



Recent developments in paraffin wax- sand/ $C_3H_8O_2-H_2O$ nanomaterial under LTNE conditions using Levenberg-Marquardt method

Muhammad Kamran^{a,b,c}, Nargis Khan^{d,e}, Saba Liaquat^f, Haitham M. Hadidi^g, Gafur Abdulakimov^h, Munawar Abbas^{i,j,*}, Ali Hasan Ali^{k,l}, Mohamed Hafez^{b,m}

^a Mathematics Research Center, Near East University, Mersin 10, Nicosia, 99138, Turkey

^b Faculty of Engineering and Quantity Surviving, INTI International University Colleges, Nilai, Malaysia

^c International Center for Interdisciplinary Research in Sciences, The University of Lahore, Lahore, Pakistan

^d Jadara University Research Center, Jadara University, Irbid, 21110, Jordan

^e Sustainable Development Research Center, Azerbaijan State Oil and Industry University, Baku, Azerbaijan

^f Department of Computer Engineering, Biruni University, Istanbul, 34010, Turkey

^g Department of Mechanical Engineering, College of Engineering and Computer Science, Jazan University, P. O. Box 706, Jazan 45142, Jazan, Kingdom of Saudi Arabia

^h School of Natural Sciences, National Pedagogical University of Uzbekistan named after Nizami, Tashkent, Uzbekistan

ⁱ Department of Mathematics, Firat University, Elazig, 23119, Turkiye

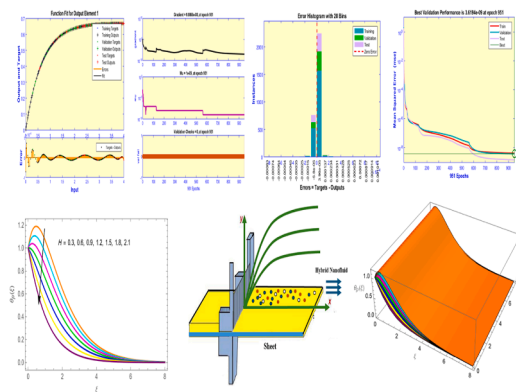
^j Mechanics and Mathematics Department of the Western Caspian University, Baku, Azerbaijan

^k Department of Business Management, Al-imam University College, Balad, Iraq

^l Technical Engineering College, Al-Ayen University, Dhi Qar, 64001, Iraq

^m Faculty of Management, Shinawatra University, Pathum Thani, Thailand

GRAPHICAL ABSTRACT



* Corresponding author at: Department of Mathematics, Firat University, 23119, Elazig, Turkiye.

E-mail address: m.abbas7478@gmail.com (M. Abbas).

ARTICLE INFO

Keywords:

LTNE conditions

Paraffin Wax-Sand/ $\text{C}_3\text{H}_8\text{O}_2\text{-H}_2\text{O}$ nanomaterial

Renewable energy

Soret and Dufour impacts

Levenberg-Marquardt method

ABSTRACT

The sand-paraffin wax/ $\text{C}_3\text{H}_8\text{O}_2\text{-H}_2\text{O}$ nanomaterial model for local thermal non-equilibrium (LTNE) circumstances has significant potential in advanced thermal energy storage and heat transfer systems. It is especially relevant for latent heat thermal energy storage units, solar thermal collectors, building energy management, electronic cooling, and industrial heat recovery systems, where paraffin wax acts as a phase change material and sand improves thermal conductivity and structural stability. The inclusion of Dufour and Soret properties allows for more accurate prediction of coupled mass and heat transmission in systems with significant temperature and concentration gradients, whereas the Levenberg-Marquardt method provides efficient and accurate prediction of the governing transport properties. The results of this model can help to build energy-efficient thermal storage devices, renewable energy technologies, and sustainable cooling and heating systems with increased thermal performance and operational reliability. This article provides a short analysis of the Dufour and Soret influence on a Paraffin Wax- Sand/ $\text{C}_3\text{H}_8\text{O}_2\text{-H}_2\text{O}$ nanomaterial under LTNE effects. The neural network with the Levenberg-Marquardt method (NN-LMA) is the foundation of an integrated numerical computing system. The liquid phase's thermal field increases as the inter-phase heat transmission characteristic increases, whereas the solid phase's thermal field declines.

Introduction

The machine learning methods like as reversion models as well as neural networks offer a computationally powerful alternative to traditional CFD (computational fluid dynamics) imitations by providing forecasts of pressure, temperature, and real-time velocity for complex liquid velocity utilization. The ML (machine learning) is therefore great for enhancing energy-saving liquid systems, aerodynamic designs as well as thermal exchangers, because it declined computational expenses without sacrificing precision. This capability is particularly transformative for renewable energy systems, where ML optimizes the performance of solar thermal collectors, wind turbine aerodynamics, and geothermal heat exchangers by predicting complex fluid behavior under varying environmental conditions. In renewable energy applications, ML-driven models enable real-time control of hybrid systems, such as solar-wind-battery microgrids, by forecasting energy demand and supply fluctuations, thereby maximizing efficiency and reducing reliance on fossil fuels. Furthermore, ML-based turbulence models can significantly improve prediction accuracy by training on high-fidelity datasets generated by DNS (Direct Numerical Simulation), outperforming traditional turbulence modelling techniques. The ML advances liquid velocity investigation in employment reaching from optimization to industrial cooling systems by classifying critical elements that increase effectiveness as well as decrease heat loss. The accuracy-cost gap is closed by this machine learning and CFD combination, transmuting fluid dynamics research and industrial uses. Liaqat et al. [1] scrutinized how LTNE affected thermophoresis in HFE 7100 NF (nanofluid) with NPs. Abdal et al. [2] used a linear regression-based ML framework to analyses three-dimensional Casson liquid velocity including the CCHFm (Cattaneo-Christov heat flux model), for industrial applications. The ANN surrogate for Ellis HNF with Soret-Dufour significance and Joule heating: Improve mass and heat transmissions is observed by Liaqat et al. [3]. Khan et al. [4] developed a novel deep learning-based knowledge-driven supervised backpropagation recurrent NN (neural network) for studying HNF flow. Abbas et al. [5] used an ANN to scrutinize the behavior of gyrotactic microbes in a water-based HNF containing CNTs, utilizing a developed HCM (Hamilton-Crosser model). Pemmaraju et al. [6] explored a machine learning-based computational framework for analyzing EMHD (electromagnetohydrodynamic) ternary nanofluid flow and thermal generation. Using magnetized bioconvection dynamics in THNF (trihybrid nanofluids), Abbas et al. [7] investigated the framework for process-level radiative design employing the ILMT (Intelligent Levenberg-Marquardt technique). The hybrid nanofluid of convection in micropolar fluids with Lorentz properties was studied by Mahboobtosi et al. [8]. Lone et al. [9] used a ML approach to inspect the thermal behavior of water-based radiative magnetized micropolar HNF velocity on an adjustable permeable stretching sheet under the influence of the CCHFm. The ML prediction of MHD velocity of a buoyancy-driven THNF in divergent and convergent channels was examined by Manai et al. [10].

The nanofluids contain nanoparticles smaller than 100 nm, which enhance heat conductivity, they are being used quickly in many different industries. Heat exchangers, solar collectors, pharmaceutical operations, reactors, absorption technologies, refrigeration chiller systems, aggregate manufacturing, and coolant applications are some of these industries. Additionally, nanofluids are being used in novel fields like safe cooling techniques, cancer treatment, heart surgery, and brain tumour therapy. In many engineering fields, such as geothermal, petroleum, and industrial engineering, fluid flow through porous media is significant in addition to nanofluids. Khan et al. [11] examined how IPN (intelligent predictive networks) affected radiative MHD-HNF velocity thermal efficiency while accounting for thermal generation and absorption. Rehman et al. [12] inspected the properties of VD (viscous dissipation) on the temperature transmission properties of mixed-convection, MHD flow past a contracting surface. By taking melting heat transfer into account, Hafeez et al. [13] examined the MHD engineering applications with radiative HNF flow over a wedge. The thermal efficiency of HNF in MHD flow on a stretched surface was examined by Mumtaz et al. [14]. The influence of MHD and entropy production on HNF convection in a staggered cavity was examined by Turabi et al. [15].

A solid matrix with organized pores that permit fluid to velocity makes up porous media. In addition to man-made materials like ceramics, composite materials, and high-porosity metal foams, it also contains natural materials like sand, wood, and human lungs.

Nomenclature

ν_f	Kinematic viscosity ($m^2 s^{-1}$)
D_B	Mass diffusivity coefficient ($m^2 s^{-1}$)
ρ_f	Fluid density (kgm^{-3})
C_p	Specific heat ($Jkg^{-1}k^{-1}$)
C	Species of the fluid phase solutal
k_f	Thermal conductivity ($Wm^{-1}k^{-1}$)
Sh	Sherwood number
T	Fluid temperature (k)
B_0	Uniform magnetic field ($kg s^{-2}A^{-1}$)
K	Porosity parameter
C_m	Solutal at the surface
Rd	Thermal radiation parameter
Pr	Prandtl number
(x, y)	Cartesian coordinates
T_∞	Fluid ambient thermal
H	Inter phase heat transfer factor
Sc	Schmidt number
Nu_p	Nusselt number (solid phase)
(u, v)	flow fields ($m s^{-1}$)
Sr	Soret number
σ_f	Electrical Conductivity
Sr	Soret number
γ	Porosity modified conductivity ratio
Nu	Nusselt number (liquid phase)
T_0	Constants
μ_f	Dynamic viscosity ($kgm^{-1}s^{-1}$)
Du	Dufour number

Microchemical reactors, the production of steel and iron, electrochemical systems, and the production of renewable fuels all employ permeable medium. permeable medium has been the subject of theoretical and computational analysis. Bhargava et al. [16] inspected the impacts of diffusion absorption, chemo radiation as well as hall current on an unstable magnetohydrodynamic flow of second-grade fluid by permeable medium in the existence of VD. Salman et al. [17] study the effect of variation on MHD flow of pseudo plastic fluid using a waveform channel. Mandal and Pal [18] studied stability for THNF velocity in a DF (Darcy-Forchheimer) permeable medium on a disk.

The LTE model as well as the LTNE model are two fundamental models that could be utilized to define convective thermal transfer in permeable medium. The LTNE model explicitly accounts for the thermal difference along the both phases. As there is a important temperature difference among the two phases, the LTE assumption is no longer valid, and the LTNE model more accurately represents the thermal transport process. The LTNE approach must now be applied, along with an extra TBCs (thermal boundary conditions). Galal et al. [19] studied the YOM and XM thermal conductivity models in three-dimensional radiative Casson THNF flow, incorporating LTNE effects. Chandel et al. [20] investigated the constancy of partially ionised plasma in a permeable media during LTNE conditions. Mokhtari and Tayebi [21] investigated second-law thermodynamics in a NF-filled enclosure divided by a non-Darcian permeable wall, accounting for LTNE conditions, internal thermal generation. Siddabasappa et al. [22] observed the thermal transmission of a Newtonian fluid across a porous medium under LTNE conditions. Ahmad et al. [23] analyzed the temperature transportation properties of a magneto-micropolar liquid flowing across a surface that included the LTNEC. The mixed convective flow of fluid in a microchannel under LTNE conditions was examined by Goli et al. [24].

The thermos diffusion mechanism describes how particles are moved utilizing multiple factors that are added together and computed using a gradient in temperature. When there is an important density difference in the flow field, the SDE (Soret-Dufour effects) become significant. Fluxes and Driving potentials have a more complicated relationship when mass conduction occurs simultaneously in fluid motion. This effect occurs when the solutal of one chemical species varies significantly from that of the other species involved in the reaction. For gases with transitional atomic weight, the SDE can aid in the combined transmission of mass and temperature in binary systems. Mass and thermal transmission in permeable media has sparked significant theoretical and practical interest due to its large number of applications, which include energy storage systems, geothermal engineering, nuclear waste repositories, thermal insulation, catalytic reactors, drying technologies, and other industrial processes. Srinivasa et al. [25] scrutinized the properties of the SDE, and MHD on HNF flow across a revolving thin needle embedded in a permeable media. Aul Matao et al. [26] mathematically examined magnetohydrodynamic flow over an inclined porous plate, including the SDE as well as an inclined MF

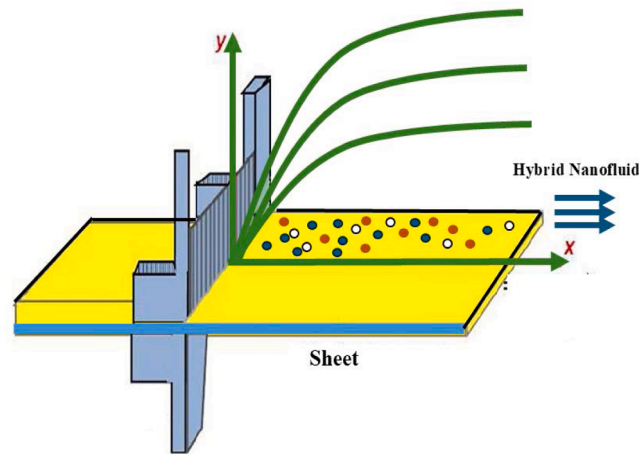


Fig. 1. Flow description.

Table 1

The basic characteristics of the NPs and base fluid [36].

Features constituents	c_p	k	ρ
Sand	730	1.5	2650
Paraffin Wax	2900	0.25	900
$C_3H_8O_2-H_2O$	3342.75	0.363	1020

Table 2

HNF's thermophysical features [36].

Features	HN
μ_{hnf}	$W_1 = \frac{\mu_{hnf}}{\mu_f} = \frac{1}{(-\varphi_1 + 1)^{2.5} (1 - \varphi_2)^{2.5}}$
ρ_{hnf}	$W_2 = \frac{\rho_{hnf}}{\rho_f} = \left[\varphi_2 \frac{\rho_{s_2}}{\rho_f} + (-\varphi_2 + 1) \left\{ \varphi_1 \frac{\rho_{s_1}}{\rho_f} + (-\varphi_1 + 1) \right\} \right]$
$(\rho c_p)_{hnf}$	$W_6 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} = \left[\left\{ \varphi_1 \frac{(\rho c_p)_{s_1}}{(\rho c_p)_f} + (-\varphi_1 + 1) \right\} \varphi_2 \frac{(\rho c_p)_{s_2}}{(\rho c_p)_f} + (1 - \varphi_2) \right]$

Table 3

Numerical results of ML technique.

Cases	Time	Epoch	Performance	Training	Validation	Testing
I	10	532	$1.87 \times E^{-08}$	$6.16 \times E^{-09}$	$8.76 \times E^{-08}$	$7.73 \times E^{-10}$
II	18	951	$3.56 \times E^{-09}$	$4.03 \times E^{-09}$	$3.61 \times E^{-09}$	$1.35 \times E^{-09}$
III	11	924	$1.40 \times E^{-08}$	$1.21 \times E^{-08}$	$2.12 \times E^{-08}$	$1.57 \times E^{-08}$

(magnetic field). Shah [27] examined how MF orientation affected the SDE in an inclined corrugated triangular cavity with a NNF. Hamida et al. [28] investigated solar-driven magnetohydrodynamic Maxwell NFF with gyrotactic microbes for thermal engineering applications, examining the SDE. Sarma et al. [29] examine the EMHD (electromagnetohydrodynamic) flow of a Carreau liquids across a cylinder while controlling for the joint properties of the Soret and Dufour phenomena, Hall current, and activation energy. Awais et al. [30] studied the consequence of Dufour and Soret on viscous fluid flow in a permeable square cavity. Chowdhury et al. [31] investigated the morphological and structural features of aluminum-doped reduced graphene oxide NPs. Roy et al. [32] investigated an updated review of carbon NMs: types, production, functionalization and uses toxicity and degradation.

The novel contribution of the examination is the study of Dufour and Soret influences on a magnetized paraffin wax-sand/ $C_3H_8O_2-H_2O$ HNF with LTNECs using Machine learning algorithm. It shows graphically how the system responds to changes in the governing parameters. The analysis's primary objectives are as follows:

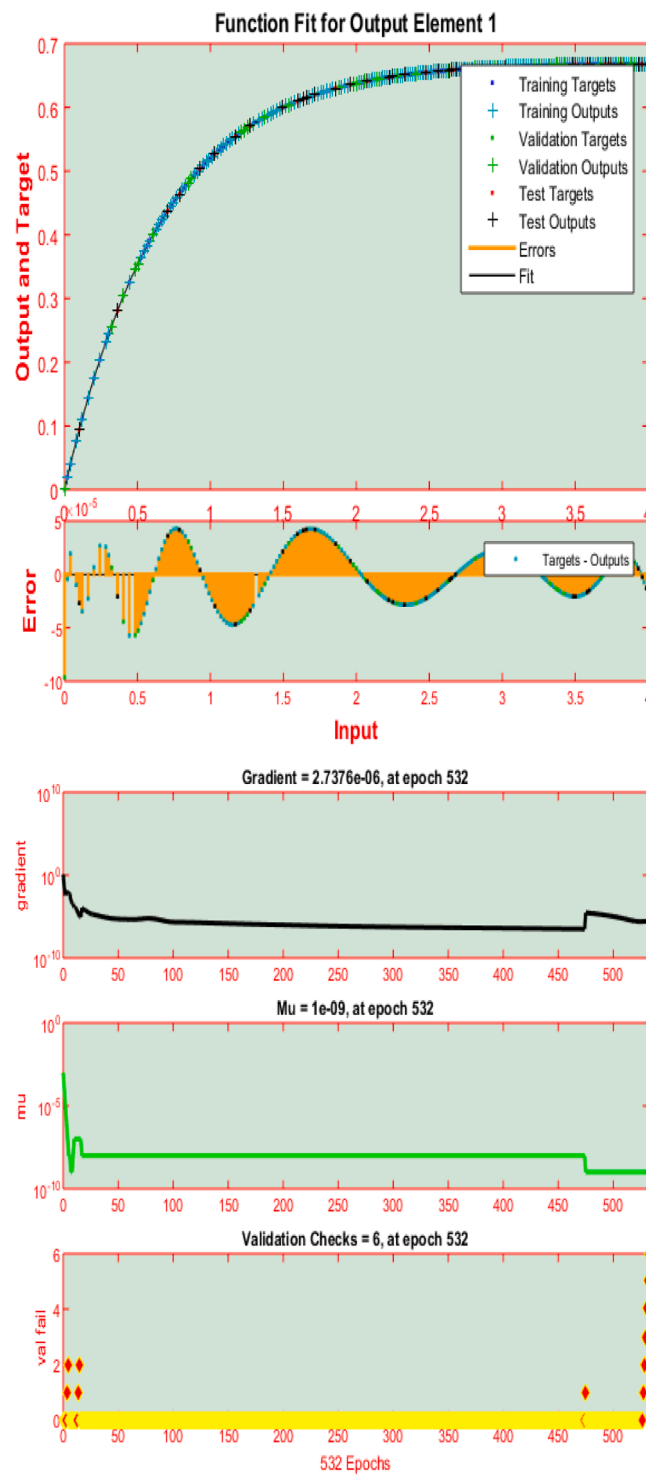


Fig. 2. Case I (a) Curve fitting example. Regression analysis (c), EH (Error histogram) (d), Transition-state analysis (b), and M-S-E (e).

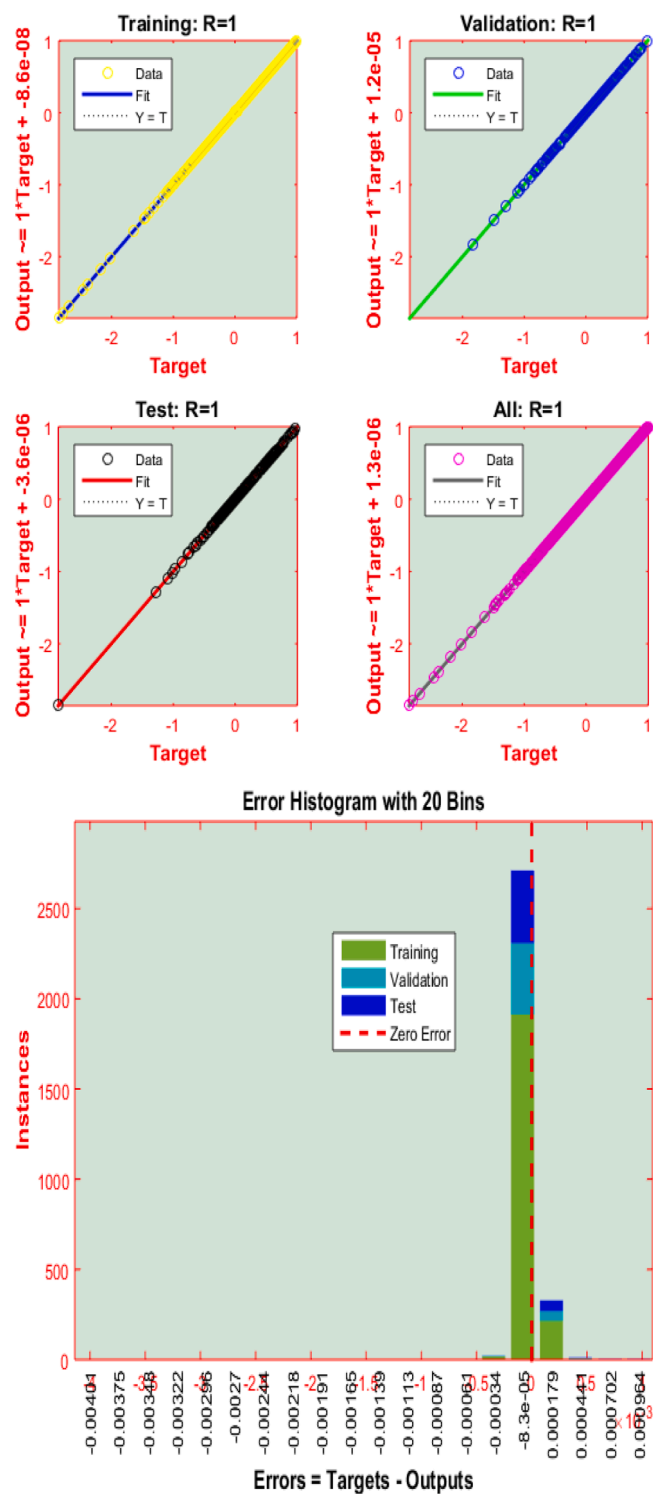


Fig. 2. (continued).

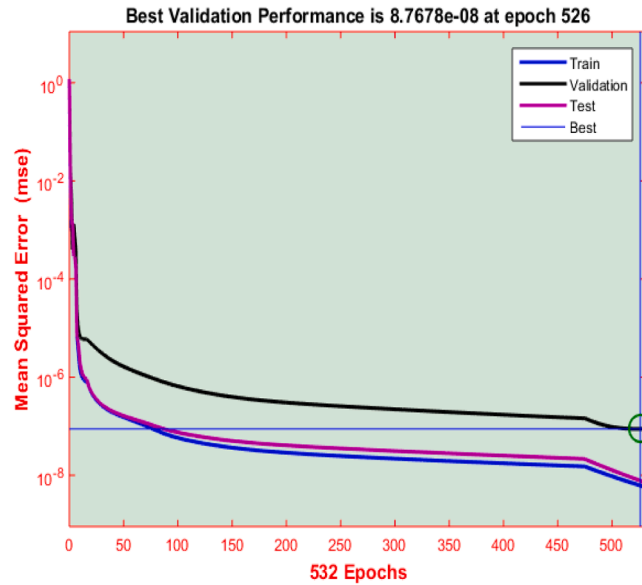


Fig. 2. (continued).

The study aims to scrutinize how the sand-paraffin wax/C₃H₈O₂-H₂O hybrid nanofluid model affects LTNE that move across a sheet.

- A detailed analysis of the ANN was used to analyse a suggested model's efficiency and stability.
- Investigating the Dufour and Soret effects on sand-paraffin wax/C₃H₈O₂-H₂O HNF using HCM is the goal of this investigation.
- The reliability and accuracy of the NN-LMA are authorized by regression analysis, MSE, and error histogram evaluations.

Mathematical formulation

Imagine the LTNE impacts on steady, two-dimensional sand-paraffin wax /C₃H₈O₂-H₂O HNF two dimensions across a stretched sheet within Dufour and Soret impacts. It is limited to the plane $y > 0$ and overlaps with the velocity (see Fig. 1(b)). Using two different equations, heat transport in the both phases is examined. The flow is assumed to be subjected to a UTMF (uniform transverse magnetic field), B_0 , this is often applied on the sheet.

The governing equations are defined as follows [33,34]:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

$$\left(\frac{\partial u}{\partial y} v + \frac{\partial u}{\partial x} u \right) \rho_{hnf} = \left(\frac{\partial^2 u}{\partial y^2} \right) \mu_{hnf} - \mu_{hnf} \frac{u}{K^*} - \sigma_{hnf} B_0^2 u, \quad (2)$$

$$\left(v \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} \right) (\rho c_p)_{hnf} = h_{fs} (-T + T_p) + \left(\frac{16 T_\infty^3 \sigma^*}{3 k^*} + k_{hnf} \right) \left(\frac{\partial^2 T}{\partial y^2} \right) + \left(\frac{D_m k_T}{C_s} \right) \frac{\partial^2 C}{\partial y^2}, \quad (3)$$

$$0 = \left(\frac{\partial^2 T_p}{\partial y^2} \right) \frac{h_p}{(\rho c_p)_p} + (-T_p + T) \frac{h_{fs}}{(\rho c_p)_p}, \quad (4)$$

$$\left(v \frac{\partial C}{\partial y} + u \frac{\partial C}{\partial x} \right) = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2}, \quad (5)$$

Boundary conditions [33,34]:

$$T_p = T_w, \quad v = 0, \quad u = cx, \quad T = T_w, \quad \text{at } y = 0, \quad (6)$$

$$C = C_w, \quad \text{at } y = 0, \quad (7)$$

$$C \rightarrow C_\infty, \quad u \rightarrow 0, \quad T_p \rightarrow T_\infty, \quad T \rightarrow T_\infty, \quad \text{at } y \rightarrow \infty. \quad (8)$$

Similarity variables [33]:

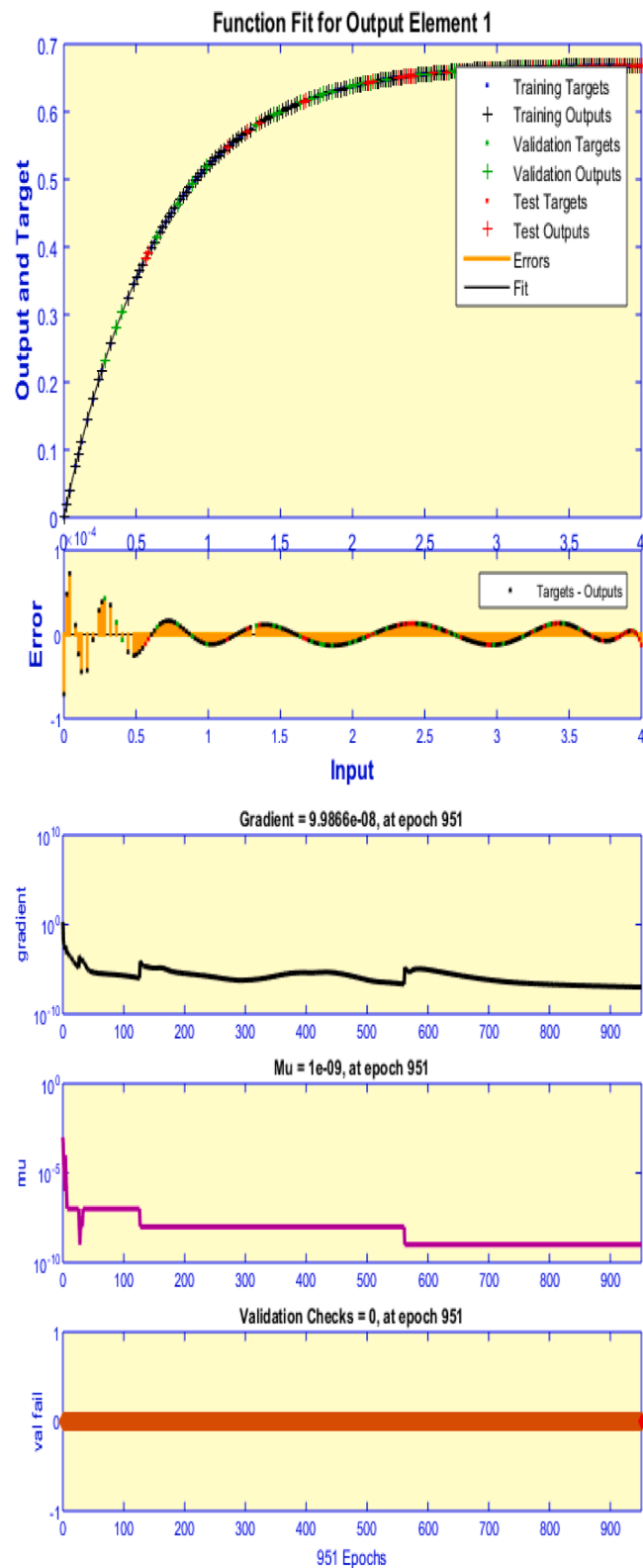


Fig. 3. Case II (a) Curve fitting example. Regression analysis (c), Transition-state analysis (b), EH (d), and M-S-E (e).

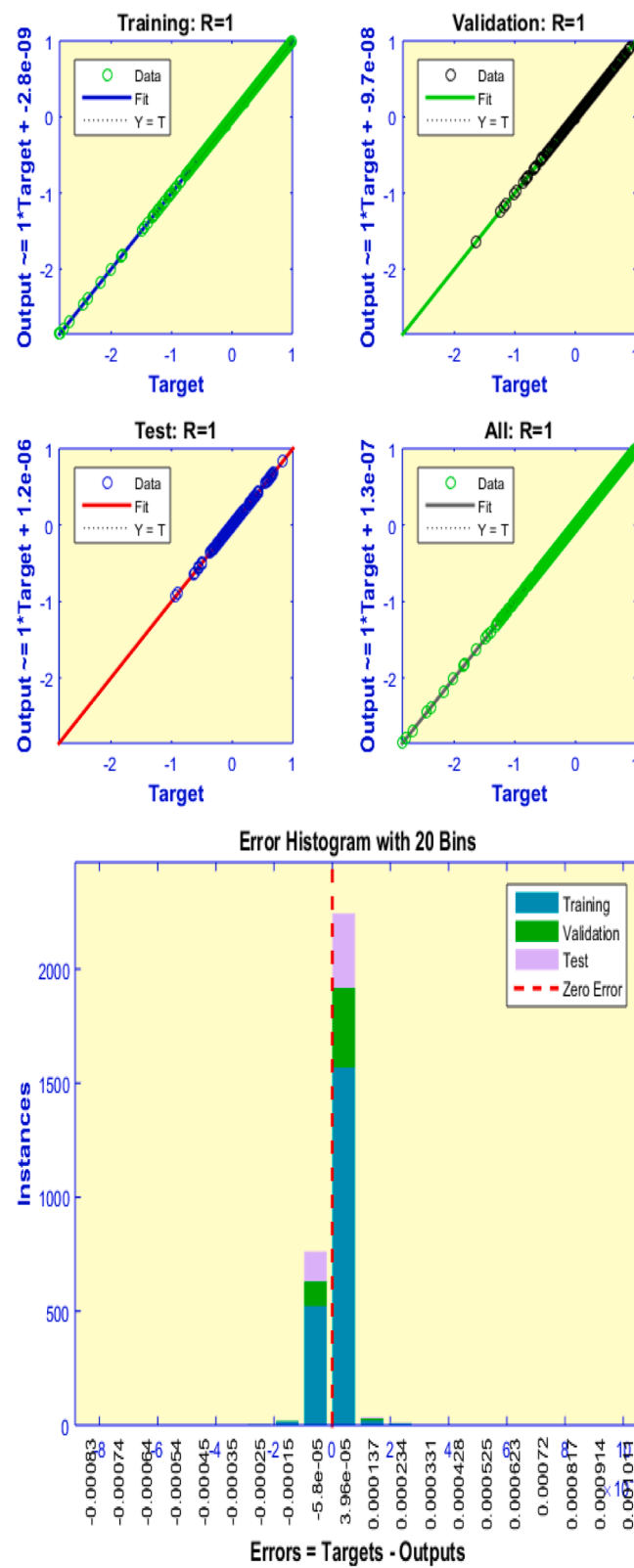


Fig. 3. (continued).

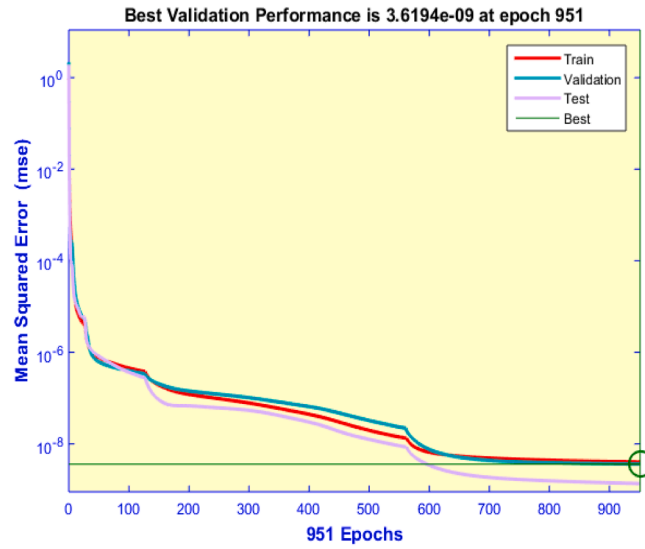


Fig. 3. (continued).

$$u = cx f'(\xi), \quad v = -\sqrt{c v_f} f'(\xi), \quad (12)$$

$$\xi = \sqrt{\frac{c}{v_f}} y, \quad \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (13)$$

$$\phi(\xi) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \theta_p(\xi) = \frac{T_p - T_\infty}{T_w - T_\infty} \quad (14)$$

ODEs.

$$Q_1 f'' + Q_2 (2ff'' - (f')^2) - Q_3 M f' - Q_4 K f' = 0, \quad (15)$$

$$(Q_4 + Rd)\theta'' + PrQ_5(f\theta') + PrH(\theta_p - \theta) + PrDu\phi'' = 0, \quad (16)$$

$$(\theta - \theta_p)Pr\gamma H + \theta_p'' = 0, \quad (17)$$

$$Scf\phi' + ScSr\theta'' + \phi'' = 0, \quad (18)$$

BCs:

$$\theta = 1, \phi = 1, f = 0, f' = 1, \xi = 0 \quad (21)$$

$$\theta_p = 1, \xi = 0, \quad (22)$$

$$f' \rightarrow 0, \phi \rightarrow 0, \theta \rightarrow 0, \theta_p \rightarrow 0, \xi \rightarrow \infty. \quad (23)$$

Where porosity-modified conductivity ratio, magnetic parameter, Dufour number, Soret number, thermal radiation, Schmidt number, Prandtl number, and interphase heat transfer parameter.

$$M = \frac{\sigma_f B_0^2}{\rho_f c}, \quad Pr = \frac{c_p \mu}{k_f}, \quad (24)$$

$$H = \frac{\alpha_f h_{fs}}{k_f}, \quad \gamma = \frac{k_f}{k_s}, \quad Sc = \frac{v_f}{D_B}, \quad (25)$$

$$Rd = \frac{4\sigma^* T_\infty^3}{k^* k}, \quad Du = \frac{D_m K_T (C_w - C_\infty)}{C_p C_s (T_w - T_\infty)}, \quad Sr = \frac{D_m K_T (T_w - T_\infty)}{T_\infty (C_w - C_\infty)}. \quad (26)$$

The Sherwood number and Nusselt numbers for fluid and solid phase:

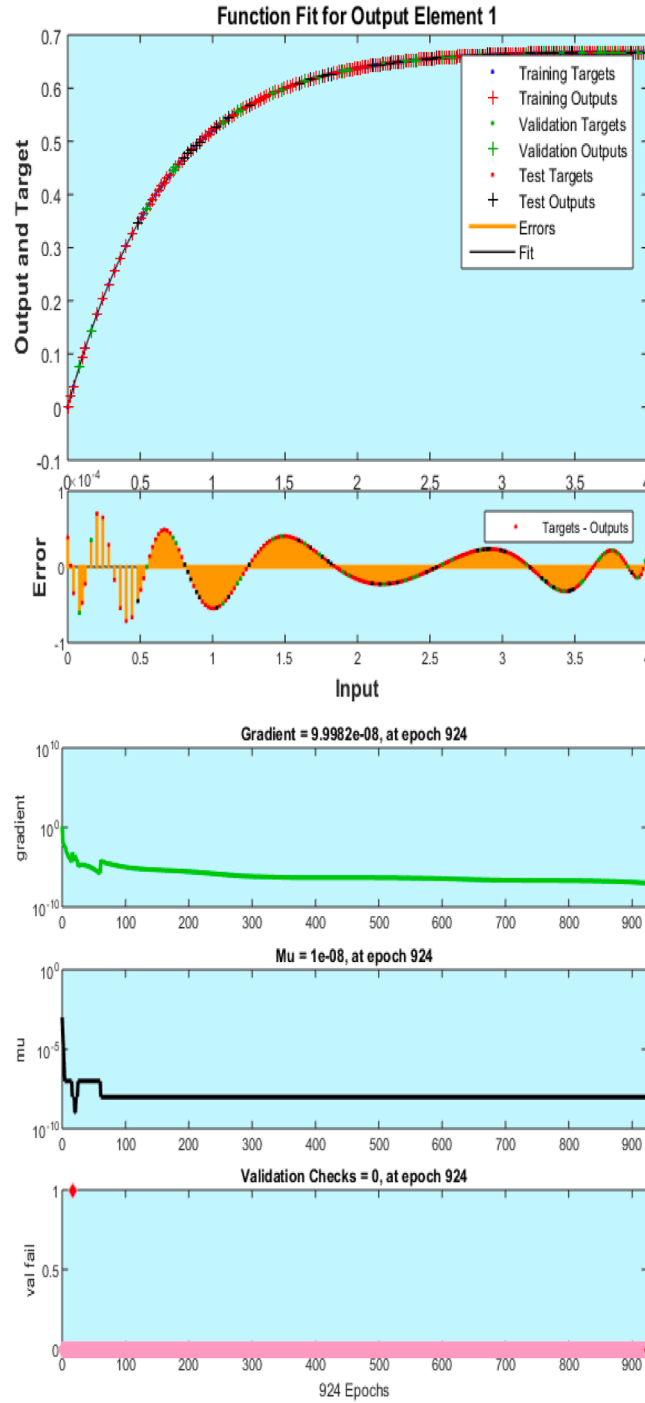


Fig. 4. Case III (a) Curve fitting example. Regression analysis (c), EH (d), Transition-state analysis (b), and M–S–E (e).

$$Sh(Re)^{-0.5} = -\phi'(0), Nu(Re)^{-0.5} = -(W_4 + Rd)\theta'(0), \quad (27)$$

$$Nu_p(Re)^{-0.5} = -\theta'_p(0), \quad (28)$$

The Hamilton–Crosser model of thermal conductivity is represented in [35].

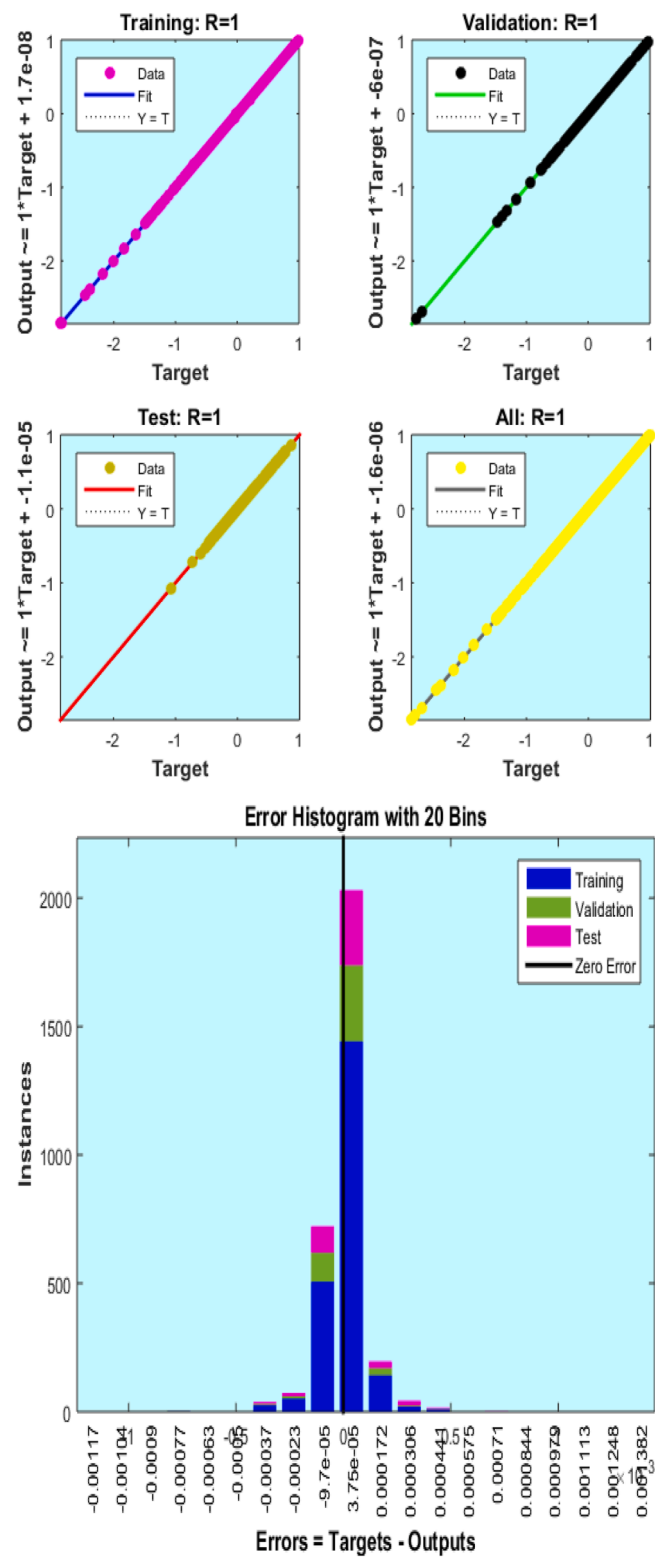


Fig. 4. (continued).

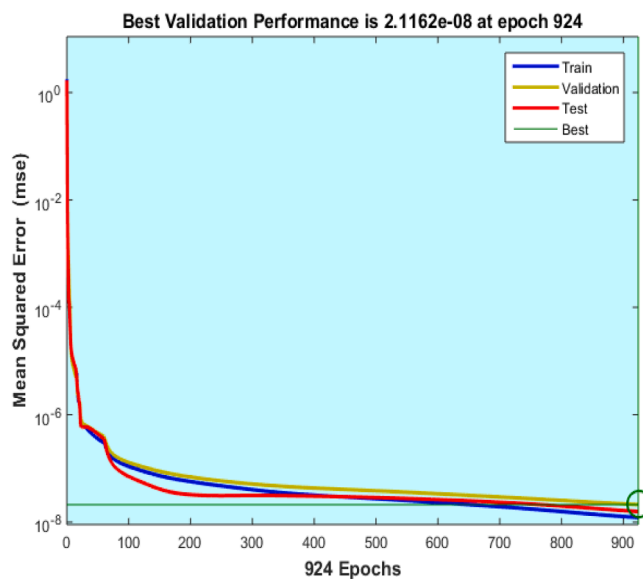
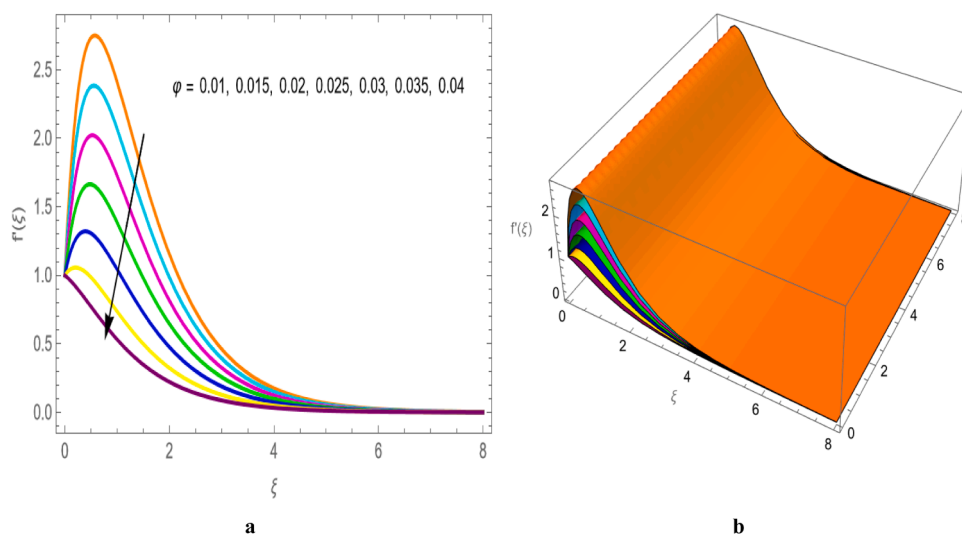


Fig. 4. (continued).

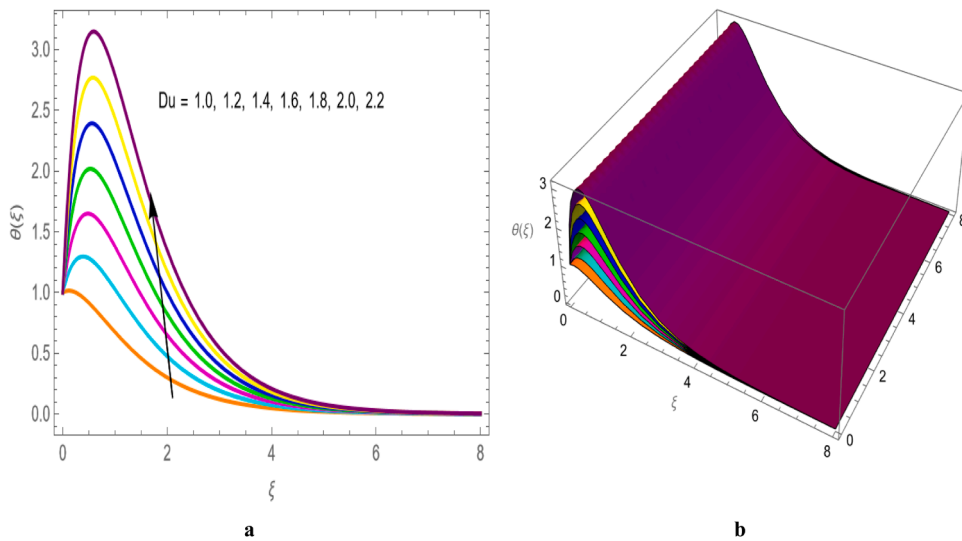
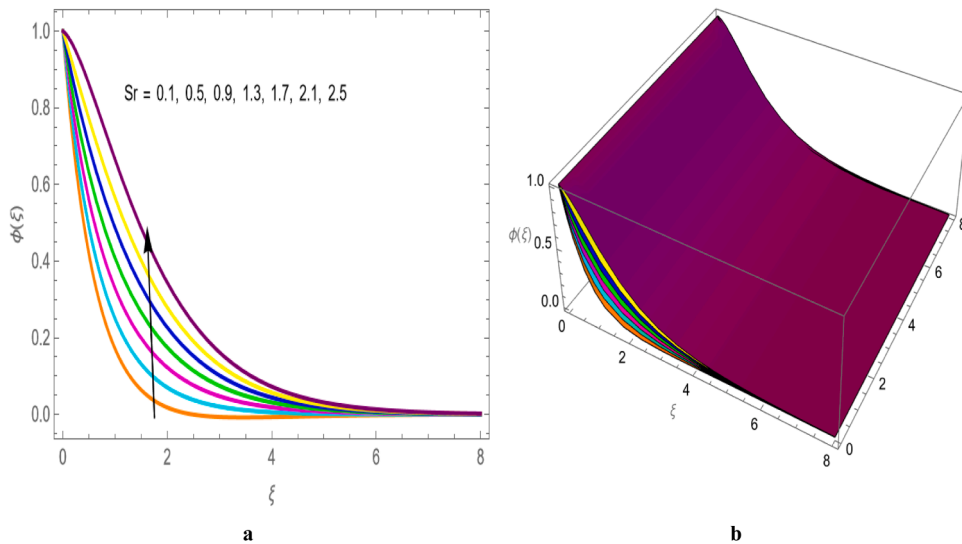
Fig. 5. (b&a) Result of φ on f' .

$$\frac{k_{\text{hnf}}}{k_{\text{nf}}} = \left(\frac{-(S-1)\varphi_2(k_{\text{nf}} - k_2) + k_2 + (S-1)k_{\text{nf}}}{\varphi_2(k_{\text{nf}} - k_2) + k_2 + (S-1)k_{\text{nf}}} \right), \quad (32)$$

$$\frac{k_{\text{nf}}}{k_{\text{f}}} = \left(\frac{-(S-1)\varphi_1(k_{\text{f}} - k_1) + k_1 + (S-1)k_{\text{f}}}{k_1 + (S-1)k_{\text{f}} + \varphi_1(k_{\text{f}} - k_1)} \right). \quad (33)$$

$$W_1 = \frac{\mu_{\text{hnf}}}{\mu_{\text{f}}}, W_3 = \frac{\sigma_{\text{hnf}}}{\sigma_{\text{f}}}, W_2 = \frac{\rho_{\text{hnf}}}{\rho_{\text{f}}}, W_5 = \frac{(\rho C_p)_{\text{hnf}}}{(\rho C_p)_{\text{f}}}, W_4 = \frac{k_{\text{hnf}}}{k_{\text{f}}}. \quad (34)$$

The basic aspects of the based liquid as well as its component NPs are displayed in Table 1. Table 2 illustrate the HNF's basic features.

Fig. 6. (b&a) Effect of Du on θ .Fig. 7. (b&a) Effect of Sr on ϕ .

Machine learning technique

Depending on the data supplied and the training strategy selected, a variety of mathematical, scientific, and technical, problems could be now solved to the development of MLAs (machine learning algorithms). Specifically, SLAs (supervised learning algorithms) closely mimic human learning through data analysis and problem-solving. SLAs are used in this study to minimize the difference between the expected and target outputs in order to update network weights. A collection of labeled data, also known as training samples or a training dataset, is utilized to train ANNs and direct the modification of network parameters. This architecture enables the model to learn and generalize effectively because the desired outcome for a given input is specified (Table 3).

An ANN optimized using the Levenberg Marquardt scheme is created to address the proposed model in scenario I. Function fit, which plots ANN prediction against target data, is shown in Fig. 2(a); Hig2h fitting precision is attested to by the close match between the predicted and actual values; The proposed model's state transition plot is shown in Fig. 2(b); Fig. 2(c) illustrate the RA (regression analysis) plots for the testing datasets, training and validation, and the EH (error histogram) plot obtained from ANN simulations is displayed in Fig. 2(d). The forecast accuracy of our developed method is confirmed by the mean square error and EH smallest values. The simulation results are displayed in Fig. 2. In this case, the mean square error value for solving the fluid model is displayed in Fig. 2 (e). At epoch 532, the ANN network's optimal training performance value is 6.16×10^{-9} . Fig. 3 depicts the simulation results for Case II obtained with the LM (Levenberg-Marquardt) algorithm. Fig. 3(a) shows the function fitting performance, with the predicted and

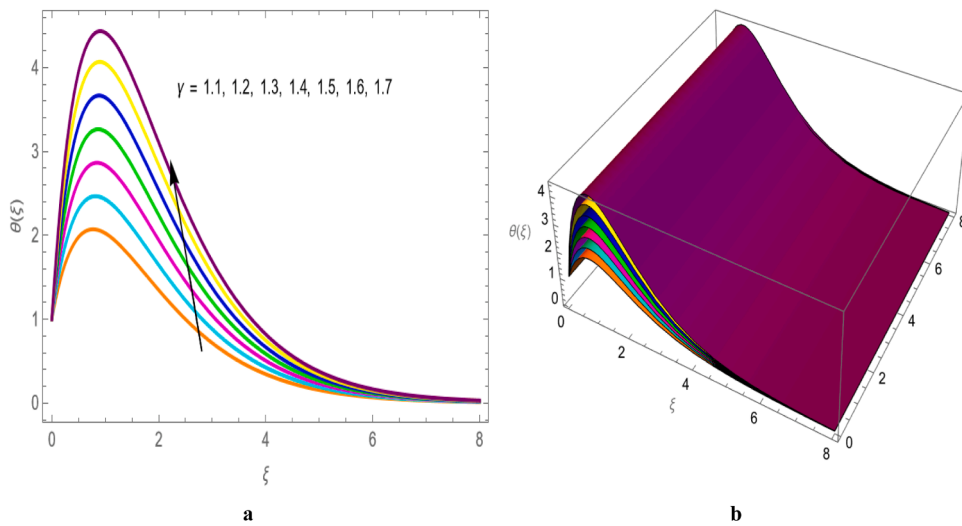


Fig. 8. (b&a) Impact of γ on θ .

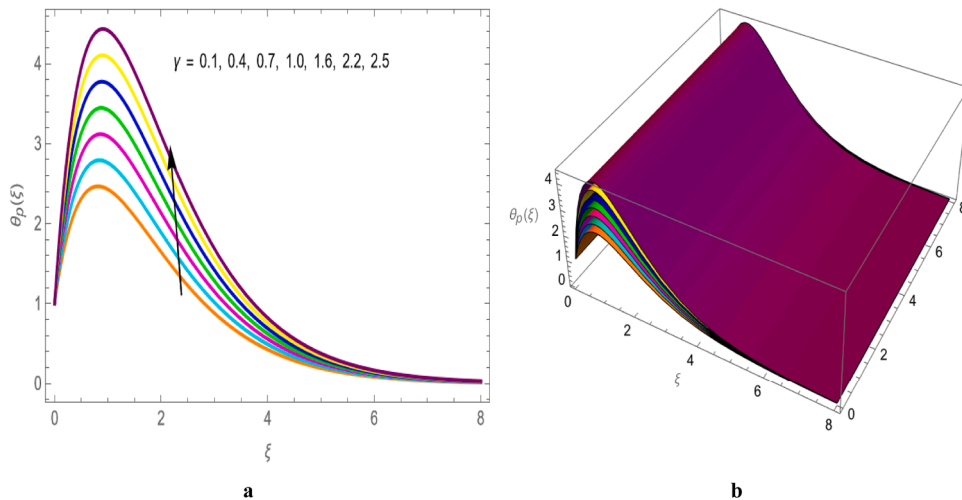


Fig. 9. (b&a) Effect of γ on θ_p .

target thermal data agreeing almost perfectly, confirming the proposed model's high accuracy in reproducing the thermal response. The state transition plot used to analyse the implications of the proposed model is shown in Fig. 3(b). The RA plots for the ANN simulations used to solve the recommended model are demonstrated in Fig. 3(c), This indicates a high correlation along the predicted and target values. The corresponding error histogram is shown in Fig. 3(d), where the clustering of prediction errors closes to zero attests to the ANN mode's exceptional correctness and robustness. Fig. 3(e) demonstrate the MSE rate for the validation, training and testing data. The optimal validation performance value for the ANN network at epochs 951 is $3.61 \times E-09$. Fig. 4(b) demonstrations the simulation results using the LM approach for scenario III. The minimum gradient value confirms the ANN's optimal convergence, validity, and predictive accuracy. The regression investigation shown in Fig. 4(c) demonstrates the ANN's effectiveness in modelling the thermal field, with the regression coefficient ($R = 1$) indicating nearly perfect agreement between predicted and target thermal outputs. Fig. 4(d) depicts the error histogram, which shows how prediction errors are distributed. The clustering of errors around zero demonstrates the ANN model's high predictive accuracy and reliability. Fig. 4(e) demonstrates the MSE curves for the validation, testing datasets and training. The training error gradually decreases with each epoch, indicating effective learning and stable convergence of the proposed ANN model. For the ANN-LMS network, the ideal testing performance value is $1.57 \times E-08$ at epochs 924. Fig. 5(a,b) show how the nanoparticle volume fraction (ϕ) affects velocity profiles in 2D and 3D (f'). The velocity profile decreases within enhanced NP volume fraction, owing to the increased resistance and effective viscosity introduced by the suspended nanoparticles. Fig. 6(a,b) show how the Dufour number (Du) affects temperature profiles in 2D and 3D ($\theta(\eta)$). Heat transfer brought on by concentration gradients is described by the Dufour effect. The TBL thickens as the Dufour number rises, improving the flow field's thermal distribution. As the Dufour number rises, it is obvious that the velocity through the BL and the TBL width increase. The rate at

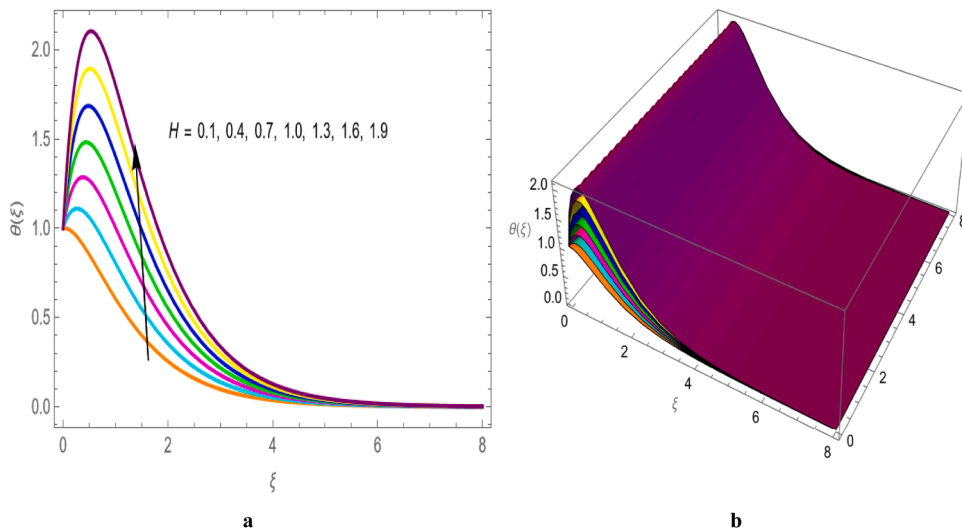


Fig. 10. (b&a) Effect of H on θ .

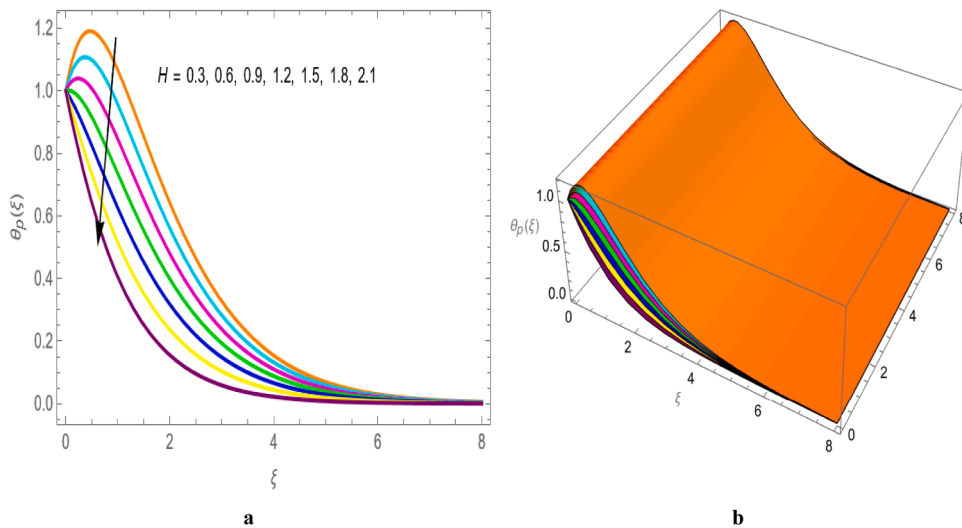


Fig. 11. (a&b) Outcome of H on θ_p .

which heat is transferred increases with increasing Du values. The properties of Sr on $\phi(\eta)$ are displayed in Fig. 7(a&b). The concentration profiles improve as Sr levels increase. With Sr , both $\phi(\eta)$ and molar mass diffusivity increase. The outcome of γ on θ and θ_p are demonstration in 9(b&a) and Fig. 8(b&a). The solid and fluid phase thermal fields upsurge as rise the values of porosity modified conductivity ratio. Figs. 10(a&b) and 11(a&b) demonstration how H is motivated by θ and θ_p . The θ increases and the θ_p decreases as H increases. It is the outcome of quick heat transfer between stages in this occurrence. The graphs' upward shift in parameter H suggests that the process is moving in the direction of altering the LTE phase. At intermediate values of the H , a discernible thermal difference exists among the both phases, which suggests that LTNE conditions are common. The interphase heat transfer is improved as H rises, encouraging the exchange of heat energy along the two phases. Consequently, the thermal differential decreases as the fluid-phase temperature increasingly methods that of the solid phase. The system gradually approaches LTE at sufficiently large values of H , though a small temperature difference may still exist before full equilibrium is reached. As H rises and the heat transfer in both phases is nearly equal, the system is completely under LTE control. Figs. 12(a&b) and 13(a&b) show the results of Du on Sr and Nu on Sh . The rate of mass transmission improvement as increase the values of Sr and Du . The result of φ on Nu are display in Fig. 14(a&b). As NPs volume fraction values upsurge, the rate of heat transmission also does. In this study, the value of thermal transmission as well as only the integer case and standard factors are taken into consideration when comparing the heat transmission of fluid and solid phases to earlier work are show in Table 4. The latest findings are in complete agreement within earlier research.

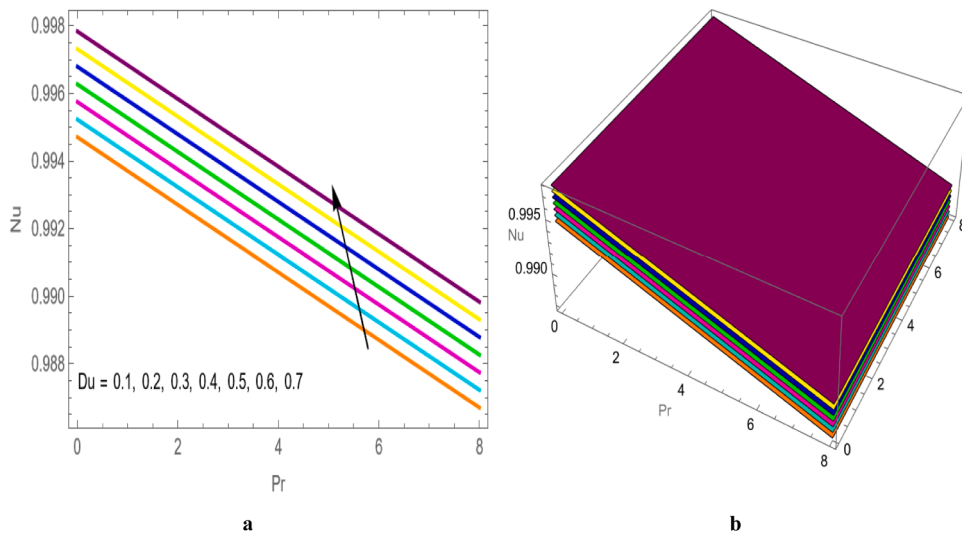


Fig. 12. (a&b) Significance of Du on Nu.

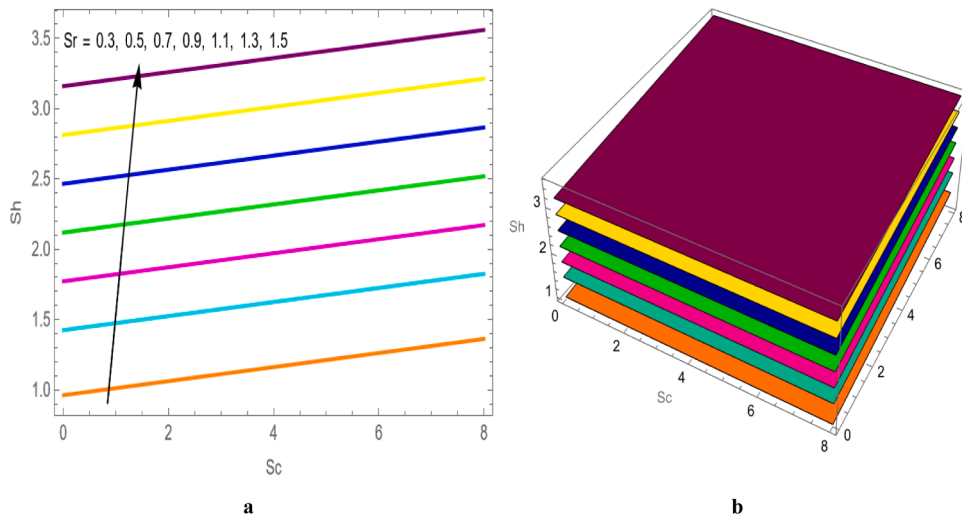


Fig. 13. (a&b) Impact of Sr on Sh.

Conclusion

The sand-paraffin wax/ $\text{C}_3\text{H}_8\text{O}_2$ - H_2O nanomaterial model for local thermal non-Equilibrium (LTNE) circumstances has significant potential in progressive thermal energy storage and heat transfer systems. It is especially relevant for latent heat thermal energy storage units, solar thermal collectors, building energy management, electronic cooling, and industrial heat recovery systems, where paraffin wax acts as a phase change material and sand improves thermal conductivity and structural stability. The inclusion of Dufour and Soret properties allows for more precise prediction of coupled mass and heat transmission in systems with significant temperature and concentration gradients, whereas the Levenberg-Marquardt method provides efficient and accurate prediction of the governing transport properties. The results of this model can help to build energy-efficient thermal storage devices, renewable energy technologies, and sustainable cooling and heating systems with increased thermal performance and operational reliability. The following are the study's main deductions:

- ❖ A Neural Network with Levenberg-Marquardt approach (NN-LMA), which offers a reliable predictive framework with enhanced noise and generalization tolerance, was trained using the outcomes of the soft computing and machine learning workflow.
- ❖ As interphase thermal transmission factors values rise, dense phase thermal transmission decreases.
- ❖ When the Dufour and Soret numbers upsurge, so does the value of thermal and mass transfer.

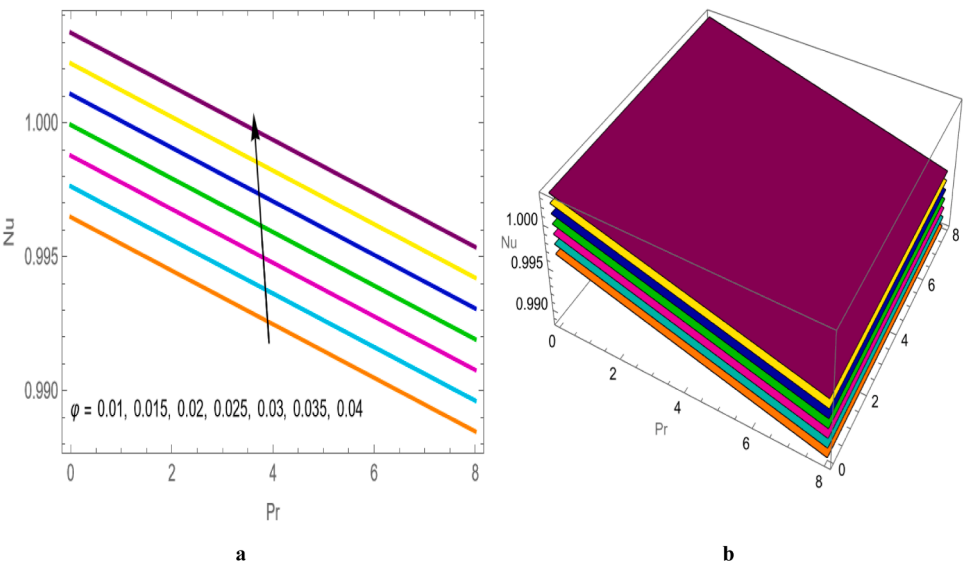


Fig. 14. (a&b) Effect of ϕ on Nu.

Table 4
The thermal gradient results are compared to published works.

Pr	$-\theta'(0)$		$-\theta_p'(0)$	
	Current outcome	Alhadhrami et al. [33]	Present results	Alhadhrami et al. [33]
0.7	0.808666	0.808625	0.808632	0.808619
1	0.999947	0.999953	0.999960	0.999945
2	1.923690	1.923685	1.923685	1.923675
10	3.720651	3.720632	3.720641	3.720616

- ❖ Liquid and solid phase thermal profile of paraffin wax-sand/ $C_3H_8O_2$ - H_2O hybrid nanofluid upsurge as upsurge the values of porosity modified conductivity ratio.
- ❖ The velocity field falls as the volume fraction NPs upsurgues.
- ❖ The Sand- paraffin wax/ $C_3H_8O_2$ - H_2O HNF's thermal and solutal fields rise as the Dufour and Soret numbers upsurge.

CRediT authorship contribution statement

Muhammad Kamran: Supervision, Methodology, Conceptualization. **Nargis Khan:** Writing – review & editing. **Saba Liaqat:** Methodology, Formal analysis. **Haitham M. Hadidi:** Supervision, Methodology, Conceptualization. **Gafur Abdulakimov:** Writing – review & editing. **Munawar Abbas:** Methodology, Data curation. **Ali Hasan Ali:** Methodology, Formal analysis. **Mohamed Hafez:** Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors have no relevant financial or non-financial interests to disclose.

Data availability

The authors do not have permission to share data.

References

[1] A. Venkateswarlu, S. Suneetha, M.J. Babu, J.G. Kumar, C.S.K. Raju, Q. Al-Mdallal, Significance of magnetic field and chemical reaction on the natural convective flow of hybrid nanofluid by a sphere with viscous dissipation: a statistical approach, *Nonlinear Eng.* 10 (1) (2021) 563–573.

[2] S. Abdal, T. Taha, N.A. Shah, S.J. Yook, Machine learning study for three-dimensional magnetohydrodynamics casson fluid flow with Cattaneo-Christov heat flux using linear regression technique: application in engineering science and technology, *Eng. Appl. Artif. Intell.* 156 (2025) 111159.

[3] S. Liaqat, H. Ullah, M. Bayram, A.H. Ali, H. Kanwal, A. Abbas, M.U. Athar, Artificial neural network surrogate for Ellis hybrid nanofluid with Soret-Dufour effects and joule heating: enhance heat and mass transfer, *Proc. Inst. Mech. Eng. Part N* (2025) 23977914261443405.

- [4] A. Khan, M.A.Z. Raja, C.Y. Chang, M.P. Khan, Z.A. Khan, M. Shoaib, Novel deep learning knowledge-driven supervised backpropagated recurrent neural networks for MHD Maxwell hybrid nanofluidic model, *Int. J. Thermophys.* 46 (10) (2025) 152.
- [5] M. Abbas, A.A. Darem, A.B. Elhag, A.A. Alhashmi, T.M. Alkhaldi, D.R. Farkhad, M. Bayram, Artificial neural network optimization of gyrotactic microbes in water hybrid nanofluid with carbon nanotubes using modified Hamilton crosser model, *Discover Nano* 21 (1) (2026) 191.
- [6] R.S. Pemmaraju, J.R. Gurejala, S.N.R. Thottampudi, A machine learning technique for computational analysis of emhd ternary nanofluid with heat generation: $Al_2O_3 + CuO + TiO_2$ /water, *Nano Trends* 12 (2025) 100150.
- [7] M. Abbas, D. Rustamova Farkhad, M. Bayram, A.H. Ali, S.L. Dilawazi, A. Abbas, A framework for process-level radiative design using magnetized bioconvection dynamics in ternary hybrid nanofluid: intelligent Levenberg-Marquardt technique, *Proc. Inst. Mech. Eng. Part N* (2025) 23977914261443409.
- [8] M. Mahboobtosi, H. Ehsani, A.M. Ganji, P. Jalili, M.H. Mohamed, B. Jalili, D.D. Ganji, Hybrid physics-machine learning modeling of unsteady magnetohydrodynamic convection in micropolar fluids with suction and Lorentz effects, *Phys. Fluids* 37 (8) (2025).
- [9] S.A. Lone, Z. Raizah, R. Bossly, F.S. Alduais, A. Al-Bossly, A. Khan, Thermal investigation of water-based radiative magnetized micropolar hybrid nanofluid flow subject to impacts of the Cattaneo-Christov flux model on a variable porous stretching sheet with a machine learning approach, *Chin. Phys. B* 34 (6) (2025) 064401.
- [10] L. Manai, N.A. Alsaif, A.M. Obalalu, S. Chebaane, S. Al Bustanji, O. Rubanenko, Machine learning forecasting of electroosmotic MHD flow of a buoyancy-driven ternary nanofluid in converging and diverging channels, *Sci. Rep.* 16 (2026).
- [11] M.H.U. Khan, W.A. Khan, T. Muhammad, M.S. Alqarni, H. Qureshi, I. Hussain, influences of intelligent predictive networks on thermal efficiency of radiative magnetohydrodynamics hybrid nanofluid flow considering heat absorption-generation aspects, *Results Chem.* 16 (2025) 102468.
- [12] A. Rehman, A.A. Saad, M. Inc, S.S.B. Abas, K. Sudarmozhi, Heat transfer characteristics of mixed-convection, couple-stress MHD ternary hybrid nanofluid flow past a stretching and shrinking surface with the impact of viscous dissipation, *Int. J. Numer. Methods Heat. Fluid. Flow* 36 (5) (2026) 1935–1960.
- [13] A. Hafeez, D. Liu, A. Khalid, MHD flow of radiative hybrid nanofluid across a wedge influenced by melting heat transfer: engineering application, *Alex. Eng. J.* 122 (2025) 18–27.
- [14] F. Mumtaz, A. Al-Zubaidi, T. Abbas, S. Saleem, Thermal performance analysis of quaternary hybrid nanofluids with radiative and joule heating effects in magnetohydrodynamic flow over a stretched surface, *Results Phys.* 71 (2025) 108176.
- [15] Y.U.U.B. Turabi, S. Munir, R. Nawaz, Entropy generation and magnetohydrodynamic influences on hybrid nanofluid convection in a staggered cavity, *Int. J. Thermofluids* 27 (2025) 101204.
- [16] K.N. Ch Bhargava, S.M. Ibrahim, R. Kodi, Effects of hall current, radiation absorption and diffusion thermo on an unsteady MHD flow of second grade fluid through porous media in the presence of joule heating and viscous dissipation, *Multisc. Multidiscip. Model. Exp. Des.* 8 (5) (2025) 260.
- [17] M.R. Salman, H.A. Ali, B. Albuohimad, A.M. Nasralla, Rotation and porous medium impact on MHD peristaltic flow of pseudoplastic fluid through a waveform channel, *J. Adv. Res. Fluid Mech. Therm. Sci.* 127 (2) (2025) 119–132.
- [18] G. Mandal, D. Pal, Stability and multiple solutions of MHD-quadratic radiative tri-hybrid nanofluid flow in a Darcy–Forchheimer porous medium over a rotating shrinking disk, *Int. J. Numer. Methods Heat. Fluid. Flow* 35 (8) (2025) 2982–3025.
- [19] A.M. Galal, A. Akgül, S.A. Idris, S. Formanova, T.K. Ibrahim, M.K. Hassani, I. Mahariq, The performance evolution of Xue and Yamada-Ota models for local thermal non equilibrium effects on 3D radiative Casson trihybrid nanofluid, *Sci. Rep.* 15 (1) (2025) 7325.
- [20] V. Chandel, Stability analysis of partially ionized plasma in a porous medium with local thermal non-equilibrium effects, *J. Plasma Phys.* 91 (1) (2025) E1.
- [21] D. Mokhtari, T. Tayebi, Magneto-thermosolutal natural convection and second law examination in a nanofluid-filled enclosure separated by a non-Darcian porous wall under the local thermal non-equilibrium condition with internal heat production and joule heating effects, *J. Therm. Anal. Calorim.* 150 (12) (2025) 9293–9313.
- [22] C. Siddabasappa, G. Kalpana, Babitha, A numerical study on heat transfer and MHD flow of a Newtonian through a porous channel—local thermal nonequilibrium model, *Heat Transf.* 52 (1) (2023) 949–967.
- [23] N. Ahmad Khan, M. Sulaiman, Heat transfer and thermal conductivity of magneto micropolar fluid with thermal non-equilibrium condition passing through the vertical porous medium, *Waves Random Complex Media* 35 (5) (2025) 9735–9759.
- [24] A. Goli, I. Zahmatkesh, S.R. Saleh, S.M. Abohasan Alavi, MHD mixed convection of developing slip flow in a vertical porous microchannel under local thermal non-equilibrium conditions, *Nanoscale Microscale Thermophys. Eng.* 28 (1) (2024) 28–45.
- [25] V.S. Reddy, J. Kandasamy, S. Sivanandam, Joule heating, Dufour and Soret effects on MHD hybrid nanofluid flow over a moving thin needle in a porous medium with thermal radiation, *World J. Eng.* 22 (4) (2025) 866–875.
- [26] M. Paul Matao, B. Prabhakar Reddy, J. Mng'ang'a, A numerical approach of hydromagnetic flow past magnetic field inclination wavering tilted porous plate with Dufour and Soret effects, *Heat Transf.* 54 (4) (2025) 2816–2836.
- [27] S.S. Shah, Magnetohydrodynamic orientation effects on Soret and Dufour phenomena in inclined corrugated triangular cavities with non-Newtonian fluids, *Int. Commun. Heat Mass Transf.* 160 (2025) 108366.
- [28] M.B.B. Hamida, M. Irfan, M.S. Anwar, M. Javed, Solar radiation-driven MHD Maxwell nanofluid flow with gyrotactic microorganisms and Soret–Dufour effects: a thermal engineering study, *J. Radiat. Res. Appl. Sci.* 18 (4) (2025) 101887.
- [29] A.K. Sarma, D. Sarma, S. Mudoi, Electromagnetohydrodynamic flow and impact of Soret, Dufour, hall current, and activation energy on Carreau fluid through a stretching cylinder, *Numer. Heat Transf. Part B* 87 (1) (2026) 2528932.
- [30] M. Awais, T. Salahuddin, A. Ehsan, M.A. Ansari, A. Kumar, Z. Mahmood, Featuring the significance of Soret and Dufour effects for the thermally radiated viscous fluid inside a porous square cavity, *J. Radiat. Res. Appl. Sci.* 18 (2) (2025) 101527.
- [31] M.S. Chowdhury, J. Vincent, K.S. Lau, A.S. Najm, A. Mebdir Holli, N. Amin, C.H. Chia, Structural and morphological properties of aluminum-doped reduced graphene oxide nanomaterials, *Mater. Sci. Technol.* (2025) 02670836251344063.
- [32] A. Roy, A. Gupta, B. Haque, A.A. Qureshi, D. Verma, K. Sharma, R. Verma, An updated review on carbon nanomaterials: types, synthesis, Functionalization and applications, degradation and toxicity, *Green Process. Synth.* 13 (1) (2024) 20240150.
- [33] A. Alhadhrami, C.S. Vishalakshi, B.M. Prasanna, B.R. Sreenivasa, H.A. Alzahrani, R.P. Gowda, R.N. Kumar, Numerical simulation of local thermal non-equilibrium effects on the flow and heat transfer of non-newtonian Casson fluid in a porous media, *Case Stud. Therm. Eng.* 28 (2021) 101483.
- [34] H. Chaudhary, N. Singh, M. Roshan, P.K. Yadav, S. Saleem, Modeling of magnetohydrodynamic Casson fluid flow due to moving permeable plate with Soret and Dufour impacts: H. Chaudhary et al, *J. Korean Phys. Soc.* 86 (8) (2025) 745–768.
- [35] L.T. Benos, N.D. Polychronopoulos, U.S. Mahabaleshwar, G. Lorenzini, I.E. Sarris, Thermal and flow investigation of MHD natural convection in a nanofluid-saturated porous enclosure: an asymptotic analysis: LT Benos et al, *J. Therm. Anal. Calorim.* 143 (1) (2021) 751–765.
- [36] A. Venkateswarlu, S. Suneetha, M.J. Babu, J.G. Kumar, C.S.K. Raju, Q. Al-Mdallal, Significance of magnetic field and chemical reaction on the natural convective flow of hybrid nanofluid by a sphere with viscous dissipation: a statistical approach, *Nonlinear Eng.* 10 (1) (2021) 563–573.