members, it is often the case that the ensemble is under-dispersive and in this way, uncalibrated (Buizza *et al.*, 2005), therefore calibration is absolutely needed to account for this deficiency.

The Bayesian model averaging (BMA) method for calibrating forecast ensembles was introduced by Raftery *et al.* (2005). The BMA predictive probability density function (PDF) of a future weather quantity is the weighted sum of individual PDFs corresponding to the ensemble members. An individual PDF can be interpreted as the conditional PDF of the future weather quantity provided the considered forecast is the best one and the weights are based on the relative performance of the ensemble members during a given training period. In this way BMA is a special, fixed parameter version of dynamic model averaging method developed by Raftery *et al.* (2010). Weights and other model parameters are usually estimated using linear regression and maximum likelihood (ML) method, where the maximum of the likelihood function is mostly found by EM algorithm. At the BMA calibration process one should also take into account whether the ensemble members can be distinguished clearly or some ensemble members are statistically exchangeable (see, e.g., Fraley *et al.*, 2010). In Raftery *et al.* (2005) the BMA method was successfully applied to obtain 48-hour forecasts of surface temperature and sea level pressure in the North American Pacific Northwest based on the 5 members of the University of Washington mesoscale ensemble (Grimit and Mass, 2002). These weather quantities can be modeled by normal distributions, so the predictive PDF is a Gaussian mixture. Later, Sloughter *et al.* (2007) developed a discrete-continuous

BMA model for precipitation forecasting, where the discrete part corresponds to the event of no precipitation, while the cubic root of the precipitation amount (if it is positive) is modeled by a gamma distribution. In Sloughter *et al.* (2010) the BMA method was used for wind speed forecasting and the component PDFs follow generalized gamma distributions. Using a von Mises distribution to model angular data, Bao *et al.* (2010) introduced a BMA scheme to predict surface wind direction. Finally, Baran (2014) suggests the use of a truncated normal mixture for modelling wind speed.

Another possible method for statistical post-processing of ensemble forecasts is the ensemble model output statistics (EMOS) introduced by Gneiting *et al.* (2005) for calibrating forecasts of weather quantities following a normal distribution (sea level pressure, temperature). For these weather variables the EMOS model produces a single normal PDF, where the mean and the variance depend on the ensemble members. Later, Thorarinsdottir and Gneiting (2010) extended this method to truncated normal distributions and used it for calibrating wind speed data, while Scheuerer (2014) developed an EMOS model for precipitation forecasting.

All models mentioned above consider only a single weather quantity and recently an increasing interest has appeared in investigating the correlation between different variables. For calibrating wind vector forecasts, which can be modeled using a bivariate normal distribution, Pinson (2012) suggested an adaptive technique, Sloughter *et al.* (2013) described a BMA model, while Schuhen *et al.* (2012) developed an EMOS approach. A different idea appears in Möller *et al.* (2013), where after performing separate univariate calibrations the authors use a Gaussian copula to preserve the dependence between the weather variables investigated. Finally, for exchangeable ensembles Schefzik *et al.* (2013) introduced the ensemble copula coupling method (ECC) which after univariate calibration uses the rank order information available in the raw ensemble.

In the present paper we develop a BMA model for joint calibration of ensemble forecasts of wind speed and temperature. In our approach the predictive PDF is a mixture of bivariate normal distributions truncated in the first (wind) coordinate from below at the origin. For parameter estimation we use the ML method and the likelihood function is maximized with the help of truncated data EM algorithm for Gaussian mixture models (Lee and Scott, 2012).

We test our model on the ensemble forecasts of maximum wind speed and daily minimum temperature produced by the eight-member University of Washington mesoscale ensemble (UWME; Eckel and Mass, 2005), and compare the results with the performances of the independent BMA calibration of these weather quantities and the Gaussian copula method suggested by Möller *et al.* (2013). Additionally, we perform tests with the operational Limited Area Model Ensemble Prediction System of the Hungarian Meteorological Service (HMS) called ALADIN-HUNEPS (Hágel, 2010; Horányi *et al.*, 2011). We remark that similarly to the UWME, univariate BMA calibration of wind speed (Baran *et al.*, 2013; Baran, 2014) and temperature (Baran *et al.*, 2014) forecasts of the ALADIN-HUNEPS system have already been investigated.

## 2 Data

### 2.1 University of Washington mesoscale ensemble

The eight-member University of Washington mesoscale ensemble covers the Pacific Northwest region of western North America providing forecasts on a 12 km grid. The ensemble members are obtained from different runs of the fifth generation Pennsylvania State University–National Center for Atmospheric Research mesoscale model (PSU-NCAR MM5) with initial conditions from different sources (Grell *et al.*, 1995). Our data base (identical to the one used in Möller *et al.* (2013)) contains ensembles of 48-hour forecasts and corresponding validation observations of 10 meter maximum wind speed (maximum of the hourly instantaneous wind speeds over the previous twelve hours, given in m/s, see, e.g., Sloughter *et al.* (2010)) and 2 meter minimum temperature (given in K) for 152 stations in the Automated Surface Observing Network (National Weather Service, 1998) in the US states of Washington, Oregon, Idaho, California and Nevada for calendar years 2007 and 2008. The forecasts are initialized at 0 UTC (5 pm local time when daylight saving time (DST) is in use and 4 pm otherwise) and the generation of the ensemble implies that its members are not exchangeable. In the present study we investigate only forecasts for calendar year 2008 with additional data from the last two months of 2007 used for parameter estimation. After removing days and locations with missing data, 90 stations remained where the number of days for which forecasts and validating observations are available varies between 141 and 290.

### 2.2 ALADIN-HUNEPS ensemble

The ALADIN-HUNEPS system of the HMS covers a large part of Continental Europe with a horizontal resolution of 12 km and it is obtained by dynamical downscaling (by the ALADIN limited area model) of the global ARPEGE based PEARP system of Météo France (Horányi *et al.*, 2006; Descamps *et al.*, 2009). The ensemble consists of 11 members, 10 initialized from perturbed initial conditions and one control member from the unperturbed analysis, implying that the ensemble contains groups of exchangeable forecasts. The data base contains 11 member ensembles of 42-hour forecasts for 10 meter instantaneous wind speed (given in m/s) and 2 meter temperature (given in K) for 10 major cities in Hungary (Miskolc, Szombathely, Győr, Budapest, Debrecen, Nyíregyháza, Nagykanizsa, Pécs, Kecskemét, Szeged) produced by the ALADIN-HUNEPS system of the HMS, together with the corresponding validating observations for the one-year period between April 1, 2012 and March 31, 2013. The forecasts are initialized at 18 UTC (8 pm local time when DST operates and 7 pm otherwise). The data set is fairly complete since there are only six days when no forecasts are available and these days have been excluded from the analysis.

### 3 Methods

#### 3.1 Bayesian model averaging

Denote by  $f_1, f_2, \dots, f_M$  the ensemble forecast of a certain weather quantity (vector)  $X$  for a given location and time. In the BMA model for ensemble forecasting (Raftery *et al.*, 2005), to each ensemble member  $f_k$  corresponds a component PDF  $g_k(x|f_k, \theta_k)$ , where  $\theta_k$  is a parameter to be estimated. The BMA predictive PDF of  $X$  is

$$p(x|f_1, \dots, f_M; \theta_1, \dots, \theta_M) := \sum_{k=1}^M \omega_k g_k(x|f_k, \theta_k), \quad (3.1)$$

where the weight  $\omega_k$  is connected to the relative performance of the ensemble member  $f_k$  during the training period. Obviously, these weights form a probability vector, that is  $\omega_k \geq 0$ ,  $k = 1, 2, \dots, M$ , and  $\sum_{k=1}^M \omega_k = 1$ . Although this choice of weights is not a real Bayesian one, BMA model (3.1) is simpler than, e.g., the fully Bayesian approach of Di Narzo and Cocchi (2010) or the combined model of Marty *et al.* (2014).

Once the predictive density (3.1) is given, one can take its mean or median as a point forecast for  $X$ . We remark, that for a  $d$ -dimensional distribution function  $F$  a multivariate median is a vector minimizing the function

$$\phi(\boldsymbol{\alpha}) := \int_{\mathbb{R}^d} \|\boldsymbol{\alpha} - \mathbf{x}\| F(d\mathbf{x}),$$

where  $\|\cdot\|$  denotes the Euclidean norm, and if  $F$  is not concentrated on a line in  $\mathbb{R}^d$  then the median is unique (Milasevic and Ducharme, 1987).

BMA model (3.1) is valid only in the cases when the ensemble members are clearly distinguishable, as for the UWME (Eckel and Mass, 2005) or for the COSMO-DE ensemble of the German Meteorological Service (Gebhardt *et al.*, 2011). However, most of the currently used ensemble prediction systems produce ensembles where some ensemble members are statistically indistinguishable. Usually, these exchangeable ensemble members are obtained with the help of perturbations of the initial conditions, which is the case for the 51 member European Centre for Medium-Range Weather Forecasts ensemble (ECMWF; Leutbecher and Palmer, 2008) or for the ALADIN-HUNEPS ensemble described in Section 2.2.

Suppose we have  $M$  ensemble members divided into  $m$  exchangeable groups, where the  $k$ th group contains  $M_k \geq 1$  ensemble members, so  $\sum_{k=1}^m M_k = M$ . Further, denote by  $f_{k,\ell}$  the  $\ell$ th member of the  $k$ th group. For this situation Fraley *et al.* (2010) suggested to use model

$$p(x|f_{1,1}, \dots, f_{1,M_1}, \dots, f_{m,1}, \dots, f_{m,M_m}; \theta_1, \dots, \theta_m) := \sum_{k=1}^m \sum_{\ell=1}^{M_k} \omega_k g_k(x|f_{k,\ell}, \theta_k), \quad (3.2)$$

where ensemble members within a given group have the same weights and parameters.

To simplify notations we give the results and formulae of this section for model (3.1), but their generalization to model (3.2) is rather straightforward.

### 3.2 Bivariate truncated normal model

As it has already been mentioned in the Introduction, for temperature observations a BMA model with normal component PDFs can be fit reasonably well, while for wind speed observations BMA methods with gamma (Slougher *et al.*, 2010) and truncated normal components (Baran, 2014) have been developed. This gives the natural idea of joint modelling wind speed and temperature with a bivariate normal distribution with first (wind) coordinate truncated from below at zero. If

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_W \\ \mu_T \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_W^2 & \sigma_{WT} \\ \sigma_{WT} & \sigma_T^2 \end{bmatrix}$$

are the location vector and scale matrix, respectively, provided  $\Sigma$  is regular, the joint PDF of this special bivariate truncated normal distribution  $\mathcal{N}_2^0(\boldsymbol{\mu}, \Sigma)$  is

$$g(\mathbf{x}|\boldsymbol{\mu}, \Sigma) := \frac{(\det(\Sigma))^{-1/2}}{2\pi\Phi(\mu_W/\sigma_W)} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^\top \Sigma^{-1}(\mathbf{x} - \boldsymbol{\mu})\right) \mathbb{I}_{\{x_W \geq 0\}}, \quad \mathbf{x} = \begin{bmatrix} x_W \\ x_T \end{bmatrix} \in \mathbb{R}^2, \quad (3.3)$$

where  $\Phi$  denotes the cumulative distribution function (CDF) of the standard normal distribution and by  $\mathbb{I}_H$  we denote the indicator function of a set  $H$ . The mean vector  $\boldsymbol{\kappa}$  and covariance matrix  $\Xi$  of  $\mathcal{N}_2^0(\boldsymbol{\mu}, \Sigma)$  are

$$\begin{aligned} \boldsymbol{\kappa} &= \boldsymbol{\mu} + \frac{\varphi(\mu_W/\sigma_W)}{\Phi(\mu_W/\sigma_W)} \begin{bmatrix} \sigma_W \\ \sigma_{WT}/\sigma_W \end{bmatrix} \quad \text{and} \\ \Xi &= \Sigma - \left( \frac{\mu_W}{\sigma_W} \frac{\varphi(\mu_W/\sigma_W)}{\Phi(\mu_W/\sigma_W)} + \left( \frac{\varphi(\mu_W/\sigma_W)}{\Phi(\mu_W/\sigma_W)} \right)^2 \right) \begin{bmatrix} \sigma_W^2 & \sigma_{WT} \\ \sigma_{WT} & \sigma_{WT}^2/\sigma_W^2 \end{bmatrix}, \end{aligned}$$

respectively, where  $\varphi$  denotes the PDF of the standard normal distribution (see, e.g., Rosenbaum, 1961).

By assuming that location vector  $\boldsymbol{\mu}_k$  of the  $k$ th component PDF of the BMA mixture (3.1) is an affine function of the corresponding ensemble member  $\mathbf{f}_k$  and that the scale matrices of all components are equal, we obtain model

$$p(\mathbf{x}|\mathbf{f}_1, \dots, \mathbf{f}_M; A_1, \dots, A_M; B_1, \dots, B_M; \Sigma) := \sum_{k=1}^M \omega_k g(\mathbf{x}|A_k + B_k \mathbf{f}_k, \Sigma), \quad (3.4)$$

where  $g$  is the PDF defined by (3.3),  $A_k \in \mathbb{R}^2$  and  $B_k$  is a two-by-two real matrix. In this way model (3.4) is a direct extension of the univariate BMA models of temperature and



wind speed investigated in Raftery *et al.* (2005) and Baran (2014) where the authors also used the assumption of a common scale parameter for all BMA components. It reduces the number of parameters and makes computations easier.

One can have an even more parsimonious model by using the same bias correction parameters for all ensemble members, resulting in the predictive PDF

$$q(\mathbf{x} | \mathbf{f}_1, \dots, \mathbf{f}_M; A; B; \Sigma) := \sum_{k=1}^M \omega_k g(\mathbf{x} | A + B\mathbf{f}_k, \Sigma). \quad (3.5)$$

We remark that a similar type of simplification is used in the wind speed model of the `ensembleBMA` package of R (Fraley *et al.*, 2009, 2011).

### 3.3 Parameter estimation

Model parameters  $A_k$ ,  $B_k$ ,  $\omega_k$ ,  $k = 1, 2, \dots, M$ , and  $\Sigma$  of PDF (3.4) and  $A$ ,  $B$ ,  $\Sigma$  and  $\omega_k$ ,  $k = 1, 2, \dots, M$ , of PDF (3.5) are estimated using training data consisting of ensemble members and validating observations from the preceding  $n$  days (rolling training period). In what follows,  $\mathbf{f}_{k,s,t}$  denotes the  $k$ th ensemble member vector for location  $s \in \mathcal{S}$  and time  $t \in \mathcal{T}$  and by  $\mathbf{x}_{s,t}$  we denote the corresponding validating observation.

In BMA modelling of uni- and multivariate weather quantities the bias correction parameters are usually estimated with linear regression of the validating observations on the corresponding ensemble members from the training period (see, e.g., Raftery *et al.*, 2005; Sloughter *et al.*, 2010, 2013), while the estimates of weights  $\omega_k$  and scale parameter  $\Sigma$  are calculated by maximizing the likelihood function of the training data using mainly the EM algorithm for mixtures (Dempster *et al.*, 1977; McLachlan and Krishnan, 1997). However, this approach assumes that the location parameter equals the mean or it can easily be derived from it, which is not the case for the truncated normal distribution. Hence, we suggest a pure maximum likelihood method (ML) for estimating all parameters (Sloughter *et al.*, 2010; Baran, 2014).

#### Full model

Under the assumption of independence of forecast errors in space and time the log-likelihood function corresponding to model (3.4) equals

$$\ell(\omega_1, \dots, \omega_M; A_1, \dots, A_M; B_1, \dots, B_M; \Sigma) = \sum_{s,t} \log \left[ \sum_{k=1}^M \omega_k g(\mathbf{x}_{s,t} | A_k + B_k \mathbf{f}_{k,s,t}, \Sigma) \right], \quad (3.6)$$

where the first summation is over all locations  $s \in \mathcal{S}$  and time points  $t$  from the training period containing  $N$  terms ( $N$  distinct values of  $(s, t)$ ).

To find the maximum of the log-likelihood (3.6) we make use of the EM algorithm for truncated normal mixtures suggested by Lee and Scott (2012). Similarly to the traditional EM algorithm for mixtures we introduce latent allocation variables  $z_{k,s,t}$  taking values one or zero according as whether  $\mathbf{x}_{s,t}$  comes from the  $k$ th component PDF or not. The complete data log-likelihood corresponding to the training data and allocations equals

$$\begin{aligned} \ell_C(\omega_1, \dots, \omega_M; A_1, \dots, A_M; B_1, \dots, B_M; \Sigma) \\ = \sum_{s,t} \sum_{k=1}^M z_{k,s,t} \left[ \log(\omega_k) + \log \left( g(\mathbf{x}_{s,t} | A_k + B_k \mathbf{f}_{k,s,t}, \Sigma) \right) \right]. \end{aligned}$$

The EM algorithm starts with initial values of the parameters then alternates between an expectation (E) step and a maximization (M) step until convergence. The coefficients of linear regression of the validating observations on the corresponding ensemble members can serve as initial values of  $A_k^{(0)}$  and  $B_k^{(0)}$ ,  $k = 1, 2, \dots, M$ , the covariance matrix of the validating observations can be taken as  $\Sigma^{(0)}$ , while the initial weights  $\omega_k^{(0)}$ ,  $k = 1, 2, \dots, M$ , might be set to be all equal.

For the truncated normal mixture model given by (3.3) and (3.4) the E step is,

$$z_{k,s,t}^{(j+1)} := \frac{\omega_k^{(j)} g(\mathbf{x}_{s,t} | A_k^{(j)} + B_k^{(j)} \mathbf{f}_{k,s,t}, \Sigma^{(j)})}{\sum_{i=1}^M \omega_i^{(j)} g(\mathbf{x}_{s,t} | A_i^{(j)} + B_i^{(j)} \mathbf{f}_{i,s,t}, \Sigma^{(j)})}, \quad (3.7)$$

where the superscript refers to the actual iteration. Observe, that the above estimates of  $z_{k,s,t}$  are usually not integers even though the true values of these latent allocation variables are either 0 or 1. Further, the first part of the M step is

$$\omega_k^{(j+1)} := \frac{1}{N} \sum_{s,t} z_{k,s,t}^{(j+1)}, \quad (3.8)$$

while the second part can be derived from equations

$$\frac{\partial \ell_C}{\partial A_k} = 0, \quad \frac{\partial \ell_C}{\partial B_k} = 0, \quad \frac{\partial \ell_C}{\partial \Sigma} = 0, \quad k = 1, 2, \dots, M. \quad (3.9)$$

As the above system of equations is nonlinear, we suggest iteration steps

$$\begin{aligned} A_k^{(j+1)} &:= \left[ \sum_{s,t} z_{k,s,t}^{(j+1)} \left( \left( \mathbf{x}_{s,t} - B_k^{(j)} \mathbf{f}_{k,s,t} \right) - \frac{1}{\sigma_W^{(j)}} \frac{\varphi\left(\mu_{W,k,s,t}^{(j)} / \sigma_W^{(j)}\right)}{\Phi\left(\mu_{W,k,s,t}^{(j)} / \sigma_W^{(j)}\right)} \begin{bmatrix} (\sigma_W^{(j)})^2 \\ \sigma_{WT}^{(j)} \end{bmatrix} \right) \right] \left[ \sum_{s,t} z_{k,s,t}^{(j+1)} \right]^{-1}, \\ B_k^{(j+1)} &:= \left[ \sum_{s,t} z_{k,s,t}^{(j+1)} \left( \left( \mathbf{x}_{s,t} - A_k^{(j+1)} \right) - \frac{1}{\sigma_W^{(j)}} \frac{\varphi\left(\tilde{\mu}_{W,k,s,t}^{(j)} / \sigma_W^{(j)}\right)}{\Phi\left(\tilde{\mu}_{W,k,s,t}^{(j)} / \sigma_W^{(j)}\right)} \begin{bmatrix} (\sigma_W^{(j)})^2 \\ \sigma_{WT}^{(j)} \end{bmatrix} \right) \mathbf{f}_{k,s,t}^\top \right] \\ &\quad \times \left[ \sum_{s,t} z_{k,s,t}^{(j+1)} \mathbf{f}_{k,s,t} \mathbf{f}_{k,s,t}^\top \right]^{-1}, \end{aligned} \quad (3.10)$$



$$\Sigma^{(j+1)} := \frac{1}{N} \sum_{s,t} \sum_{k=1}^M z_{k,s,t}^{(j+1)} \left( \left( \mathbf{x}_{s,t} - \boldsymbol{\mu}_{k,s,t}^{(j+1)} \right) \left( \mathbf{x}_{s,t} - \boldsymbol{\mu}_{k,s,t}^{(j+1)} \right)^\top + \boldsymbol{\mu}_{k,s,t}^{(j+1)} \frac{1}{\sigma_W^{(j)}} \frac{\varphi\left(\mu_{W,k,s,t}^{(j+1)}/\sigma_W^{(j)}\right)}{\Phi\left(\mu_{W,k,s,t}^{(j+1)}/\sigma_W^{(j)}\right)} \begin{bmatrix} (\sigma_W^{(j)})^2 & \sigma_{WT}^{(j)} \\ \sigma_{WT}^{(j)} & (\sigma_{WT}^{(j)}/\sigma_W^{(j)})^3 \end{bmatrix} \right),$$

where  $\mu_{W,k,s,t}^{(j)}$  and  $\tilde{\mu}_{W,k,s,t}^{(j)}$  denote the first (wind) coordinates of  $\boldsymbol{\mu}_{k,s,t}^{(j)} := A_k^{(j)} + B_k^{(j)} \mathbf{f}_{k,s,t}$  and  $\tilde{\boldsymbol{\mu}}_{k,s,t}^{(j)} := A_k^{(j+1)} + B_k^{(j)} \mathbf{f}_{k,s,t}$ , respectively.

### Parsimonious model

For the parsimonious model (3.5) the log-likelihood function is obviously

$$\ell(\omega_1, \dots, \omega_M; A; B; \Sigma) = \sum_{s,t} \log \left[ \sum_{k=1}^M \omega_k g(\mathbf{x}_{s,t} | A + B \mathbf{f}_{k,s,t}, \Sigma) \right],$$

which is maximized using the same type of EM algorithm as before. The E step, and the iterations corresponding to  $\omega_k^{(j+1)}$  and  $\Sigma^{(j+1)}$  are obvious modifications of (3.7), (3.8) and of the last iteration of (3.10), respectively, while the first two iterations of (3.10) should be replaced by

$$\begin{aligned} A^{(j+1)} &:= \frac{1}{N} \sum_{s,t} \sum_{k=1}^M z_{k,s,t}^{(j+1)} \left( \left( \mathbf{x}_{s,t} - B^{(j)} \mathbf{f}_{k,s,t} \right) - \frac{1}{\sigma_W^{(j)}} \frac{\varphi\left(\mu_{W,k,s,t}^{(j)}/\sigma_W^{(j)}\right)}{\Phi\left(\mu_{W,k,s,t}^{(j)}/\sigma_W^{(j)}\right)} \begin{bmatrix} (\sigma_W^{(j)})^2 \\ \sigma_{WT}^{(j)} \end{bmatrix} \right), \\ B^{(j+1)} &:= \sum_{s,t} \sum_{k=1}^M z_{k,s,t}^{(j+1)} \left( \left( \mathbf{x}_{s,t} - A^{(j+1)} \right) - \frac{1}{\sigma_W^{(j)}} \frac{\varphi\left(\tilde{\mu}_{W,k,s,t}^{(j)}/\sigma_W^{(j)}\right)}{\Phi\left(\tilde{\mu}_{W,k,s,t}^{(j)}/\sigma_W^{(j)}\right)} \begin{bmatrix} (\sigma_W^{(j)})^2 \\ \sigma_{WT}^{(j)} \end{bmatrix} \right) \mathbf{f}_{k,s,t}^\top \\ &\quad \times \left[ \sum_{s,t} \sum_{k=1}^M z_{k,s,t}^{(j+1)} \mathbf{f}_{k,s,t} \mathbf{f}_{k,s,t}^\top \right]^{-1}. \end{aligned}$$

In this case  $\mu_{W,k,s,t}^{(j)}$  and  $\tilde{\mu}_{W,k,s,t}^{(j)}$  denote the first coordinates of  $\boldsymbol{\mu}_{k,s,t}^{(j)} := A^{(j)} + B^{(j)} \mathbf{f}_{k,s,t}$  and  $\tilde{\boldsymbol{\mu}}_{k,s,t}^{(j)} := A^{(j+1)} + B^{(j)} \mathbf{f}_{k,s,t}$ , respectively.

### 3.4 Multivariate scores

To check the goodness of fit of bivariate probabilistic forecasts and the corresponding point forecasts we apply the methods suggested by Gneiting *et al.* (2008).

For inspecting calibration of univariate ensemble forecasts a popular tool is the verification rank histogram (or Talagrand diagram) which is the histogram of ranks of validating

observations with respect to the corresponding ensemble forecasts (see, e.g., Wilks, 2011, Section 8.7.2). In case of proper calibration the ranks follow a uniform distribution on  $\{1, 2, \dots, M+1\}$ , and the deviation from uniformity can be quantified by the reliability index  $\Delta$  defined by

$$\Delta := \sum_{r=1}^{M+1} \left| \rho_r - \frac{1}{M+1} \right|, \quad (3.11)$$

where  $\rho_j$  is the relative frequency of rank  $r$  (Delle Monache *et al.*, 2006). In the multivariate case the usual problem is the proper definition of ranks – in the present work we use the multivariate ordering proposed by Gneiting *et al.* (2008). For a probabilistic forecast one can calculate the reliability index from a preferably large number of ensembles (we use 100) sampled from the predictive PDF and the corresponding verifying observations.

In the univariate case, sharpness of an ensemble forecast or of a predictive distribution can be quantified by its standard deviation. An obvious generalization of this idea to  $d$ -dimensional quantities is the determinant sharpness DS defined as

$$\text{DS} := (\det(\Sigma))^{1/(2d)}, \quad (3.12)$$

where  $\Sigma$  is the covariance matrix of an ensemble or of a predictive PDF.

For evaluating density forecasts of univariate quantities the continuous ranked probability score (CRPS) is a widely accepted and used proper scoring rule (Gneiting and Raftery, 2007; Wilks, 2011). A direct multivariate extension of the CRPS is the energy score introduced by Gneiting and Raftery (2007). Given a CDF  $F$  on  $\mathbb{R}^d$  and a  $d$ -dimensional vector  $\mathbf{x}$ , the energy score is defined as

$$\text{ES}(F, \mathbf{x}) := \mathbb{E} \|\mathbf{X} - \mathbf{x}\| - \frac{1}{2} \mathbb{E} \|\mathbf{X} - \mathbf{X}'\|, \quad (3.13)$$

where  $\mathbf{X}$  and  $\mathbf{X}'$  are independent random vectors having distribution  $F$ . However, for a mixture of truncated bivariate normal distributions considered in the present work the energy score cannot be given in a closed form, so it is replaced by a Monte Carlo approximation

$$\widehat{\text{ES}}(F, \mathbf{x}) := \frac{1}{n} \sum_{j=1}^n \|\mathbf{X}_j - \mathbf{x}\| - \frac{1}{2(n-1)} \sum_{j=1}^{n-1} \|\mathbf{X}_j - \mathbf{X}_{j+1}\|, \quad (3.14)$$

where  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_n$  is a (large, we use  $n = 10000$ ) random sample from  $F$  (Gneiting *et al.*, 2008). Finally, if  $F$  is a CDF corresponding to a forecast ensemble  $\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_M$  then (3.13) reduces to

$$\text{ES}(F, \mathbf{x}) = \frac{1}{M} \sum_{j=1}^M \|\mathbf{f}_j - \mathbf{x}\| - \frac{1}{2M^2} \sum_{j=1}^M \sum_{k=1}^M \|\mathbf{f}_j - \mathbf{f}_k\|. \quad (3.15)$$

Finally, for point forecasts (mean and median) one can consider the mean Euclidean distance (EE) of forecasts from the corresponding validating observations. For multivariate

forecasts the ensemble median can be obtained using the Newton-type algorithm given in Dennis and Schnabel (1983), the algorithm of Vardi and Zhang (2000), or any other method implemented, e.g., in the R package `pcaPP` (Fritz *et al.*, 2012). For a predictive distribution  $F$  one may apply the same algorithm on a preferably large sample from  $F$ .

## 4 Results

As it has been mentioned in the Introduction, the predictive skills of the bivariate BMA models (3.4) and (3.5) are tested on the eight-member UWME and ensemble forecasts produced by the ALADIN-HUNEPS system of the HMS. We quantify the goodness of fit of the predictive PDFs and point forecasts with the help of multivariate scores described in Section 3.4 and compare the results to the performances of independent BMA calibration of wind speed (Baran, 2014) and temperature (Raftery *et al.*, 2005) and the Gaussian copula method proposed by Möller *et al.* (2013). For the case study conducted in Möller *et al.* (2013), the univariate BMA post-processing of the copula margins is performed at each considered station individually, as the performance of the method at specific stations as well as the structure of correlations were investigated. For the analysis in this paper the copula margins are formed by applying a global BMA model to have a better comparability to the bivariate truncated normal BMA model. This leads to the estimation of only one single correlation matrix over all considered stations instead of station specific correlation matrices.

### 4.1 University of Washington mesoscale ensemble

#### Raw ensemble

Earlier studies dealing with statistical calibration of the UWME (see, e.g., Thorarinsdottir and Gneiting, 2010; Fraley *et al.*, 2010) found that both wind speed and temperature forecasts are strongly under-dispersive and in this way they are uncalibrated. This under-dispersive character can clearly be observed in Figure 1 showing the univariate verification rank histograms of wind speed and temperature as well as their joint multivariate rank histogram. All three histograms are far from the desired uniform distribution and in many cases the ensemble members either underestimate or overestimate the validating observations. The ensemble ranges contain the observed maximum wind speed and minimum temperature only in 45.43% and 35.69% of the cases, respectively. The reliability index  $\Delta$  computed from the multivariate ranks equals 0.550, while the  $\Delta$  values corresponding to the univariate ranks of wind speed and temperature observations are given as 0.647 and 0.842, respectively. Verifying observations of wind speed and temperature for calendar year 2008 taken along all dates and locations show a positive correlation of 0.125, while the correlations of forecast errors of the ensemble median and mean are 0.187 and 0.189, respectively. These correlation values justify the need of a bivariate model for the investigated weather variables.

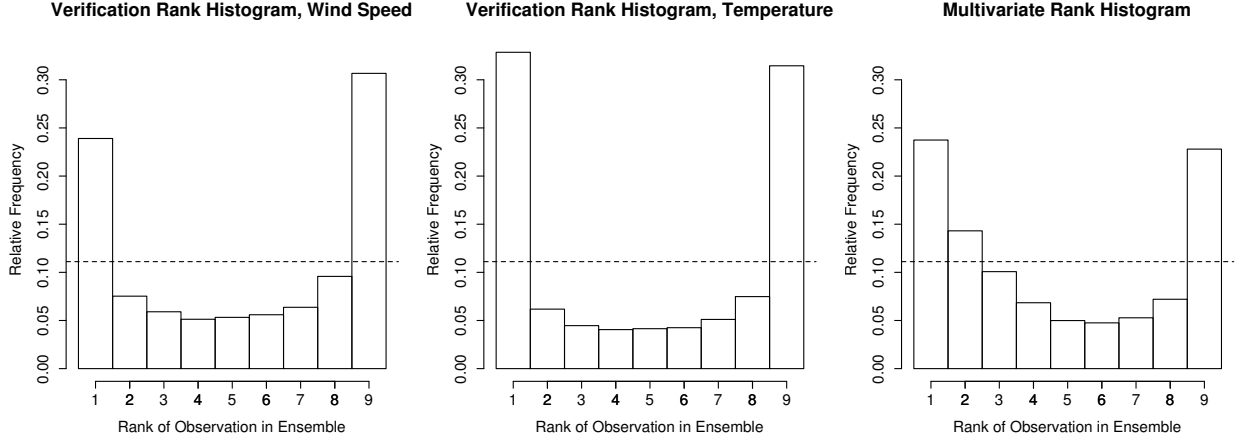


Figure 1: Verification rank histograms of the 8-member UMWE forecasts of maximum wind speed (*left*) and minimum temperature (*center*) and the multivariate rank histogram (*right*). Period: January 1, 2008 – December 31, 2008.

	Probabilistic forecasts			Median forecasts			Mean forecasts		
	ES	$\Delta$	DS	EE	$\varrho$	$\varrho_{err}$	EE	$\varrho$	$\varrho_{err}$
BMA	2.110	0.015	2.250	2.973	0.154	0.182	2.972	0.155	0.183
Pars. BMA	2.117	0.033	2.286	2.967	0.180	0.182	2.967	0.171	0.182
Indep. BMA	2.124	0.048	2.320	2.977	0.163	0.175	2.977	0.151	0.177
Copula	2.089	0.030	2.272	2.977	0.160	0.176	2.978	0.152	0.177
Raw ensemble	2.562	0.550	0.773	3.087	0.017	0.187	3.072	0.007	0.189

Table 1: Mean energy score (ES), reliability index ( $\Delta$ ) and mean determinant sharpness (DS) of probabilistic forecasts, mean Euclidean error (EE) of point forecasts (median/mean), empirical correlation ( $\varrho$ ) and empirical correlation of errors ( $\varrho_{err}$ ) of wind speed and temperature components of point forecasts for the UWME. Empirical correlation of observations corresponding to the forecast cases: 0.125.

As the eight members of the UWME are non exchangeable, in what follows we consider BMA models (3.4) and (3.5) with  $M = 8$ .

### Bivariate ensemble calibration

In the present work we apply the same training period length of 40 days as in Möller *et al.* (2013) which was determined with the help of an exploratory data analysis on a subset of the data set. Since in our rolling training periods for estimating the BMA parameters we can also use data from calendar year 2007, BMA models can be produced for the whole calendar year 2008. This means 291 calendar days (after excluding dates with missing data) and a total of 24302 individual forecast cases. Similar to the case study performed in Möller

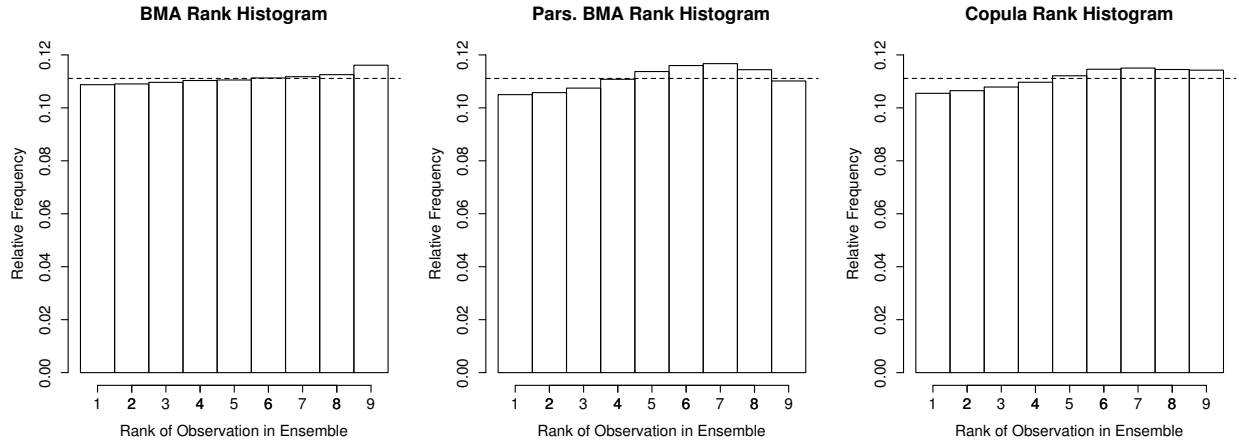


Figure 2: Multivariate rank histograms for BMA (*left*), parsimonious BMA (*center*) and Gaussian copula (*right*) post-processed UWME forecasts of maximum wind speed and minimum temperature.

*et al.* (2013) involving the UWME ensemble, the data from 2007 are utilized for correlation estimation. The resulting global correlation matrix is then employed for the analysis of the 2008 data.

Table 1 shows the mean energy score (ES), reliability index ( $\Delta$ ) and mean determinant sharpness (DS) of probabilistic forecasts, the mean Euclidean error (EE) of point forecasts (median/mean) and the empirical correlation of their wind speed and temperature components together with the correlation of the forecast errors, calculated using both bivariate BMA models considered, independent BMA models of wind speed and temperature, the copula model of Möller *et al.* (2013) and the raw ensemble. All four post-processing methods result in substantial improvement in calibration of the probabilistic forecasts, quantified by the decrease of the mean energy score and reliability index, and in a slight improvement in accuracy of median and mean forecasts (see the corresponding EE values). The improvement in calibration can clearly be observed on the difference between the U-shaped multivariate rank histogram of the raw ensemble (see Figure 1) and the rank histograms of post-processed forecasts plotted in Figure 2, which are almost uniform. Furthermore, the empirical correlations of wind speed and temperature components of all post-processed point forecasts are close to the correlation of 0.125 of the verifying wind speed and temperature observations, while the corresponding correlations of ensemble median and mean are smaller by a magnitude, which is a weakness of the raw ensemble. However, one should also remark that the correlations of errors of all point forecasts (including ensemble median and mean) are very similar to each other. The DS of the predictive PDFs is much higher than that of the raw ensemble, however, this is a direct consequence of the small dispersion of the latter (see Figure 1). Comparing the different post-processing techniques it is noticeable that the bivariate methods outperform the independent BMA approach and the largest difference among the various calibration methods appears in the reliability indices.

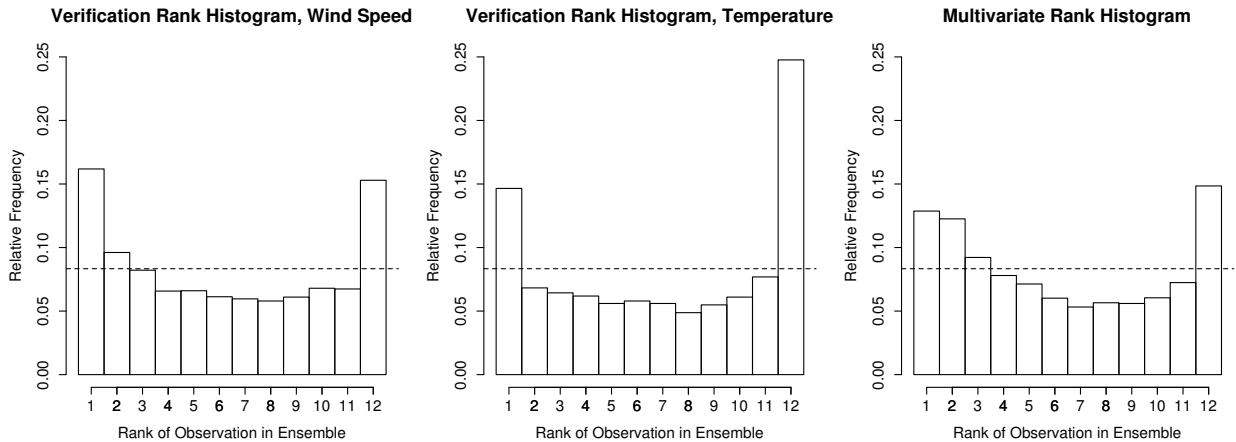


Figure 3: Verification rank histograms of the 11-member ALADIN-HUNEPS ensemble forecasts of wind speed (*left*) and temperature (*center*) and the multivariate rank histogram (*right*). Period: April 1, 2012 – March 31, 2013.

The smallest  $\Delta$  value corresponds to the full BMA model (3.4) which is in line with the shapes of the corresponding multivariate rank histograms in Figure 2. The rank histogram of this model is closest to uniformity, while the rank histograms of the parsimonious BMA and the copula model both exhibit similar deviations from the uniform distribution. The smallest mean energy score belongs to the copula method, while the bivariate BMA models produce slightly more accurate point forecasts. The full BMA model (3.4) outperforms its parsimonious counterpart (3.5) in terms of ES,  $\Delta$ , and DS and from the competing methods this has the best overall performance.

## 4.2 ALADIN-HUNEPS ensemble

### Raw ensemble

Similar to the UWME, wind speed and temperature forecasts of the ALADIN-HUNEPS ensemble are found to be under-dispersive (Baran *et al.*, 2013, 2014). The univariate verification rank histograms of wind speed and temperature as well as their joint multivariate rank histogram plotted in Figure 3 are strongly U-shaped and the ensemble ranges contain the observed wind speed and temperature only in 68.52% and 60.61% of the cases, respectively. The reliability index computed from the multivariate ranks equals 0.317, while the reliability indices obtained from the univariate ranks of wind speed and temperature observations are 0.322 and 0.455, respectively. Observations of wind speed and temperature taken along all dates and locations show a slight negative correlation of  $-0.029$ , while the correlations of forecast errors of the ensemble median and mean are 0.119 and 0.123, respectively. This smaller correlation of the observed values, compared to the UWME, might be explained by the different types of wind and temperature quantities being analyzed, while the forecast

error correlations of ensemble median and mean support the idea of bivariate modelling.

Following Baran *et al.* (2013, 2014) we consider two different groupings of ensemble members. In the first case we have two exchangeable groups. One contains the control denoted by  $\mathbf{f}_c$  while the other group contains the remaining 10 ensemble members corresponding to the different perturbed initial conditions denoted by  $\mathbf{f}_{p,1}, \dots, \mathbf{f}_{p,10}$ . This leads us to the predictive PDF

$$p_{AH2}(\mathbf{x} | \mathbf{f}_c, \mathbf{f}_{p,1}, \dots, \mathbf{f}_{p,10}; A_c, A_p; B_c, B_p; \Sigma) = \omega g(\mathbf{x} | A_c + B_c \mathbf{f}_c, \Sigma) + \frac{1-\omega}{10} \sum_{\ell=1}^{10} g(\mathbf{x} | A_p + B_p \mathbf{f}_{p,\ell}, \Sigma), \quad (4.1)$$

which is a particular case of model (3.4), and to its parsimonious version

$$q_{AH2}(\mathbf{x} | \mathbf{f}_c, \mathbf{f}_{p,1}, \dots, \mathbf{f}_{p,10}; A, B; \Sigma) = \omega g(\mathbf{x} | A + B \mathbf{f}_c, \Sigma) + \frac{1-\omega}{10} \sum_{\ell=1}^{10} g(\mathbf{x} | A + B \mathbf{f}_{p,\ell}, \Sigma) \quad (4.2)$$

corresponding to model (3.5), where  $\omega \in [0, 1]$ , and  $g$  is defined by (3.3).

In the second case the odd and even numbered exchangeable ensemble members form two separate groups  $\{\mathbf{f}_{p,1}, \mathbf{f}_{p,3}, \mathbf{f}_{p,5}, \mathbf{f}_{p,7}, \mathbf{f}_{p,9}\}$  and  $\{\mathbf{f}_{p,2}, \mathbf{f}_{p,4}, \mathbf{f}_{p,6}, \mathbf{f}_{p,8}, \mathbf{f}_{p,10}\}$ , respectively. This idea is justified by the method of generating their initial conditions. To obtain the initial conditions for the ALADIN-HUNEPS forecasts only five perturbations are calculated and then they are added to (odd numbered members) and subtracted from (even numbered members) the unperturbed initial conditions (Horányi *et al.*, 2011; Baran *et al.*, 2013, 2014). In this way we obtain the following PDFs for the forecasted vector of wind speed and temperature corresponding to models (3.4) and (3.5), respectively,

$$p_{AH3}(\mathbf{x} | \mathbf{f}_c, \mathbf{f}_{p,1}, \dots, \mathbf{f}_{p,10}; A_c, A_o, A_e; B_c, B_o, B_e; \Sigma) = \omega_c g(\mathbf{x} | A_c + \mathbf{f}_c B_c, \Sigma) + \sum_{\ell=1}^5 \left( \omega_o g(\mathbf{x} | A_o + B_o \mathbf{f}_{p,2\ell-1}, \Sigma) + \omega_e g(\mathbf{x} | A_e + B_e \mathbf{f}_{p,2\ell}, \Sigma) \right), \quad (4.3)$$

$$q_{AH3}(\mathbf{x} | \mathbf{f}_c, \mathbf{f}_{p,1}, \dots, \mathbf{f}_{p,10}; A, B; \Sigma) = \omega_c g(\mathbf{x} | A + \mathbf{f}_c B, \Sigma) + \sum_{\ell=1}^5 \left( \omega_o g(\mathbf{x} | A + B \mathbf{f}_{p,2\ell-1}, \Sigma) + \omega_e g(\mathbf{x} | A + B \mathbf{f}_{p,2\ell}, \Sigma) \right), \quad (4.4)$$

where for weights  $\omega_c, \omega_o, \omega_e \in [0, 1]$  we have  $\omega_c + 5\omega_o + 5\omega_e = 1$ .

## Bivariate ensemble calibration

Based on a preliminary data analysis (univariate BMA and EMOS calibration of wind speed and temperature forecasts) we use a 40 days training period. In this way ensemble members, validating observations and BMA models are available for the period 12.05.2012–31.03.2013



		Probabilistic forecasts			Median forecasts			Mean forecasts		
		ES	$\Delta$	DS	EE	$\varrho$	$\varrho_{err}$	EE	$\varrho$	$\varrho_{err}$
Two groups	BMA	1.434	0.031	1.539	2.004	−0.032	0.129	2.007	−0.041	0.129
	Pars. BMA	1.428	0.021	1.534	1.999	−0.031	0.131	1.998	−0.035	0.128
	Indep. BMA	1.454	0.015	1.573	2.033	−0.018	0.119	2.032	−0.030	0.119
	Copula	1.393	0.063	1.526	2.032	−0.021	0.119	2.031	−0.030	0.119
Three groups	BMA	1.442	0.031	1.529	2.017	−0.035	0.129	2.020	−0.038	0.128
	Pars. BMA	1.428	0.021	1.530	1.999	−0.030	0.128	1.997	−0.035	0.126
	Indep. BMA	1.452	0.013	1.567	2.031	−0.016	0.113	2.029	−0.028	0.114
	Copula	1.389	0.066	1.521	2.029	−0.019	0.114	2.028	−0.028	0.114
Raw ensemble		1.623	0.327	0.935	2.102	−0.068	0.122	2.083	−0.060	0.124

Table 2: Mean energy score (ES), reliability index ( $\Delta$ ) and mean determinant sharpness (DS) of probabilistic forecasts, mean Euclidean error (EE) of point forecasts (median/mean), empirical correlation ( $\varrho$ ) and empirical correlation of errors ( $\varrho_{err}$ ) of wind speed and temperature components of point forecasts for the ALADIN-HUNEPS ensemble. Empirical correlation of observations corresponding to the forecast cases: −0.033.

(just after the first 40 days training period having 318 calendar days, since on six days all ensemble members are missing). In line with the case study performed in Möller *et al.* (2013), additional data of the period 01.10.2010–25.03.2011 are utilized to estimate the correlation matrix of the Gaussian copula. For the BMA fits that are employed to estimate this correlation structure, a 40 days training period was used as well. The resulting (global) correlation matrix is then carried forward into the analysis of the 2012/2013 data.

The verification scores quantifying the effects of ensemble post-processing are given in Table 2. Considering first the probabilistic forecasts one can observe that compared to the raw ensemble the BMA and copula predictive PDFs are smaller in energy score and reliability index and higher in determinant sharpness. Again, the latter fact comes from the under-dispersive character of the raw ensemble illustrated by Figure 3, while the rank histograms of the post-processed forecasts, plotted in Figure 4, clearly illustrate the improvement in calibration. Regarding median/mean forecasts calculated from the above mentioned predictive PDFs, they all produce smaller EE values than the ensemble median/mean vectors. Furthermore, the correlations of wind speed and temperature components of the bivariate BMA post-processed median/mean forecasts are close to the correlation of −0.033 of the validating observations, the empirical correlations of the ensemble median and mean are around the double of this value, while the estimated correlations of the independent univariate BMA and copula median forecasts are much closer to zero. Finally, both the raw ensemble and all calibrated point forecasts exhibit almost the same forecast error correlations.

In accordance with the results for the UWME, the most substantial difference between the BMA and copula approaches appears in the reliability indices. Bivariate BMA post-

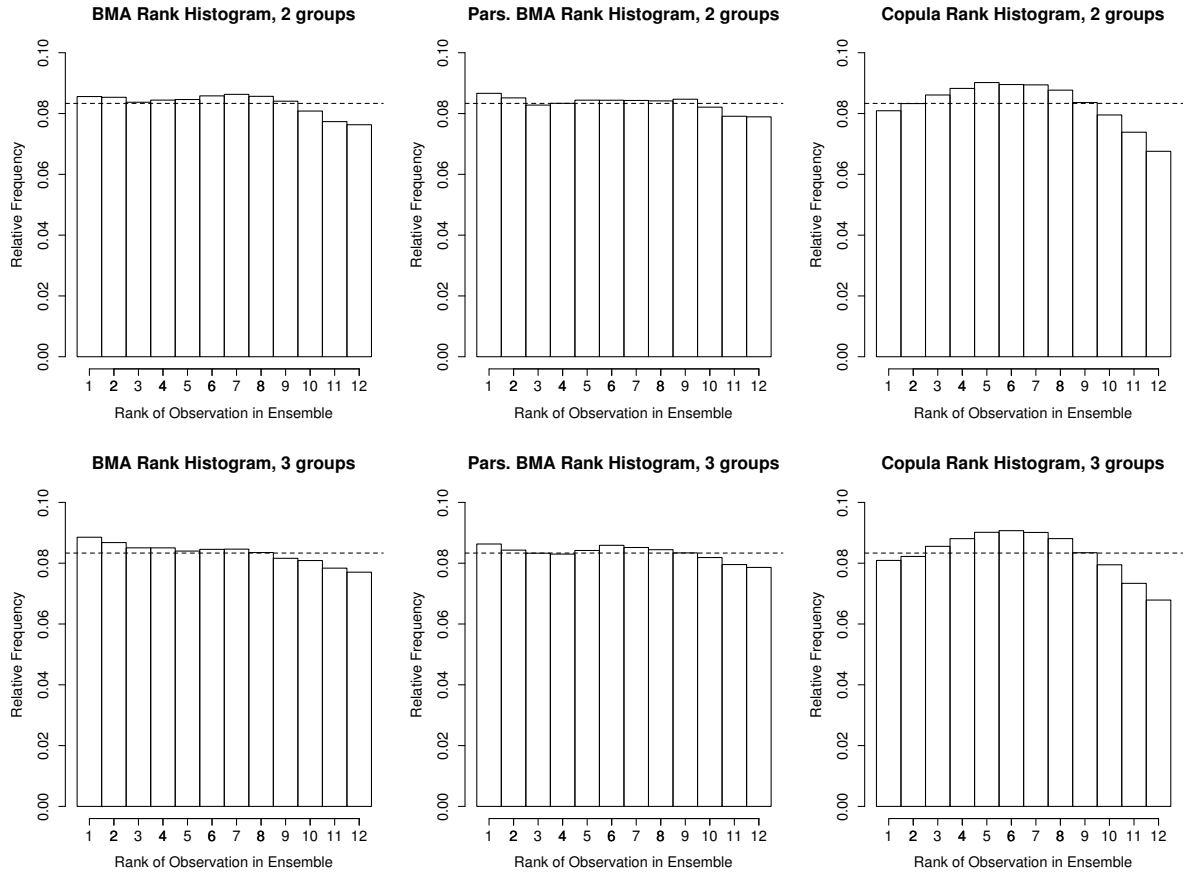


Figure 4: Multivariate rank histograms for BMA (*left*), parsimonious BMA (*center*) and Gaussian copula (*right*) post-processed ALADIN-HUNEPS forecasts of wind speed and temperature using two- and three-group models.

processing results in much smaller  $\Delta$  values than the corresponding reliability indices of the copula method, which is in line with the multivariate rank histograms in Figure 4. The histograms of both bivariate BMA methods are closer to uniformity than the one of the copula method, for the two- as well as for the three-group model. The rank histograms of the copula method are slightly hump-shaped and skewed, as the last bins are less filled than the others, indicating some over-dispersion and bias of the post-processed forecasts (see, e.g., Gneiting *et al.*, 2008). This phenomenon is also present in the histograms of the bivariate BMA models, but much less pronounced. Further, the bivariate BMA post-processing models yield slightly smaller Euclidean errors, while the copula models produce sharper predictive PDFs and better mean energy scores. The independent univariate BMA approach results in the lowest reliability indices, but the bivariate post-processing methods lead to smaller mean energy scores, mean Euclidean errors and mean determinant sharpness values.

In general, three-group models result in better calibrated and more accurate forecasts than their two-group counterparts and the parsimonious BMA models (4.3) and (4.4) outper-

form (in terms of ES,  $\Delta$  and EE) the full bivariate BMA models (4.1) and (4.2), respectively.

## 5 Discussion

In the present study we introduce a new bivariate BMA model for joint calibration of ensemble forecasts of wind speed and temperature providing a predictive PDF which is a mixture of bivariate normal distributions truncated from below at zero in their first (wind) coordinates. Two approaches are presented: a full and a parsimonious one, differing only in the number of parameters to be estimated. The models are tested on the eight-member UWME and on the eleven-member ALADIN-HUNEPS ensemble of the HMS. The two ensemble prediction systems differ both in generation of ensemble members and in wind speed and temperature quantities being forecasted. The predictive performances of both BMA post-processing methods, quantified by the energy score, reliability index and determinant sharpness of probabilistic and Euclidean error of point forecasts (median and mean), are compared to the forecast skills of the independent BMA calibration of wind speed and temperature and the Gaussian copula method suggested by Möller *et al.* (2013).

In case of the UWME forecast vectors of maximum wind speed and minimum temperature a 40 days rolling training period is used. Compared to the raw ensemble all four post-processing methods substantially improve the calibration of probabilistic and accuracy of point forecasts and bivariate models outperform the independent BMA approach. Further, the correlations of the calibrated point forecasts of wind speed and temperature are very close to the empirical correlation of the validating observations of these weather quantities, while the components of the ensemble mean and median vectors are practically uncorrelated. From the three competing bivariate post-processing methods the full bivariate BMA model has the best overall performance.

For calibrating ensemble forecast vectors of instantaneous wind speed and temperature produced by the ALADIN-HUNEPS system a training period of length 40 days and two different grouping of exchangeable ensemble members are considered: one assumes two groups (control and forecasts from perturbed initial conditions), while the other considers three (control and forecasts from perturbed initial conditions with positive and negative perturbations). According to the verification scores investigated, the overall performances of the three-group models are slightly better than those of their two-group counterparts, which is in accordance with the results of univariate BMA calibration. The comparison of the raw ensemble and the post-processed forecasts again shows the positive effect of calibration resulting in smaller ES,  $\Delta$  and EE values. Moreover, the components of the ensemble mean and median vectors show some negative correlation, while the small correlations of the post-processed wind speed and temperature forecasts give back the lack of correlation of the verifying observations. Further, bivariate calibration methods outperform the independent BMA approach in all scores but the reliability index, while compared to the copula

model of Möller *et al.* (2013) both bivariate BMA methods yield slightly lower Euclidean errors, slightly higher mean energy scores, higher DS values and definitely lower reliability indices. Among the investigated post-processing methods the overall performance of the parsimonious bivariate BMA model seems to be the best.

Based on these two case studies we conclude that compared to the raw ensemble joint BMA post-processing of ensemble predictions of wind speed and temperature results in a substantial improvement in forecast skills, the bivariate BMA model outperforms the independent BMA approach and its predictive performance is at least as good as the performance of the Gaussian copula method of Möller *et al.* (2013).

Finally, one should remark that the Gaussian copula approach can be applied for any desired type and number of weather quantities, while the current version of the bivariate BMA model is applicable only for a bivariate weather quantity vector where the components can be assumed to be normal and truncated normal. However, an extension to a higher dimensional setting is possible, although it might be computationally challenging. As the ECC methodology proposed by Schefzik *et al.* (2013) is even more flexible (and computationally more efficient) than the copula method, a comparison of the bivariate BMA to ECC on exchangeable ensemble forecasts (such as the ECMWF ensemble) might yield further improvement of bivariate ensemble calibration.

**Acknowledgments.** Essential part of this work was made during the visiting professorship of Sándor Baran at the Institute of Applied Mathematics of the University of Heidelberg. Annette Möller gratefully acknowledges support by the German Research Foundation (DFG) within the program “Spatio-/Temporal Graphical Models and Applications in Image Analysis” grant RTG 1653 in Heidelberg. Sándor Baran was supported by the Hungarian Scientific Research Fund under Grant No. OTKA NK101680 and by the TÁMOP-4.2.2.C-11/1/KONV-2012-0001 project. The project has been supported by the European Union, co-financed by the European Social Fund. The authors are indebted to Tilmann Gneiting and András Horányi for their useful suggestions and remarks, to the University of Washington MURI group for providing the UWME data, to Mihály Szűcs from the HMS for the ALADIN-HUNEPS data and to Thordis Thorarinsdottir and Alex Lenkoski for their help with the R codes for the copula method. Last but not least the authors are very grateful to the Editor and Reviewers for their valuable comments.

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