PhD Thesis

GeoGebra in Teaching and Learning Mathematics in Albanian Secondary Schools

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1. Introduction

1.1 Short History

I am working in the University of Elbasan – Albania, partner of the CEEPUS network which includes the University of Debrecen and the University of Miskolc as well. Having the opportunity of participating in the existing CEEPUS cooperation and network activities, starting in February 2007 in the University of Miskolc, one year later, during the CEEPUS Summer University in Miskolc, Hungary, 2008 August 8-20, I heard for the first time about GeoGebra software and program, presented by Zsolt Lavicza who introduced GeoGebra program for the participants. I was very much impressed by this program and immediately I became an active part of the teaching by helping the students, present there, to use the software. I shared my concern about this program and asked associated Prof. Dr. Péter Körtesi (Miskolc GeoGebra Institute) to act as scientific advisor in my planned doctoral studies of how to make this software available for the teachers and students in Albania. Our University has been visited by Prof. Peter Körtesi and Prof. Imre Juhasz, both members of the Miskolc GeoGebra Institute who have offered us direct consultation for my research. Also, I have been able to visit the University of Debrecen and the University of Miskolc several times.

- Due to the existing research interest in Elbasan University I received a supporting letter by them to plan and act in making up a large cooperation in Albania, in order to create the Albanian GeoGebra. The work was started and I translated the five (5) properties files of GeoGebra using Attesoro software for the translation. The translation was reviewed after the remarks and suggestions of Judith Hohenwarter (codesigner of **GeoGebra** with **Dr. Markus Hohenwarter)** and, now waiting we have GeoGebra version in Albanian.
- Also, are translated some basics of GeoGebra based on the Introductory Book of Markus and Judith Hohenwarter.

1.2 Training on GeoGebra. I led the teacher training with GeoGebra - the first in Albania. The training schedule was consisted of two hours teaching and practice every month. There were 20 teachers participating in this first training and there will be others in the future. The initiated teacher training program in Albania served as an impetus for other teachers' trainings and for the dissemination of the program for secondary and elementary mathematics teachers, and it will create a large community of users of the program, cooperating with the international user community.
*** The conclusions of my first talk were: - none of the teachers had heard about GeoGebra (watching some simple applications of GeoGebra in constructing geometrical figures they appreciated it very much and were moved to immediately start the training); - they considered GeoGebra as a mean to be involved and used in a program for <u>further qualification of the math teachers of the secondary schools .</u>
After this proposal of teachers, I discussed my research project on GeoGebra software with the Head of the Department of Mathematics and Informatics, prof. Agron Tato, and with the chief inspector of the mathematics in the Education Directorate of our District and I got full support by them and other mathematics specialists to firstly start the math teachers training and later with students of secondary schools. They were very positive and considered this program of great benefit in improving the teaching and learning methods and, was agreed to build such a program including GeoGebra

and Maple, Analysis and Algebra.
So far is created the Albanian language version and later build related internet pages.

2. Pedagogical Experiment, Goals, Techniques and Methods

2.1 Platform: Comparative Experimental Study

Theme: Teaching with GeoGebra versus the traditional teaching

The treatment under study: Effect of using GeoGebra software in teaching and learning process.

Purpose: investigate and determine the "cause and effect" of an action (determine if the treatment caused a change in the individuals' responses).
Decide whether the mathematical course taught by using GeoGebra software is as effective as more traditional methods of instruction.

Methodology: particular treatment of a class of students. Are selected two classes of the same secondary school and of the same level. One class receives traditional teaching and instruction, while the other takes the course by using GeoGebra software. At the end of the course, each group takes the same comprehensive exam.

Population: The entire group of the students of the secondary school “Dhaskal Todri”, in Elbasan, ALBANIA we want information about by examining a portion of the population, two classes – which are representatives of the population (the relevant characteristics of the sample members are generally the same as the characteristics of the population).

Samples: Two Classes of the 3d year of the secondary school “Dhaskal Todri”.

Sample sizes: Class A (28 students), class B (29 students).

Experimental text-book and chapter: Mathematics 3 (text-book for the secondary school); Chapter of Derivatives.

The variable to be measured: one characteristic shared by two samples of one population (the math level represented by the marks in a chapter in two classes treated in different ways).

The data for comparative experimental studies consist of two sets of measurements. Although there may be more than one variable in a study, we will restrict our attention to the analysis of data collected on one variable for now. We will use Five-Number Summaries and comparative box plots to analyze and interpret data from several different comparative studies.

Sampling practice: random selection in order to remove the bias caused by human involvement in the selection process (there are 6 classes in the third grade, randomly chosen two of them by number marking.

Venue: The computer laboratory of the school (class XI-A) and the classroom (class XI-D).

Year of experimental study: 2010

2.2 Data organizing of classes XI-A (control group) and XI-D (experimental group). INTERPRETATION OF DATA RESULTS

When two groups are compared an alternative to superimposition is to draw their two histograms back-to-back (in a similar way to back-to-back stem and leaf plots). I have grouped the data as in Tables 3.6 and 3.7 in accordance with the points the students have got in the test, which are taken out of the Table 3.4 of the experimental group and out of the Table 3.3 of the control group (look at appendices of Thesis).

Comparison at the beginning of the experiment

To compare the two classes I have taken into consideration the marks of the previous chapter, because that was what I had available from the control class in carrying out the experiment. Also, I use the marks of the test done at the end of the chapter on Derivatives. So, from the previous chapter data I got the following distribution of marks and percentages for the experimental and control class (Table 2.1):

Range of points	Mark	Experimental Group Frequencies	Percentage	Control Group Frequencies	Percentage
<35					
35 - < 45	4	1	3.40 %		
45 - < 55	5	3	10.34	2	7.14 %
55 - < 65	6	4	13.79	1	3.57
65 - < 75	7	2	6.90	6	21.43
75 - < 85	8	9	31.03	7	25
85 - < 95	9	9	31.03	6	21.43
95+	10	1	3.40	6	21.43
		Sum = 29		Sum = 28	

Table 2.1 Frequency and percentage distribution table.

As can be seen from the percentages in the table the control group has a better distribution of percentages and a higher level in mathematics (for higher marks, starting with the grade 7, it has higher accumulative percentage). This is the state of the groups in the beginning of the experiment. There is a considerable difference between the two groups. However this is not a problem for making conjectures and drawing conclusions because we do comparisons of the results between the two groups and between the two results of the experimental group got at the beginning of the chapter and at the end. This comparison is more important to judge regarding the new method used in teaching and learning process.

Notice that, in the above table we know about the ranges of points for the experimental group only. For the control group I had available only the marks, not the points. In my case, that is enough to do comparisons between the two groups. Anyone knows for a certain mark what range is of. The points help in better understanding the order of the students regarding their knowledge and skills in mathematics within their class. My issue is to compare the classes. The same frequency distribution I have depicted in a graph that is often referred to as a histogram or bar chart as shown in Figure 2.1. I have constructed back to back histograms (called bihistogram) for the two groups, using Geogebra tools, and I have put them back to back to make easier the reading and interpretation of data. The histogram above the horizontal axis, is of the experimental group, the histogram below the horizontal axis is of the control group.

The computed means are: for the experimental group….$\overline{X}$ = 7.6, for the control group…$\overline{X}$ = 8.14. For the experimental group the *median is 8* which is the mark dividing into two equal parts the given sequence. For the control group the median is 8, as well. The two groups have the same median and mode. **Summarizing:** for the experimental group the mean, median and mode are 7.6, 8, 8 and 9, respectively; for the control group they are 8.14, 8, 8, respectively.

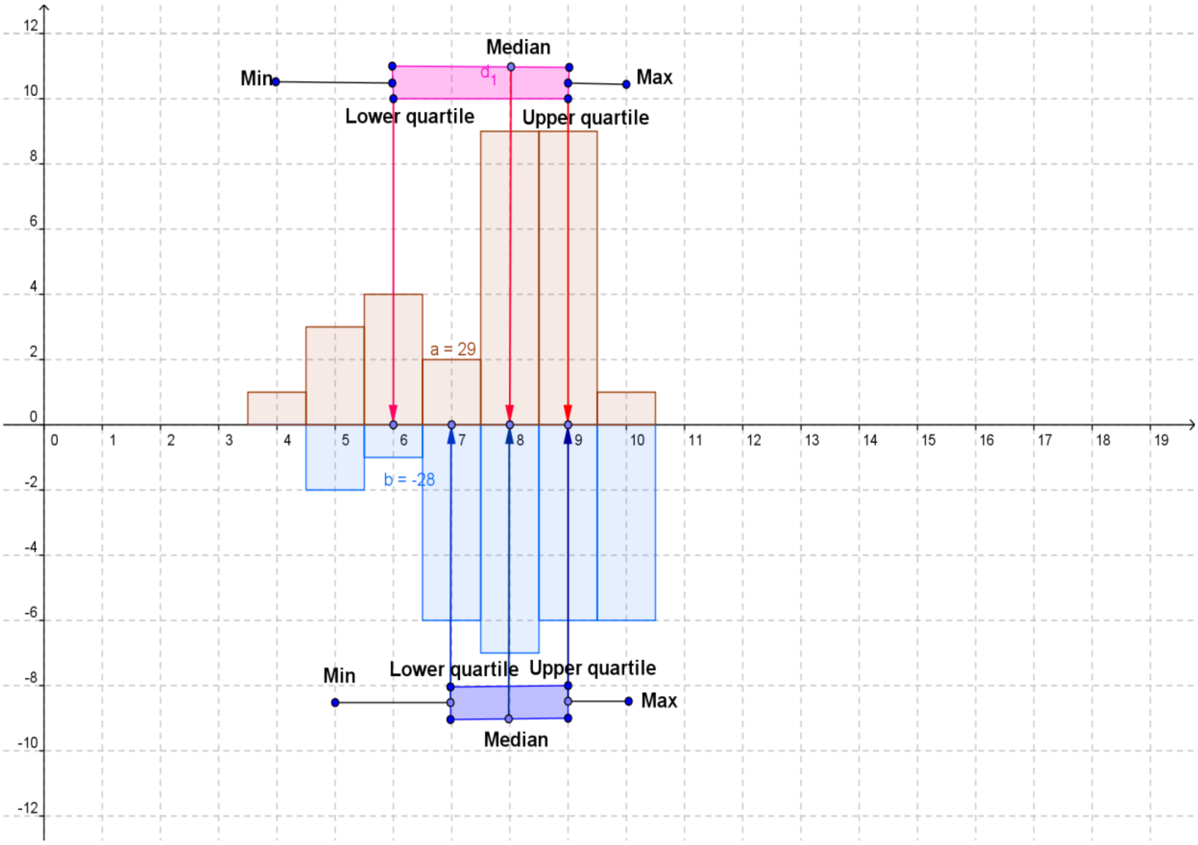

Figure 2.1 Histograms and their respective box plots
(exported by GeoGebra applet)

Five-number summary

The distribution of values in many data sets can be effectively summarized by a few numerical values called **summary statistics** by using a graphical display that is based on five summary statistics called the **5-number summary**. They are:

- The two **extremes of data set** (i.e. the minimum and maximum values).
- Three other values that split the data set into groups that contain (as closely as possible) equal numbers of values are: the **lower quartile**, the **median** and the **upper quartile**.

Here above, in Fig 2.1, are the box plots showing the relation of the five values with the respective back to back histograms, called bihistogram.

In summary:

Experimental group five values
Min = 4 ; Q1 = 6; Med = 8; Q3 = 9; Max = 10

Control group five values
Min = 5; Q1 = 7; Med = 8; Q3 = 9; Max = 10

Shape: the high density of the values is found in the part of histogram having the median and the upper quartile. Comparing the box plots clearly shows that the difference between the two groups stands for the lower quartiles, minimums and inter-quartiles. The difference between these three values is the same: 1 unit. The area of that part of the histogram above the horizontal axis is 5.92. For the histogram below the horizontal axis, corresponding to the control group, the area is 2.50. So, the number of the students of the experimental group and belonging to the poor category is more than two times greater than the number of the students of the control group and belonging to the same category. Also, the number of the students of the experimental group and belonging to the superior category is smaller than the number of the students of the control group and belonging to the this category. <u>Conclude that, at the initial stage or state, the control group appears to be better than the experimental group</u>.

Comparison at the end of the experiment

To compare the distributions of the two groups of values (e.g. measurements for the experimental group and control group) at the end of the chapter, histograms for the two groups were superimposed on the same axes. We got this way the so-called bihistogram. Color or shading are used to help distinguish the two histograms -- in ordinary black-and-white histograms it can be difficult to tell which lines belong to which histograms. In the Figure 2.2 below, the light rose histogram, above the horizontal axis, corresponds to the experimental group and the blue histogram, below the horizontal axis, corresponds to the control group. In the same picture we have displayed the box plot and the five summary numbers (two extremes, median and two quartiles).

Looking at the histogram above the horizontal axis we see that:
Centre: the vertical line inside the box (the median) gives an indication of the centre of the distribution: the centre is 8. **Spread:** the width of the box (the interquartile range) gives an indication of the spread of values in the distribution. It is 3.
Shape: the high density of the values is found in the part of histogram having the median and the upper quartile.

The histograms show that at the end of the chapter the difference between the two groups regarding the level in mathematics is reduced a lot, a confirmation of the positive influence of GeoGebra in the teaching and learning mathematics process. The right extreme and quartile are closer to the median than the left extreme and quartile, this shows that the distribution is right skew and this fact is interpreted as a shift toward higher results in math.

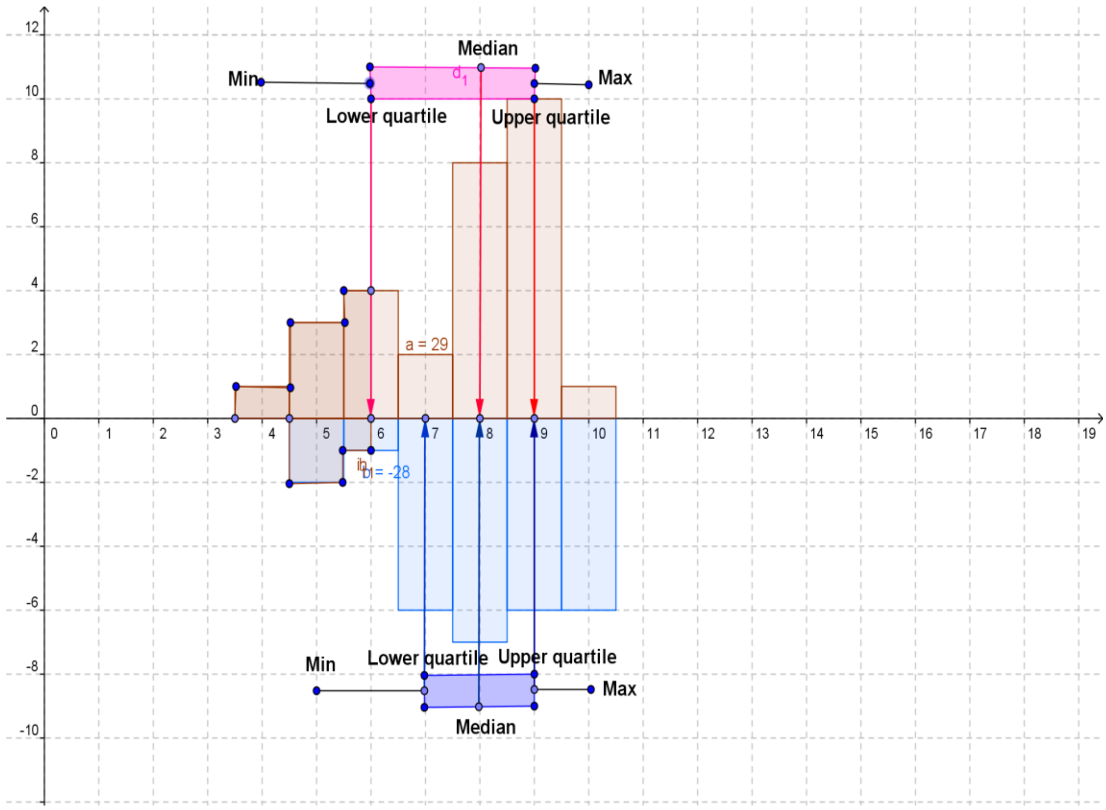

Figure 2.2 Histograms (bihistogram) and their respective box plots
(exported by GeoGebra applet)

In summary:

Experimental group five values
Min = 4; Q1 = 6; Med = 8; Q3 = 9; Max = 10

Control group five values
Min = 5; Q1 = 7; Med = 8; Q3 = 9;   Max = 10

2.3 Statistical Comparison of the Two Data Sets of the Experimental Group

The two most widely used statistical techniques for comparing two groups, **where the measurements of the groups are *normally distributed*,** are the **Independent Group t-test and the Paired t-test**.

Our purpose is to study the effect of using GeoGebra software in teaching and learning process, investigate and determine if the treatment with GeoGebra software in teaching and learning process caused a change in the individuals' math knowledge and skills. We have to investigate and decide whether the mathematical course taught by using GeoGebra software is as effective as more traditional methods of instruction. Our testing hypothesis is related to the means of two methods of instruction: Do two methods have the same mean? In my case the subjects for the two groups are the same or matched. That is, the same subjects are observed twice: at the beginning of the chapter and at the end of it. The intervention taking place between the two measures is the use of GeoGebra software in teaching and learning mathematics.

The basis of the paired t-test

There are three types of questions regarding the true means linked with the two methods that are often asked.

1. Are the means from the two methods the same?

2 + (3). Is the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional method?

We choose to test hypothesis 3 (H_0: μ_1 > or equal to μ_2), hoping that we will reject this null hypothesis and thereby feel we have a strong degree of confidence that the new method of using GeoGebra software is an improvement worth implementing.

Our test, one-sided test, at the 5% significance level, resulted with hypothesis (3) rejected because the test statistic ($t = -2.8585$) is smaller than 1.673 and, therefore, we conclude the new method of using GeoGebra software in teaching and learning math has increased the level of math knowledge and skills over the traditional method used in teaching and learning process.

Another way: Test of Differences

W*e have* measured the level of the group in mathematics b*efore applying the new method*. At the end of the chapter, in which is used GeoGebra software, we have measured the level in mathematics again. We have to do a comparison with an average increase in the level of mathematics of this group in this chapter by using the traditional method. But, this is impossible because we have a state program for the schools that must be fulfilled and rigorously observed, so there is no room for repeating the chapter. For this reason, we use the average increase in the level of mathematics of the control group in this chapter where is used the traditional method.

Again, this test provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes much more increase in the level of knowledge and skills in mathematics than the traditional method used in this process.

Correlation between the variable linked with the marks at the end of the chapter and the variable linked with the marks in GeoGebra test.

The correlation is one of the most common and most useful statistics. A correlation is a single number that describes the degree of relationship between two variables. In the Table 3.12 at appendices of Thesis is made the data up to illustrate the meaning of correlation and compute it. X is variable of the students' results in GeoGebra test, Y is the variable of the students result at the end of the chapter. Plugging the values calculated in the respective table into the formula given above, we get the following:

$$r = \frac{29 \cdot 2079 - 244 \cdot 239}{\sqrt{(29 \cdot 2124 - 244^2) \cdot (29 \cdot 2143 - 239^2)}} = \frac{1975}{3218} = 0.61$$

For our problem the correlation is positive, meaning that the increased level of knowledge and skills in GeoGebra is accompanied with an increase of the level of knowledge and skills in mathematics. The value calculated shows that there is correlation. It is not perfect, however it is considered a strong positive correlation. I believe that there is a relationship between the computer programs for math (Geogebra software also) and mathematics in regard with mastering them. Using the computer programs the students will not be delved in long time performing calculations, solving equations or systems of equations and plotting graphs, or other algorithms.

2.4 Conjectures and Conclusions on the results' tests

Our testing hypothesis is related to the means of two methods of instruction: Do two methods have the same mean?

In my case the subjects for the two groups are the same or matched. That is, the same subjects are observed twice: at the beginning of the chapter and at the end of it. The intervention taking place between the two measures is the use of GeoGebra software in teaching and learning mathematics.

1. Comparing the histograms of the two groups at the end of the chapter is seen that the right extreme and quartile are closer to the median than the left extreme and quartile for both of them. For the experimental group the comparison of the histogram at the end of the chapter with the one at the beginning shows that the distribution is more right skew at the end of the chapter, and this fact is interpreted as a shift toward higher results in math.
2. The comparison of the main statistics at the end of the chapter for the two groups shows that there is a little difference between them:

   Experimental group five values (Min = 4, Q1 = 6, Med = 8, Q3 = 9, Max = 10)

   Control group five values (Min = 5, Q1 = 7, Med = 8, Q3 = 9, Max = 10)

3. In our first test hypothesis (3): Is the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional

method? is rejected because the test statistic ($t = -2.8585$) is smaller than 1.673(the critic value), therefore, we conclude that the new method of use of GeoGebra software in teaching and learning math has increased the level of math knowledge and skills over the traditional method used in teaching and learning process.

4. In the test of differences where each "before" measurement is paired with the corresponding "after" measurement, we achieved the same conclusion. *The test has provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes increase in the level of knowledge and skills in mathematics.*
5. Regarding the correlation, for our problem the correlation is positive, meaning that the increased level of knowledge and skills in GeoGebra is associatied with an increase of the level of knowledge and skills in mathematics. The value calculated shows that there is correlation. It is not perfect, however it is considered a strong positive correlation. Closer to the value 1 be it, much stronger is the correlation between the two variables. The three test performed are a confirmation to one another. It counts that GeoGebra be used in the teaching and learning process.
6. Comparing the stem and leaf plots (the first and the third) it is clear that there is a shift toward higher marks within the experimental group.
7. Another indication of progress for the experimental class is the mean. In the beginning of the experiment they were: the computed means for the experimental group: $\overline{X} = 7.6$, for the control group: $\overline{X} = 8.14$. At the end of the chapter they were: the computed means are for the experimental group….$\overline{X} = 8.24$, for the control group…$\overline{X} = 8.25$. Almost equal!!!

3. Repetition of Experiment

School year 2010 – 2011, ALBANIA

Three classes are involved in the experimental study: Class X – A (Secondary school "Dhaskal Todri", Elbasan – teacher Shpetim Sulanjaku); Class X-A (Secondary school "Jani Kilica" , Fier – teacher Lefter Leka); Class X-B (The General Secondary School, Librazhd - teacher: Luljeta Blloshmi). One class is the control class: Class X – B (Secondary school "Dhaskal Todri", Elbasan - teacher Shpetim Sulanjaku). The four classes are considered groups and labeled respectively A, B, C and D (D – the control class)

3.1 Criterions for the selection of the classes and preliminary work with data

1. The selection of the classes is based on the known relationships between the experimenter (Pellumb Kllogjeri) and the teachers and the availability and willingness of the teachers to be involved with the experiment.

2. The experiment is performed in the same chapter "Systems of equations and inequations". This chapter is taught in the second year of the middle school.

3. The classes have not much difference in their means regarding the quality in math.

4. The classes have totally different backgrounds, coming from different towns (Elbasan, Fier and Librazhd). The control class is from Elbasan, as well. They have different teachers, experimenting for the first time – teaching with GeoGebra.

5. The groups are independent from one another and he experiment range is wider.

Because our task is to compare the means of several groups (four) and get conclusion about which teaching and learning method is better we perform ANOVA test. One-way **AN**alysis **O**f **VA**riance (**ANOVA**) is used when we want to compare more than two means. It is a technique that generalizes the two-sample *t* procedure which compares two means to a situation with more than two sample means. The statistic corresponding to ANOVA test is F-statistic (Fisher statistic) defined by the ratio $F = \frac{MS_B}{MS_W}$, where the nominator is the between-groups mean square, whereas the denominator is the within-groups mean square. More precise they are defined below the following table.

The preliminary work having all is needed to perform the test is in the following table, containing all the by hand computations. Next are the other figures necessary for the estimation of F-statistic.

Summary table: of the sizes of the sums, of the sum of squares for each group and the totals.

A	B	C [	D	All groups combined
N_A=36	N_B=29	N_C=29	N_D=38	N_T=132
$\sum X_{Ai}$=291	$\sum X_{Bi}$=223	$\sum X_{Ci}$=218	$\sum X_{Di}$=276	$\sum X_{Ti}$=1007
$\sum X^2_{Ai}$ = 2473	$\sum X^2_{Bi}$ = 1793	$\sum X^2_{Ci}$= 1639	$\sum X^2_{Di}$ =2080	$\sum X^2_{Ti}$= 7985
$\overline{X_A}$=8.08	$\overline{X_B}$=7.69	$\overline{X_C}$=7.52	$\overline{X_D}$=7.26	

SS_{BG} - Sum of Squares Between Groups; $\boldsymbol{SS_{WG}}$ - Sum of Squares Within Groups

$$MS_B = 9.423333 \;;\; MS_W = 2.1478; \; F = \frac{MS_B}{MS_W} = \frac{9.423333}{2.1478} = 4.39$$

The summary table is:

	SS	df	MS	F
Between groups(B)	**28.27**	**3**	**9.423333**	**4.39**
Within groups(W)	**274.54**	**128**	**2.1478**	
Total	**302.80**	**131**		

F test: One-way test (ANOVA) is used to test whether the means of all groups are equal. In the ANOVA test, samples are drawn from each population and the data is used to test the null hypothesis that the populations are all equal against the alternative that not all are equal. If rejected the null, we need to perform some further analysis to draw conclusions about which population means do differ.

3.2 Assumptions of the ANOVA and the test of null hypothesis

1. The data is normally distributed.
2. The population standard deviations are equal.

In our experiment, the random variable representing the marks has approximately a normal distribution. It is confirmed by many statistical studies and this fact is considered in many text books. In many problems, the population standard deviations are considered equal, but what we do with our experiment?

Recall: Our second assumption in the ANOVA model was that our population standard deviations are all equal. The official test is quite complicated and not practical, also statistical official data do not help, so we use the following rule of thumb:

We compare 2 x smallest std. dev to the largest std. dev. We want that 2 x smallest std. dev > largest std. dev.

The two hypotheses tested by ANOVA procedure are:
$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4,$ all the means are the same.

H_a: Not all the sample means are equal (at least one is different).

I have tested these hypotheses at three significance levels: α = 0.050, α = 0.025 and α = 0.010 (Fig 3.1). The tabled values of $F = \frac{MS_B}{MS_W}$, in accordance with the degrees of freedom of nominator and denominator of the ratio, taken out from the respective tables are:

$$F(3,128) = \begin{cases} 2.67 & for\ \alpha = 0.050 \\ 3.21 & for\ \alpha = 0.025 \\ 3.94 & for\ \alpha = 0.010 \end{cases}$$

In the experiment carried out, the estimated value of F, called an F statistic, is 4.39 (look at the summary table, above). It tells us how much more variability there is between treatment groups than within treatment groups. The larger that ratio, the more confident we feel in rejecting the null hypothesis, which is that all means are equal and meaning that there is no effect. The estimated value of F falls in the three areas of rejection of H_0 for the three levels of α. Therefore we **reject H_0 and accept H_a**, concluding that **the means of the four groups are not equal**.

3.3 Comparing the Means and Interpretation

Although the ANOVA F test may be significant, (i.e. we reject H_0) it does not tell us specifically which means differ from each other. We can look at the difference graphically or by formal inference. We use the method of: **Simultaneous Confidence Intervals for Differences between Means**. This method is used **ONLY AFTER the rejection of H_0** with the F test. All combinations of means are compared to know not just **which means differ, but by how much they differ** in order to see the **effect size.** The easiest thing is to compute the confidence interval first, and then interpret it for a significant difference in means.

The assumptions of Tukey's test:

1. The observations being tested MUST BE INDEPENDENT
2. The means come from normally distributed populations
3. Observations have almost equal variations

These assumptions in our case are met. The random variable representing the marks has approximately a normal distribution (It is confirmed by many statistical studies and this fact is considered in many text books). The variances differ slightly from one another. The value of q is function of the number of treatments, of the total number of data points and α level. The estimation for the differences of the means and their respective confidence intervals is as in the following table:

	$\bar{x}_i - \bar{x}_j$	**Critical q** $q(\alpha, r, df_W)$	**Standardized error**	**95% Conf. Interval for** $\mu_i - \mu_j$		**Signif. at 0.05?**
A - B	0.39	3.6805	0.258	−0.56	1.34	
A − C	0.56	3.6805	0.258	−0.39	1.51	
A − D	0.82	3.6805	0.2406	−0.055	1.705	**YES**
B – C	0.17	3.6805	0.272	- 0.83	1.170	
B – D	0.43	3.6805	0.255	- 0.51	1.370	
C – D	0.26	3.6805	0.255	- 0.673	1.20	

Explanation about the table:

1. The first column shows **which group means are being compared.**
2. The next column gives the **point estimate of difference**, which is the difference of the two sample means. The sample means of A and B are 8.08 and 7.69, so their difference is 0.39 and so on.
3. Third column belongs to **critical q**. Looking at formula is understood that q (α, r, df_W) depends on the number of treatments and total number of data points, not on the individual treatments, so it's the same for all rows in any

given experiment. In the experiment carried out in Albania regarding the effect of GeoGebra in teaching and learning math, there are four groups. Choosing $\alpha = 0.05$, we find on the table of critical values for the studentized range that $q(0.05, 4, 129) = 3.6805$.

4. Fourth column contains the **standardized error** given from Tukey's formula for confidence interval.

In the experiment we are talking, the sample sizes are unequal so, the standardized error varies for comparing different pairs of groups. For the first difference, A – B, we have:

$$\sqrt{[(MS_W/2)\cdot(1/N_A+1/N_B)]} = \sqrt{[(2.1478/2)\cdot(1/36 + 1/29)]} = 0.258$$

and, so on…

5. Fifth column contains the two **endpoints of the confidence interval** computed for each difference.

6. The last column applies to the relation between confidence interval and significance test in order to see **whether there's a significant difference** between the two groups. If the confidence interval includes the value 0, then that pair of means will not be declared significantly different, and vice versa. Looking at the difference of the means of groups A and D, that is A – D , the left endpoint of the interval is almost 0. Consequently, we don't make a big mistake saying that the endpoints of the confidence interval are both positive. This means that 0 is not in this interval and ***we reject the null hypothesis*** of equality of the respective means (by noting YES in the respective row of that difference). In this table, only groups A and D have a significant difference.

Interpretation: The means of the groups A and D (the respective classes) are not equal. Moreover, the mean of the experimental class is greater than that of the control class and we are 95% confident that teaching with GeoGebra gives higher results than the teaching of the traditional way.

The confidence intervals of the other differences go from a negative to a positive, so they do include zero. That means that the two respective means might be equal or different, so we can't say whether there is a difference between them. However, the interval center of each one is a positive number (0.9 approx.), leading us to say that there are differences. For each pair of the groups the tendency is the same. The effectiveness of the method ″teaching with GeoGebra ″ is easily obvious when compare group A with group D. But, it is not so when we compare the groups B, C and D. **I believe that one of the causes is the lack of experience of the teachers with GeoGebra software.** The first experimental class (group A) is from Elbasan, a city in which there is a 3-year experience using GeoGebra: training with teachers and diploma themes for the students of Elbasan University. The other two experimental classes are from schools of other towns where Geogebra was introduced and used for the first time. Teachers themselves of these schools have faced difficulties in teaching with GeoGebra. Even of such difficulties the respective experimental classes have higher results (higher means) than that of the control class.

4. Summary Conclusions of the Experiments, Problems and Suggestions

4.1 The First Experiment, February 2010

In **the first way** for comparing the two groups at the end of the chapter we used the main statistics like, the mean and median of the two groups, also displaying their results (scores) with bihistogram and using box-plot. At the end of the chapter the experimental group showed to be better than the control one. This was the first evidence that teaching with GeoGebra is more effective than the traditional method of teaching.

We used the average increase in the level of mathematics of the control group in this chapter where was used the traditional method. The paired sample *t*-test, the test statistic used to test for the difference of two means before and after a treatment, *provided also evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes much more increase in the level of knowledge and skills in mathematics than the traditional method used in this process.*

4.2 The Second Experiment: March, 2011 (repetition)

The main reasons of repeating the experiment were:

1. In the first experiment the teacher of the experimental class was the experimenter as well.
2. The experiment was based in two classes only, so there was not sufficient evidence of drawing right and trusted conclusions.

The statistic corresponding to ANOVA test is **F-statistic** (Fisher statistic) rejected H_0 but it does not tell us specifically which means differ from each other. Therefore, we looked at their differences using the method of **Simultaneous Confidence Intervals for Differences between Means** which is used **ONLY AFTER the rejection of $\mathbf{H_0}$** with the F test. Looking at the difference of the means of groups A and D, that is A – D (D is the control class), the left endpoint of the interval was almost 0. This means that 0 is not in this interval and ***we reject the null hypothesis*** of equality of the respective means. In this table, only groups A and D had a significant difference.
Interpretation: The means of the groups A and D (the respective classes) are not equal. Moreover, the mean of the experimental class is greater than that of the control class and we are 95% confident that teaching with GeoGebra gives higher results than the teaching of the traditional way. The effectiveness of the ″teaching with GeoGebra ″ method was easily obvious when compared group A with group D. But, it was not so when we compared the groups B, C and D. **I believe that one of the causes is the lack of experience of the teachers with GeoGebra software.**

Problems, lessons and suggestions

**** The first problem regarding the results and inferences of the first experiment was the teacher of the experimental class (who was me). I was the experimenter and the teacher, so there are strong reasons of no believing in the results and the inferences got at the end of the experiment. It is right to think that, in getting conclusions is not missing subjectivism.*

**** Another questionable topic is: if there is a good positive difference between the results at the end of a chapter and the results at the end of the previous chapter, is this an evidence that the improving scores are result of the new teaching and learning method?? My opinion is that the conclusions about the new method not be depended on this kind of comparison (by comparing the scores in different chapters).*

**** The case of making comparison with an average increase in the level of mathematics of a class in the same chapter by using the traditional method one time and by using the GeoGebra teaching the second time cannot happen. T<u>his is impossible because we have a state program for the schools that must be fulfilled and rigorously observed, so there is no room for repeating the chapter</u>. The other problem is that by repeating a second time the chapter it is expected and believed that the results must be higher (the results are correlated with the repetition process) . For this reason, we used the average increase in the level of mathematics of the experimental group where was used "teaching with GeoGebra" and of the control group (in the same chapter) where was used the traditional method.*

**** The second (repeated) experiment showed again that the <u>training of the math teachers with Geogebra</u> is very important for the implementation of "teaching and learning with GeoGebra" method in the teaching and learning process and, to draw right inferences about the experiment. The new method of teaching was based in one class only, in the first try, because the only person in Albania who was able to do such teaching was me. The need of training the teachers showed up again, this year, in the other two towns where GeoGebra was introduced the first time where no experience with GeoGebra was there.*

**** The F-test: One-way test (ANOVA), used to test whether the means of several groups are equal, is more trustful than the t-test of any kind. The F-test is based on the measurements done in at least three groups (more many groups better the conclusions).*

**** As mentioned above, the selection of the classes was based on the known relationships between the experimenter and the teachers and the availability and willingness of the teachers to be involved with the experiment. This is a violation of the important requirement and principal on randomness in carrying out the experiment. Therefore, when carried out an experiment special attention must be paid to the randomness (each member of the population must have the same chance of being member of the sample). In the case of experimenting with the teaching process must be thought well about what kind of test perform and how to independently select classes involved in the experiment. My suggestion is that, a good solution is the cooperation with the ministry of Education and with the Regional Directorates of Education.*

LIST OF PUBLICATIONS

UNIVERSITY OF DEBRECEN

UNIVERSITY AND NATIONAL LIBRARY

Registry Number: … FI 17198…………….

Subjects: Ph.D. List of Publications

Candidate: Pellumb Kllogjeri

Neptun ID:……..

Doctoral School: Matematika és Számítástudomány Doktori Iskola

I. List of Publications in Journals and Conference Proceedings

Publications included in the thesis

1. Kllogjeri P. (2008), THE POWER OF DOUBLE REPRESENTATION OF GEOGEBRA, published in the scientific bulletin DOKTORANDUSZOK FURUMA, 2008, Miskolci Egyetem, November 5 (pp. 117-122)
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3. Kllogjeri P., Shyti B. (2010), GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND USING THE SYNERGY OF MATHEMATICIANS – extended version, published in Inderscience Journals, *Int. Journal of Teaching and Case Studies (IJTCS), Vol. 2, Nos. 3/4, 2010* (pp. 225-236). **DOI:** 0.1504/IJTCS.2010.033318.
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7. Körtesi P., Kllogjeri P. (2008), Extended Help Files in GeoGebra – short version, published in the *International Journal for Technology in Mathematics Education*, Volume 16, No 1. pp. 37-41
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II. List of Published Books

1. An Introduction To Quasi-Quadrilaterals, ISBN-13: 978-3-659-42488-5, ISBN-10:3659424889, EAN:9783659424885, August 2013;
   Authors: Pellumb Kllogjeri and Adrian Kllogjeri**,** Number of pages:140,
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