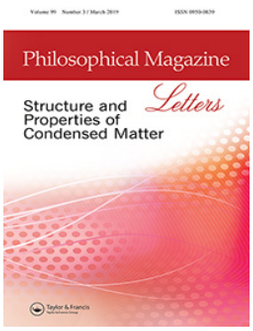

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Itinerant surfaces with spin-orbit couplings, correlations and external magnetic fields: exact results

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ABSTRACT

We analyze, in exact terms, multiband 2D itinerant correlated fermionic systems with many-body spin-orbit interactions, and in-plane external magnetic fields. Even if such systems with broad applicability in leading technologies are non-integrable, we set up an exact solution procedure for them, which is described in details. Casting the Hamiltonian in positive semidefinite form, the technique leads to the ground state, and also characterises the low lying excitation spectrum.

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I. Introduction

Surfaces with spin-orbit interactions (SOI) are the subject for a broad area of current research (see the review [1]), SOI providing essential effects in various phenomena of large interest today, ranging from quantum magnets [2], topological phases [3], ultracold atom experiments [4], to Majorana fermions [5]. The applications appear mostly in low dimensional systems [6–12], and during processing, often external fields are as well present, the most interesting applications being related to strongly correlated systems. Contrary to its importance, although exact treatments of 2D strongly correlated systems with spin-orbit coupling are being developed [12], studies including applied external magnetic fields are absent. Our aim in this Letter is to fill up this gap by setting up the details of a calculation procedure for such situations, considering Hamiltonians describing realistic correlated systems.

The main difficulty encountered is that the here studied 2D systems are non-integrable, so special techniques must be used in order to describe them in exact terms. For this reason we use the method based on positive semidefinite operator properties whose applicability does not depend on dimensionality and integrability [13–16]. The method has been previously applied in conditions unimaginable befor in the context of exact solutions in 1-3D, even in the presence of the disorder [17–24].

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II. The system analysed

The Hamiltonian of the system has the form $\hat{H} = \hat{H}_{kin} + \hat{H}_{int} + \hat{H}_h$,

$$\hat{H} = \sum_{p,p'} \sum_{\mathbf{i},\mathbf{r}} \sum_{\sigma,\sigma'} (k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p';\sigma,\sigma'} \hat{c}_{p,\mathbf{i},\sigma}^{\dagger} \hat{c}_{p',\mathbf{i}+\mathbf{r},\sigma'} + H.c.) + \sum_{p} \sum_{\mathbf{i}} U_{p,\mathbf{i}} \hat{n}_{p,\mathbf{i},\uparrow} \hat{n}_{p,\mathbf{i},\downarrow} + \sum_{p,\mathbf{i}} \sum_{\sigma,\sigma'} \vec{h}_{p,\mathbf{i}} \hat{c}_{p,\mathbf{i},\sigma}^{\dagger} \vec{\sigma}_{\sigma,\sigma'} \hat{c}_{p,\mathbf{i},\sigma'}. \quad (1)$$

where the first term represents the kinetic part of the Hamiltonian ($\hat{H}_{kin}$), the second term is the interaction part ($\hat{H}_{int}$), while the last term describes the interaction with the external magnetic field ($\hat{H}_h$). At the level of $\hat{H}_{kin}$, in order to have a realistic 2D surface description, two bands are considered, denoted hereafter by $p, p' = a, b$. However we note, that this choice not diminishes the applicability of the deduced results, since usually, the theoretical description of muliband systems is given by projecting the multiband structure in a few-band picture [25], projection which is stopped here only for its relative simplicity at two-bands level. Again in order to approach a real systems, besides on-site one particle terms ($\mathbf{r} = 0$), one takes into consideration nearest-neighbour ($\mathbf{r} = \mathbf{x}_1, \mathbf{x}_2$, where $\mathbf{x}_1, \mathbf{x}_2$ are the Bravais vectors), and next nearest-neighbour ($\mathbf{r} = \mathbf{x}_2 + \mathbf{x}_1, \mathbf{x}_2 - \mathbf{x}_1$) contributions. Furthermore, note that the $k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p';\sigma,\sigma'}$ coefficient represents for ($p = p'$, $\mathbf{r} = 0$), ($p = p'$, $\mathbf{r} \neq 0$) on-site potential, (hopping matrix element); while for ($p \neq p'$, $\mathbf{r} = 0$), ($p \neq p'$, $\mathbf{r} \neq 0$) on-site hybridisation, (inter-site hybridisation). Concerning $\hat{H}_{int}$, since in itinerant many-body systems strong screening effects are present, we consider at this stage only the on-site Coulomb repulsion (Hubbard interaction term) in the correlated band (p=b, $U_b > 0$), the second band being considered non-correlated (p=a, $U_a = 0$). The many-body spin-orbit interactions being of one-particle type, are introduced in the kinetic part of the Hamiltonian, explicitly in the nearest neighbour spin-flip hopping terms, i.e. coefficients $k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p;\sigma,-\sigma}$, $\mathbf{r} = \mathbf{x}_1, \mathbf{x}_2$. These terms are of Rashba (λ_R^p, $p = a, b$) and Dresselhaus (λ_D^p, $p = a, b$) type [26]. Consequently, one has for $\mathbf{r} = \mathbf{x}_1$, the structure $k_{\mathbf{i},\mathbf{i}+\mathbf{x}_1}^{p,p;\uparrow,\downarrow} = \lambda_R^p - i\lambda_D^p$, $k_{\mathbf{i},\mathbf{i}+\mathbf{x}_1}^{p,p;\downarrow,\uparrow} = -\lambda_R^p - i\lambda_D^p$, while for $\mathbf{r} = \mathbf{x}_2$ the expressions $k_{\mathbf{i},\mathbf{i}+\mathbf{x}_2}^{p,p;\uparrow,\downarrow} = \lambda_D^p - i\lambda_R^p$, $k_{\mathbf{i},\mathbf{i}+\mathbf{x}_2}^{p,p;\downarrow,\uparrow} = -\lambda_D^p - i\lambda_R^p$. We underline that even if usually the SOI contributions are small, they introduce essential effects since they break the double spin-projection degeneracy of each band. Hence, in the presence of strong correlations, the essential effects introduced cannot be obtained by standard perturbation approximations [12]. We note that other spin-flip terms are not present in $\hat{H}_{kin}$, and one has for all considered $\mathbf{r}$ values $k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p';\uparrow,\uparrow} = k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p';\downarrow,\downarrow} = k_{\mathbf{i},\mathbf{i}+\mathbf{r}}^{p,p'}$. Furthermore, in order to not diminish the effect of the spin-flip nearest-neighbour hopping terms produced by SOI, the external fields are only applied in-plane, hence without the z-component ($h_{p,\mathbf{i}}^z = 0$, $h_{p,\mathbf{i}}^x, h_{p,\mathbf{i}}^y \neq 0$). We underline, that the in-plane $h_{p,\mathbf{i}}^x$, $h_{p,\mathbf{i}}^y$

contributions will additively renormalise the $k_{\mathbf{i},\mathbf{i}}^{p,p;\sigma,-\sigma}$ contributions as $\bar{k}_{\mathbf{i},\mathbf{i}}^{p,p;\uparrow,\downarrow} = k_{\mathbf{i},\mathbf{i}}^{p,p;\uparrow,\downarrow} + h^x - ih^y$, $(\bar{k}_{\mathbf{i},\mathbf{i}}^{p,p;\downarrow,\uparrow})^* = \bar{k}_{\mathbf{i},\mathbf{i}}^{p,p;\uparrow,\downarrow}$.

III. The Hamiltonian cast in pozitive semidefinite form

A. The transformation of the Hamiltonian

Now we turn back to (1), and present the transformation of $\hat{H}$ in exact terms. On this line we introduce two block operators Q=A,B for each site $\mathbf{i}$, which for a fixed Q value are defined as

$$\hat{Q}_{\mathbf{i}} = \sum_{p=a,b} \sum_{n=1,2,3,4} \sum_{\alpha=\uparrow,\downarrow} q_{Q,p,n,\alpha} \hat{c}_{p,\mathbf{i}+\mathbf{r}_n,\alpha}. \tag{2}$$

Here, in order $\mathbf{r}_1 = 0$, $\mathbf{r}_2 = \mathbf{x}_1$, $\mathbf{r}_3 = \mathbf{x}_1 + \mathbf{x}_2$, and $\mathbf{r_4} = \mathbf{x}_2$, see Figure 1. At a given lattice site $\mathbf{i}$, for a fixed Q and p value, the $\hat{Q}_{\mathbf{i}}$ operator has 8 contributions, 4 for spin $\alpha =\uparrow$, and other 4 for spin $\alpha =\downarrow$. For fixed α the mentioned 4 values denoted by n=1,2,3,4 are placed in the four corners of an elementary plaquette connected to the lattice site $\mathbf{i}$. Using (2), the starting system Hamiltonian $\hat{H}$ in (1) becomes of the form

$$\hat{H} = \hat{P} + S_c, \tag{3}$$

where $\hat{P}$ represents a positive semidefinite operator, while S_c a scalar. Taking into account that $\hat{P} = \hat{P}_Q + \hat{P}_U$ where $\hat{P}_U = U_b \sum_{\mathbf{i}} \hat{P}_{\mathbf{i}}$, for $U_b > 0$ one has

$$\hat{P}_Q = \sum_{\mathbf{i}} \sum_{Q=A,B} \hat{Q}_{\mathbf{i}} \hat{Q}_{\mathbf{i}}^{\dagger}, \quad \hat{P}_{\mathbf{i}} = \hat{n}_{b,\mathbf{i},\uparrow} \hat{n}_{b,\mathbf{i},\downarrow} - (\hat{n}_{b,\mathbf{i},\uparrow} + \hat{n}_{b,\mathbf{i},\downarrow}) + 1,$$

$$S_c = \eta N - U_b N_\Lambda - \sum_{\mathbf{i}} \sum_{Q=A,B} d_{\mathbf{i},Q}, \quad d_{\mathbf{i},Q} = \{\hat{Q}_{\mathbf{i}}, \hat{Q}_{\mathbf{i}}^{\dagger}\}, \tag{4}$$

where $N(N_\Lambda)$ represents the number of electrons (lattice sites).

The corresponding matching equations which allows the transformation of the starting Hamiltonian from (1) into the form described by $\hat{H}$ in (3), (4),

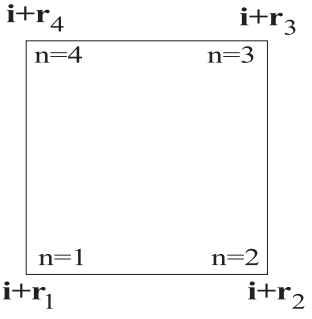

Figure 1. Unit cell defined at the lattice site $\mathbf{i}$ with in-cell notations of sites n=1,2,3,4.

are as follows: One has 32 equations for nearest-neighbour contributions m=1,2, namely 16 for a fixed m

$$-k^{p,p';\sigma,\sigma'}_{\mathbf{i},\mathbf{i}+\mathbf{x}_m} = \sum_{Q=A,B} (q^*_{Q,2m,p,\sigma} q_{Q,1,p',\sigma'} + q^*_{Q,3,p,\sigma} q_{Q,6-2m,p',\sigma'}), \tag{5}$$

and similarly one has 32 equations for the next nearest-neighbour contributions, as previously 16 for a fixed $m = \pm 1$

$$-k^{p,p';\sigma,\sigma'}_{\mathbf{i},\mathbf{i}+\mathbf{x}_2+m\mathbf{x}_1} = \sum_{Q=A,B} q^*_{Q,3+(1-m)/2,p,\sigma} q_{Q,1+(1-m)/2,p',\sigma'}. \tag{6}$$

Finally local (e.g. $\mathbf{r} = 0$) contributions give rise to 16 equations which can be written as

$$\begin{aligned} &- k^{p,p';\sigma,\sigma'}_{\mathbf{i},\mathbf{i}} [(1-\delta_{p,p'}) + (1-\delta_{\sigma,\sigma'})\delta_{p,p'} + \delta_{p,p'}\delta_{\sigma,\sigma'}] + (\eta - U_p)\delta_{p,p'}\delta_{\sigma,\sigma'} \\ &- [h^x - ik(\sigma)h^y]\delta_{p,p'}(1-\delta_{\sigma,\sigma'}) = \sum_{Q=A,B} \sum_{n=1,2,3,4} q^*_{Q,n,p,\sigma} q_{Q,n,p',\sigma'}, \end{aligned} \tag{7}$$

where $k(\sigma) = \delta_{\uparrow,\sigma} - \delta_{\downarrow,\sigma}$. One has here totally 80 non-linear equations, whose unknown are the 32 numerical prefactors $q_{Q,n,p,\sigma}$ called 'block operator parameters', and the parameter η entering in the ground state energy ($E_g = S_c$). The total number of Hamiltonian parameters (taking into account all possible spin dependences as well) is 76, so a proper description for a real material can be provided. But taking into account the conditions presented below (1) and used in this description, besides SOI couplings and U, one remains with only 10 $\hat{H}_{kin}$ parameters per one band in both $\mathbf{x}_1$, $\mathbf{x}_2$ directions.

B. Solution of the matching equations

In order to start the deduction of the exact ground states, first we should deduce the numerical prefactors $q_{Q,p,n,\alpha}$ of the block operators from (2) from the matching Equations (5)–(7). Starting this job, first we observe from (5)–(7) that all $q_{Q=A,p,n,\alpha}$ components can be given in function of the $q_{Q=B,p,n,\alpha}$ coefficients via the relation $q_{A,p,n,\alpha} = d_{n,\alpha} q_{B,p,n,\alpha}$, where the coefficiens $d_{n,\alpha}$ have the expression

$$\begin{aligned} d_{n,\alpha} = &-\left(\frac{\delta_{\alpha,\uparrow}}{y} + \frac{\delta_{\alpha,\downarrow}}{x}\right)\delta_{n,1} - \left(\frac{\delta_{\alpha,\uparrow}}{v} + \frac{\delta_{\alpha,\downarrow}}{z}\right)\delta_{n,2} + (x^*\delta_{\alpha,\uparrow} + y^*\delta_{\alpha,\downarrow})\delta_{n,3} \\ &+ (z^*\delta_{\alpha,\uparrow} + v^*\delta_{\alpha,\downarrow})\delta_{n,4}, \end{aligned} \tag{8}$$

where x,y,v,z are numerical prefactors. After this step it results that the remaining $q_{B,p,n,\alpha}$ unknowns with p=a can be given in term of the $q_{B,p,n,\alpha}$ coefficients containing p=b via the relation $q_{B,a,n,\alpha} = \alpha_n q_{B,b,n,\alpha}$, where one has for the

numerical coefficients α_n the expression

$$\alpha_n = \alpha_1 \delta_{n,1} + \alpha_2 \delta_{n,2} + \frac{\gamma_0}{\alpha_1^*} \delta_{n,3} + \frac{\gamma_0}{\alpha_2^*} \delta_{n,4} \tag{9}$$

where γ_0 is an arbitrary real and positive parameter, while α_1, α_2 are two further numerical prefactors. In this manner, up to (9) only 8 unknown coefficients remain, namely $q_{B,b,n,\alpha}$ with n=1,2,3,4 and $\alpha = \uparrow, \downarrow$. But it turns out that these eight unknown coefficients are interdependent, and all can be expressed in function of one block operator parameter, namely $q_{B,b,n=1,\uparrow}$, via

$$\begin{aligned}
&q_{B,b,1,\downarrow} = \frac{1}{w^*} q_{B,b,1,\uparrow}, \quad q_{B,b,3,\downarrow} = -\frac{u^*}{w^*} q^*_{B,b,1,\uparrow}, \quad q_{B,b,3,\uparrow} = \frac{|\alpha_1|^2}{\gamma_0} \frac{1}{uyx^*} q_{B,b,1,\uparrow}, \\
&q_{B,b,2,\downarrow} = \omega q_{B,b,2,\uparrow}, \quad q_{B,b,4,\downarrow} = -\frac{\alpha_2^*}{\alpha_1^*} \frac{u^* y^*}{v^* w^*} q^*_{B,b,2,\uparrow}, \quad q_{B,b,4,\uparrow} = \frac{\alpha_2^* \alpha_1}{\gamma_0} \frac{\omega^*}{uyz^*} q^*_{B,b,2,\uparrow},
\end{aligned} \tag{10}$$

where $|q_{B,b,2,\uparrow}| = |\mu| |q_{B,b,1,\uparrow}|$, $\omega = [zwx^*(1+vy^*)]/[vy^*(1+zx^*)]$, $|w| = (|u||y| \sqrt{\gamma_0})/|\alpha_1|, \sigma = yv^*$. Taking $\sigma, k, \phi_1, \phi_2$ as arbitrary parameters, one obtains three coupled equations in $X = vx^*$, $Z = vz^*$, $V = |v|^2$

$$\begin{aligned}
&k\sigma^*(1+\sigma) = Z\left[k(1+\sigma) + X^* - \frac{|X|^2}{V}\right] + (V - X), \\
&\frac{X - V}{1+X} e^{i\phi_1} = V \frac{\sigma^* - Z}{V + \sigma Z}, \\
&\frac{V + ZX^*}{X - Z} e^{i\phi_2} = V \frac{(1+\sigma^*)}{\sigma - V}.
\end{aligned} \tag{11}$$

from where, together with the σ expression, the remaining unknown x, y, z, v parameters can be deduced, and based on them, starting from the relation $k_2 = f\, q_{B,b,n=1,\uparrow}$, where $k_2 = |q_{B,b,n=2,\uparrow}|$ is a free parameter, and $f \equiv f(x,y,z,v)$ is a known function, see (12),

$$f = \frac{V(|\alpha_1|^2 - \gamma_0)}{|y|^2(\gamma_0 - |\alpha_2|^2)} \frac{k(1+|y|^2) - (1+|x|^2)}{k(1+V) - (1+|z|^2)\dfrac{|v-x|^2}{|y-z|^2}}, \tag{12}$$

$q_{B,b,n=1,\uparrow}$ can be determined. Then, $q_{B,b,n=2,\uparrow} = |q_{B,b,n=2,\uparrow}| \exp(i\theta_2)$, where θ_2 is a free parameter, is given by $k_2 \exp i\theta_2$. The Hamiltonian parameters expressed in $k^{a,a}_{\mathbf{i},\mathbf{i}+\mathbf{x}_1}$ units, enter in the 12 free parameters $k, \mathrm{Re}(\sigma), \mathrm{Im}(\sigma), \phi_1, \phi_2, \mathrm{Re}(\alpha_1), \mathrm{Im}(\alpha_1), \mathrm{Re}(\alpha_2), \mathrm{Im}(\alpha_2), \gamma_0, k_2, \theta_2$ of the solution presented above (e.g. $k^{a,b,\sigma,\sigma}_{\mathbf{i},\mathbf{i}+\mathbf{x}_1} = \frac{2\alpha_2^*}{\alpha_1\alpha_2^* + \gamma_0}$, $k^{b,a,\sigma,\sigma}_{\mathbf{i},\mathbf{i}+\mathbf{x}_1} = \frac{2\alpha_1}{\alpha_1\alpha_2^* + \gamma_0}$, etc.). We further note that when the presented solution appears, the relation $k^{b,b,\sigma,\sigma}_{\mathbf{i},\mathbf{i}+\mathbf{r}} = (1/\gamma_0) k^{a,a,\sigma,\sigma}_{\mathbf{i},\mathbf{i}+\mathbf{r}}$ fixes for all possible $\mathbf{r}$ [see the discussion following (1)] the magnitudes ratio of diagonal $\hat{H}_{kin}$ overlap elements from the two bands.

IV. The ground state wave functions

The first deduced ground state wave function corresponding to the transformed Hamiltonian from (3) connected to the matching Equations (4)–(7) is of the form

$$|\Psi_{1,g}\rangle = \prod_{\mathbf{i}} \left(\prod_{Q=A,B} \hat{Q}_{\mathbf{i}}^{\dagger} \right) \hat{Q}_{1,\mathbf{i}}^{\dagger} |0\rangle \tag{13}$$

where $\prod_{\mathbf{i}}$ extends over all N_Λ lattice sites, one has $\hat{Q}_{1,\mathbf{i}}^{\dagger} = \sum_{\sigma} \alpha_{\sigma,\mathbf{i}} \hat{c}_{b,\mathbf{i},\sigma}^{\dagger}$, where $\alpha_{\sigma,\mathbf{i}}$ are numerical prefactors, and $|0\rangle$ is the bare vacuum with no fermions present. This $|\Psi_{1,g}\rangle$ solution corresponds to 3/4 system filling.

The presented wave vector from (13) represents the ground state for the following reason: a) As seen from (2) the block operators $\hat{Q}_{\mathbf{i}}^{\dagger}$ are linear combinations of canonical Fermi creation operators acting on the finite number of sites of the given block, consequently the $\hat{Q}_{\mathbf{i}}^{\dagger} \hat{Q}_{\mathbf{i}}^{\dagger} = 0$ equality is satisfied. Hence the relation $\hat{P}_Q |\Psi_{1,g}\rangle = 0$ automatically holds. Furthermore, b) The $\hat{P}_{\mathbf{i}}$ positive semidefinite operators from the expression of the $\hat{P}_U$ operators in (4) (note that because of $U_b > 0$, also $\hat{P}_U$ is a positive semidefinite operator) attain their minimum eigenvalue zero when at least one b-electron is present on the site $\mathbf{i}$. Hence, for the minimum eigenvalue zero of $\hat{P}_U$, at least one b-electron is needed to be present on all lattice sites. But $\prod_{\mathbf{i}} \hat{Q}_{1,\mathbf{i}}^{\dagger}$ introduces a b-electron on each site, consequently also $\hat{P}_U |\Psi_{1,g}\rangle = 0$ holds. As a summary of the above presented arguments, also for $\hat{P} = \hat{P}_Q + \hat{P}_U$ one has $\hat{P} |\Psi_{1,g}\rangle = 0$, i.e. $|\Psi_{1,g}\rangle$ represents the ground state. The uniqueness of this ground state at 3/4 system filling can also be demonstrated on the line of the Appendix 2 of Ref. [16].

We note that the ground state (13) can be extended also above 3/4 system filling as follows:

$$|\Psi_{2,g}\rangle = \prod_{\mathbf{i}} \left(\prod_{Q=A,B} \hat{Q}_{\mathbf{i}}^{\dagger} \right) \hat{Q}_{1,\mathbf{i}}^{\dagger} \left(\prod_{j=1}^{N_1} \hat{c}_{b,\mathbf{k}_j,\alpha_{\mathbf{k}_j}}^{\dagger} \right) |0\rangle \tag{14}$$

where $N_1 < N_\Lambda$, $\hat{c}_{b,\mathbf{k},\alpha}^{\dagger}$ is the Fourier transformed $\hat{c}_{b,\mathbf{i},\alpha}^{\dagger}$, $\alpha_{\mathbf{k}_j}$ being an arbitrary spin projection for each $\mathbf{k}_j$, and $\prod_{j=1}^{N_1}$ is taken over N_1 arbitrary $\mathbf{k}_j$ values. The filling corresponding to (14) corresponds to $3/4 + N_1/N_\Lambda$ system filling. The demonstration of the ground state nature follows the line presented above in the case of (13), and is based on the observation that the supplementary product $(\prod_{j=1}^{N_1} \hat{c}_{b,\mathbf{k}_j,\alpha_{\mathbf{k}_j}}^{\dagger})$ not alters the properties $\hat{P}_Q |\Psi_{z,g}\rangle = \hat{P}_U |\Psi_{z,g}\rangle = 0$, for both z=1,2.

In similar manner we have deduced ground state wave vectors also below system half filling. On this line one has

$$|\Psi_{3,g}\rangle = \prod_{j}^{N_s} \hat{C}_j^{\dagger}|0\rangle \tag{15}$$

where $\hat{C}_j^{\dagger}$ represent block operators which on their turn are linear combinations of $\hat{c}_{p,\mathbf{i},\sigma}^{\dagger}$ creation operators, and must satisfy the anti-commutation relations $\{\hat{Q}_{\mathbf{i}}, \hat{C}_j^{\dagger}\} = 0$ for all lattice sites $\mathbf{i}$, and both Q=A,B values. The j index here denotes different (independent) $\hat{C}_j^{\dagger}$ terms. The number of carriers described by (15) is given by N_s. We underline that in the case of the ground state (15) the starting Hamiltonian (1) is transformed in the expression

$$\hat{H} = \hat{P}_{Q,1} + \eta N, \quad \hat{P}_{Q,1} = \sum_{\mathbf{i}} \sum_{Q=A,B} \hat{Q}_{\mathbf{i}}^{\dagger} \hat{Q}_{\mathbf{i}}. \tag{16}$$

The corresponding matching Equations (5)–(7) remain unaltered in their right hand side, but their left hand side gains a minus sign, and supplementary, in (7) the renormation $\eta \to \eta + U_p$ emerges. The energies $E_{n,g}$ corresponding to the ground states $|\Psi_{n,g}\rangle$ for n=1,2,3 become

$$E_{n,g} = \left[\eta(N + N_1\delta_{n,2}) - U_b N_\Lambda - \sum_{\mathbf{i}} \sum_{Q=A,B} d_{\mathbf{i},Q}\right](\delta_{n,1} + \delta_{n,2}) + \eta N_s \delta_{n,3}. \tag{17}$$

V. Summary and conclusions

We started with the observation that surfaces and interfaces have a broad application spectrum in leading technologies, and such two dimensional systems have by their nature in the case of real materials potential gradients (∇V) at their surfaces. These gradients are generating many-body spin-orbit coupling ($\vec{\sigma} \cdot (\nabla V \times \vec{k})$), which even if small, produces essential effects since breaks the double spin projection degeneracy of each band. In applications, the correlated electron surfaces are often exposed to in-plane magnetic fields. Such processes, contrary to their importance, have not been analysed till now in exact terms mainly due to the non-integrable nature of the systems. In this Letter we fill up this gap by working out and describing a procedure which, using positive semidefinite operator properties, deduces exact results in 2D for such systems. The technique merits attention since provide solution for the matching equations consisting of 80 coupled nonlinear complex-algebraic equations. Extended results presenting how this technique works in concrete cases will be published in a forthcoming detailed paper.

Disclosure statement

No potential conflict of interest was reported by the authors.

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