International Journal of Heat and Mass Transfer 227 (2024) 125535

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Role of the volume-specific surface area in heat transfer objects: A critical thinking-based investigation of Newton's law of cooling

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ARTICLE INFO

*Keywords:*
Thermal energy
Heat storage
Energy efficiency
Thermal insulation
Critical thinking
Volume-specific area

ABSTRACT

In this research, the process whereby objects are cooled (or heated) is reconsidered by studying Newton's law of cooling. The findings of the study highlight the important relationship between the volume-specific surface area of objects and the dynamics of heat content change; that is, the rate at which the temperature of an object decreases is shown to additionally depend on the size of the volume-specific surface area of the object. If the surface area in contact with the environment is small compared to the volume of the object, then the effect of the environment (e.g., heat exchange, heat loss) on the object will also be small. Consequently, when attempting to enhance the energy efficiency in heat storage design, apart from increasing the thickness of the thermal insulating layer, the specific heat loss of an object could be lowered by reducing the volume-specific surface area as an additional option. This hypothesis was based on observations, and the observed phenomenon has been confirmed by a computational example following the renewed interpretation of Newton's law of cooling, which has become more understandable by applying critical thinking. Newton's law of cooling has been newly expressed for use in engineering design, practice, and education. The interpretation of this law presented in this study can be used to reduce specific heat loss and intensify heat transfer. The article draws attention to the importance of the volume-specific surface area, which is an important variable in many engineering fields other than the case presented here. The method described here can be viewed as an additional alternative to the traditional education methods considered in heat transfer textbooks.

## 1. Introduction

Thermal energy storage (TES) offers a good solution to traverse the delay between the time of energy availability and the time of consumption. TES facilitates the use of thermal solar energy for solar district heating (SDH) systems and for industrial heat recovery systems. TES should be implemented such that (1) the energy efficiency of the equipment is as high as possible, that is the heat loss of the heat storage equipment is as low as possible, and (2) the stored energy is charged and discharged as fast as possible, and (3) the volume-specific thermal energy capacity is as high as possible.

The integration of large-scale seasonal thermal energy storage (STES) facilities into solar district heating (SDH) systems is rapidly expanding in practice because this solution offers good thermal efficiency. Pan et al. [1] created an XST model in TRNSYS for a large-scale, 60.000 $m^3$ water pit thermal energy storage (PTES) facility that has been operational since 2014. The results of the model are in good agreement with practical measurements. Analysis showed that the storage cycle (yearly total energy discharge divided by the maximum energy capacity of PTES) has a significant impact on storage efficiency. By increasing the storage cycle from 1.1 to 2.16, the storage efficiency improved from 66% to 90.1% [1]. In fact, they achieved superior energy efficiency by reducing the storage time. However, because this only entailed an operational change, it will not improve the PTES. Enhancing energy efficiency could be achieved by increasing the size or applying a thicker thermal insulation layer. Storage facilities are constructed in ever larger sizes (a facility sized 200,000 $m^3$ has been developed) for a good reason.

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https://doi.org/10.1016/j.ijheatmasstransfer.2024.125535
Received 12 December 2023; Received in revised form 19 March 2024; Accepted 2 April 2024
Available online 17 April 2024

## Nomenclature

| | |
|---|---|
| $A$ | area of the surface of the object, $[m^2]$ |
| a | constant of the differential equation of energy balance |
| Bi | Biot number, $Bi = \frac{h \cdot L}{k_{object}}$ |
| b | constant, $[s^{-1}]$, $b = \frac{h\ A}{C_{object}}$ |
| $C_{object}$ | heat capacity of the object at constant pressure $\left[\frac{kJ}{^{\circ}C}\right]$ |
| $c_p$ | specific heat capacity of the object at constant pressure, $\left[\frac{kJ}{kg\ ^{\circ}C}\right]$ |
| $c_p^V$ | volume-specific heat capacity of the object at constant pressure, $\left[\frac{kJ}{m^3\ ^{\circ}C}\right]$ |
| D | diameter, $[m]$ |
| Fo | Fourier number, $Fo = \frac{\alpha \cdot \tau}{L^2}$ |
| H | height, $[m]$ |
| h | heat transfer coefficient, $\left[\frac{W}{m^2\ ^{\circ}C}\right]$ |
| $h^*$ | convection heat transfer coefficient corrected with volume-specific heat capacity, $\left[\frac{m}{s}\right]$, $h^* = \frac{h}{c_p \cdot \rho} = \frac{h}{c_p^V}$ |
| $k$ | thermal conductivity, $\left[\frac{W}{m\ ^{\circ}C}\right]$ |
| L | characteristic length of the object, $[m]$, $L = V/A = 1/\omega$ in general, $L = thickness/2$ for a slab, $L = R/2$ for a cylinder, and $L = R/3$ for a sphere |
| $m$ | mass of the object, $[kg]$ |
| $m$ | slope (see Fig. 5) |
| $M_1$, $M_2$ | specific points of the object, $M_1(x_1,y_1,z_1)$, $M_2(x_2,y_2,z_2)$ |
| Nu | Nusselt number, $Nu = \frac{h \cdot L}{k_{fluid}}$ |
| Q | enthalpy (heat), $[kJ]$ |
| T | temperature, $[^{\circ}C]$ or $[K]$ |
| $T_1$ | time factor of the differential equation of energy balance |
| V | volume of the object, $[m^3]$ |

*Greek Symbols*

| | |
|---|---|
| α | thermal diffusivity of the object, $\left[\frac{m^2}{s}\right]$, $\alpha = \frac{k_{object}}{\rho \cdot c_p}$ |
| $\delta$ | thickness (of thermal insulation material), $[m]$ |
| η | energy efficiency |
| Θ | deviation variable of the temperature, $[^{\circ}C]$ or $[K]$ |
| $\overline{\Theta}_S$ | average excess temperature of the surface of the object (deviation variable), [°C] or [K] |
| $\overline{\Theta}_V$ | average excess temperature of the volume of the object (deviation variable), [°C] or [K] |
| $\rho$ | density of the object, $\left[\frac{kg}{m^3}\right]$ |
| $\tau$ | time, $[s]$ |
| Ψ | correction factor characterises the inequality of temperature field in the object, $\Psi = \frac{\overline{\Theta}_S}{\overline{\Theta}_V}$ |
| ω | volume-specific surface area of object, $\left[\frac{m^2}{m^3}\right]$, $\omega = \frac{A}{V}$ |

*Abbreviation*

| | |
|---|---|
| CTES | Concrete Thermal Energy Storage |
| ETS | Electro-Thermal Storage |
| HTF | Heat Transfer Fluid |
| LHTES | Latent Heat Thermal Energy Storages |
| LHTSP | Latent Heat Thermal Storage Pool |
| PCM | Phase Change Material |
| PTES | Pit Thermal Energy Storage |
| SDH | Solar District Heating |
| STES | Seasonal Thermal Energy Storage |
| SWHP | Seawater-source Heat Pump system |
| TES | Thermal Energy Storage |
| TRNSYS | Transient System Simulation Tool |
| UTES | Underground Thermal Energy Storage |
| 2D | Two-dimensional |

Liu and Yang carried out a numerical investigation for a concrete thermal energy storage (CTES) facility [2], which uses hot air as the heat transfer fluid (HTF). They performed a complex optimisation of the operating parameters of the CTES. The results indicated that the HTF velocity is the most important factor affecting the charging time and charging energy efficiency. Closely related to this research is the work of Borbély [3], who investigated heat storage by way of a cascade system arranged in the form of a honeycomb structure with a spiral flow-path, where the HTF was also air. Borbély proposed a mathematical model for heat storage and optimised the structure and working parameters required for maximal thermal efficiency. Furthermore, Wang et al. used air as the heat transfer fluid (HTF), which was allowed to flow through a packed-bed of ceramic spheres ($Al_2O_3$) [4] to enable the phase contact between the air and the heat storage material to be investigated. They established that a long flow path and faster air flow rate in tanks with a small diameter resulted in more effective heat transfer than in tanks with a reduced height and a large diameter. Instead of using spheres in the packed bed, heat storage objects with other shapes and thus different volume-specific surface areas could be considered. Yu et al. [5] conducted a real-scale technical experiment with an innovative heat pump system using seawater as the heat source, after which they conducted a numerical sensitivity study of its thermal performance by investigating the effects of five parameters. Their studies are useful for designing a natural heat store. However, they used a heat exchanger of a given size and design on the heat-absorbing side, thus the effect of changing the specific surface area on the dynamics was not investigated [5]. Two geometrically different designs of geothermal heat exchanger (GHE) were investigated by Barua et al. and it was shown that the coiled tube design provides better performance compared to the straight tube design of GHEs [6].

Naturally, it is not only large-scale heat stores that are needed but also smaller ones. These stores are suitable for heat storage for shorter periods, and are explained in this article by analysing Newton's law of cooling. Luo et al. developed small heat storage equipment containing the Al-12.6%Si alloy as PCM for heating electric cars [7]. This alloy undergoes a phase change at 577 °C. The shape of the TES tank was correctly chosen as a cylinder with a diameter-to-height (260 mm) ratio close to 1 to reduce the heat dissipation area. The thermal store was heated by two electric heaters. Inside the tank, air was circulated through a U-tube while being heated. Nano ceramic insulation felt with a thickness of 10 mm was used for external insulation. The TES could supply heat for more than 3 h under the discharge power of 1.5 kW, and the heat efficiency was higher than 80%. The developers correctly recognised the importance of the geometrical shape of the tank, however, this was not explained, and the impact of the size thereof was not discussed. The design calculations of the size of the tank and the thickness of the insulating layer were not reported.

Khimenko et al. carried out highly interesting dynamic experimental investigations on heat storage equipment with electric heating (electric-thermal storage (ETS)) from a practical point of view [8]. The described work presents contradictory results. Two types of ETS structures were investigated and compared, an existing structure and the newly proposed one. As heat storage material, the old structure contained magnesite bricks in which slot-shaped air channels were created for internal heat transfer to the air, whereas the new structure contained chamotte bricks as heat storage material, with round-shaped air channels for internal heat transfer. The objective of their research was to prove that the new version of the equipment constructed with the chamotte heat storage bricks was more effective than the old structure with the magnesite bricks. The authors' conclusion that the new equipment is superior is questionable, given that magnesite is a more effective heat storage material because of its higher volume-specific heat capacity and higher coefficient of thermal conductivity [9]. Furthermore, based on our estimate, the heat transfer surface on the air side of the old construction is considerably larger than that of the new construction. Why would the round-shaped channel with a smaller surface be more effective than the slot-shaped channel with a larger surface for similar airflow characteristics? The average values of the heat transfer coefficients were determined by empirical formulas, which may cause deviations from reality. Khimenko et al. (2011) subsequently used the calculation method for the regular thermal regime, suggested by Kondratiev and Isachenko, to determine the cooling (heating) rates of TES equipment [10]. This calculation method is a logarithmic form of Newton's law of cooling, with an additional factor to account for heat conduction inside the body. Unfortunately, the same value was employed for both TES structures, which was not the correct approach. Khimenko and co-workers subsequently worked on the same subject and performed numerical 2D simulations [11]. Here, it should be noted that, in his book, Mihejev explained Kondratiev's efforts to generalize further and simplify [12]. He developed a calculation method to eliminate the size of the object, which is an erroneous approach, and our work demonstrates that size affects the volume-specific surface area.

Many studies on the intensification of heat transport between two phases have been reported in the professional literature [13–22]. The direction in which these studies developed was to find ways in which to increase the heat transfer performance. One technical area of this endeavour involved improving the internal heat transfer of heat storage equipment and heat exchangers. Another technical area is the development of cooling efficiency for technical equipment.

Zonouzi & Dadvar numerically investigated ways to enhance the internal heat transfer of thermal storage [13]. Convection was influenced by fins in different positions and of various constructions (straight and spiral). The heat transfer surfaces were the same size in the two constructions. The different heat transfer coefficients affect the heat transport intensity at each temperature level owing to the different flows. Here, it would be beneficial to investigate the effect of heat transfer surfaces with different sizes on the heat transport performance, for example, to determine more effective ways to influence the system by modifying the construction design or by increasing the heat transfer surface. Although the case without fins was investigated, this case was not analysed. Wu et al. investigated the effect of natural convection on the melting process of a horizontal shell-and-tube phase change ice storage unit [14]. The change in natural convection was induced by modifying the position of the electrical heater in the storage unit. Had they used not only simple copper tubes as heat transfer material on the electric heater, but also different types of finned heat transfer tubes, they could also have investigated the effect of the volume-specific surface area on the thermal performance because the aim was to achieve a higher heat transfer rate. Zhang et al. studied and modelled heat exchangers with different designs of smooth and finned tubes for geothermal energy recovery [19]. The fins naturally improved heat transfer. Huang et al. investigated the effects resulting from increasing the specific surface of the internal heat transfer system in latent heat thermal storage pools (LHTSPs). The new tree-shaped fins significantly improved the dynamics of heat storage [15]. Kumar et al. modified and investigated by numerical simulation the effect of natural convection by changing the location of the heat transfer fins [16]. The contact surface and volume of the fins were identical in all cases. Therefore, the effect of the change in the volume-specific surface area was eliminated. Ren et al. investigated the heat transfer to the phase change material by fins fabricated of different metals by varying several parameters such as the sizes, shapes, and angular positions [17]. The results could also have been evaluated in relation to the volume-specific surface area. The results of Kocak et al. also showed that heat transfer can be increased by modifying the geometric parameters of the fin structure [20]. Ghalambaz et al. performed simulation studies to improve the dynamics of latent heat thermal energy storages (LHTES) by applying different types and concentrations of metal nanoparticles, porous metal foam, and different geometries. The parameters have been optimised (Taguchi approach) to achieve the lowest charge and discharge time and the lowest charge leakage [23–25]. In their paper, Quan et al. presented the improvement of a thermoelectric generator (TEG). In order to improve the heat transfer performance of the heat exchanger, fins of different widths, lengths, intersection angles and spacings were placed in the heat exchanger [18]. It would also have been useful to present the effect of the variation in the volume-specific surface area of the fins. A highly comprehensive review article was written by Moradikazerouni on the design of flat-plate, pin-fin, and microchannel heat sinks [26]. Within this review article, Moradikazerouni mentioned the article by Soliman and Hassan, in which results related to the actual topic were reported [27]. Soliman and Hassan investigated the effect of the surface area ratio, which they increased to significantly improve the cooling, although they did not conduct an in-depth study of the phenomenon.

These studies show that the impact of the size of the volume-specific surface area on the results obtained is usually not properly analysed and that it is ignored as an important design parameter. The purpose of this study is to demonstrate and scientifically explain the importance of the volume-specific surface area of objects that undergo cooling and heating. Studies that directly address this issue have not yet been reported in the professional literature, and this work aims to fill this gap.

## 2. Methods

Lomonosov's research guide is used as a method to address the problem.

### *2.1. Research guidance*

> "To build a theory from observations and to correct an observation with the aid of theory, it is the best way of searching for truth." M. V. Lomonosov [28]

### *2.2. Observations*

- In winter, a snowman and a snow pile melt at a slower rate when the weather warms up. Therefore, heat transfer here is slower than with a thin layer of snow.
- A blacksmith can shape (hammer) larger glowing pieces of iron for a longer time without reheating than the smaller pieces of glowing iron. The smaller pieces of iron need to be reheated more often during forming because they cool faster.

### *2.3. The hypothesis (the theory)*

When objects undergo cooling or heating (sensible heat change), the rate of change of the temperature depends not only on

- the temperature difference (ΔT) between the objects and their environment,
- the convection heat transfer coefficient, h [W/(m$^2$ °C)],
- the specific heat capacity of the object, $c_p$ [kJ/(kg °C)], and
- the thermal conductivity within the objects, k [W/(m °C)] (internal thermal resistance), however,
- it also depends on the size of the objects, more specifically on the ratio of the surface area of the objects (the 'flow' cross-section) to the volume of the object, the volume-specific surface area ($\omega = A/V$ [$m^2/m^3$])

If the surface area (the energy flow cross section) of objects is small compared to the size of the objects, that is, if the volume-specific surface area (ω [m$^2$/m$^3$]) is small, then the effect of the process (heat exchange or heat loss) will be minor for the objects. Therefore, to decrease, for example, the cooling rate (the specific heat loss), it is necessary to reduce the volume-specific surface area, whereby the specific heat loss will be reduced as well.

### 2.4. Factors affecting the volume-specific surface area

This parameter depends on the geometric shape (this is taught), and volume (however, this is not usually taught).

a) The volume-specific surface area of objects with the same volume but different geometrical shapes

The geometrical object with the smallest volume-specific surface area is a sphere (Fig. 1).

a) The volume-specific surface area of objects with different volumes but the same geometrical shape

The volume-specific surface areas of objects with different sizes (volumes) but the same geometrical shape are not the same. The larger the size of the object, the smaller the volume-specific surface area of the object (Fig. 2).

#### 2.4.1. relationship between volume and volume-specific surface area of objects

Let the independent variable ‘V [$m^3$]’ be the volume and the dependent variable ‘ω [$m^2/m^3$]’ be the volume-specific surface area (ω=A/V, where A is the surface area of the object [$m^2$]). Table 1 lists the relationship between V and ω of the geometric bodies with different shapes. It is clear from Table 1 that the sphere has the smallest volume-specific surface area and is followed by the cylindrical body (H/D=1), and then the volume-specific surface area of the cube is even larger, if the volumes of bodies are the same (Fig. 1, Table 1). Moreover, the formulas demonstrate that the larger the volume is, the smaller the specific surface area becomes (Table 1, Fig. 3). The smaller the volume-specific surface area (ω) is, the less the extent to which the environment can influence the thermal storage.

### 2.5. Possible ways to reduce the specific heat loss

- Using thermal insulation material (thermal resistance) and/or
- reducing the specific energy transfer surface (this is the hypothesis).

## 3. Results and discussion

In this section, the correctness of the hypothesis is demonstrated.

### 3.1. Creation of the mathematical model using Newton’s law of cooling

The question: How does the temperature of a solid change over time when it enters a colder or warmer environment?

The literature, e.g. [29,30] discusses this issue from the perspective of heat conduction (the Fourier perspective), whereas now it is being examined from the perspective of heat convection (the Newton perspective).

Simplification criteria used to solve the problem:

- Let the isobaric specific heat ($c_p$) and density (ρ) of the object be constant.
- Let the value of the heat transfer coefficient (h) between the object and the environment (e.g., ambient air) during the process be constant.
- Let the temperature of the fluid or medium ($T_{environment}$) surrounding the object be constant during the process.
- Assume that the object (the solid body) is a body without internal thermal resistance. That is, the object has high thermal conductivity. For example, metals or systems with Bi < 0.1 can be considered to have such properties [31,32]. In this case, the temperature of the object ($T_{object}$) changes over time during the process ($T_{object}$ (τ)), however, the temperature on its surface and inside the object everywhere at a given time is practically the same (thermally homogeneous). The temperature of the entire object can be

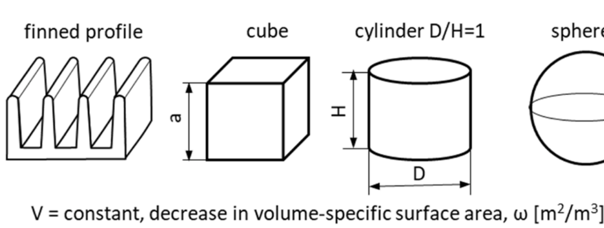


**Fig. 1.** Order of decrease of the volume-specific surface area of different geometrical objects with the same volume.

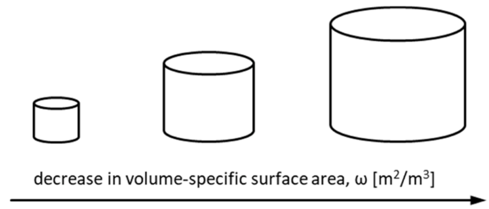


**Fig. 2.** Order of decrease of the volume-specific surface area of objects with the same geometrical shape and different volumes.

**Table 1**
Relationship between V and ω of geometric bodies with different shapes.

| | sphere | cylinder D/H=1 | cube |
|---|---|---|---|
| „V” volume, [$m^3$] | $V = \frac{D^3\pi}{6}$ | $V = \frac{D^2\pi}{4}\cdot H = \frac{D^3\pi}{4}$ | $V = a^3$ |
| „A” surface area, [$m^2$] | $A = D^2\pi$ | $A = 2\cdot\frac{D^2\pi}{4} + D\pi\cdot H = \frac{3\cdot D^2\pi}{2}$ | $A = 6\cdot a^2$ |
| „ω” volume-specific surface area (A/V), [$m^2/m^3$] | $\omega = \frac{6}{D} = \frac{6}{\sqrt[3]{\frac{6}{\pi}\cdot V}}$ $\omega \approx \frac{4.84}{\sqrt[3]{V}}$ | $\omega = \frac{6}{D} = \frac{6}{\sqrt[3]{\frac{4}{\pi}\cdot V}}$ $\omega \approx \frac{5.54}{\sqrt[3]{V}}$ | $\omega = \frac{6}{a} = \frac{6}{\sqrt[3]{V}}$ |

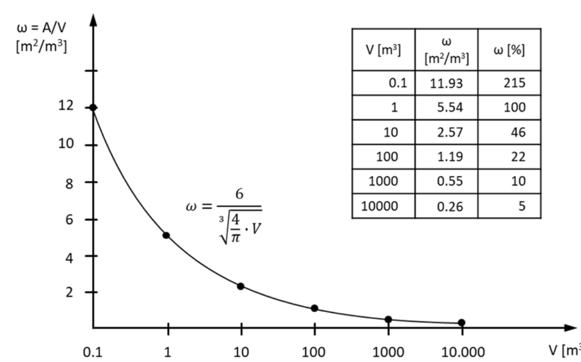


**Fig. 3.** Rate of reduction of the volume-specific surface area as the volume increases in the case of a cylinder (H/D=1)

characterised by a single time-dependent temperature. The temperature of the object is independent of location ($T_{object} = T_{surface} = T$). This case is the so-called 'lumped-heat-capacity system' in thermodynamics, and it is analogous to the 'perfectly stirred tank reactor model' in chemical engineering science.

#### 3.1.1. Newton's law of cooling

The reduction in the internal energy of object (1) is equal to the energy transferred to environment (2):

$$dQ_{object} = C_{object} \cdot dT_{object}, \tag{1}$$

where $C_{object} = m \cdot c_p$.

$$dQ_{loss} = -h \cdot A \cdot (T_{object}(\tau) - T_{environment}) \cdot d\tau, \tag{2}$$

$$C_{object} \cdot dT_{object} = -h \cdot A \cdot (T_{object}(\tau) - T_{environment}) \cdot d\tau, \tag{3}$$

$$\frac{dT_{object}}{(T_{object}(\tau) - T_{environment})} = -\frac{h \cdot A}{C_{object}} \cdot d\tau, \tag{4}$$

$$\frac{dT_{object}}{(T_{object}(\tau) - T_{environment})} = -b \cdot d\tau, \tag{5}$$

where

$$b = \frac{h \cdot A}{C_{object}}. \tag{6}$$

Usually, in practice, a specific body is tested under specific conditions, thus in this case this quotient can be considered a 'constant.'

Let us consider the environmental temperature as a reference level and only observe the deviation from this level. This is the so-called deviation variable. Let us apply this deviation variable:

$$\Theta(\tau) = T_{object}(\tau) - T_{environment}. \tag{7}$$

Subsequently, the differential equation can be described as follows:

$$\frac{d\Theta}{\Theta} = -b \cdot d\tau. \tag{8}$$

Accordingly, for a given object or body under investigation, 'b' is also considered to be constant in the literature, thus the differential equation is solved as:

$$\Theta(\tau) = \Theta(0) \cdot e^{-b \cdot \tau}, \tag{9}$$

where

$\Theta(\tau)$ is the deviation from the environmental temperature of the object at time $\tau$,

$\Theta(0)$ is the deviation from the environmental temperature of the object at time $\tau$=0.

Eq. (9) is Newton's law of cooling.

The temperature difference changes exponentially over time and disappears in the boundary case (in equilibrium):

$$\lim_{\tau \to \infty} T_{object}(\tau) = T_{environment}, \tag{10a}$$

$$\lim_{\tau \to \infty} \Theta(\tau) = 0. \tag{10b}$$

The following expression, which is often used in practice, is expressed by the same Newton's law of cooling:

$$\Theta(\tau) = \Theta(0) \cdot e^{-Bi \cdot Fo}. \tag{9b}$$

#### 3.1.2. A new expression for Newton's law of cooling

Based on critical thinking, the surface area of an object should not consider a given value (constant), because the surface area of the object depends on the volume (size) and the shape of the object. Therefore, 'b' will not be constant, and it can be written as:

$$b = \frac{h \cdot A}{C_{object}} = \frac{h \cdot A}{c_p \cdot m} = \frac{h \cdot A}{c_p \cdot \rho \cdot V} = \frac{h}{c_p \cdot \rho} \cdot \frac{A}{V} = \frac{h}{c_p^V} \cdot \omega = h^* \cdot \frac{A}{V} = h^* \cdot \omega, \tag{11}$$

where

$\omega = \frac{A}{V}$ is the volume-specific surface area of the object, a characteristic geometric parameter,

$h^* = \frac{h}{c_p^V}$ is the convection heat transfer coefficient corrected with the volume-specific heat capacity. The correction is required to take into account the properties of the object (solid material) ($c_p$, $\rho$).

Substituting this into Newton's law of cooling yields the new expression (Fig. 4):

$$\Theta(\tau) = \Theta(0) \cdot e^{-h^* \cdot \omega \cdot \tau}. \tag{12}$$

The exponential function of the temperature change, that as, the rate of cooling, depends not only on the time (and the corrected heat transfer coefficient) but also on the volume-specific surface area. Eq. (12) expresses Newton's law of cooling such that it highlights and emphasizes the importance and extent to which the volume-specific surface area affects the rate of temperature change. In fact, during cooling and heating, the rate of temperature change depends on the heat transfer coefficient, characteristics of the material, and volume-specific surface area. Inspection of the equation reveals an extremely interesting phenomenon that must be considered in experiments and during design. It is meaningless to set the heat transfer coefficient 'h' to the same value, e.g., using the same Nusselt (Nu) number for an object with the same geometry and environment, if the object consists of a different material. In this case, the thermal properties of the system are different, because the heat transfer is also affected by the volume specific heat capacity of the object. In fact, h*, the corrected convection heat transfer coefficient, would have to be assigned the same value.

Now, suppose our system is not a lumped-heat-capacity system. In this case, the rate of change in the temperature of the object also depends on the thermal conductivity of the object, and the system will be even slower. This is the case if $0.1 \leq Bi \leq 40$ [32]. Another study considered a larger range: $0.1 \leq Bi \leq 100$ [10]. For practical purposes, Kondratiev's calculation method, which was developed for the regular thermal regime, can be used (Fig. 5). Furthermore, Kondratiev started from Eq. (9) and modified the exponent by a factor Ψ [12,33]:

$$\Theta(\tau) = \Theta(0) \cdot e^{-m \cdot \tau}, \tag{13}$$

$$m = \Psi \cdot \frac{h \cdot A}{C_{object}}. \tag{14}$$

If ln Θ is plotted as a function of time, then m is the slope of the line (Fig. 5).

The correction factor Ψ characterises the inequality of the tempera-

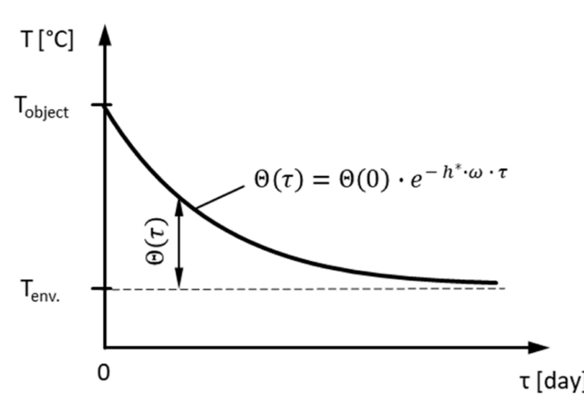


**Fig. 4.** Cooling of the object.

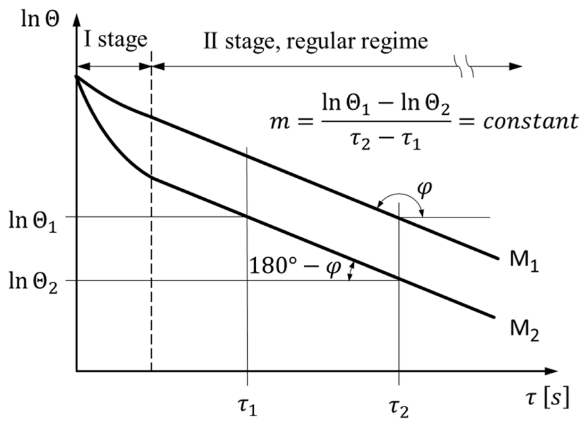


**Fig. 5.** Dependence of ln Θ on time for cooled object [10,12] $M_1$, $M_2$ are specific points in the object: $M_1(x_1,y_1,z_1)$, $M_2(x_2,y_2,z_2)$.

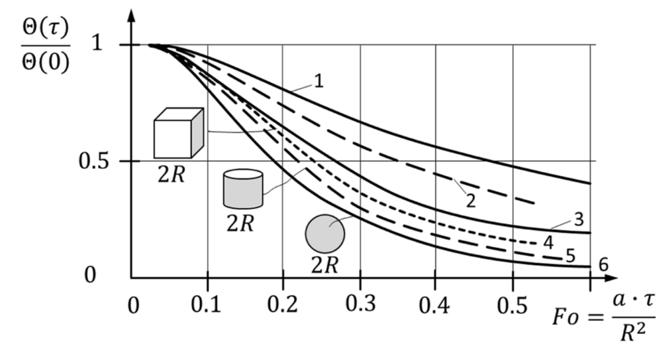


**Fig. 6.** Rate of cooling in the centre of various bodies with equal reference dimension L: 1 – infinite plate; 2 – square beam of infinite length; 3 – cylinder of infinite length; 4 – cube; 5 – cylinder of length equal to diameter; 6 – sphere (Authors' note to the diagram [10]: the sphere, cylinder, and cube with their dimensions were drawn by the authors for improved visualisation and understanding.).

ture field in the object. Ψ, the coefficient of non-uniform temperature distribution, is equal to the ratio of the average excess temperature of the surface of the object, $\overline{\Theta}_S$, to the average excess temperature of the volume of the object, $\overline{\Theta}_V$:

$$\Psi = \frac{\overline{\Theta}_S}{\overline{\Theta}_V}. \tag{15}$$

Its value is $0 < \Psi \leq 1$, where Ψ is a function of the Biot number, Ψ(Bi).

It can be written using the new expression for Newton's law of cooling (12) as follows

$$\Theta(\tau) = \Theta(0) \cdot e^{-\Psi \cdot h^* \cdot \omega \cdot \tau}, \tag{16a}$$

$$\Theta(\tau) = \Theta(0) \cdot e^{-\Psi \cdot \mathrm{Bi} \cdot \mathrm{Fo}}. \tag{16b}$$

If $Bi < 0.1$, then $\Psi = 1$ and the system becomes a lumped-heat-capacity system.

### 3.2. Critical comment on the description of the cooling process in the literature

Chapter 3–9 of the book by Isachenko et al. [10] is titled 'Dependence of Cooling (Heating) on the Form and Size of Bodies'. At the beginning of the chapter, it is correctly written that, the larger the surface-to-volume ratio, the faster the rate at which the cooling process proceeds. However, this is not followed by a correct interpretation and conclusion. Subsequently, they state that 'this is true at any Bi number, ... and an identical reference dimension and other identical conditions the greatest rate of temperature variation in time will be observed in the sphere.' The following figure (Fig. 3.19. in the book, Fig. 6 in this study) is then presented:

Finally, the authors of the book come to the following conclusion with reference to Figs. 3–19.: 'The rate of cooling is faster for a sphere than for any other body (at an equal Biot number)'.

The same statement appears in Chapter 32.b of Mihejev's book [12]. The source of the descriptions in the books by Mihejev and Isachenko, although no reference is given, is the original 1955 edition of Gröbers' book (in Chapter C), because the description corresponds word for word to that in Gröbers' book and the figure (abb. 32.) is also the same [34]. English, Japanese, Spanish, and Russian translations of Gröbers' book have also been published.

Although the cooling temperature charts cited here are correct, they should have been explained and should not have led the reader to the general conclusion that, in the same environment, among objects of the same material but with different shapes, the sphere cools the fastest.

The original English-language literature, articles, and textbooks refer to Heisler's temperature charts [35], the Gurney-Lurie charts, and the Williamson-Adams chart [36]. The Williamson-Adams chart is the same as that in Fig. 6 (diagram in Gröbers' book), and summing up the individual diagrams (sphere, cylinder, etc.) gives the same result. For example, a comparison of the temperature charts in Figs. 3, 4, and 5 in the paper by Colakyan et al. [29] illustrates the descriptions in the text.

Thus, Fig. 6 is neither satisfactorily explained nor interpreted in the literature, and the general conclusion, with reference to the figure, that the sphere cools the fastest, is not correct. Therefore, the following explanation can be added to the literature:

Under the same conditions, cooling (heating) objects consisting of the same material could be divided into three different categories:

1) Objects with the same volumes, but different geometrical shapes. In this case, the object with the smallest volume-specific surface area cools the slowest. Comparing the sphere, cylinder, and cube, the sphere has the smallest volume-specific surface area (see Table 1 and Fig. 1). Thus, the sphere cools the slowest, followed by the cylinder and, finally the cube.
2) Objects with the same geometrical shapes, but different volumes (see Fig. 2). In this case, the object with the largest volume has the smallest specific surface area (see Fig. 3), hence the object with the largest volume cools the slowest. This was the observation and the hypothesis, and it has been verified.
3) The third case is a special case and includes objects with different geometrical shapes and volumes, but the same volume-specific surface areas. Only this special case was presented in the literature (see Fig 6). In this case, the object with the smallest volume cools the fastest. Determination of the volumes of the sphere, cylinder, and cube illustrated in Fig. 6 reveals that the sphere has the smallest volume, followed by the cylinder, and then the cube with the largest volume, whereas their volume-specific surface areas are the same. Thus, if Bi>0.1, the sphere cools the fastest, followed by the cylinder, and finally the cube, as shown in the figure. If $\mathrm{Bi} \leq 0.1$, the objects cool at the same rate.

These comments should be considered when designing heat accumulators and heat transfer objects.

### 3.3. Practical benefits

Eq. (12) shows that the cooling time can also be changed by changing the volume-specific surface area of an object. The value of this area can be modified not only by changing the shape of the object but also by changing the size. This equation also demonstrates that the initial hypothesis was correct. This expression is highly advantageous in technical design, as the required heat storage time or the desired energy efficiency can be designed by appropriately selecting the volume-specific surface area (size and shape of an object), and of course, the use of a thermal insulation layer. Therefore, the practical benefits manifest themselves in the design of heat storage equipment/facilities, equipment for the chemical industry, buildings, etc., where the heat loss and energy efficiency are not calculated retrospectively, however, the required size and shape are already determined in this way. It is immediately understandable that, with the same thermal insulation, large multi-family urban apartment blocks are much more energy efficient than small single-family houses, as the volume-specific surface area of large buildings is much smaller. Engineers now have a new design variable to take into account in their heat loss calculations: the volume-specific surface area.

## 4. Calculation example for heat storage

Consider an underground thermal energy storage (UTES) facility [37], which is a buried cylindrical (H/D=1) container for heat storage. We consider two cases. In the first, heat is extracted from the heat storage container at a continuous constant output of 4 kW, and in the other case at 1 MW. In both cases, the thermal storage tank must be designed (sized) such that its temperature changes from 80 °C to 15 °C in 6 months (180 d). The temperature of the surrounding soil is assumed to remain constant at 10 °C during winter. The tank is assumed to be empty when its temperature reaches 15 °C. The thermal conductivity of the applied thermal insulation material is 0.04 W/(m°C). Let the mean volume-specific heat capacity of the heat storage material (moist sand) be 2.5 MJ/($m^3$ °C).

The size of the thermal storage tank and thickness of the thermal insulation can be estimated, which enables the energy efficiency of the thermal storage container to be calculated as follows.

Simplification criteria used to solve the problem:

- The thermal insulation of the heat storage container is considered to be part of the environment and the resistance of the environment; thus, we consider the heat in the insulation layer as heat loss and do not take into account the increase in the size of heat storage container because of the thickness of the thermal insulation.
- Let the temperature of the environment ($T_{environment}$, or $T_{soil}$) surrounding the thermal insulation of the object be constant during the process. Therefore, let the temperature of the soil be constant.
- Consider the thermal insulation layer to be a flat wall for simplicity. For large heat storage facilities, this does not introduce significant calculation errors.
- The model with the lumped-heat-capacity system is applied to the object (i.e., to the thermal storage container).

Known data:

$$\Theta_{object}(0) = 80^{\circ}C - 10^{\circ}C = 70^{\circ}C(initial\ condition)$$

$$\Theta_{object}(180) = 15^{\circ}C - 10^{\circ}C = 5^{\circ}C$$

$$k_{insulation} = 0.04\ W/(m\ ^{\circ}C)$$

$$c_p^V = c_p \cdot \rho_{object} = 2.5\ MJ\Big/\left(m^3\ ^{\circ}C\right)$$

$$P_1 = 4\ kW; P_2 = 1\ MW$$

The energy balance:

$$\frac{dQ_{object}}{dt} = -\dot{Q}_{loss} - P \tag{17}$$

$$dQ_{object} = c_p \cdot m \cdot d\Theta_{object} = c_p \cdot \rho_{object} \cdot V \cdot d\Theta_{object} = c_p^V \cdot V \cdot d\Theta_{object} \tag{1a}$$

$$\dot{Q}_{loss} = \frac{k_{insulation}}{\delta_{insulation}} \cdot A \cdot \left(T_{object} - T_{environment}\right) = \frac{k_{insulation}}{\delta_{insulation}} \cdot A \cdot \Theta_{object}(\tau) \tag{18}$$

$$c_p^V \cdot V \cdot \frac{d\Theta_{object}}{dt} = -\frac{k_{insulation}}{\delta_{insulation}} \cdot A \cdot \Theta_{object} - P$$

$$\frac{d\Theta_{object}}{dt} = -\frac{k_{insulation}}{c_p^V \cdot \delta_{insulation}} \cdot \frac{A}{V} \cdot \Theta_{object} - \frac{P}{c_p^V \cdot V}$$

The equation rearranged:

$$T_1 \cdot \frac{dy(t)}{dt} + y(t) = a$$

where $T_1 = \frac{c_p^V \cdot \delta_{insulation}}{k_{insulation} \cdot \omega}$, $y(t) = \Theta_{object}(\tau)$, $a = -\frac{\delta_{insulation}}{k_{insulation} \cdot \omega} \cdot \frac{P}{V}$, and $\omega = \frac{A}{V} = \frac{5.54}{\sqrt[3]{V}}$ (see Table 1).

The differential equation is solved using the Laplace transform:

$$T_1 \cdot (s \cdot y(s) - y(0)) + y(s) = \frac{a}{s}$$

where $y(0) = 70$.

The equation rearranged:

$$y(s) = a \cdot \frac{1}{s \cdot (T_1 \cdot s + 1)} + T_1 \cdot 70 \cdot \frac{1}{(T_1 \cdot s + 1)}$$

The inverse Laplace transformation is used to obtain the time function:

$$y(t) = a \cdot \left(1 - e^{-\frac{t}{T_1}}\right) + 70 \cdot e^{-\frac{t}{T_1}},$$

$$\Theta_{object}(t) = -\frac{\delta_{insulation}}{k_{insulation} \cdot \frac{5.54}{\sqrt[3]{V}}} \cdot \frac{P}{V} \cdot \left(1 - e^{-\frac{k_{insulation} \cdot \frac{5.54}{\sqrt[3]{V}}}{c_p^V \cdot \delta_{insulation}} \cdot t}\right) + 70 \cdot e^{-\frac{k_{insulation} \cdot \frac{5.54}{\sqrt[3]{V}}}{c_p^V \cdot \delta_{insulation}} \cdot t}$$

Substituting $\Theta_{object}(180) = 5\ ^{\circ}C$, and the different thicknesses of the thermal insulation material in the equation gives the corresponding heat storage volumes. The results of this calculation are presented in Table 2.

The results clearly show that the energy efficiency of the larger storage container is considerably higher than that of the smaller storage container. This indicates that ‘economies of scale’ apply to energy storage. In addition, it can be seen that size ‘insulates’, and reduces the specific heat transfer. In the small size range, the system is highly

**Table 2**
Calculated solutions of the example.

| | $P_1 = 4$ kW | | | | $P_2 = 1$ MW | | | |
|---|---|---|---|---|---|---|---|---|
| $\delta_{insulation}$ [m] | V [$m^3$] | H or D [m] | ω [$m^2/m^3$] | η [%] | V [$m^3$] | H or D [m] | ω [$m^2/m^3$] | η [%] |
| 0.2 | 646 | 9.4 | 0.640 | 31.3 | 104 300 | 51.0 | 0.118 | 91.0 |
| 0.3 | 546 | 8.9 | 0.677 | 57.4 | 101 400 | 50.5 | 0.119 | 94.0 |
| 0.4 | 500 | 8.6 | 0.697 | 69.4 | 99 900 | 50.3 | 0.119 | 95.6 |
| 0.5 | 475 | 8.5 | 0.709 | 75.9 | 99 100 | 50.2 | 0.120 | 96.5 |
| 0.6 | 458 | 8.4 | 0.718 | 80.4 | 98 500 | 50.1 | 0.120 | 97.1 |
| 0.7 | 447 | 8.3 | 0.724 | 83.2 | 98 100 | 50.0 | 0.120 | 97.5 |
| 0.8 | 438 | 8.2 | 0.729 | 85.6 | 97 800 | 49.9 | 0.120 | 97.8 |

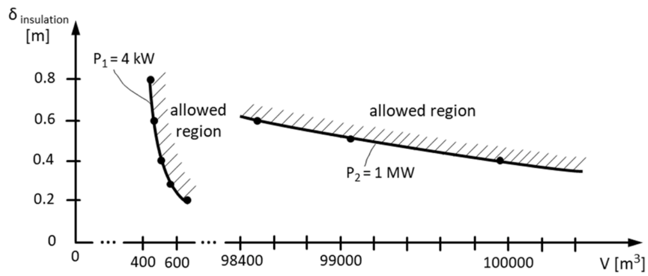


**Fig. 7.** Ranges of possible solutions.

sensitive to changes in the volume, which can only be compensated for by a significant change in the insulation thickness. The reverse is also true in that small changes in the insulation thickness result in large variations in the specific heat loss (Table 2 and Fig. 7).

## 5. Summary and conclusion

The storage of thermal energy generated from the utilisation of industrial waste heat or concentrated solar energy is of great importance for sustainable development and energy economics [38]. Therefore, further considerations and improvements are needed in the field of heat storage. This article also draws attention to the thermal design of objects and technical components, which has become another important topic.

The most important statement regarding the line of thought presented in this study is that the rate at which objects are cooled also depends on the volume-specific surface area, which is influenced by the shape and size of an object. Although there are publications in the literature on this topic [39–44], they have not been discussed in such depth, and they have not reached the interpretation presented here. The volume-specific surface area would also have to be included in Newton's heat transfer law because this is the only way to interpret it correctly.

(1) This study presented a new expression for Newton's law, which is also applicable to engineering education and practice. The new expression helps to interpret the cooling (heating) of objects. This can be seen as a new alternative to traditional teaching methods and is recommended for science and engineering education.
(2) The descriptions of the cooling (heating) of objects in the literature were examined and critical comments were considered to enhance their interpretability.

The environment always affects the object. However, for the desired effect to be small, the volume-specific surface area must be reduced and/or the object must be insulated. Increasing the volume has the same effect as increasing the thickness of thermal insulation. However, we also showed that the relationship between the volume-specific surface area and volume is nonlinear, such that the same size increase would decrease the volume-specific surface area differently depending on the size range; conversely, the same size decrease would not result in the same increase in the volume-specific surface area in all size ranges. This should be taken into account when designing heat transfer objects. For large sizes, the system is not as sensitive to changes in the volume or insulation thickness as for small sizes.

(3) The results of the calculation we presented show that the larger the heat store is, the higher is the energy efficiency. Thus, the principle of 'economies of scale' also applies to heat storage.

The "lumped-heat-capacity system" model is appropriate for preliminary engineering designs because of its simplicity. This can also be done because the uncertainty of the environmental parameters is also high [30].

The most significant outcome of this article would be the expansion of the literature with the presented approach. Besides this, the study can be linked to many practical applications. For example, it confirms that the energy efficiency of larger buildings, such as multi-apartment blocks, is much better than that of small detached houses with the same thickness and material of insulation, as size also influences specific surface area. It is also confirmed and explained by the theory presented here that it is a much faster process to melt smaller particles of the same total mass than larger bodies. Grinding, which increases the specific surface area, therefore plays a major role in the melting of ores and rocks. Another result is that the article also draws attention to the importance of what material is used for cooling fins. This is indicated by h* and the practical applications and possibilities of interpretation presented in the article could be listed extensively.

However, the reasoning we presented here is not only interesting with respect to the topic of heat transfer but also the topic of mass (component) transfer because the volume-specific surface area is also an important parameter in the latter case. Transport phenomena is an important field of chemical engineering science and practice. One of the significant tasks of engineering education is the integration of natural science and engineering practice. In this regard, our work aims to make a valuable contribution.

## CRediT authorship contribution statement

**István W. Árpád:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Investigation, Formal analysis, Conceptualization. **Judit T. Kiss:** Writing – review & editing, Validation, Supervision, Investigation. **Dénes Kocsis:** Writing – review & editing, Visualization, Supervision, Formal analysis.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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