

## Article

# A Dynamic Systems Approach to Integrated Sustainability: Synthesizing Theory and Modeling Through the Synergistic Resilience Framework

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**Abstract:** Sustainability research encompasses diverse theories and frameworks focused on promoting sustainable economic ( $E$ ), social ( $S$ ), and environmental ( $Env$ ) systems. However, integrated approaches to sustainability challenges have been impeded due to the absence of a unified analytical framework in the field. This study investigated how foundational and emerging theories, including resilience thinking, systems theory, and planetary boundaries, could be synthesized to develop an Integrated Sustainability Model (ISM) that captures nonlinear feedback, adaptive capacities  $A_i(t)$ , and threshold effects across these domains. The ISM model employs a system dynamics approach, where the rates of change for  $E$ ,  $S$ , and  $Env$  are governed by coupled differential equations, each influenced by cross-domain feedback ( $\alpha_i$  and  $\beta_i$ ), adaptive capacity functions, and depletion rates ( $\gamma_i$ ). The model explicitly incorporates boundary constraints and adaptive capacity, operationalizing the dynamic interplay and co-evolution of sustainability dimensions. Grounded in an integrative perspective, this research introduces the Synergistic Resilience Theory (SRT), which proposes optimal sustainability arises from managing economic, social, and environmental systems as interconnected, adaptive components of a resilient system. Theoretical analysis and conceptual simulations demonstrated that high adaptive capacity and positive cross-domain reinforcement foster resilient, synergistic growth, while reduced capacity or breaches of critical thresholds ( $Env_{min}$  and  $S_{min}$ ) can lead to rapid decline and slow recovery. These insights illuminate the urgent need for integrated, preventive policy interventions that proactively build adaptive capacity and maintain system resilience. This research, by advancing a mathematically robust and conceptually integrative framework, provides a potent new lens for developing empirically validated, holistic sustainability strategies within sustainability research.



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**Keywords:** integrated sustainability model; systems dynamics; resilience thinking; planetary boundaries; adaptive capacity

## 1. Introduction

Sustainability research has become increasingly vital in response to urgent global challenges such as climate change, biodiversity loss, resource depletion, and widening social inequalities [1,2]. Since its early conceptualization in the late 20th century, rooted in environmental conservation and economic development, sustainability has evolved into a complex, multidimensional paradigm encompassing ecological, social, and economic dimensions [3,4]. However, the concept of sustainability has risen to the forefront of global discourse, reflecting a growing awareness of the interconnected challenges facing humanity and the planet. At its core, sustainability is defined by the principle of meeting

the needs of the present without compromising the ability of future generations to meet their own needs, a foundational idea articulated by the Brundtland Commission and commonly operationalized through the three interdependent pillars of economic, social, and environmental sustainability [5].

Recent decades have witnessed an explosion of sustainability-related theories, ranging from the Social-Ecological Systems framework and Resilience Theory to Ecological Modernization, Degrowth, Circular Economy, Doughnut Economics, Strong vs. Weak Sustainability, and beyond [6–9]. These dimensions interact in dynamic and often nonlinear ways, yet despite the proliferation of sustainability discourses and frameworks, a coherent integration of these diverse perspectives remains elusive. Moreover, while each of these perspectives provides valuable understanding, they often lack integration or tend to concentrate on particular dimensions of the holistic sustainability equation. As sustainability challenges such as climate change, biodiversity loss, social inequality, and economic instability intensify, there is an urgent need to synthesize and extend existing theories into more cohesive, dynamic models capable of capturing the complex realities of global development.

The existing literature comprises a rich variety of theories that address different facets of sustainability. Economic theories such as the Triple Bottom Line, Green Growth, and Circular Economy emphasize the integration of economic viability with environmental stewardship and social equity. Social sustainability theories, including the Capabilities Approach, Environmental Justice, and Social Capital, highlight equity, inclusion, and community resilience [10–12]. Environmental frameworks such as Resilience Thinking, Planetary Boundaries, and Industrial Ecology focus on ecological limits, system adaptability, and resource efficiency [9,11,13]. These dimensions interact in dynamic and often nonlinear ways, yet despite the proliferation of sustainability discourses and frameworks, a coherent integration of these diverse perspectives remains elusive. The current state of sustainability research, characterized by a lack of synthesis across various theoretical perspectives, limits its transformative potential and obstructs the development of unified strategies. This lack of cohesion within the theoretical landscape exposes a critical research gap: the need for a comprehensive, dynamic framework that synthesizes these diverse theories into an integrated model capable of representing the multifaceted and evolving nature of sustainability. Existing approaches, including the Nexus framework and Integrated Assessment Models, have advanced systemic thinking but often remain constrained in scope and lack a unified theoretical structure that incorporates technological innovation, governance, social dynamics, and ecological thresholds within a feedback-oriented dynamic system [14–18].

To further understand this unaddressed gap, the current study formulates the following research questions: How can key sustainability theories be systematically synthesized into a coherent, dynamic framework that captures the nonlinear interactions among economic performance, social equity, and environmental integrity? How can adaptive capacities and planetary boundaries be formally integrated into such a framework to reflect the realities of sustainability transitions? What mathematical and conceptual structures best represent the feedback loops, threshold effects, and path dependencies inherent in sustainability?

The objectives of this research are threefold: first, to synthesize key sustainability concepts across economic, social, and environmental domains as categorized in Table 1; second, to develop the Integrated Sustainability Model (ISM), a mathematically formalized framework that explicitly incorporates nonlinear feedback loops, adaptive capacity functions, and boundary constraints; and third, to provide a conceptual and analytical platform that supports empirical validation and informs policy and practice toward sustainability.

The ISM conceptualizes sustainability as an emergent property arising from the dynamic interplay among economic viability, social inclusivity, and environmental integrity, mediated by technological innovation, stakeholder engagement, policy coherence, and systemic resilience. The ISM moves beyond simple linear or sectoral approaches by incorporating complexity, feedback loops, and evolving path dependencies, captured mathematically through tailored differential equations. This rigorous yet flexible structure offers a foundation for empirical research and practical application, enabling scholars and practitioners to explore complex sustainability trajectories and interventions.

By targeting a broad audience of sustainability scholars, policymakers, educators, and practitioners, this article aspires to provide a widely applicable theoretical resource. The ambition of this research is to contribute to the extant efforts aimed at unifying the current theoretical landscape characterized by limited scope and overlooked complexities in sustainability science, thereby offering a platform conducive to more coherent, systemic, and impactful scholarly inquiry. Through this integrative and innovative approach, the article aims to catalyze further dialogue, critical reflection, and empirical investigation within the dynamic and rapidly evolving field of sustainability science.

## 2. Literature Review

### 2.1. Economic Sustainability Theories

Economic sustainability theories have evolved from growth-centric frameworks to those emphasizing biophysical limits and systemic integration. Traditional approaches, such as Neoclassical Growth Theory [14,15], focus on technological progress and capital accumulation, often assuming that natural and human-made capital are substitutable (“weak sustainability”). In contrast, Ecological Economics [19–21] and Steady-State Economics [22] challenge this assumption, arguing for “strong sustainability” and the recognition of non-substitutable natural capital and planetary boundaries.

More recent concepts, such as the Circular Economy [8,23] and Green Growth [7,24,25], seek to reconcile economic development with resource efficiency and environmental protection, often through innovation and closed-loop systems. The Triple Bottom Line [6] and Ecological Modernization Theory [26–28] further expand the scope by embedding social and environmental considerations within economic decision-making, though critics caution against superficial adoption (“greenwashing”) and over-reliance on technological fixes.

Collectively, these theories underscore the ongoing tension between economic growth and ecological limits, and the need for models that can integrate technological, institutional, and social innovations to achieve sustainable outcomes.

Table 1 synthesizes these diverse economic sustainability theories, highlighting their core principles, key proponents, relevance to sustainability research, and critical limitations. As shown in Table 1, these theories represent an evolution from growth-focused models to those embracing circularity and integration.

**Table 1.** Conceptual theories of economic sustainability.

Theory Name	Core Principles/Detailed Concept	Key Researchers/Originators	Relevance to Sustainability Research	Key Critiques
Neoclassical Growth Theory	Explains long-term economic expansion as a function of labor, capital accumulation, and exogenous technological progress, emphasizing the role of innovation in productivity growth. Assumes substitutability between forms of capital and posits that sustained growth is possible if the total capital stock is maintained or increased.	Robert Solow [14], Trevor Swan [15]	Provides foundational insights into economic growth dynamics and resource allocation, serving as a baseline for evaluating the integration of environmental and social factors in sustainability models.	May not fully account for the finite nature of natural resources and environmental externalities; the emphasis on continuous growth can present challenges for long-term ecological sustainability.

Table 1. Cont.

Theory Name	Core Principles/Detailed Concept	Key Researchers/Originators	Relevance to Sustainability Research	Key Critiques
Ecological Economics	Views the economy as a subsystem embedded within the finite biosphere, governed by biophysical and thermodynamic limits. Advocates for “strong sustainability”, recognizing critical natural capital as non-substitutable and emphasizing the necessity of maintaining ecological integrity for long-term wellbeing.	Herman Daly [19], Robert Costanza [20], Nicholas Georgescu-Roegen [21]	Addresses environmental justice; promotes equal opportunities; addresses the wellbeing of marginalized communities; and is interconnected with economic/environmental equity.	The strong sustainability paradigm may be perceived as restrictive in growth-oriented contexts; operationalizing theoretical concepts into practical policy remains complex.
Steady-State Economics	Advocates for an economic system characterized by stable or mildly fluctuating population and consumption levels, prioritizing ecological balance and quality of life over continuous GDP growth. Emphasizes maintaining resource throughput within the regenerative and assimilative capacities of the planet.	Herman Daly [22]	Suggest a conceptual basis for exploring alternatives to growth-centric paradigms, stimulating debate on sustainable wellbeing and resource throughput in policy and research.	Implementation in predominantly growth-driven economies may be challenging; questions remain regarding how to foster innovation and maintain wellbeing without traditional growth.
Circular Economy	Proposes a restorative and regenerative economic model that designs out waste and pollution, keeps products and materials in use for as long as possible, and regenerates natural systems. Focuses on closed-loop systems, resource efficiency, and extending product lifecycles through reuse, repair, and recycling.	Ellen MacArthur Foundation [23], Walter Stahel [8]	Supplies practical strategies for resource efficiency and waste minimization, supporting innovation in sustainable production and consumption systems within both research and policy contexts.	There is a risk that circular strategies may primarily delay, rather than fundamentally resolve, environmental pressures; ensuring equitable distribution of benefits and addressing social dimensions requires further attention.
Triple Bottom Line (TBL)	Expands traditional financial performance metrics to include environmental stewardship and social responsibility, urging organizations to balance profit, planet, and people. Promotes integrated reporting and holistic assessment of sustainability impacts across all three dimensions.	John Elkington [6]	Encourages the holistic assessment of organizational and societal performance by integrating economic, environmental, and social dimensions, influencing sustainability metrics and reporting standards.	Effective integration of all three pillars can be difficult in practice; without robust metrics and enforcement, there is potential for superficial adoption (“greenwashing”).
Green Growth Theory	Seeks to reconcile economic growth with environmental sustainability by promoting investment in clean technologies, energy efficiency, and green industries. Emphasizes decoupling economic activity from environmental degradation through innovation and policy reform.	OECD [7], United Nations Environment Programme (UNEP) [25], World Bank [24]	Informs policy and investment in green technologies and industries, fostering research on the compatibility of economic prosperity and environmental stewardship.	The approach may underemphasize absolute biophysical limits; ensuring that green growth translates into genuine environmental improvements requires careful monitoring and policy rigor.
Ecological Modernization Theory	Argues that technological innovation, market-based instruments, and institutional reforms can harmonize industrial development with environmental protection. Highlights the potential for win-win solutions where economic modernization drives ecological improvement.	Arthur Mol [28], Gert Spaargaren [27], David Sonnenfeld [26]	Provides a framework for understanding the role of technological innovation and institutional reform in advancing sustainability, with relevance to policy design and evaluation.	A strong reliance on technological and market solutions may overlook deeper structural or behavioral changes needed for sustainability; the approach may not fully address social equity considerations.
Cradle to Cradle Design	Advocates for designing products and systems so that all materials are either safely returned to nature or perpetually cycled as high-value technical resources, thereby eliminating the concept of waste. Emphasizes regenerative design and closed material loops at every stage of production and consumption.	William McDonough, Michael Braungart [29]	Inspires research and practice in sustainable product and process design, promoting closed-loop systems that minimize environmental impact and support regenerative development.	System-wide implementation can be complex and resource-intensive; widespread adoption may be limited by current economic structures and regulatory frameworks.

Note: Table 1 synthesizes foundational and emerging theories in economic sustainability from multiple academic and conceptual sources.

## 2.2. Social Sustainability Theories

Social sustainability theories converge on the importance of equity, inclusion, and collective agency. Social Equity Theory [10,30] and Human Rights Theory [31,32] foreground justice, fairness, and the protection of marginalized groups, while Social Capital Theory [12,33,34] emphasizes the role of trust, networks, and community engagement in fostering resilience and adaptive capacity. Stakeholder Theory [35] and Collective Stewardship [16,36,37] extend these ideas to organizational and global scales, advocating for participatory governance and ethical responsibility.

Despite their shared focus on social wellbeing, these theories differ in their approaches to operationalizing equity, balancing diverse stakeholder interests, and addressing structural inequalities. The challenge of measuring social sustainability and translating principles into practice remains a persistent gap, highlighting the need for integrative frameworks that can bridge normative ideals with actionable strategies.

These social sustainability theories are systematically compared in Table 2, which illustrates their conceptual foundations, leading researchers, applications in sustainability, and potential weaknesses. The theories summarized in Table 2 collectively emphasize the human dimensions essential for truly sustainable systems.

**Table 2.** Conceptual theories of social sustainability.

Theory Name	Core Principles/ Detailed Concept	Key Researchers /Originators	Relevance to Sustainability Research	Key Critiques
Social Equity Theory	Centers on fairness, justice, and impartiality in the distribution of resources and opportunities, emphasizing the need to address systemic inequalities and ensure that all individuals and groups can participate fully in society. Recognizes that equitable outcomes may require differentiated approaches based on context and need.	John Rawls [10], H. George Frederickson [30]	Provides a critical lens for examining fairness and justice in sustainability transitions, guiding research on inclusive policies and equitable resource distribution.	The concept of fairness is inherently subjective and can vary across cultural and social contexts; translating equity principles into measurable and actionable policies remains challenging.
Human Rights Theory	Provides a universal ethical and legal framework grounded in the inherent dignity and equal rights of all people, emphasizing principles such as universality, indivisibility, equality, non-discrimination, participation, and accountability. Integrates human rights into the foundation of social and environmental policy and practice.	Eleanor Roosevelt [32], John Locke [31]	Establishes a normative and ethical foundation for sustainability, supporting research on rights-based approaches to development and environmental protection.	Application of universal rights may be influenced by cultural and contextual differences; addressing deep-rooted structural inequalities requires ongoing adaptation and engagement.
Social Capital Theory	Focuses on the value derived from social networks, trust, norms of reciprocity, and collective action, positing that strong social capital enhances community resilience, adaptive capacity, and the effectiveness of sustainability initiatives. Differentiates between bonding, bridging, and linking forms of social capital.	Pierre Bourdieu [34], James Coleman [33], Robert Putnam [12]	Enhances understanding of how networks, trust, and collective action contribute to community resilience and the success of sustainability initiatives.	Benefits of social capital may not be evenly distributed, and, in some cases, strong networks can reinforce existing inequalities; measuring social capital and its impacts remains complex.

Table 2. Cont.

Theory Name	Core Principles/ Detailed Concept	Key Researchers /Originators	Relevance to Sustainability Research	Key Critiques
Stakeholder Theory	Proposes that organizations should create value for all stakeholders—not just shareholders—by considering the interests and wellbeing of employees, customers, communities, and the environment. Encourages participatory governance and ethical decision-making in sustainability transitions.	R. Edward Freeman [35]	Underpins participatory approaches in sustainability research and practice, facilitating the integration of diverse perspectives in decision-making processes.	Balancing diverse and sometimes conflicting stakeholder interests can be difficult; the absence of clear prioritization frameworks may complicate decision-making in practice.
Collective Stewardship Theory	Emphasizes the shared ethical responsibility of all individuals and groups to care for planetary resources, advocating for cooperation, moral responsibility, and the collective management of common goods to ensure long-term sustainability. Highlights the importance of trust and collaboration in resource governance.	Davis et al. [36], Chapin et al. [16], Robinson [37]	Promotes the ethical and cooperative management of shared resources, informing research on governance structures and collaborative sustainability efforts.	Assumptions of universal cooperation and ethical behavior may not always align with real-world dynamics; fostering collective action across diverse groups requires sustained effort and trust-building.

Note: Table 2 synthesizes major social sustainability theories, integrating conceptual clarifications and key critiques.

### 2.3. Environmental Sustainability Theories

Environmental sustainability theories are united by their focus on ecological limits, system resilience, and the interdependence of human and natural systems. The Planetary Boundaries framework [11] provides a science-based assessment of Earth system thresholds, while Ecological Footprint [38] and Carrying Capacity theories [39–43] offer metrics for quantifying human impact and resource limits. Deep Ecology [13,44] promotes a biocentric ethic, challenging anthropocentric paradigms and calling for a fundamental realignment of human–nature relations.

These theories are complemented by newer approaches, such as Extinction Avoidance [45–47] and Resource-Resilient World [9] theories, which emphasize the urgency of conservation and adaptive management. However, critiques persist regarding the practical quantification of boundaries, integration of social dimensions, and the translation of philosophical principles into policy. This underscores the necessity of models that can dynamically integrate environmental, social, and economic factors.

Table 3 provides a structured overview of these environmental sustainability theories, categorizing their fundamental concepts, originators, contributions to sustainability research, and limitations. As evident in Table 3, these frameworks offer complementary perspectives on ecological boundaries and human–nature relationships.

Table 3. Conceptual theories of environmental sustainability.

Theory Name	Core Principles/ Detailed Concept	Key Researchers/ Originators	Relevance to Sustainability Research	Key Critiques
Planetary Boundaries Theory	Identifies nine critical Earth system processes and proposes quantifiable boundaries that define a safe operating space for humanity. Emphasizes the interconnectedness of biophysical systems and the risks of crossing thresholds that could lead to large-scale, irreversible environmental change.	Johan Rockström, Will Steffen [11]	Provides a science-based framework for defining safe operating spaces for humanity, guiding research on global limits and risk assessment in sustainability.	Determining precise global thresholds is scientifically complex; the framework may benefit from greater integration of social, economic, and governance dimensions.

Table 3. Cont.

Theory Name	Core Principles/Detailed Concept	Key Researchers/Originators	Relevance to Sustainability Research	Key Critiques
Ecological Footprint Theory	Provides a quantitative measure of human demand on nature by comparing the biological resources required to support consumption and absorb waste with the Earth's biocapacity. Highlights ecological deficits and overshoot as key indicators of unsustainable resource use.	Mathis Wackernagel, William Rees [38]	Propose quantitative tools for assessing human demand on ecosystems, supporting comparative research and policy evaluation in sustainability studies.	Primarily focuses on renewable resources and may not capture all environmental impacts; the static nature of the metric can limit its ability to reflect dynamic system changes.
Carrying Capacity Theory	Defines the maximum population or level of resource use that an environment can sustainably support over time, considering the availability of essential resources and the assimilative capacity of ecosystems. Used to assess limits to growth and inform resource management strategies.	Thomas Malthus [39], Aldo Leopold [40], Eugene Odum [41], Garrett Hardin [42], Paul Ehrlich [43]	Informs research on the limits of resource use and population growth, contributing to scenario analysis and long-term sustainability planning.	The concept can oversimplify the complexity of human–environment interactions; estimates are sensitive to assumptions and may risk misapplication in policy contexts.
Deep Ecology	Advocates a biocentric worldview that recognizes the intrinsic value of all living beings, promoting biospheric egalitarianism, diversity, and reduced human interference in natural systems. Calls for profound ethical and societal transformation to realign human–nature relationships.	Arne Naess [13], George Sessions [44]	Challenges anthropocentric perspectives and encourages research on intrinsic ecological values, fostering holistic and ethical approaches to sustainability.	The philosophical orientation may be perceived as idealistic or difficult to operationalize; practical pathways for societal transformation are not always clearly articulated.
Extinction Avoidance Theory	Stresses the importance of safeguarding both natural and human-made resources to prevent irreversible loss, advocating for responsible use, conservation, and the preservation of biodiversity as central goals of sustainability.	Tsur and Zemel [45], Stephan, Wissel and Zschke [46], Colin W. Clark [47]	Emphasizes the importance of biodiversity and resource preservation, guiding research on conservation strategies and long-term ecosystem viability.	The focus on preventing resource extinction may overlook opportunities for sustainable transformation and adaptation; practical implementation can be constrained by economic and institutional factors.
Resource-Resilient World Theory	Argues that societies must build adaptive capacity and resilience by conserving, regenerating, and managing natural resources to withstand environmental shocks and long-term change. Focuses on proactive adaptation and ecosystem-based strategies for sustainability.	Peterson Kitakogelu Ozili [9]	Advances understanding of adaptive capacity and resilience in socio-ecological systems, supporting research on strategies for coping with environmental change and uncertainty.	Achieving resilience at scale may require significant systemic change; translating theoretical resilience into actionable strategies can be challenging in diverse contexts.

Note: Table 3 presents a consolidated overview of leading environmental sustainability theories, combining conceptual clarifications and critiques from various sources.

#### 2.4. Integrative and Systems Approaches

Sustainability challenges are inherently complex, characterized by nonlinear interactions, feedback loops, and emergent properties across economic, social, and environmental systems. Recognizing these complexities, integrative and systems approaches have become central to contemporary sustainability scholarship. Systems Theory [48] provides a foundational lens, emphasizing that real-world problems rarely have linear solutions and that interventions in one domain often generate cascading effects in others. This perspective is further developed in the Social-Ecological Systems (SES) framework [49], which conceptualizes human and natural systems as deeply intertwined, co-evolving entities whose resilience depends on adaptive cycles and cross-scale interactions.

Transition Management and the Multi-Level Perspective (MLP) [50] extend systems thinking to socio-technical change, exploring how innovations emerge, diffuse, and reshape regimes through interactions among niches, established systems, and broader landscapes. These frameworks highlight the importance of agency, governance, and institutional learning in steering sustainability transitions.

Despite these advances, many existing models remain sectoral or linear, often failing to capture the full spectrum of adaptive capacities, feedback mechanisms, and boundary constraints that define sustainability transitions. Recent integrative frameworks, such as the Nexus Approach and Integrated Assessment Models (IAMs) [17,18], have begun to bridge disciplinary divides by linking water, energy, food, and climate systems [51–54]. However, they often lack a unified theoretical structure that incorporates technological innovation, governance, social dynamics, and ecological thresholds within a dynamic, feedback-oriented system. Building upon the disciplinary theories outlined in Tables 1–3, integrative and systems approaches attempt to address complex sustainability challenges through holistic frameworks that capture cross-domain interactions and feedback mechanisms.

Overall, integrative and systems approaches underscore the necessity of moving beyond reductionist and siloed thinking toward frameworks that can accommodate complexity, uncertainty, and the dynamic interplay of multiple drivers. This systems-oriented paradigm sets the stage for the development of more robust, adaptive, and holistic models—such as the Integrated Sustainability Model (ISM) advanced in this study—that can more effectively inform sustainability science and policy.

### *2.5. Limited Scope and Overlooked Complexities in Sustainability Theory*

Despite the theoretical landscape presented in Tables 1–3, significant gaps exist in the comprehensive representation of sustainability, and crucial interactions remain unaddressed across these domains. A limitation of many models in these tables is their predominant focus on a narrow set of dimensions, often neglecting the dynamic feedback and threshold effects crucial to real-world systems. This disciplinary compartmentalization limits the capacity of sustainability science to generate actionable insights for complex, interconnected challenges.

Adaptive capacity and governance dynamics, though increasingly recognized as critical, are often treated as static or exogenous rather than as endogenous, evolving properties of systems. Furthermore, the operationalization of planetary boundaries and social foundations remains inconsistent, with many frameworks lacking mechanisms to enforce or respond to critical thresholds and regime shifts.

Recent efforts, such as the Nexus Approach and Integrated Assessment Models, have made important strides toward systemic integration, yet they frequently remain constrained in scope or lack the mathematical rigor to capture nonlinearities and emergent behaviors. The absence of a comprehensive, dynamic theoretical architecture that fully incorporates technological, institutional, environmental, and social factors within a feedback-oriented system continues to impede progress.

The inherent limitations in scope and the overlooking of complexities within current sustainability theory highlight the critical need for a unified, dynamic framework that can effectively synthesize diverse perspectives and represent the multifaceted and evolving realities of sustainability. The development of the Integrated Sustainability Model (ISM) presented in this article is motivated by this gap. The ISM aims to bridge disciplinary divides by formalizing nonlinear interactions, adaptive capacities, and boundary constraints, providing a rigorous foundation for empirical research, scenario analysis, and policy application in the pursuit of sustainability.

## **3. Methods and Methodology**

### *3.1. Conceptual Foundation*

The Integrated Sustainability Model (ISM) is grounded in a coupled socio-ecological systems perspective that formalizes the dynamic interactions among three core dimensions of sustainability: economic activity (*E*), social equity (*S*), and environmental integrity (*Env*).

These dimensions do not operate in isolation; rather, they influence and are influenced by one another through complex feedback mechanisms. The ISM captures three fundamental realities observed in sustainability transitions.

First, nonlinear feedback effects mean that improvements in one dimension can either amplify or constrain progress in others. For example, enhanced social equity can stimulate economic growth, while environmental degradation can limit social wellbeing. Second, time-dependent adaptation reflects that system responses evolve as technological innovations, policy frameworks, and societal values change over time. Third, threshold effects acknowledge that crossing certain environmental or social boundaries can trigger irreversible regime shifts, fundamentally altering system dynamics.

### 3.2. Core Dynamical System

To mathematically represent these interdependencies, the ISM employs a system of coupled differential equations that describe the rates of change of economic, social, and environmental sustainability over time. The rate of change of economic sustainability,  $\frac{dE}{dt}$ , is modeled as the sum of positive contributions from social equity and environmental health, minus losses due to resource depletion and economic inefficiencies. Formally, this is expressed as follows:

$$\frac{dE}{dt} = \alpha_1 S + \beta_1 Env - \gamma_1 E \quad (1)$$

The rate of change of economic sustainability is given by Equation (1), where the coefficient  $\alpha_1$  quantifies the social equity dividend, representing how improvements in social inclusivity and fairness bolster economic stability and productivity. The term  $\beta_1 Env$  captures the environmental contribution to economic sustainability, such as ecosystem services that support industries and livelihoods. The decay term  $\gamma_1 E$  models resource depletion and economic losses that reduce sustainability if not counterbalanced.

Similarly, the rate of change of social sustainability,  $\frac{dS}{dt}$ , depends on economic inclusion and ecosystem services, offset by social inequalities and systemic challenges:

$$\frac{dS}{dt} = \alpha_2 E + \beta_2 Env - \gamma_2 S \quad (2)$$

The coefficient  $\alpha_2$  represents economic inclusion, indicating how economic growth and wealth distribution enhance social wellbeing. The term  $\beta_2 Env$  reflects the role of healthy ecosystems in supporting social cohesion, public health, and community resilience. The decay term  $\gamma_2 S$  accounts for inequality propagation and the resulting lack of social cohesion, which can erode social sustainability.

The environmental dimension rate of change,  $\frac{dEnv}{dt}$ , is influenced by green investments and community stewardship, counteracted by degradation pressures:

$$\frac{dEnv}{dt} = \alpha_3 E + \beta_3 S - \gamma_3 Env \quad (3)$$

Here,  $\alpha_3$  quantifies the impact of green investment—economic activities directed toward renewable energy, pollution control, and sustainable resource management. The coefficient  $\beta_3$  captures community stewardship, reflecting how social engagement and local governance contribute to environmental conservation. The decay term  $\gamma_3 Env$  models degradation pressure, including pollution, habitat loss, and overexploitation.

These interconnected equations create a dynamic system where each facet of sustainability acts as both a catalyst and a consequence of change, underscoring the intricate, two-way relationships inherent in sustainability.

### 3.3. Adaptive Capacity Modulation

Real-world sustainability systems are not static; they adapt over time through technological innovation, institutional reforms, and social learning. To incorporate this adaptive capacity, the ISM introduces time-dependent modulation functions  $A_i(t)$  for each dimension  $i \in E, S, Env$ . These functions adjust the effective strength of the coupling and decay coefficients, reflecting how improvements in innovation, governance, and public awareness enhance or constrain sustainability trajectories.

The adaptive capacity function is modeled as follows:

$$A_i(t) = \frac{\theta_i I(t) + \phi_i G(t)}{1 + e^{-\kappa[D(t) - D_0]}} \quad (4)$$

In this expression,  $I(t)$  represents an innovation index, capturing metrics such as patent activity, R&D investment, or technological diffusion rates.  $G(t)$  denotes governance effectiveness, reflecting policy coherence, institutional quality, and regulatory frameworks.  $D(t)$  is the level of public sustainability awareness, which influences societal support for sustainable practices. The parameters  $\theta_i, \phi_i$ , are calibration constants that tune the sensitivity and shape of the adaptive response, while  $D_0$  is a threshold awareness level.

These adaptive functions modify the core equations by scaling the positive and negative terms. For example, the economic sustainability equation becomes the following:

$$\frac{dE}{dt} = (\alpha_1 A_E(t))S + (\beta_1 A_E(t))Env - \frac{\gamma_1}{A_E(t)}E \quad (5)$$

This formulation means that higher adaptive capacity amplifies the beneficial effects of social and environmental factors on the economy, while mitigating decay effects. Analogous adaptive modulation applies to the social and environmental sustainability equations, allowing the ISM to capture evolving resilience and responsiveness driven by human agency and institutional change.

### 3.4. Boundary Constraints

The ISM explicitly incorporates planetary boundaries and social foundations as hard limits to sustainability. Environmental sustainability must remain above a critical threshold  $Env_{min}$ , often defined as a fraction (e.g., 70%) of pre-industrial ecosystem health to ensure biophysical safety. Similarly, social sustainability must exceed a minimum social floor  $S_{SDG}$ , aligned with indicators such as the Sustainable Development Goals (SDGs) for equity, health, and education. Mathematically, these constraints are expressed as follows:

$$Env(t) \geq Env_{min}, S(t) \geq S_{SDG} \quad (6)$$

When these thresholds are violated, the system experiences regime shifts modeled using Heaviside step functions  $\Theta(x)$ . For instance, if environmental integrity falls below the critical limit, the capacity for economic growth is curtailed:

$$\frac{dE}{dt} \rightarrow \Theta(Env - Env_{min}) \cdot \Theta(S - S_{SDG}) \cdot \frac{dE}{dt} \quad (7)$$

Here,  $\Theta(x)$  equals 1 if  $x \geq 0$  and 0 otherwise, effectively halting economic growth when boundaries are breached. This mechanism captures tipping points and irreversible changes documented in ecological and social systems, emphasizing the necessity of maintaining sustainability within safe operating spaces.

### 3.5. Integrated Sustainability Index (ISI)

To provide a comprehensive, time-sensitive measure of overall sustainability performance, the ISM defines an Integrated Sustainability Index (ISI). This index aggregates the economic, social, and environmental dimensions using weighted elasticities and incorporates an intertemporal discount factor to reflect the urgency and intergenerational equity of sustainability efforts. The ISI is formulated as follows:

$$ISI(t) = \int_{t_0}^t [\lambda_E E(\tau) + \lambda_S S(\tau) + \lambda_{Env} Env(\tau)] e^{-\rho(\tau-t_0)} d\tau \quad (8)$$

In this integral,  $\lambda_E$ ,  $\lambda_S$ , and  $\lambda_{Env}$  are dimension-specific weights derived through analytic hierarchy processes or participatory stakeholder engagement, allowing the model to adapt to different contexts and priorities. The discount rate  $\rho$  encodes preferences for present versus future sustainability outcomes, emphasizing the importance of timely action.

The ISI thus synthesizes multidimensional sustainability trajectories into a single metric that can inform policy evaluation, scenario analysis, and comparative assessments.

### 3.6. Visualization and Software

All figures, including the Integrated Sustainability Model (ISM) diagrams and graphical abstract, were programmatically generated using Python (version 3.11) with *Matplotlib* (version 3.8) and *NetworkX* (version 3.1). This approach ensured consistency, scalability, and reproducibility across visualizations.

## 4. Formulation of the Integrated Sustainability Model (ISM)

The Integrated Sustainability Model (ISM) visualizes sustainability as a dynamic system where economic, social, and environmental dimensions interact through feedback mechanisms, as mathematically formalized in Equations (1)–(3). Unlike conventional frameworks that treat these domains as separate pillars, the ISM explicitly models their feedback and co-evolution, recognizing that progress or setbacks in one dimension can profoundly influence others. The model's architecture is grounded in the foundational principles of the Triple Bottom Line [6], Ecological Modernization [26,28], and Social Sustainability Theory [10,30,33] but advances beyond these by formalizing their interactions mathematically.

The ISM is structured around three core state variables: economic sustainability ( $E$ ), social sustainability ( $S$ ), and environmental sustainability ( $Env$ ). Their evolution is governed by a system of nonlinear differential equations:

$$\frac{dE}{dt} = \alpha_1 A_E(t) S + \beta_1 A_E(t) Env - \frac{\gamma_1}{A_E(t)} E \quad (9)$$

$$\frac{dS}{dt} = \alpha_2 A_S(t) E + \beta_2 A_S(t) Env - \frac{\gamma_2}{A_S(t)} S \quad (10)$$

$$\frac{dEnv}{dt} = \alpha_3 A_{Env}(t) E + \beta_3 A_{Env}(t) S - \frac{\gamma_3}{A_{Env}(t)} Env \quad (11)$$

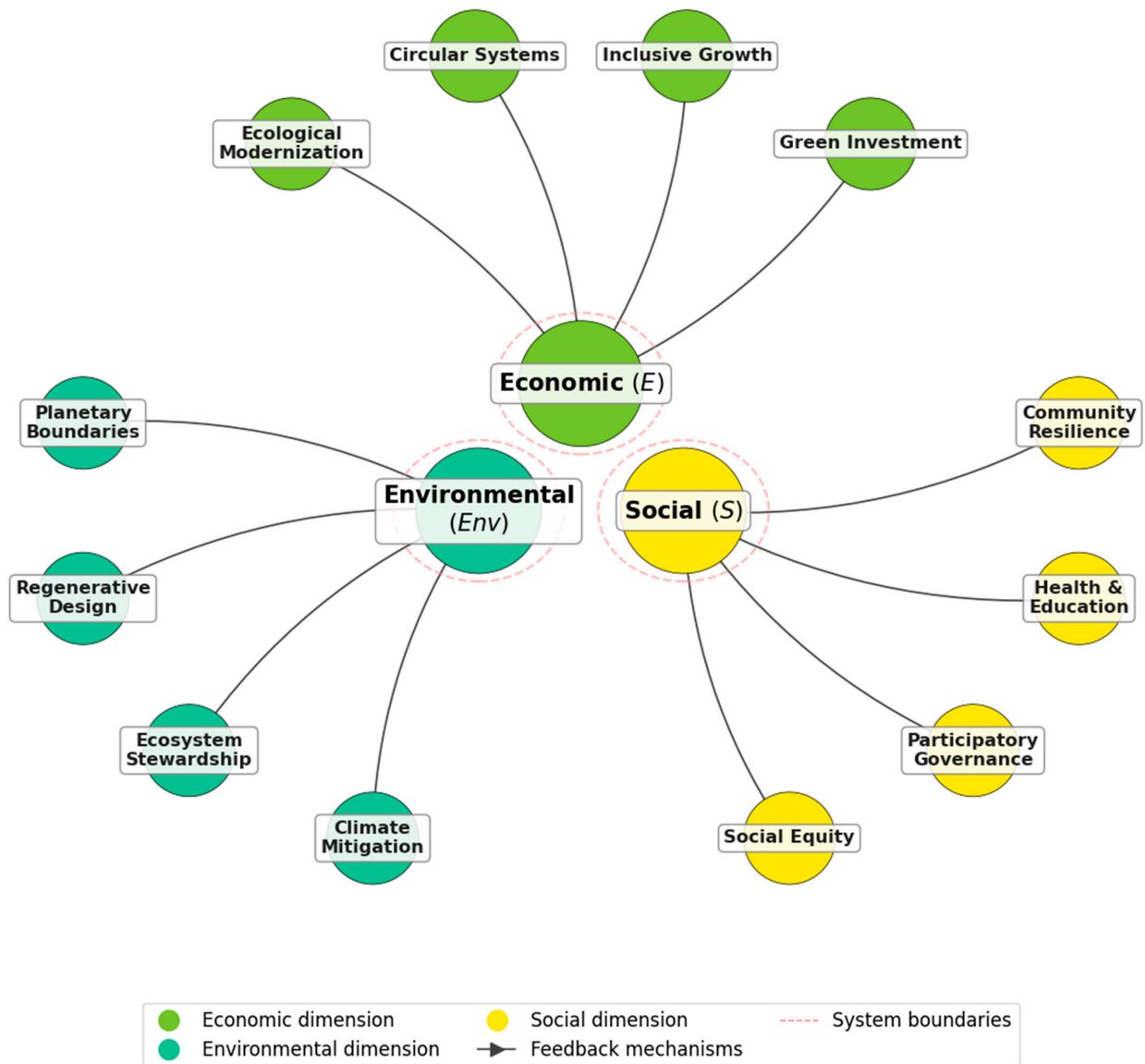
Here,  $\alpha$  and  $\beta$  are coupling coefficients that quantify the strength of positive feedback between domains, while  $\gamma$  represents decay or depletion rates. Adaptive capacity, represented by the time-dependent modulation functions  $A_i(t)$  in Equation (4), modulates these interactions. When adaptive capacity is high, the positive feedback between domains is amplified, and decay effects are mitigated, allowing the system to respond dynamically to internal and external shocks. Conversely, when adaptive capacities are weakened, arising from factors such as political instability, technological stagnation, or social instability, the

system’s resilience is compromised. This is reflected mathematically by a decrease in  $A_i(t)$ , which amplifies the impact of the decay terms ( $\gamma$ ) and reduces the positive influence of the coupling terms in Equations (1)–(3). For example,  $A_E(t)$  may be defined as follows:

$$A_E(t) = \frac{\theta I(t) + \phi G(t)}{1 + e^{-\kappa[D(t)-D_0]}} \tag{12}$$

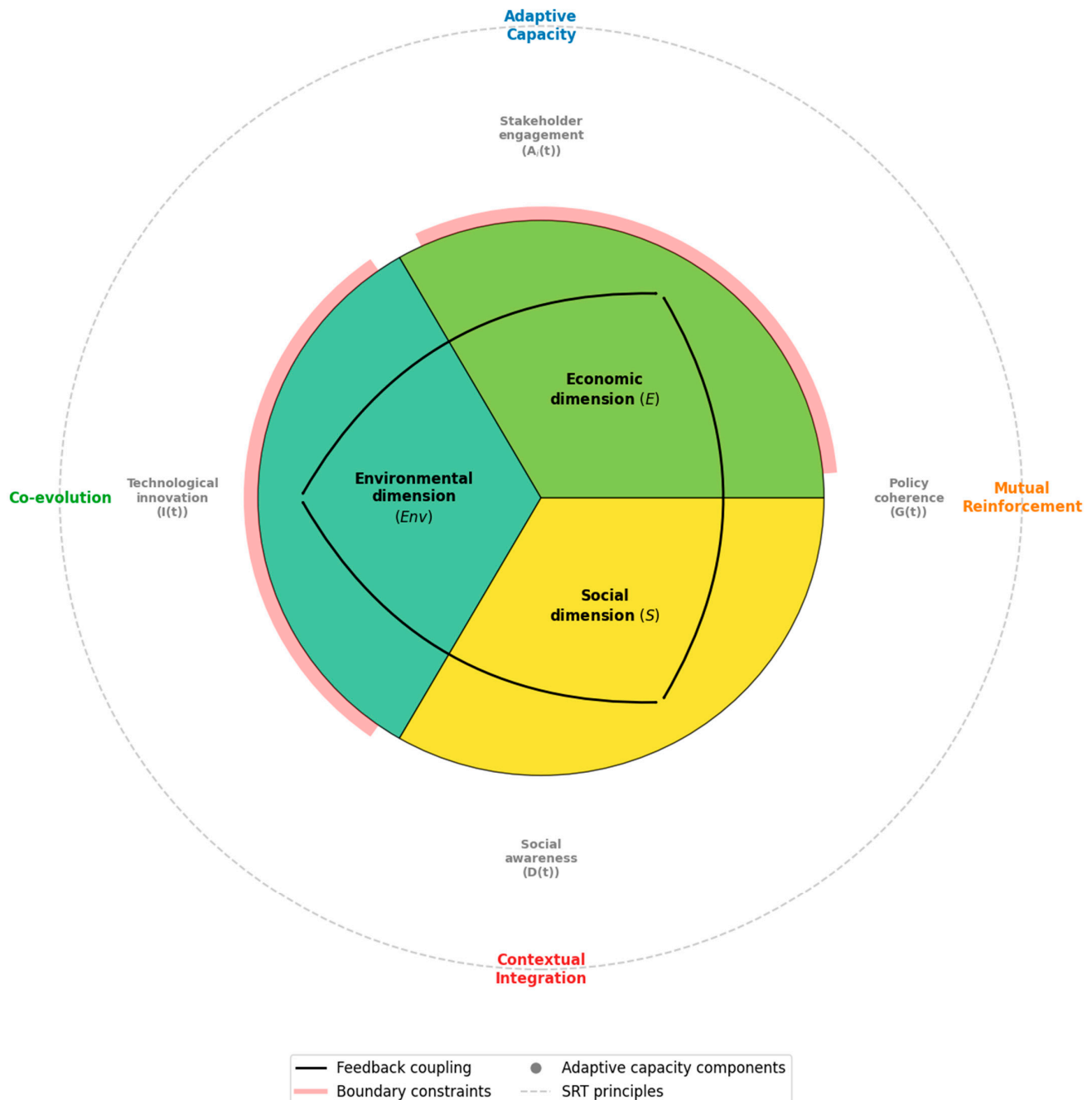
where  $I(t)$  is a technological innovation index,  $G(t)$  is governance effectiveness, and  $D(t)$  is public awareness. These adaptive capacities allow the system to respond dynamically to internal and external shocks. The sigmoid form of Equation (12) captures threshold effects in adaptive responses, where capacity increases rapidly once awareness exceeds a critical level, but changes more gradually otherwise. This mathematically formalizes the observation that sustainability transitions often exhibit nonlinear dynamics, with tipping points in social and technological adoption.

To operationalize these equations and connect them to actionable policies, Figure 1 presents a conceptual map of how the three core dimensions relate to specific thematic drivers.



**Figure 1.** Conceptual map of the three core dimensions of sustainability (economic, environmental, and social) and their primary thematic drivers. Note: Arrows indicate how each dimension influences

supporting factors such as green investment, inclusive growth, circular systems, ecological modernization (economic); planetary boundaries, regenerative design, ecosystem stewardship, climate mitigation (environmental); and social equity, participatory governance, health and education, community resilience (social). This figure categorizes foundational theories and key areas addressed in the literature. Integrative principles and dynamic interactions among these dimensions are further developed in Figure 2.



**Figure 2.** Integrated Sustainability Model (ISM) illustrating the dynamic interactions among economic (E), social (S), and environmental (Env) dimensions. Note: The inner ring denotes adaptive capacity drivers (policy coherence, stakeholder engagement, technological innovation, and resilience), while the new outer ring highlights the four core principles of the Synergistic Resilience Theory (Mutual Reinforcement, Adaptive Capacity, Co-evolution, and Contextual Integration). Red arcs represent planetary and social boundary constraints.

Figure 1 illustrates the thematic network of the ISM, showing how each core dimension is linked to actionable sub-drivers that reflect the latest theoretical synthesis. For the eco-

conomic dimension, these include green investment, inclusive growth, circular systems, and ecological modernization; for the environmental dimension, planetary boundaries, regenerative design, ecosystem stewardship, and climate mitigation; and for the social dimension, social equity, participatory governance, health and education, and community resilience. This updated structure operationalizes the ISM's mathematical framework by connecting high-level sustainability variables to specific, theory-driven policy levers and interventions. The arrows between dimensions in Figure 1 correspond to the coupling coefficients ( $\alpha$ ,  $\beta$ ) in Equations (9)–(11), providing a visual representation of mathematical relationships.

The ISM explicitly incorporates planetary boundaries and social foundations as hard limits to sustainability, as described in Equation (5). Environmental sustainability must remain above a critical threshold ( $Env_{min}$ ), and social sustainability must exceed a minimum social floor ( $S_{SDG}$ ). When environmental sustainability falls below the critical threshold ( $Env < Env_{min}$ ), a Heaviside function is triggered in the equations (Equation (13)), simulating regime shifts or collapse dynamics:

$$\frac{dE}{dt} \rightarrow \Theta(Env - Env_{min}) \cdot \Theta(S - S_{SDG}) \cdot \frac{dE}{dt} \quad (13)$$

$$\text{where } \Theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

This effectively halts economic growth when boundaries are breached. This mechanism captures tipping points and irreversible changes documented in ecological and social systems, emphasizing the necessity of maintaining sustainability within safe operating spaces. The boundary constraints in the ISM (Equations (5) and (6)) recognize that thresholds may vary contextually. For instance, environmental thresholds ( $Env_{min}$ ) reflect both global planetary boundaries and regionally specific ecological capacities. In water-scarce regions, the threshold for sustainable water use will be lower than in water-abundant areas. Similarly, social foundation thresholds ( $S_{SDG}$ ) must be adapted to reflect local socioeconomic conditions while maintaining universal principles of human dignity and wellbeing. The model accommodates this contextual variation through parameter calibration, allowing for application across diverse geographical and socioeconomic settings.

The economic dimension ( $E$ ) within the ISM prioritizes inclusive growth, ensuring that economic development benefits all segments of society and reduces inequality. This dimension emphasizes green investment, which channels capital toward environmentally sound projects and technologies; the adoption of circular systems to minimize waste and maximize resource efficiency; and the principles of ecological modernization, which integrate policy innovation and market-based solutions to advance sustainable prosperity. These economic drivers interact dynamically with other dimensions through the feedback coefficients in Equation (9), where economic sustainability both influences and is influenced by social and environmental conditions.

The environmental dimension ( $Env$ ) builds on Ecological Modernization Theory and the Planetary Boundaries framework, recognizing that environmental protection and economic growth can be mutually reinforced through technological innovation, regulatory reform, and systemic shifts in production and consumption. The ISM incorporates this perspective by maintaining planetary boundaries to ensure the biosphere's capacity to support human activities, advancing regenerative design to restore ecosystems, promoting ecosystem stewardship for biodiversity conservation, and driving climate mitigation to reduce emissions and safeguard long-term planetary health. As formalized in Equation (11), environmental health depends on positive inputs from economic and social systems, while simultaneously providing vital ecosystem services that support both domains.

The social dimension (S) draws from Social Sustainability Theory, emphasizing community resilience, social cohesion, cultural diversity, and the fair distribution of resources and services. The model centers on promoting social equity to address disparities and ensure fair access to opportunities, strengthening participatory governance to embed diverse perspectives in decision-making, investing in health and education to empower individuals and communities, and building community resilience to adapt to social and environmental challenges. These dynamics are captured in Equation (10), where social sustainability evolves based on economic inclusion and environmental support, while simultaneously contributing to both economic and environmental domains.

To enhance the dynamic interactions among these three core dimensions, the ISM integrates four supporting principles: policy coherence, stakeholder engagement, technological innovation, and resilience. Policy coherence ensures harmonized strategies across sectors and governance levels; stakeholder engagement legitimizes policies and incorporates diverse perspectives; technological innovation enables the decoupling of economic growth from environmental degradation and strengthens adaptive capacities; and resilience theory underscores the importance of systems that can adapt and transform in response to shocks and long-term changes. These principles influence the adaptive capacity functions (Equation (12)) and modulate the strength of interactions among the three dimensions, providing a comprehensive framework for understanding sustainability as an emergent property of complex, adaptive systems.

## 5. Results and Discussion

The Integrated Sustainability Model (ISM) develops sustainability as a dynamic system where economic, social, and environmental dimensions interact through feedback mechanisms, as illustrated in Figure 2. Adaptive capacity, represented by the inner ring (stakeholder engagement, policy coherence, technological innovation, and resilience), modulates these interactions, while boundary constraints (red arcs) enforce critical thresholds to prevent system collapse. This structure directly operationalizes the differential equations described in Section 4 and the conceptual map in Figure 1, where coupling coefficients ( $\alpha$ ,  $\beta$ ) and decay rates ( $\gamma$ ) govern interdependencies.

The rationale for this model is further supported by multiple theoretical frameworks. Resilience theory [55] and adaptive management [56] underscore the necessity of flexibility and learning in complex systems. Systems theory [48] advocates holistic thinking to understand leverage points. Justice theories [57] insist on procedural and substantive fairness in transitions. Innovation systems theory [50] demonstrates how socio-technical change emerges. Each of these pillars independently provides valuable insights, but only through their synthesis, as outlined in Tables 1–3, that unlocks the potential for a genuinely transformative sustainability paradigm.

Furthermore, Table 4 illustrates how specific elements from diverse sustainability theories are operationalized within the ISM framework, demonstrating the practical integration of previously siloed theoretical perspectives. The table maps key concepts from economic, social, environmental, and integrative theories to their corresponding mathematical expressions and functional relationships in the model, providing a clear visualization of how theoretical principles translate into operational model components.

**Table 4.** Integration of sustainability theories within the ISM framework.

Theory/Framework	Key Concept(s)	ISM Element(s)/Equation(s)	Operational Expression(s)
Economic Sustainability Theories			
Ecological Economics	Biophysical limits, strong sustainability	Environmental boundary constraint	$Env \geq Env_{min}$ threshold in Equations (5) and (6), Heaviside function in Equation (13)
Circular Economy	Resource efficiency, closed loops	Economic–environmental feedback, adaptive capacity	$\frac{dE}{dt} = f(\alpha_1 \times S, \beta_1 \times Env, A_i(t))$ (Equations (9) and (12))
Triple Bottom Line	Integrated performance	Three core ISM dimensions (E, S, and Env)	$ISI = w_1 E + w_2 S + w_3 Env$ (Equation (7))
Social Sustainability Theories			
Social Equity Theory	Fairness, inclusion	Social threshold, social feedback	$S \geq S_{SDG}$ (Equation (6)), $\frac{dS}{dt} = f(\alpha_2 \cdot E, \beta_2 \cdot Env, A_i(t))$ (Equations (10) and (12))
Social Capital Theory	Networks, trust, collective action	Adaptive capacity, stakeholder engagement	$A_i(t) = f(G(t), I(t), D(t))$ (Equation (12))
Stakeholder Theory	Participatory governance	Stakeholder engagement principle	$A_i(t)$ component influencing social feedback mechanisms
Environmental Sustainability Theories			
Planetary Boundaries	Critical system thresholds	Environmental boundary, regime shift	$\Theta(Env - Env_{min})$ (Equation (13))
Resilience Thinking	System adaptability, recovery	Adaptive capacity, feedback modulation	$A_i(t)$ (Equation (12)), nonlinear feedback in Equations (9)–(11)
Carrying Capacity	Maximum sustainable resource use	System limits and decay rates	$\gamma$ parameters (depletion rates) in Equations (9)–(11)
Integrative Approaches			
Systems Theory	Feedback mechanisms, nonlinearity	Coupled differential equations	Bidirectional arrows and feedback loops in Figure 2
Transition Theory	Regime shifts, path dependence	Heaviside function, threshold effects	$\Theta(\cdot)$ (Equation (13))
Adaptive Governance	Learning, institutional flexibility	Governance quality component	$G(t)$ parameter in adaptive capacity function (Equation (12))

Note.  $A_i(t)$  is the adaptive capacity function;  $\alpha, \beta$  are coupling coefficients;  $\Theta$  is the Heaviside function for regime shifts; and ISI is the Integrated Sustainability Index.

Table 4 demonstrates how diverse sustainability theories are operationalized within the ISM framework, mapping theoretical concepts to specific mathematical elements and functional relationships. Moving from this theoretical integration to visual representation, Figure 2 illustrates how these integrated theories manifest in the dynamic model. The coupling coefficients ( $\alpha, \beta$ ) that operationalize co-evolution theory in Table 4 appear as bidirectional arrows connecting the economic, social, and environmental dimensions. The adaptive capacity functions ( $A_i(t)$ ) identified in the table as mathematical expressions of Social Capital Theory and Resilience Thinking are represented by the inner ring of drivers (policy coherence, stakeholder engagement, technological innovation, and resilience). Similarly, the planetary boundaries and social thresholds described in the Planetary Boundaries Theory and Social Equity Theory columns are visualized as red constraint arcs in Figure 2. This visualization transforms abstract theoretical concepts into a functional model where sustainability emerges from the dynamic interactions among dimensions, moderated by adaptive capacity and constrained by critical thresholds. The core principles of Synergistic Resilience Theory that emerge from this integration are subsequently detailed in Table 5.

**Table 5.** Core principles of the proposed Synergistic Resilience Theory (SRT).

Principle	Description	Mechanism/Operationalization	Illustrative Application
Mutual Reinforcement	Economic, social, and environmental systems are managed as interdependent and mutually strengthening components of sustainability.	Positive feedback loops and cross-domain synergies are explicitly modeled to amplify resilience and sustainability outcomes.	Policies that simultaneously address green investment, social equity, and ecosystem restoration.
Adaptive Capacity	The ability of systems to learn, innovate, and reorganize in response to shocks and changing conditions is central to long-term resilience.	Adaptive capacity is quantified through dynamic functions (e.g., $A_i(t)$ ) reflecting innovation, governance, and public awareness.	Investment in education, technological R&D, and participatory governance to enhance system flexibility.
Co-evolution	Sustainability transitions are shaped by the co-evolution of technological, institutional, and behavioral change across domains.	The model incorporates time-dependent feedback and path dependencies, allowing for emergent dynamics and nonlinear responses.	Integrated climate, energy, and social policy reforms that evolve in tandem.
Contextual Integration	Strategies are tailored to local, regional, and global contexts, recognizing diversity in challenges, capacities, and priorities.	Theoretical parameters and policy levers are calibrated to context-specific data and stakeholder input, ensuring relevance and effectiveness.	Locally adapted resilience planning and scenario analysis.
Threshold Sensitivity	Recognizes the existence of critical boundaries (e.g., planetary or social thresholds), beyond which system collapse or regime shifts may occur.	Boundary constraints are operationalized mathematically (e.g., Heaviside functions) to trigger regime shifts when limits are breached.	Early warning systems and preventive interventions to avoid crossing ecological or social thresholds.
Preventive and Transformative Action	Emphasize proactive, early interventions and transformative change rather than reactive measures, as recovery after collapse is slower and more costly.	Policy frameworks prioritize investments and actions that build resilience and address vulnerabilities before thresholds are approached.	Pre-emptive investment in renewable energy, biodiversity conservation, and social safety nets.

Note: Table 5 synthesizes the foundational principles, mechanisms, and applications of the Synergistic Resilience Theory, as developed in this study. Each principle is operationalized within the Integrated Sustainability Model (ISM) to guide adaptive, context-sensitive, and preventive strategies for sustainability transitions.

ISM demonstrates dynamic interactions among economic (E), social (S), and environmental (Env) dimensions. Bidirectional arrows represent feedback loops, the outer ring denotes adaptive capacity drivers, and red arcs signify planetary and social boundary constraints.

In a baseline scenario with favorable adaptive capacities—meaning higher  $A_i(t)$  values and strong cross-sectoral feedback ( $\alpha, \beta$ ), and lower decay rates ( $\gamma$ )—the ISM predicts a stable trajectory towards equilibrium, where all three domains can achieve simultaneous growth. This dynamic is mathematically captured by the positive terms in the differential equations outweighing the decay terms, resulting in reinforcing cycles of improvement. For example, an increase in *Env* not only directly benefits *E* and *S* through the  $\beta$  terms but also reduces the effective decay through higher adaptive capacity.

Political instability, technological stagnation, and social disintegration (among other factors shown in Figure 2 as weakening adaptive capacities) ultimately weaken the system's resilience. In these scenarios, even small shocks can lead to large deviations, pushing one or more dimensions into decline. Mathematically, this is seen when  $A_i(t)$  decreases, amplifying the impact of the  $\gamma$  decay terms and reducing the positive influence of the coupling terms. If the environmental variable drops below  $Env_{min}$ , the boundary constraint in Equation (6) is activated, leading to rapid system collapse—a regime shift that is difficult to reverse, as the feedback intended to drive recovery is now insufficient.

A key insight from the ISM is the temporal asymmetry between degradation and recovery. The equations show that collapse can occur swiftly after boundary exceedance (Equations (5) and (6)), while restoration, even with substantial policy interventions, proceeds at a much slower rate due to the erosion of adaptive capacity (Equation (4)). This mirrors real-world observations in ecological and institutional resilience, where rebuilding takes significantly longer than collapse. To illustrate the model's behavior, the researcher conducted preliminary simulations using hypothetical parameter values ( $\alpha^1 = 0.4$ ,  $\beta^1 = 0.3$ ,  $\gamma^1 = 0.2$ ) calibrated to align with observed patterns in sustainability

transitions. Simulations of the ISM under a policy intervention that increases technological innovation ( $I(t)$ ) by 25% at year 3 resulted in continued gradual decline across all sustainability dimensions. Results show that, over the five years following the policy intervention, environmental resilience declined by approximately 13.7%, while economic and social dimensions decreased by about 6.5% and 5.5%, respectively. The intervention produced only marginal changes in these trajectories, and all indicators remained above their critical thresholds throughout the simulation. However, when environmental integrity ( $Env$ ) approaches the critical threshold ( $Env_{min} = 0.3$ ), the system exhibits reduced responsiveness to interventions, requiring significantly larger investments to achieve equivalent outcomes. These simulations, while conceptual, underscore the model's capacity to capture nonlinear dynamics and threshold effects.

The ISM also reveals trade-offs between short-term economic gains and long-term sustainability is mathematically encoded in Equation (1), where high  $\alpha_1$  values (aggressive economic growth) can initially boost  $E$ , but if  $\beta_1$  and  $\beta_2$  (the positive contributions to environmental and social domains) are low, these gains come at the expense of environmental degradation and social inequality. This dynamic, encoded in Equations (1)–(3), supports the critique of “weak sustainability” and highlights the need to prioritize strong sustainability principles.

Importantly, the ISM identifies policy leverage points. By enhancing adaptive capacity through technological innovation ( $I(t)$ ), governance improvements ( $G(t)$ ), and social awareness ( $D(t)$ ), as represented in Equation (4), the model demonstrates that it is possible to shift system trajectories towards recovery, even after partial degradation. Increasing  $A_i(t)$  strengthens positive feedback and dampens decay, improving the system's resilience.

Furthermore, the ISM proposes that these dimensions and supporting principles are not static but are dynamically linked through feedback loops. Positive changes in one sphere, such as technological innovation enhancing resource efficiency, can create cascading benefits across economic, social, and environmental domains. Conversely, neglect in one area, such as environmental degradation, can undermine social stability and economic viability. This systems-oriented view is consistent with the sustainability science paradigm [58], as summarized in Tables 1–3, which advocates for interdisciplinary approaches that recognize complexity, uncertainty, and the necessity for adaptive governance.

The overall sustainability performance is evaluated using the Integrated Sustainability Index (ISI), as defined in Equation (7). The ISI aggregates the economic, social, and environmental dimensions using weighted elasticities and incorporates an intertemporal discount factor to reflect the urgency and intergenerational equity of sustainability efforts. For scenario comparison and aggregation, normalization is performed following Equation (8), enabling a direct comparison of sustainability performance across regions and time periods.

The ISM underscores that achieving sustainability demands a deep systemic perspective, wherein economic growth must not undermine environmental integrity, and social cohesion must be nurtured to maintain adaptive capacity. Furthermore, the ISM proposes that these dimensions and supporting principles are not static but are dynamically linked through feedback loops (Equations (1)–(3)). This study's results suggest that short-term economic gains at the expense of environmental or social degradation are likely to result in systemic collapse, a finding that resonates strongly with both theoretical predictions and empirical evidence from historical case studies. Conversely, strategic investment in technological innovation, green infrastructure, inclusive governance, and social empowerment can reinforce positive feedback that drives long-term resilience and prosperity. The model's dynamic structure also reveals the temporal asymmetry between system collapse and recovery, emphasizing the preventive importance of maintaining system resilience before critical thresholds are crossed.

Collectively, these results show that the ISM not only addresses the limitations of previous models identified in the literature review but also fills the critical research gap by providing a robust, integrative approach to sustainability science. The model's structure and simulations confirm that the research objectives have been met, offering both theoretical advancement and practical tools for empirical research and policy design.

Building on the strengths and addressing the limitations of existing sustainability theories (see Tables 1–3), this study proposes the Synergistic Resilience Theory. This integrative framework contends that sustainability is most effectively achieved when economic, social, and environmental systems are managed as mutually reinforcing and adaptive components of a resilient whole. Unlike existing models that prioritize one dimension or treat others as constraints, this theory emphasizes co-evolution, mutual reinforcement, and context-sensitive adaptation. This approach aligns with recent calls in sustainability science for multidimensional, integrative frameworks capable of addressing complex, real-world challenges. Table 5 summarizes its core principles, dimensions, mechanisms, and practical applications, offering a framework for integrated, adaptive, and context-sensitive sustainability strategies.

While existing frameworks such as Social-Ecological Systems (SES) and Transition Management capture aspects of sustainability dynamics, the Synergistic Resilience Theory (SRT) proposed here differs in three fundamental ways. First, unlike SES's primarily descriptive approach, SRT provides a mathematically formalized representation of feedback mechanisms through coupled differential equations. Second, whereas the Multi-Level Perspective emphasizes socio-technical transitions without explicit environmental thresholds, SRT integrates planetary boundaries as operative constraints that trigger regime shifts when breached. Third, SRT uniquely quantifies adaptive capacity as a dynamic function of governance quality, technological innovation, and social awareness (Equation (12)), rather than treating it as a static property or exogenous variable.

The six core principles outlined in Table 5 collectively articulate the foundation of the Synergistic Resilience Theory (SRT). These principles are not isolated; rather, they interact dynamically to foster robust and adaptive sustainability transitions. Mutual reinforcement ensures that economic, social, and environmental objectives are addressed in an integrated manner, amplifying positive outcomes across sectors. Building on this, adaptive capacity emphasizes the system's ability to respond and reorganize in the face of shocks, which is essential for long-term resilience. The principle of co-evolution highlights the importance of feedback and interdependencies among technological, institutional, and behavioral domains, enabling systems to adapt to emergent and often nonlinear changes.

Contextual integration ensures that resilience strategies are tailored to the unique characteristics and priorities of local, regional, and global contexts, thereby enhancing their relevance and effectiveness. Threshold sensitivity introduces a precautionary perspective by recognizing critical boundaries and operationalizing early warning mechanisms to prevent undesirable regime shifts. Finally, preventive and transformative action shifts the focus from reactive responses to proactive investments that build resilience and address vulnerabilities before they escalate.

Together, these principles provide a comprehensive framework for designing and implementing sustainability policies that are adaptive, context-sensitive, and capable of navigating complex, interconnected challenges. By operationalizing these principles, decision-makers can move beyond incremental adjustments and pursue transformative pathways that enhance resilience across scales and sectors.

Building on the six core principles summarized in Table 5, these results highlight several policy implications. First, policymakers must recognize the interconnectedness of sustainability dimensions and design interventions that address economic, environmental,

and social systems simultaneously rather than in isolation. This aligns with the principle of Mutual Reinforcement, which emphasizes the need for integrated policy portfolios that support green innovation, environmental restoration, and social justice to reinforce positive feedback and enhance adaptive capacity. Second, the principle of Preventive and Transformative Action underscores the importance of early action; system recovery after boundary exceedance is both slower and more resource-intensive compared to preventive sustainability measures. Investments in education, participatory governance, decentralized energy systems, and biodiversity conservation exemplify pre-emptive strategies that strengthen societal resilience and reduce vulnerability to collapse. Third, the Threshold Sensitivity principle calls for policy frameworks that explicitly acknowledge and safeguard critical thresholds in environmental and social systems, operationalizing concepts from Planetary Boundaries Theory to define safe operating spaces for humanity.

Overall, the results and theoretical contributions presented here demonstrate that the ISM successfully answers the research questions posed at the outset, fulfills the stated objectives, and bridges the gap in sustainability theory by unifying economic, social, and environmental dimensions within a dynamic, systems-based framework. This integrative approach provides a foundation for future empirical validation and supports the development of actionable strategies for sustainability.

## 6. Conclusions

This study advances sustainability science by formalizing an Integrated Sustainability Model (ISM) that captures dynamic interdependence among economic, social, and environmental systems. The system of nonlinear differential equations (Equations (9)–(11)), adaptive capacity functions (Equation (12)), and boundary constraints (Equation (13)) operationalizes the theoretical synthesis of resilience thinking, systems theory, and planetary boundaries outlined in Tables 1–3. Unlike static frameworks, the ISM reveals how feedback loops, threshold effects, and adaptive capacities co-evolve, offering a mathematically rigorous tool to analyze sustainability transitions.

The ISM demonstrates that sustainability is an emergent property of complex interactions rather than a static equilibrium. For instance, the coupling coefficients ( $\alpha$ ,  $\beta$ ) quantify how social equity amplifies economic resilience, while the decay terms ( $\gamma$ ) model the erosion of environmental health under unsustainable practices. Policy simulations derived from Equation (13) show that breaching planetary boundaries triggers irreversible regime shifts, underscoring the urgency of preventive action.

This dynamic systems approach addresses critical gaps in sustainability research by providing:

- A unified framework to analyze trade-offs and synergies across dimensions;
- Quantifiable metrics for adaptive capacity and boundary constraints;
- A pathway to operationalize the SDGs through systems thinking.

Despite these contributions, several limitations should be acknowledged. The ISM relies on parameter estimates that may vary across regions and sectors, and its theoretical structure, while comprehensive, requires empirical calibration and validation with real-world data. The model's abstraction may not capture all contextual or cultural factors influencing sustainability outcomes. Future research should focus on empirical testing of the ISM, refinement of its parameters, and application to diverse case studies to enhance its predictive power and policy relevance.

At a theoretical level, the Integrated Sustainability Model invites further research into the specific parameterizations and causal mechanisms that govern sustainability dynamics across different regions, sectors, and governance scales. While this research remains primarily theoretical, future empirical studies could calibrate the ISM to real-world data,

thereby enabling predictive simulations and scenario planning exercises that enhance policy relevance. The model also encourages a move beyond traditional sectoral silos towards a systems-thinking approach that aligns with emerging global paradigms such as the United Nations' Integrated SDG Framework.

However, the contribution of this article lies not merely in synthesizing past theories but in advancing a coherent dynamic systems model that captures the essence of sustainability as an ongoing, adaptive, and deeply interconnected process. By proposing a structured yet flexible theoretical foundation, it provides a valuable tool for scholars, practitioners, and policymakers striving to navigate the complexities of global sustainability. As the global community faces mounting environmental, social, and economic challenges, models such as the ISM are essential for envisioning sustainable futures and designing pathways to achieve them.

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## Abbreviations

The following abbreviations are used in this manuscript:

AG	Adaptive Governance
CE	Circular Economy
E	Economic (dimension/variable)
Env	Environmental (dimension/variable)
GDP	Gross domestic product
GHG	Greenhouse gas
IAM	Integrated Assessment Model
ISI	Integrated Sustainability Index
ISM	Integrated Sustainability Model
MLP	Multi-Level Perspective
PB	Planetary Boundaries
RT	Resilience Theory
S	Social (dimension/variable)
SDGs	Sustainable Development Goals
SES	Social-Ecological Systems
SRT	Synergistic Resilience Theory
TBL	Triple Bottom Line
TT	Transition Theory
UN	United Nations
UNEP	United Nations Environment Programme
OECD	Organisation for Economic Co-operation and Development

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