# INVESTIGATION OF A PRODUCTION PROCESS UNDER UNCERTAINTY 

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#### Abstract

A key role of production managers at manufacturing companies is to make economy-based decisions related to production scheduling. If the production is subject to uncertain factors, like human resource or lack of standardization, production planning becomes difficult and calls for advanced models that are tailored to the manufacturing process. This research investigates a real furniture manufacturing system from both managerial and materialflow points of view. Statistical simulation was run on the manufacturing process, where the possible production structures were given. ANOVA analysis was calculated in order to identify those activities that have the most significant influence on the profit.


Keywords: Production planning, Network models, Pull system, Multi period planning, Material flow

## 1. Introduction

One of the greatest challenges that a manufacturing company may face is the material flow optimization. If the material flow is not balanced within a manufacturing process, it is possible that high work-in-process will be accumulated in the production, which always results in extra costs [1]. A major task under these circumstances is to determine an optimal or near the optimal production schedule that takes logistics 5R into account.

[^0]Another challenging problem is the stochastic behavior of activity durations. Due to the explosion of uncertain and stochastic factors, both the prediction of total lead time as well as making decisions on accepting or rejecting orders can be difficult. Taking these factors into consideration, a stochastic multi period production planning system should be applied [2].

## 2. Literature review

### 2.1. Operations research models for scheduling

Every company aims to satisfy their customer orders [3]. It can only be feasible if all the necessary raw materials and components are available in harmony with the 5Rs of the logistics. If one of these requirements is not adequate, extra costs should be paid [2].

Another important issue in the capability and the capacity of the manufacturing process: the decision about accepting or rejecting purchases depends on these indicators [3]. In the case of this kind of decision making problem, the objectives are the time and cost, in which balance should be found [4].

An effective way of production representation is the application of network models [5]. The representation of a network usually occurs with the use of a $G(N, A)$ graph, where $N$ displays nodes and $A$ represents connections between the nodes. Nodes can symbolize anything: the meanings of these elements depend on the problems themselves and on analysts [6].

As far as production scheduling is concerned, there are numerous existing models: when it is a project under process, critical path method and process network methods can be used [7], [8], or Wagner-method is suitable for multi-period production planning [5]. Other cases, for example the material flow optimization, a generalized network flow model can be applied [9], and the list could be expanded.

Furthermore, the use of deterministic optimization cannot be the best choice, because in an always changing environment input values are not fixed. A good solution for this problem is the integration of operations research method with Monte-Carlo simulation technique [10]. In the recent years, several articles were published related to the combination of optimization and simulation, for instance [11], [12]. Investigating a system with the use of this method, can result in getting more reliable information which make decisions more grounded [12].

## 3. Methodology

### 3.1. Process presentation

The examined company deals with furniture manufacturing-to-order. Their major products are corpus and kitchen furniture. In this process, the use of corpus is optional: it can be either sold individually, or it can be built into the ready-made kitchen furniture. Corpus manufacturing consists of two activities: the preparation and the assembly phases. The kitchen furniture manufacturing involves several two phases: different sawing and wood planning activities, plus screwing, gluing, hinging activities. The full
process map can be seen in Fig. 1. Purchase orders are based on the combination of these products. In order to analyze how much the profit is achievable in each of the product combinations, it is important to determine what kinds of order combinations can be feasible in this process environment.


Fig. 1. Graph representation of the process

### 3.2. Uncertain elements in the production

There are two elements in the manufacturing system those are considered uncertain: the activity times and the order combination. As far as the former indicator is concerned, 25 measurements were executed, and based on the measured data; a probability distribution was assigned to each activity - in this case, the theoretical distributions were applied to the activity on the basis of 11 measurements. The result can be seen in Table I.

Table I
Uncertain activity times

| Activity <br> $\left(x_{i}\right)$ | Minimum <br> duration | Maximum <br> duration | Manufacturing <br> cost | Distribution |
| :---: | :---: | :---: | :---: | :---: |
| x1 | 60 TMUs | 120 TMUs | $690 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x2 | 60 TMUs | 204 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x3 | 30 TMUs | 60 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x4 | 12 TMUs | 60 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x5 | 13.2 TMUs | 66 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x6 | 6 TMUs | 21 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x7 | 20.4 TMUs | 60 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x8 | 12 TMUs | 30 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x9 | 8.4 TMUs | 21.6 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |
| x10 | 60 TMUs | 228 TMUs | $300 \mathrm{P} \$$ | $\beta[3 ; 5]$ |

where $x_{i}$ is the activity ID; Minimum/maximum durations: the minimal and maximal cycle time of a certain activity (time duration values are indicated with Time Measurement Unit (TMU)); Manufacturing cost includes all the labor cost related to a carry out a certain activity (excluding the cost of the raw material); Distributions: the values in the brackets determine the shape of the distribution value ( $\alpha, \beta$ values).

### 3.3. Proposed model

The proposed model is a stochastic multi-period production scheduling model with integer constraints.

## Objective:

Just like at every manufacturing company, the examined objective is the maximization of the profit:

$$
\begin{equation*}
z=\text { Profit } \rightarrow \text { MAX! } \tag{1}
\end{equation*}
$$

## Constraints.

1) Time constraints: Every order is considered as a project. An order has to be completed within 1 week, which equals 40 hours ( 2400 sec ) based on the work schedule of the company. Therefore, the following equation can be drawn:

$$
\begin{equation*}
\sum_{i=0}^{n} \sum_{j=1}^{m} t_{i, j} \leq 2400 \mathrm{~min} \tag{2}
\end{equation*}
$$

where $i$ means the product type and $j$ is the process step.
2) Demand constraints: There are two products in this project. These products are usually ordered in combinations. The following equations show the possible intervals of the ordered products:

$$
\begin{array}{ll}
x_{1} \in \mathbb{Z}, & x_{1}=\{0,1,2,3\}, \\
x_{2} \in \mathbb{Z}, & x_{2}=\{3,4,5,6\} \tag{4}
\end{array}
$$

Furthermore, based on the product features, sold quantities have to be integer. In addition, the following equations were applied:

Number of produced product $\geq$ given demand for product $A$,
Number of produced product $\geq$ given demand for product $B$.
3) Raw materials: Due to the limited space of the warehouse, some resources are considered constraints in this case. Only those types of raw materials are listed here that have their effect on the possible solution set:
$0 \leq U S e_{\text {Wood } 1} \leq 5$,
$0 \leq U s e_{\text {Wood } 2} \leq 7$,
where Wood1 and Wood2 are different raw materials for kitchen furniture manufacturing. Nevertheless, all of the raw material was built into the model in order to see the consumptions of them.

## Decision variables.

In this case study, the decision variables represent the material flow. They indicate how much raw or semi processed material travels from one activity to another: these are called Work-In-Process (WIP). WIP shows all the connection between process activities, that is why it is a key factor in a production system.

In addition, it displays how many products are sold in a given week. Based on the model, the goal was to maximize the profit, which was the objective of the built-up network model.

## 4. Results

### 4.1. Results of the simulation

A deterministic model was constructed on the basis of the previously presented data (1)-(8). Furthermore, MS Excel's Solver add-in was used for optimization. After creating the deterministic model, the environment for that of the stochastic was also elaborated. The simulation was programmed in Visual Basic Application programming language. The simulation was run 10,000 times, and the results were exported to an analyzable database. The descriptive statistics of the simulation can be seen in Table II.

Table II
Results of the simulation - Descriptive statistics of time durations

|  | N | Median | Std. <br> Deviation | Range | Minimum | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Duration-x1 | 10000 | 82 | 9.528 | 54 | 61 | 115 |
| Duration-x2 | 10000 | 113 | 23.248 | 129 | 63 | 192 |
| Duration-x3 | 10000 | 41 | 4.804 | 26 | 31 | 57 |
| Duration-x4 | 10000 | 30 | 7.759 | 43 | 13 | 56 |
| Duration-x5 | 10000 | 33 | 8.550 | 50 | 14 | 64 |
| Duration-x6 | 10000 | 12 | 2.462 | 14 | 7 | 21 |
| Duration-x7 | 10000 | 35 | 6.419 | 35 | 22 | 57 |
| Duration-x8 | 10000 | 19 | 2.895 | 16 | 13 | 29 |
| Duration-x9 | 10000 | 14 | 2.168 | 12 | 9 | 21 |
| Duration-x10 | 10000 | 122 | 27.297 | 149 | 63 | 212 |

Focusing on the order combinations, the following data were simulated. The first figure of the product combination is used for the number of corpus, while the second figure means the number of the kitchen furniture. It can be seen in Table III.

These production combinations represent most of the possible orders. The orders of extreme amounts were excluded from the simulation, because they must be dealt with individually. These orders usually request for high amounts, however, their frequency is quite low. The results of the simulation on purchasing orders reflect real data.

The most demanded product combinations are:

- 1 corpus +4 furniture pieces (hereinafter $1 \_4$ );
- 2 corpuses +4 furniture pieces(hereinafter 2_5);
- 4 kitchen furniture pieces (hereinafter $0 \_4$ );
- 5 kitchen furniture pieces (hereinafter $0 \_5$ ).

Table III
Results of the simulation - Product combinations and their frequencies

| Product combination | 1_4 | 0_5 | 2_4 | 0_4 | 1_5 | 2_3 | 2_5 | 3_4 | 1_3 | 0_6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3636 | 2069 | 2045 | 1552 | 480 | 111 | 50 | 29 | 20 | 8 |
| Percentage <br> (\%) | $\begin{aligned} & \stackrel{\circ}{\circ} \\ & \stackrel{\text { ®}}{0} \end{aligned}$ | $\begin{aligned} & \text { ¿̀ } \\ & \stackrel{\text { Nu}}{ } \end{aligned}$ | $\begin{aligned} & \text { ì } \\ & \text { ì } \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{n} \\ & \stackrel{n}{n} \end{aligned}$ | $\stackrel{\circ}{\stackrel{\circ}{+}}$ | $\stackrel{\circ}{\square}$ | ì | $\stackrel{\circ}{0}$ | તે̀ | $\frac{\square}{\circ}$ |
| Cumulative percentage (\%) | $\begin{aligned} & \stackrel{\circ}{\dot{\gamma}} \\ & \stackrel{\rightharpoonup}{\circ} \end{aligned}$ | $\frac{0}{i}$ | $\stackrel{i}{n}$ | $\begin{aligned} & \text { oे } \\ & \text { oi } \end{aligned}$ | $\stackrel{\infty}{\stackrel{\infty}{\circ}}$ | $\begin{aligned} & \text { oे } \\ & \dot{\circ} \end{aligned}$ | $\begin{aligned} & \circ \circ \\ & \dot{\sigma} \end{aligned}$ | $\stackrel{\rightharpoonup}{\mathrm{O}}$ | $\begin{aligned} & \text { oे } \\ & \text { ลे } \end{aligned}$ | $\begin{aligned} & \text { Ò } \\ & \text { O. } \end{aligned}$ |

### 4.2. Indicators under investigation

## Work-time utilization

Work-time utilization is a very important part of the production: it sheds light on time puffers and helps to handle unplanned obstacles in the production, for instance delivery delays or machine repairs. Work-time utilization can be calculated with the following equation:

Utilization $=\frac{\text { Total process time }}{\text { Available time for production in a certain week }}$.
Furthermore, the efficiency of the production can be analyzed by this calculation. An important standpoint is to find the balance value between utilization and work-time.

As far as the production is concerned, Analysis Of Variance (ANOVA) analysis was carried out after the completion of normality test in order to see if there are any differences between product combinations in respect of work utilization. The grouping variable was the production structure, while the measured indicator was the utilization.

The ANOVA test was significant ( $p \leq 0.05 ; n=10,000$ ).
The ANOVA test was significant ( $p \leq 0.05$ ), and the completion of Tukey-b posthoc test proved that the work utilizations of the order combinations are statistically different (Table IV).

Table IV
Results of Tukey-b Post-hoc analysis

| Time utilization |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Subset for alpha $=0.05$ |  |  |  |  |
| Order combinations | N | 1 | 2 | 3 | 4 |
| 0_4 | 1552 | 94.15\% |  |  |  |
| 1_4 | 3636 |  | 95.64\% |  |  |
| 0_5 | 2069 |  |  | 97.03\% | 97.03\% |
| 2_4 | 2045 |  |  |  | 97.67\% |

## Profit calculation

Profit is the subtraction of the income and costs:

$$
\begin{equation*}
\text { Unit profit }=\text { unit price }- \text { unit cost } . \tag{10}
\end{equation*}
$$

Small and medium sized enterprises employing human workforce can only make rough estimations about their profit due to the unpredictability of total process times in the production. Furthermore, additional machine set-ups or repairs may arise that can extend the duration of activities. This stochastic background does not guarantee the even nature of profit generation. Based on the values gained by the simulation, more valid estimations can be calculated about purchase orders.

It is a very informative part of the research, because by being aware of the most profitable order combinations, companies can control and affect their customers' needs. The descriptive statistics on profits generated by the most frequently ordered product combinations is displayed in Fig. 2 and Table V.


Fig. 2. Probable profits by order combinations

Table V
Descriptive statistics about the probable profit by order combinations

|  | $0-4$ | $0 \_5$ | $1 \_4$ | $2 \_4$ |
| :---: | :---: | :---: | :---: | :---: |
| Mean (TMU) | 331623 | 416087 | 350905 | 370276 |
| Median (TMU) | 331618 | 416105 | 350924 | 370291 |
| Std. Deviation (TMU) | 530 | 590 | 585 | 613 |
| Minimum (TMU) | 330022 | 414333 | 348738 | 368282 |
| Maximum (TMU) | 333176 | 418018 | 352515 | 372413 |

As it can be seen in Fig. 2 and Table $V$, the range of probable profits is around 4,000 in the case of each order combinations. It helps the company to make plans for the future. In the model the cost of raw material and that of employment are indicated separately. The model does not include the occurrence of defected products; therefore, the estimated cost of human workforce is presented in the following chart, see Fig. 3.


Fig. 3. Probable cost of human workforce by order combinations
The cost of raw material is directly proportional with the produced quantity; the occurrence of the defects and their effects are not modeled.

## Correlations between profit and activity time durations

As it was mentioned in the previous section more human work results in higher costs. With the use of correlation analysis, relationships are revealed between profit and activities, that is, which activities have the strongest influence on the profit. It can also highlight activities that must be improved first. The result of the analysis can be seen in the Table VI.

Table VI
Results of the simulation - Product combinations and their frequencies

|  | 荡 | $\begin{aligned} & \tilde{x} \\ & .0 \\ & \tilde{H} \\ & \tilde{Z} \end{aligned}$ |  |  |  | $\begin{aligned} & \bar{x} \\ & \text { E } \\ & \text { E } \\ & \text { In } \end{aligned}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Profit | - | $\underset{i}{\sim}$ | $\frac{0}{i}$ | $\underset{i}{\infty}$ | ${ }_{c}^{\circ}$ | $\stackrel{\sim}{i}$ | $\hat{B}_{i}$ | તે | $\stackrel{\stackrel{0}{4}}{\substack{4 \\ \hline}}$ | $\stackrel{n}{n}$ | $\stackrel{\bullet}{\text { n }}$ |
| Duration x3 |  | - | $\bigcirc$ | $\bigcirc$ | §̀ | $\stackrel{\rightharpoonup}{0}$ | $\overbrace{i}^{n}$ | \% | ou | $\stackrel{\square}{8}$ | $\bigcirc$ |
| Duration x4 |  |  | - | $\bigcirc$ | $8$ | $\stackrel{0}{8}$ | $\stackrel{\infty}{\circ}$ | $\frac{0}{0}$ | ${\underset{i}{0}}_{0}^{2}$ | $\stackrel{3}{8}$ | $\stackrel{\sim}{\square}$ |
| Duration x5 |  |  |  | - | $\underset{i}{0}$ | $\bar{\sigma}_{i}$ | તે તે | $\stackrel{\partial}{i}_{i}$ | $\hat{8}$ | O. | $\stackrel{\square}{8}$ |
| Duration x6 |  |  |  |  | - | n | $\underset{i}{8}$ | $\hat{8}$ | $\stackrel{\infty}{8}$ | $\bigcirc$ | \%ิ. |
| Duration x7 |  |  |  |  |  | - | $\stackrel{0}{0}_{\infty}^{\circ}$ | $\hat{o}_{i}$ | ô | $\vec{o}_{i}$ | $\stackrel{\text { or }}{\text { or }}$ |
| Duration x8 |  |  |  |  |  |  | $-$ | $8$ | $\underset{i}{0}$ | $8$ | $\stackrel{\%}{8}$ |
| Duration x9 |  |  |  |  |  |  |  | - | ơ | No | $\vec{\square}$ |
| Duration x1 |  |  |  |  |  |  |  |  | - | $\frac{2}{0}$ | $\stackrel{\rightharpoonup}{8}$ |
| Duration x2 |  |  |  |  |  |  |  |  |  | - | \%ิ. |
| Duration x10 |  |  |  |  |  |  |  |  |  |  | - |

The result of the analysis is evident: negative correlation can be identified between the activity times (plus the costs) and the profit. Based on the results in the table above, there are strong negative correlations between the profit and (Duration-x2; Durationx 10 ) activities, while the other activities show weaker or zero correlations with the profit. In other words, the improvement of activities (Activity2; Activity10) can
generate higher profit values, as well as it may lead to either lower costs, or the production of extra pieces (it also means extra profit for the company).

## 5. Conclusion

The aim of each company is to earn profit, while they are trying to optimize the utilization of all their resources. In this case study, stochastic operations of a manufacturing system were modeled through a stochastic multi-period production scheduling model. Based on the gathered and measured data from a real life furniture manufacturing system, indicators like raw material usage, probable working hours, work utilization, expected profit and costs can be calculated. With the application of this analysis, the most profitable product combinations were determined and the most crucial activities were revealed to see where the process improvement should be applied.

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