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Optimal control strategies for taming TikTok addiction: a mathematical model and analysis

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Abstract Optimal control theory is an extension of the calculus of variations. It is a mathematical optimization method for deriving control strategies for a dynamic system. In this paper, the system of differential equations for which we aim to utilize control theory is TikTok, which is one of the most attractive internet platforms. TikTok has garnered immense popularity, surpassing other social media platforms. However, its addictive nature has raised concerns about mental health, including depression, eating disorders, anxiety, self-obsession, and narcissistic personality disorder among its users. This paper introduces a mathematical model for TikTok, considering the usage of this app as an epidemic. The model is rigorously validated through stability analysis of both local and global equilibrium. Moreover, disease-free and non-trivial equilibrium scenarios are discussed by calculating their reproduction numbers. This study aims to raise awareness of TikTok's potential misuse and explore control theory solutions to mitigate addiction. Additionally, statistical data is used to visualize the numerical results and analyze the impact of control parameters on the TikTok model.

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1 Introduction

Mathematical control theory is a branch of applied mathematics that focuses on the behavior of dynamical systems [18]. It aims to leverage well-established mathematical principles as controllers. Introduced in the mid-20th century, this field has become integral to engineering and economics, and its applications extend across diverse disciplines, including biology [14]. The fundamental premise of control theory involves employing a model to unveil the nature of a system and its variables. Subsequently, a controller is designed to regulate these variables as needed. This can entail reducing errors, controlling variables, maximizing performance, or any combination thereof, placing humans in the control loop. Control theory and stability analysis are grounded in mathematical principles, drawing from differential equations, linear algebra, and optimization [1,2,4–7,10,15]. In practical terms, mathematical control theory can address a nearly universal issue in the real world. Its relevance to social media addiction becomes apparent in today's pervasive online environment, where limiting social media use has become a widespread concern [3]. Moreover, it examines human reactions as influenced by models, seeking to employ them in the design of control systems. It becomes applicable when user behavior patterns on social media can be modeled, suggesting the potential to intervene through measures such as designing manageability or algorithms. This intervention could involve establishing individual thresholds for screen time, adjusting content exposure based on contextual factors, or implementing feedback loops to encourage healthier online behavior. Embracing control theory concepts empowers individuals, allowing them to regain autonomy over their digital lives. Consequently, the adverse effects of social media use can be minimized or even eliminated, creating a more balanced digital experience. Beyond social media, control theory holds promise in various other social arenas, including addiction prevention [11]. By addressing issues related to addicts and impulse-addicted individuals, control theory can mitigate the spread of negative influences in society at large.

In recent years, to alleviate boredom, achieve fame, and gain the attention of people worldwide, Millennials, Gen Z, and Gen Alpha have increasingly utilized apps such as YouTube, Snapchat, Instagram, and TikTok, among others. The COVID-19 lockdown also played a significant role in the surge in usage of these apps. The TikTok app was initially launched in September 2016 in China under the name "Douyin," and it is owned by Byte Dance. Later, in September 2017, it was made available globally under the name "TikTok," having previously gone by the name Musically. Following its launch, TikTok unexpectedly gained immense popularity, with 693 million people downloading the app in 2019, making it the most popular app of the year. Since then, its user base has continued to grow despite bans and controversies associated with its name or the presence of clone apps [19].

It has just been a few years since the launch of TikTok, and it has users of all ages and ethnicities. This app allows its users to create short videos ranging from 15 to 60 s in 75 languages, incorporating music, songs, soundbites, or effects of their choice. The app is available in 150 marketplaces, and what makes it more interesting is the benefit of collaboration. Users can 'duet' by responding to someone's video, creating a chain of unlimited responses. The app utilizes artificial intelligence and image capture technology, providing content creators with more options than other competitive apps in the market, such as Snapchat, Vine, or Dubsmash, among others.

Studies conducted on this app show that many users also exploit it, and therefore, its impact tends to be more negative than positive [12]. As a result of its excessive usage, teenagers perform poorly in educational institutes and achieve low grades/CGPA, while people of all ages develop beauty complexes, inferiority, or superiority complexes [9]. There have also been numerous incidents where people lost their lives while creating videos for TikTok. One such incident involves a college student who died in 2019 in Tamil Nadu. His scooter rammed into a bus while he was riding with two friends, making a TikTok video. Their bike lost balance and smashed into the bus [16]. Another incident involves a farmer who, while making a TikTok video, attempted to climb a tractor with a cultivator attached. Unfortunately, his foot slipped, and he ended up being crushed under the cultivator machine [16]. Content creators often engage in dangerous and ridiculous activities to gain popularity, putting their lives at risk. Unfortunately, young viewers who watch such content may be tempted to imitate these acts, putting themselves at risk as well.

Several studies have been conducted to understand why TikTok is so famous and addictive for users, and one of these studies proves that, essentially, this app is not just a social media platform but also effectively conveys the users' imagination. It is purely based on the user's interests and understanding [21]. Numerous research articles have been published about TikTok, covering a range of topics, including its impact on mental health, user engagement, content creation, and more.

These are just a few examples of the many research articles that have been published about TikTok. All these studies involve psychological theories and statistical analysis [13]. However, in this paper, a mathematical model is developed for the first time to analyze the addictive behavior of TikTok users. Users of all types are considered in this work, and then, based on control parameters, the paper discusses addiction control for TikTok.

The structure of this paper is as follows: The introduction, literature review, and purpose of the study are outlined in Sect. 1. The TikTok app's mathematical model is explained in Sect. 2, along with a discussion of the solutions' boundedness. The reproduction number and equilibrium analysis of the TikTok mathematical model are covered in Sect. 3; the stability analysis is presented in Sect. 4; the optimal control analysis for this model is contained in Sect. 5, along with some control strategies to address TikTok addiction. The conclusion and discussion of this paper are included in the last section.

2 Mathematical formulation

TikTok, like many other social media platforms, has the potential to become addictive for some users. Addiction is typically characterized by compulsive and excessive use of a substance or activity despite adverse consequences. TikTok addiction can manifest in several ways. For example, some users may spend hours scrolling through the app, neglecting other important activities such as work, school, or personal relationships. Others may feel a strong compulsion to create and post content, constantly checking their likes and followers. One reason why TikTok can be addictive is its algorithm, which is designed to keep users engaged by showing them content tailored to their interests. This creates a feedback loop in which users spend more and more time on the app, constantly exposed to new and exciting content. Moreover, the TikTok short-form video format is particularly well-suited for capturing and retaining users' attention, allowing for quick and easy consumption of content. This can make it difficult for some users to disengage from the application, even when they intend to.

Therefore, if some users feel they may be developing a TikTok addiction or struggling to control their usage of the application, it may be helpful for them to take a break or limit their usage. They can also consider seeking support from a mental health professional who can help them develop healthy habits and coping strategies. TikTok can be used for a variety of purposes beyond just entertainment. Many businesses, both small and large, use TikTok to promote their products or services and create brand awareness. TikTok algorithm can be employed to reach a wider audience and increase engagement, leading to more sales or conversions. For instance, Trianasari et al. [20] described the role of TikTok in social media marketing for building brand image and effectively selling more products. Additionally, Sim [17] explained the role of TikTok in increasing the interest of youngsters in entrepreneurship.

In addition, educators and experts in various fields can use TikTok to share educational content in a fun and engaging way. The short-form video format is well-suited for breaking down complex concepts into bite-sized pieces that are easy to understand and remember. This can be especially useful for reaching younger audiences who may not respond as well to traditional classroom teaching methods. TikTok can also be used to raise awareness about important social issues, such as mental health, environmentalism, or social justice. Many advocacy groups and non-profits use TikTok to spread their message and connect with supporters [8]. Overall, TikTok can be a powerful tool for marketing, education, and advocacy, in addition to being a popular social media platform for entertainment. All such categories have been studied in this mathematical model.

The group of TikTok users who use the platform for business, educational, or awareness purposes is denoted as educators, entrepreneurs, and entertainers, represented by $E(t)$. $S(t)$ represents the population group susceptible to getting addicted to TikTok. $R_L(t)$ and $R_H(t)$ denote the groups of people who are minimally or highly exposed, respectively. $I(t)$ stands for the infected or addicted TikTok users, while $H(t)$ represents those users who have realized their addiction and have been cured after seeking professional help. Table 1 provides information about the variables and their names.

The mathematical model formed is given as follows:

$$S'(t) = aT - aS(t) - \frac{bS(t)I(t)}{T},$$

$$R_L'(t) = \frac{cbS(t)I(t)}{T} - (d + e + a)R_L(t),$$

Table 1 Mathematical formulation for model and defined variables

Symbol	Name of variable
$S(t)$	Susceptible
$R_L(t)$	Minimally exposed
$R_H(t)$	Highly exposed
$I(t)$	Infected
$E(t)$	Educationist/entrepreneurs/entertainers
$H(t)$	Healing population

Table 2 Parameters in Eq. (1) are defined here

Symbol	Name of parameter
T	Total population
a	Birth rate and death rate
b	Transmission rate
c	Rate at which an individual becomes R_L
d	Rate at which R_L becomes I
e	Rate at which R_L becomes E
f	Rate at which R_H becomes I
g	Rate at which R_H becomes E
h	Rate at which I becomes H
j	Rate at which I becomes E
k	Rate at which E heals

$$
\begin{aligned}
R_H'(t) &= \frac{(1-c)bS(t)I(t)}{T} - (f+g+a)R_H(t),\\
I'(t) &= dR_L(t) + fR_H(t) - (h+j+a)I(t),\\
E'(t) &= eR_L(t) + gR_H(t) + jI(t) - (k+a)E(t),\\
H'(t) &= hI(t) + kE(t) - aH(t).
\end{aligned} \tag{1}
$$

2.1 Boundedness of the solutions

Since $\{S, E, R_L, R_H, I, H\}$ should be bounded and should lie in a positive region, therefore, we have:

$$
T = \{(S, E, R_L, R_H, I, H) : S, E, R_L, R_H, I, H \geq 0\}. \tag{2}
$$

3 Reproduction number and equilibrium analysis

In this section, for the theoretical stability of the TikTok mathematical model, its reproduction number and equilibrium points will be discussed. By examining these parameters, it can be understood under which circumstances the survival is possible and what measures should be taken to attain a disease free equilibrium. For equilibrium points, let us write Eq. (1) as

$$
\begin{aligned}
&aT - aS(t) - \frac{bS(t)I(t)}{T} = 0,\\
&\frac{cbS(t)I(t)}{T} - (d+e+a)R_L(t) = 0,\\
&\frac{(1-c)bS(t)I(t)}{T} - (f+g+a)R_H(t) = 0,\\
&dR_L(t) + fR_H(t) - (h+j+a)I(t) = 0,\\
&eR_L(t) + gR_H(t) + jI(t) - (k+a)E(t) = 0,\\
&hI(t) + kE(t) - aH(t) = 0.
\end{aligned} \tag{3}
$$

3.1 Trivial equilibrium

To obtain the equilibrium points, there are two points; one is trivial, that is, disease-free equilibrium $D_f = (T, 0, 0, 0, 0, 0)$. To find the reproduction number, we use the next-generation method. Assume that

$$y = \begin{pmatrix} S \\ E \\ R_L \\ R_H \\ I \\ H \end{pmatrix},$$

then Eq. (1) becomes

$$\frac{dy}{dt} = v(y) - w(y), \tag{4}$$

where $w(y) = \begin{pmatrix} -a(T-S) + \frac{bSI}{T} \\ (d+e+a)R_L \\ (f+g+a)R_H \\ -dR_L - fR_H + (h+j+a)I \\ -eR_L - gR_H - jI + (k+a)E \\ -hI - kE + aH \end{pmatrix}$, $v(y) = \begin{pmatrix} 0 \\ \frac{cbSI}{T} \\ \frac{(1-c)bSI}{T} \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

Therefore the Jacobian matrix for $v(y)$ and $w(y)$ are given as:

$$J^w_{(D_f)} = \begin{pmatrix} 0 & 0 & b & 0 & 0 & a \\ d+e+a & 0 & 0 & 0 & 0 & 0 \\ 0 & f+g+a & 0 & 0 & 0 & 0 \\ -d & -f & h+j+a & 0 & 0 & 0 \\ -e & -g & -j & k+a & 0 & 0 \\ 0 & 0 & -h & -k & a & 0 \end{pmatrix}, \tag{5}$$

$$J^v_{(D_f)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & cb & 0 & 0 & 0 \\ 0 & 0 & (1-c)b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

The basic reproduction number from the matrices above could be written as:

$$R_0 = \frac{b\big((1-c)f(a+d+e) + cd(a+f+g)\big)}{(h+j+a)(d+e+a)(f+g+a)}. \tag{7}$$

3.2 Nontrivial equilibrium

$D_d = (S^*(t), R^*_L(t), R^*_H(t), I^*(t), E^*(t), H^*(t))$ of Eq. (1) is determined by Eq. (3) as:

$$\begin{aligned} S^*(t) &= \frac{T}{R_0}, \\ R^*_L(t) &= \frac{acT(R_0-1)}{R_0(d+e+a)}, \\ R^*_H(t) &= \frac{(1-c)aT(R_0-1)}{R_0(f+g+a)}, \\ I^*(t) &= \frac{Ta(R_0-1)}{b}, \end{aligned}$$

$$E*(t) = \frac{(R_0-1)}{k+a}\left(\frac{ecaT}{R_0(d+e+a)} + \frac{agT(1-c)}{R_0(f+g+a)} + \frac{jaT}{b}\right),$$
$$H^*(t) = \frac{hT(R_0-1)}{b} + \frac{k(R_0-1)}{k+a}\left(\frac{ecT}{R_0(d+e+a)} + \frac{g(1-c)T}{R_0(f+g+a)} + \frac{jT}{b}\right). \tag{8}$$

As we can clearly observe, D_d exists only if $R_0 > 1$.

4 Stability analysis

The Jacobian matrix about y considered in Sect. 4 is given as:

$$J = \begin{pmatrix} \frac{cbI}{T} & -(d+e+a) & 0 & \frac{cbS}{T} & 0 & 0 \\ \frac{(1-c)bI}{T} & 0 & -(f+g+a) & \frac{(1-c)bS}{T} & 0 & 0 \\ 0 & d & f & -(h+j+a) & 0 & 0 \\ 0 & e & g & j & -(k+a) & 0 \\ 0 & 0 & 0 & h & k & -a \\ -\frac{bI}{T}-a & 0 & 0 & -\frac{bS}{T} & 0 & 0 \end{pmatrix}. \tag{9}$$

Theorem 4.1 *For* system (1)*, D_f is locally asymptotically stable if $R_0 < 1$.*

Proof

$$J_{(D_f)} = \begin{pmatrix} 0-\lambda & -(d+e+a) & 0 & 0 & 0 & 0 \\ 0 & 0-\lambda & -(f+g+a) & 0 & 0 & 0 \\ 0 & d & f-\lambda & -(h+j+a) & 0 & 0 \\ 0 & e & g & j-\lambda & -(k+a) & 0 \\ 0 & 0 & 0 & h & k-\lambda & -a \\ -a & 0 & 0 & 0 & 0 & 0-\lambda \end{pmatrix}. \tag{10}$$

Let us assume $m_1 = d+e+a$, $m_2 = f+g+a$, and $m_3 = h+j+a$. Also, we analyzed that all the real parts of the auxiliary equation are negative based on the roots. Therefore, it is locally asymptotically stable where $R_0 < 1$. Since m_i where $(i = 1, 2, 3)$ and $0 < m_i < 1$ with the condition $0 < R_0 < 1$, therefore

$$\begin{aligned} R_0 &= \frac{b(1-c)f(a+d+e)+cd(a+f+g)}{(h+j+a)(d+e+a)(f+g+a)}, \\ &= \frac{b(1-c)fm_1+cdm_2}{m_3m_1m_2}, \\ &= \frac{b(1-c)f}{m_3m_2} + \frac{cd}{m_3m_1}. \end{aligned} \tag{11}$$

From this, we get $m_2m_3 + m_1m_2 + m_1m_3 - f(1-c)b - dcb > 0$. Once solved, it becomes $m_1m_2m_3 - m_1 f(1-c)b - m_2dcb > 0$. Consequently, the real part of all eigenvalues of λ is negative. Therefore, D_f is locally asymptotically stable. Hence, the proof is complete. □

Theorem 4.2 *For* system (1)*, D_f is globally asymptotically stable if $R_0 < 1$.*

Consider a subsystem of Eq. (1) as:

$$\begin{aligned} R'_L(t) &= \frac{cbS(t)I(t)}{T} - (d+e+a)R_L(t), \\ R'_H(t) &= \frac{(1-c)bS(t)I(t)}{T} - (f+g+a)R_H(t), \\ I'(t) &= dR_L(t) + fR_H(t) - (h+j+a)I(t). \end{aligned} \tag{12}$$

Proof For $S \leq T$, we have

$$\begin{pmatrix} R_L'(t) \\ R_H'(t) \\ I'(t) \end{pmatrix} \leq \begin{pmatrix} \frac{cbS(t)I(t)}{T} - (d+e+a)R_L(t) \\ \frac{(1-c)bS(t)I(t)}{T} - (f+g+a)R_H(t) \\ dR_L(t) + fR_H(t) - (h+j+a)I(t) \end{pmatrix}, \tag{13}$$

$$\begin{pmatrix} R_L'(t) \\ R_H'(t) \\ I'(t) \end{pmatrix} \leq \begin{pmatrix} -(d+e+a) & 0 & cb \\ 0 & -(f+g+a) & (1-c)b \\ d & f & -(h+j+a) \end{pmatrix} \begin{pmatrix} R_L(t) \\ R_H(t) \\ I(t) \end{pmatrix}. \tag{14}$$

As we can observe, the real part of eigenvalues of this matrix are all < 0; therefore, system (12) is stable when $R_0 > 1$. Also, it can be observed that when $t \to \infty$, $(R_L(t), R_H(t), I(t)) \to (0, 0, 0)$. Then by the comparison theorem, we can also get $(S(t), R_L(t), R_H(t), I(t), E(t), H(t)) \to (T, 0, 0, 0, 0, 0)$, when $t \to \infty$. Hence, it is proven that if $R_0 < 1$, only then D_f is globally asymptotically stable. □

Theorem 4.3 *D_d is globally asymptotically stable when $t \to \infty$.*

Proof Since we know that $T = S(t) + R_L(t) + R_H(t) + I(t) + E(t) + H(t)$ is a constant; therefore, we can write it as

$$s = \frac{S}{T}, \quad r_l = \frac{R_L}{T}, \quad r_h = \frac{R_H}{T}, \quad i = \frac{I}{T}, \quad e_1 = \frac{E}{T}, \quad h_1 = \frac{H}{T}, \tag{15}$$

where $s + r_l + r_h + i + e_1 + h_1 = 1$. Now from system (1) at D_d^*, it becomes

$$\begin{aligned} a &= bs^*i^* + as^*, \\ cbs^*i^* &= (d+e+a)r_l^*, \\ (1-c)bs^*i^* &= (f+g+a)i^* \\ dr_l^* + fr_h^* &= (h+j+a)i^*, \\ er_l^* + gr_h^* + ji^* &= (k+a)e_1^*, \\ hi^* + ke_1^* &= ah_1^*. \end{aligned} \tag{16}$$

Let us define the Lyapunov function V as:

$$\begin{aligned} v = {} & x_1\left(s - s^* - s^* \ln \frac{s}{s^*}\right) + x_2\left(r_l - r_l^* - r_l^* \ln \frac{r_l}{r_l^*}\right) + x_3\left(r_h - r_h^* - r_h^* \ln \frac{r_h}{r_h^*}\right) \\ & + x_4\left(i - i^* - i^* \ln \frac{i}{i^*}\right) + x_5\left(e_1 - e_1^* - e_1^* \ln \frac{e_1}{e_1^*}\right) + x_6\left(h_1 - h_1^* - h_1^* \ln \frac{h_1}{h_1^*}\right), \end{aligned} \tag{17}$$

where x_i are the undetermined coefficients. Applying system (16), we get

$$\begin{aligned} v' = {} & x_1\left(1 - \frac{s^*}{s}\right)s' + x_2\left(1 - \frac{r_l^*}{r_l}\right)r_l' + x_3\left(1 - \frac{r_h^*}{r_h}\right)r_h' + x_4\left(1 - \frac{i^*}{i}\right)i' \\ & + x_5\left(1 - \frac{e_1^*}{e_1}\right)e_1' + x_6\left(1 - \frac{h_1^*}{h_1}\right)h_1' \\ = {} & x_1\left(bs^*i^* + as^* - bsi - as - \frac{s^*}{s}bs^*i^* + \frac{s^*}{s}as^* + bs^*i + as^*\right) \\ & + x_2\left(cbsi - (d+e+a)r_l - \frac{r_l^*}{r_l}cbsi + (d+e+a)r_l^*\right) \\ & + x_3\left((1-c)bsi - (f+g+a)r_h - \frac{r_h^*}{r_h}(1-c)bsi + r_h^*(f+g+a)\right) \end{aligned}$$

$$+x_4\left(dr_l+fr_h-(h+j+a)i-\frac{i^*}{i}dr_l-\frac{i^*}{i}fr_h+i^*(h+j+a)\right)$$
$$+x_5\left(er_l+gr_h+ji-(k+a)e_1-\frac{e_1^*}{e_1}er_l-\frac{e_1^*}{e_1}gr_h-\frac{e_1^*}{e_1}ji+e_1^*(k+a)\right)$$
$$+x_6\left(hi+ke_1-ah_1-\frac{h_1^*}{h_1}hi-\frac{h_1^*}{h_1}ke_1+h_1^*a\right), \tag{18}$$

$$v'=x_1as^*\left(2-\frac{s}{s^*}-\frac{s^*}{s}\right)+si\left(-x_1b+x_2cb+x_3(1-c)b\right)$$
$$+i\left(x_1bs^*-x_4(h+j+a)+x_5j+x_6h\right)+er_l\left(-x_2(d+e+a)+x_4d+x_5e\right)$$
$$+r_h\left(-x_3(f+g+a)+x_4f+x_5g\right)+e_1\left(-x_5(k+a)+x_6k\right)+\left(x_1bs*i*\right.$$
$$+x_2(d+e+a)er_l^*+x_3r_h^*(f+g+a)+x_4i^*(h+j+a)+x_5e_1^*(k+a)$$
$$\left.+x_6h_1^*a\right)-\left(x_1\frac{s^*}{s}bs^*i^*+x_4\frac{i^*}{i}dr_l+x_4\frac{i^*}{i}fr_h+x_5\frac{e_1^*}{e_1}gr_h+x_5\frac{e_1^*}{e_1}ji+x_6\frac{h_1^*}{h_1}hi\right.$$
$$\left.+x_6\frac{h_1^*}{h_1}ke_1+x_2\frac{r_l^*}{r_l}cbsi+x_3\frac{r_h^*}{r_h}(1-c)bsi\right). \tag{19}$$

By eliminating the parameters s, i, r_l, r_h, e_1, h_1, we get

$$\begin{aligned}-x_1b+x_2cb+x_3(1-c)b&=0,\\ x_1bs^*-x_4(h+j+a)+x_5j+x_6h&=0,\\ -x_2(d+e+a)+x_4d+x_5e&=0,\\ -x_3(f+g+a)+x_4f+x_5g&=0,\\ -x_5(k+a)+x_6k&=0,\\ -x_6a&=0,\end{aligned} \tag{20}$$

so we have

$$\begin{aligned}x_1&=1,\\ x_2&=\frac{dbs^*}{(d+e+a)(h+j+a)},\\ x_3&=\frac{fbs^*}{(f+g+a)(h+j+a)},\\ x_4&=\frac{bs^*}{(h+j+a)},\\ x_5&=0,\\ x_6&=0.\end{aligned} \tag{21}$$

Thus

$$v'=as^*\left(2-\frac{s}{s^*}-\frac{s^*}{s}\right)+v_1'-v_2', \tag{22}$$

where

$$\begin{aligned}v_1'&=bs^*i^*+x_2(d+e+a)r_l^*+x_3r_h^*(f+g+a)+x_4i^*(h+j+a)\\ &=bs^*i^*+x_2cbs^*i^*+x_3(1-c)bs^*i^*+bs^*i^*\\ &=3bs^*i^*,\end{aligned} \tag{23}$$

and

$$v_2' = \left[x_2c\frac{s^*}{s}bs^*i^* + x_2c\frac{r_l^*}{r_l}bsi + \frac{i^*}{i}r_lx_2(d+e+a)\right] + \left[x_3(1-c)\frac{s^*}{s}bs^*i^* + x_3(1-c)\frac{r_h^*}{r_h}bsi + \frac{i^*}{i}r_hx_3(f+g+a)\right]. \quad (24)$$

By arithmetic–geometric mean inequality, we get

$$v_2' \geq 3x_2cbs^*i^* + 3x_3(1-c)bs^*i^*. \quad (25)$$

Therefore, $v_2' \geq 3\beta s^*i^*$ and hence we conclude that $v' \leq 0$. Accordingly, by using the principle of LaSalle Invariance, D_d is globally asymptotically stable. □

5 Optimal control analysis

In optimal control analysis, we apply four control variables to our system with two main objectives: to control the addiction and to minimize the control cost. For this, we introduce the following objective function:

$$J = \int_0^{t_n}\left[w_1R_L + w_2R_H + w_3I + \frac{c_1}{2}u_1^2 + \frac{c_2}{2}u_2^2 + \frac{c_3}{2}u_3^2 + \frac{c_4}{2}u_4^2\right]dt, \quad (26)$$

where w_1, w_2, w_3 are the weight constants of R_L, R_H, I and c_1, c_2, c_3, c_4 are the weight constants of control variables of u_1, u_2, u_3, u_4. Also, these control variables are defined as u_1 for I as the ratio of isolation, u_2 & u_3 are the control variables for educating R_L and R_H populations, and u_4 is the ratio of treatment of I. These parameters have been assigned to only those terms that can benefit from these controls, such as the term $\frac{bS(t)I(t)}{T}$, which describes the population among susceptible interacting users with infected ones. Therefore, the control parameter of isolation will work on these variables efficiently compared to other populations. Therefore, the system with control variables becomes

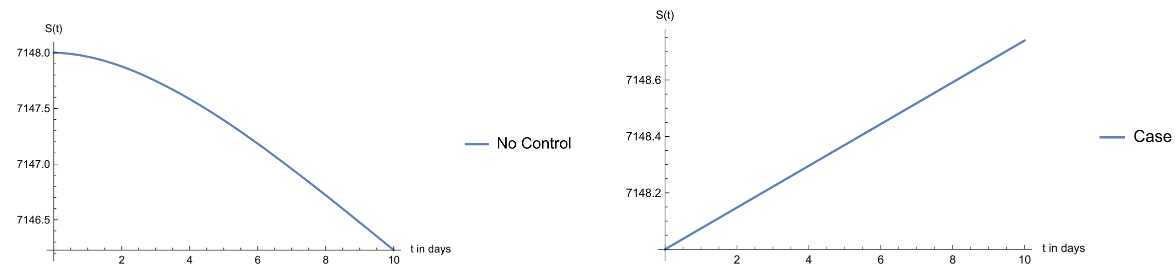
$$\begin{aligned}
S'(t) &= aT - aS(t) - (1-u_1)\frac{bS(t)I(t)}{T},\\
R_L'(t) &= (1-u_1)\frac{cbS(t)I(t)}{T} - ((1-u_2)d + e + a)R_L(t),\\
R_H'(t) &= (1-u_1)\frac{(1-c)bS(t)I(t)}{T} - ((1-u_3)f + g + a)R_H(t),\\
I'(t) &= (1-u_2)dR_L(t) + (1-u_3)fR_H(t) - (h + j + a + u_4)I(t),\\
E'(t) &= eR_L(t) + gR_H(t) + jI(t) - (k+a)E(t),\\
H'(t) &= (h+u_4)I(t) + kE(t) - aH(t).
\end{aligned} \quad (27)$$

For $u_i(t) \in [0,1] \forall t \in [0, t_n]$, $i = 1, 2, 3, 4$ and $U = (u_1, u_2, u_3, u_4)|u_i(t), i = 1, 2, 3, 4$ is the Lebesgue measurable on [0, 1]. Now, by using the maximum principle and by introducing the adjoint variables $\alpha_i (i = 1, 2, 3, 4, 5, 6)$, the Hamiltonian function becomes

$$\begin{aligned}
H &= w_1R_L + w_2R_H + w_3I + \frac{c_1}{2}u_1^2 + \frac{c_2}{2}u_2^2 + \frac{c_3}{2}u_3^2 + \frac{c_4}{2}u_4^2 + \alpha_1\Big[aT - aS(t)\\
&\quad - (1-u_1)\frac{bS(t)I(t)}{T}\Big] + \alpha_2\Big[(1-u_1)\frac{cbS(t)I(t)}{T} - ((1-u_2)d + e + a)R_L(t)\Big]\\
&\quad + \alpha_3\Big[(1-u_1)\frac{(1-c)bS(t)I(t)}{T} - ((1-u_3)f + g + a)R_H(t)\Big] + \alpha_4\Big[(1-u_2)dR_L(t)\\
&\quad + (1-u_3)fR_H(t) - (h + j + a + u_4)I(t)\Big] + \alpha_5\Big[eR_L(t) + gR_H(t) + jI(t)\\
&\quad - (k+a)E(t)\Big] + \alpha_6\Big[(h+u_4)I(t) + kE(t) - aH(t)\Big].
\end{aligned} \quad (28)$$

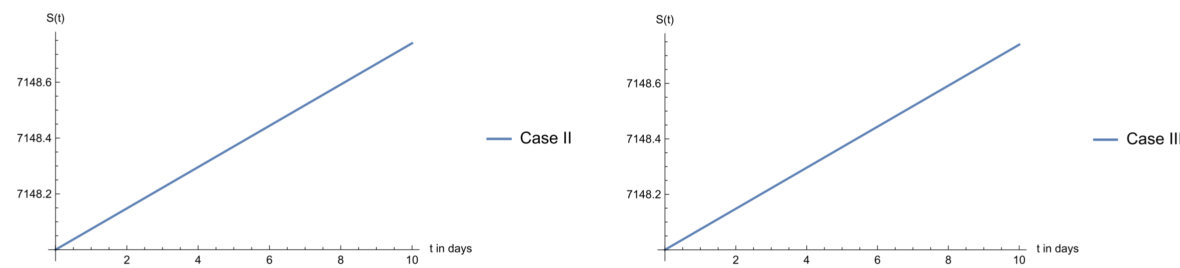
Table 3 Numerical values of parameters in Eq. (1) for numerical simulation

Symbol	Numerical value
a	0.0001
b	0.002
c	0.0003
d	0.1
e	0.0008
f	0.02
g	0.5
h	0.0001
j	0.07
k	0.09



(A) (B)

Fig. 1 Choice behavior of $S(t)$ for no control i.e. $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$



(A) (B)

Fig. 2 Graphical behavior of choice behavior of $S(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$ and case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$

In this paper, all possible controls have been applied to the TikTok model to identify the most suitable combination of control parameters for managing this epidemic and making the use of this app productive. Numerical simulations have been performed using the initial conditions $S(0) = 7148$, $R_L(0) = 362.6$, $R_H(0) = 81.4$, $I(0) = 44.4$, $E(0) = 111$, $H(0) = 140.6$, $T = 7888$ (in millions), and parameters in Table 3. The graphical results obtained from the TikTok model with and without control parameters are presented in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. To understand the best and worst choices available for avoiding TikTok app addiction, let us discuss the behavior of each population for the six options considered through the four control parameters u_1, u_2, u_3, u_4. For the case of no control, which mathematically means $u_1 = u_2 = u_3 = u_4 = 0$, Figs. 1, 4, and 7 show a decrease in the susceptible population $S(t)$, minimally exposed $R_L(t)$, and highly exposed $R_H(t)$ over time. If there is no control, then eventually, all susceptible populations will become infected. Therefore, in the absence of control, an exponential increase in the infected population $I(t)$ (see Fig. 10) and the education/entertainment population $E(t)$ (see Fig. 13) can be observed. As the number of viewers and users increases, it becomes challenging to distinguish among them. Without any control parameters applied, there will inevitably be a point where a significant portion of the susceptible, minimally exposed, and highly exposed populations will transition to the infected and education/entertainment categories. This is because the total population remains constant, and when one population increases, the others

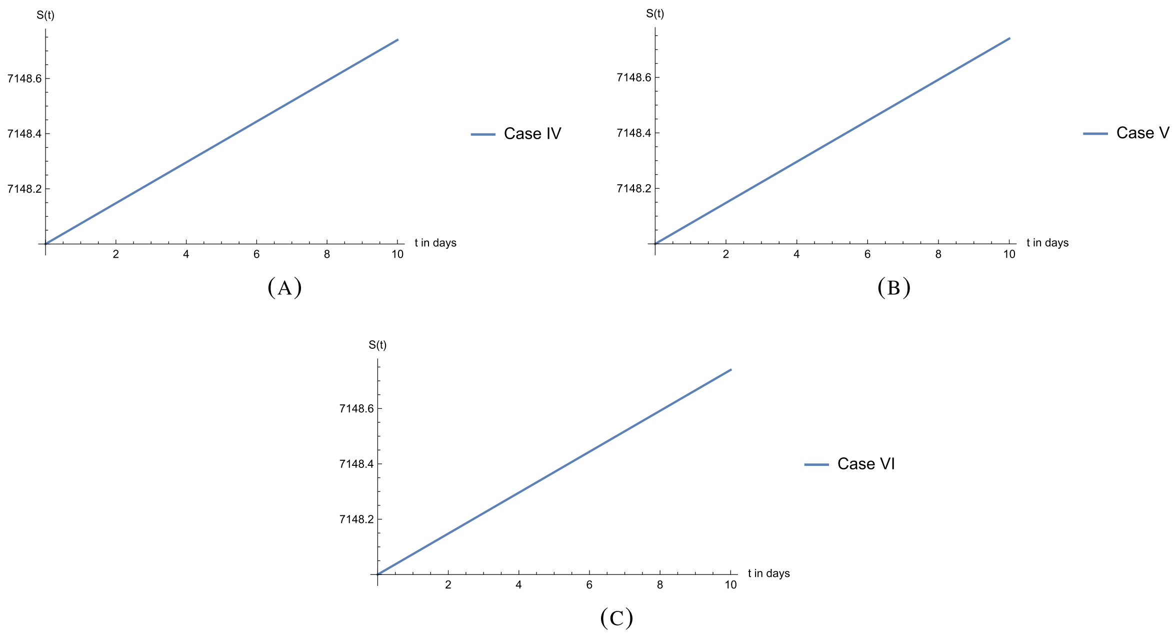


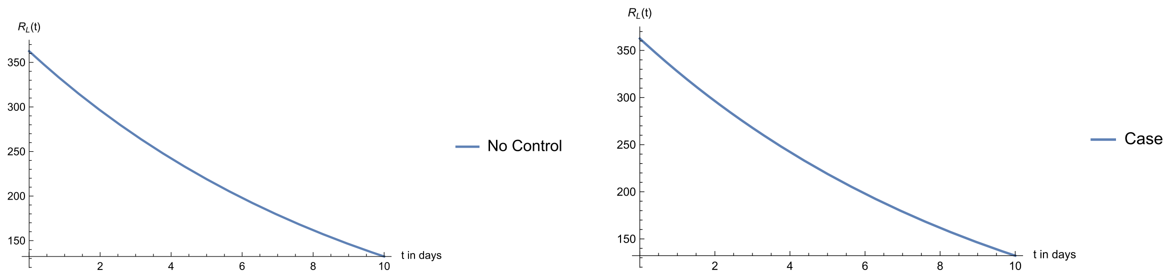
Fig. 3 Graphical representation of $S(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$ and Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$

decrease. By examining the graphs for no control parameters, it becomes evident that this is the worst choice for addressing the problem without taking any action.

Now, the first case of the control parameter considered is $u_1 \neq 0, u_2 = u_3 = u_4 = 0$, which implies controlling the usage of your device and keeping a check on the TikTok application's usage time or applying parental control to the device used by children. It is the first and foremost advice from doctors to avoid excessive phone or internet usage for pleasure and to alleviate boredom. The TikTok model for this control parameter shows that the susceptible population in Fig. 1 is increasing, which is a positive sign indicating that the control parameter is effective. The susceptible population can start using the TikTok application if inspired by others, but currently, it is not being used. By applying control to watch time, it means this population is spending time on something other than TikTok; therefore, there is a very low chance of exposing its addiction to other populations. Consequently, a decrease in the minimally and highly exposed populations can be observed with time. The TikTok model justifies this observation, and hence, in Figs. 4 and 7, a drastic decrease in $R_L(t)$ and an exponential decrease in $R_H(t)$ can be seen over time. In this control parameter, only isolation has been applied, and no other control parameters for users addicted to TikTok have been implemented. Therefore, an increase over time in the infected population $I(t)$ (see Fig. 10) and educators $E(t)$ (see Fig. 13) can be observed. Additionally, there is always a number of people trying to heal through isolation $H(t)$, and hence, this increase can be seen in the case of I in Fig. 16.

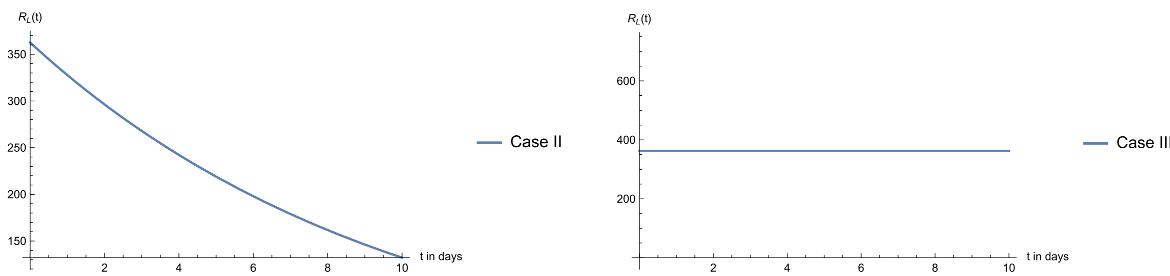
As observed in the first case, isolation alone is not sufficient to control the infected population. Since statistical algorithms of TikTok app can only identify the "user", it can't differentiate between an educator, addicted, or an entertainer therefore in results when $I(t)$ increases with time, $E(t)$ also increases. For our second case in control theory, let us consider counseling and medical therapy along with isolation time, i.e., $u_1 = u_4 \neq 0, u_2 = u_3 = 0$. Now, for these parameters, the expected increase in $S(t)$ in Fig. 3 and decrease in $R_H(t)$ and $R_L(t)$ in Figs. 5 and 8, respectively, are observed, as seen in case I due to isolation or u_1. However, the impact of u_4 can be observed in Fig. 11, where $I(t)$ and $E(t)$ in Fig. 14 decrease rapidly, and in Fig. 17, $H(t)$ increases with time.

The third case considers all other options except medical therapy, namely isolation, social awareness, and educating the minimally and highly exposed populations. This translates to considering $u_1 = u_2 = u_3 \neq 0, u_4 = 0$ as control parameters mathematically. In Fig. 3, it is evident that for Case III, the susceptible population is increasing over time, indicating that the control parameters are working effectively. Now, the minimally exposed and highly exposed populations remain constant for this case. As these control parameters also take time in real life, one cannot expect a sudden change in populations right after education through media, lectures, pamphlets, etc. The TikTok model depicts the same behavior in Figs. 6 and 9 as in real-life



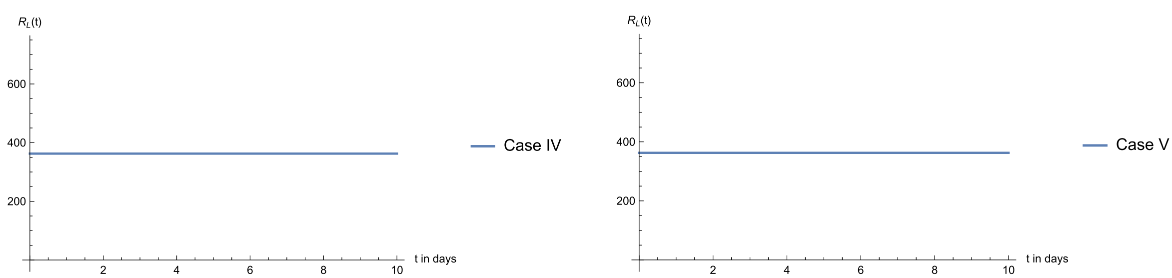
(A) (B)

Fig. 4 Choice behavior of $R_L(t)$ for no control i.e. $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$

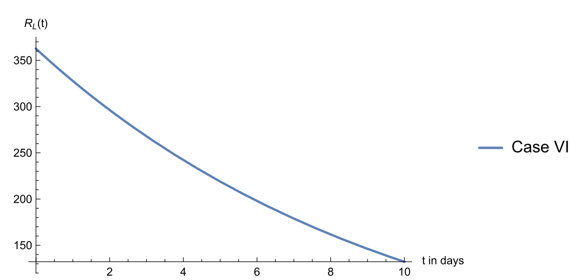


(A) (B)

Fig. 5 Choice behavior of $R_L(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$ and case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$



(A) (B)



(C)

Fig. 6 Choice behavior of $R_L(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$, and Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$

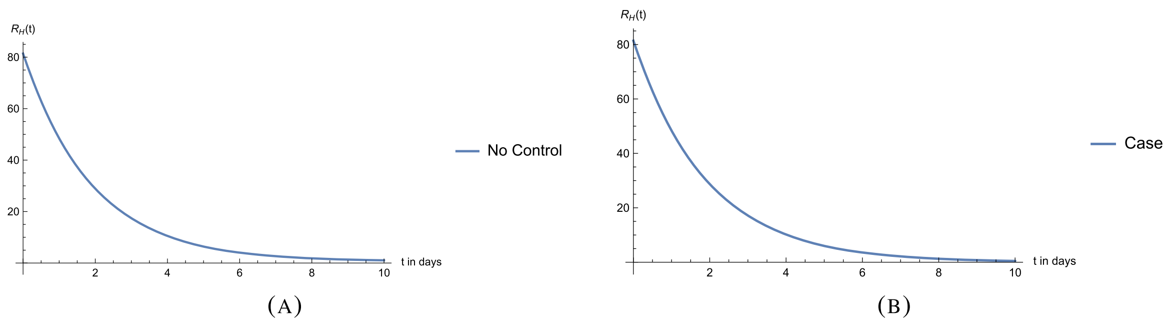


Fig. 7 Choice behavior of $R_H(t)$ for no control i.e. $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$

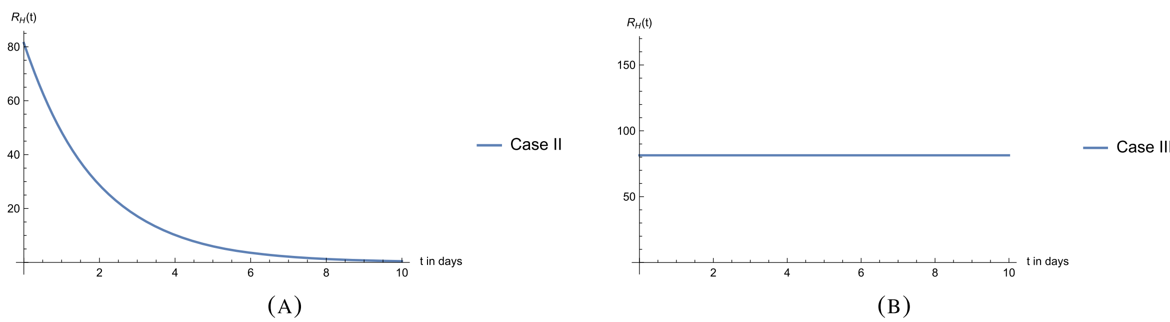


Fig. 8 Choice behavior of $R_H(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$ and case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$

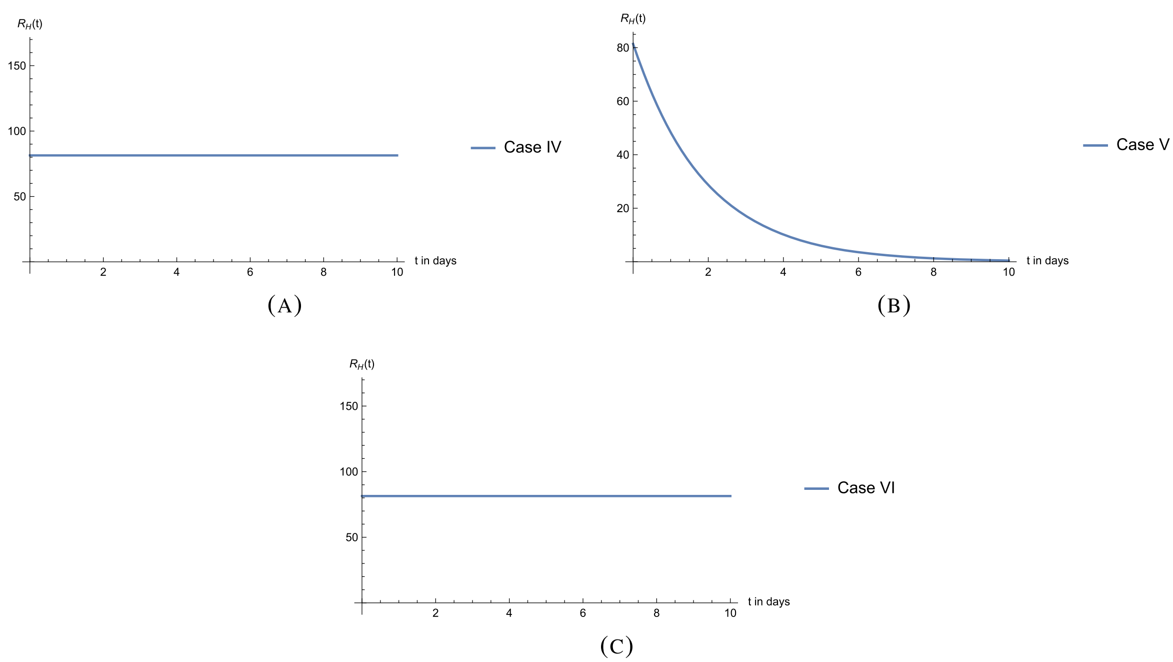


Fig. 9 Choice behavior of $R_H(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$, and Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$

cases. By examining the infected population, we observe a rapid decrease in the addicted population in Fig. 12 and an increase in the healing and educator populations in Figs. 15 and 18. This depicts another best choice for avoiding TikTok addiction over time.

In Case IV, the control parameters have been considered as $u_1 = u_2 = u_3 = u_4 \neq 0$, which means that, along with isolation, awareness, and educating the population, medical treatment needs to be applied. As a result, the susceptible population will keep increasing, but highly exposed and minimally exposed populations

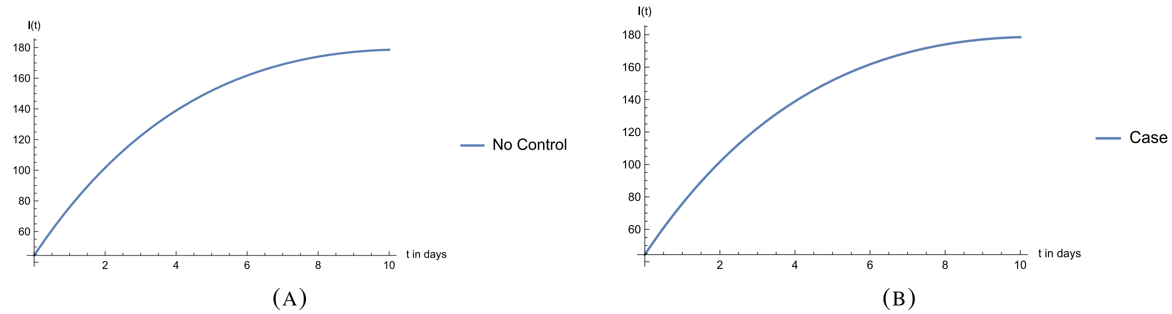


Fig. 10 Choice behavior of $I(t)$ for no control i.e. $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$

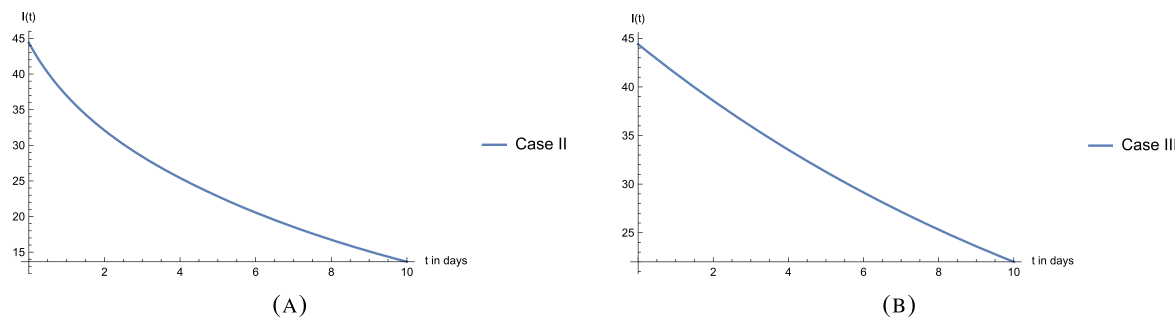


Fig. 11 Choice behavior of $I(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$, case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$

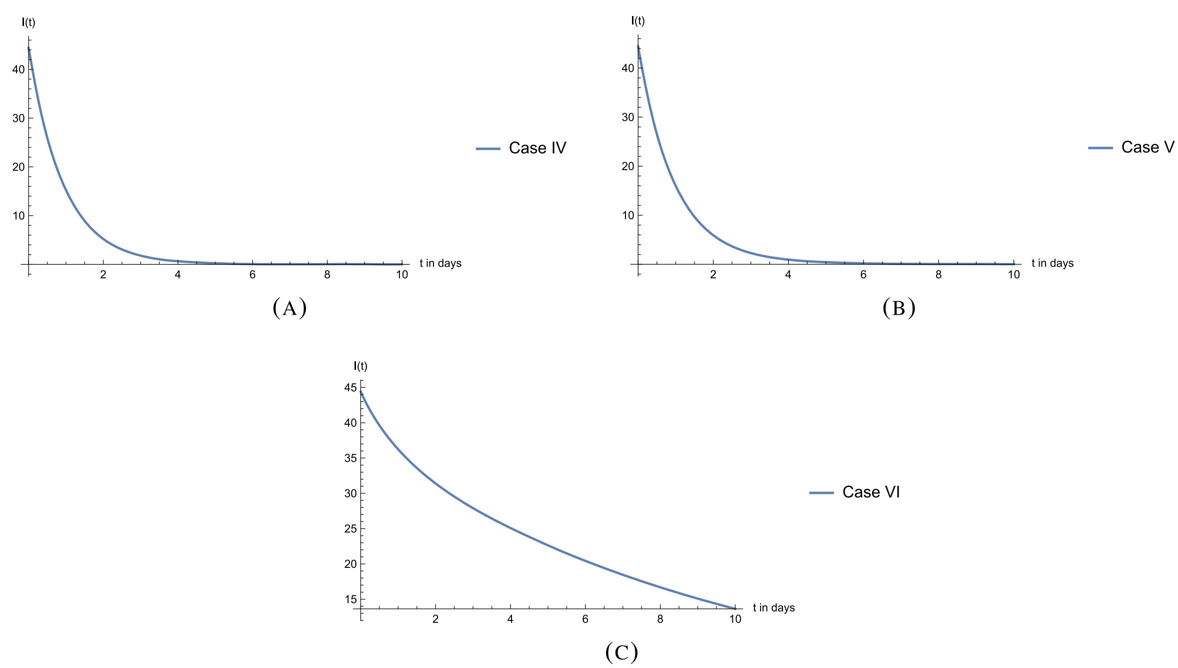


Fig. 12 Choice behavior of $I(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$, and Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$

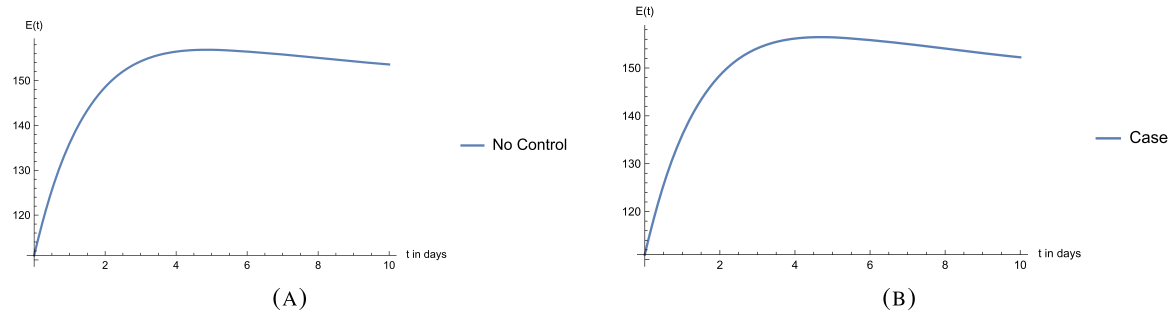


Fig. 13 Choice behavior of $E(t)$ for no control i.e., $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$

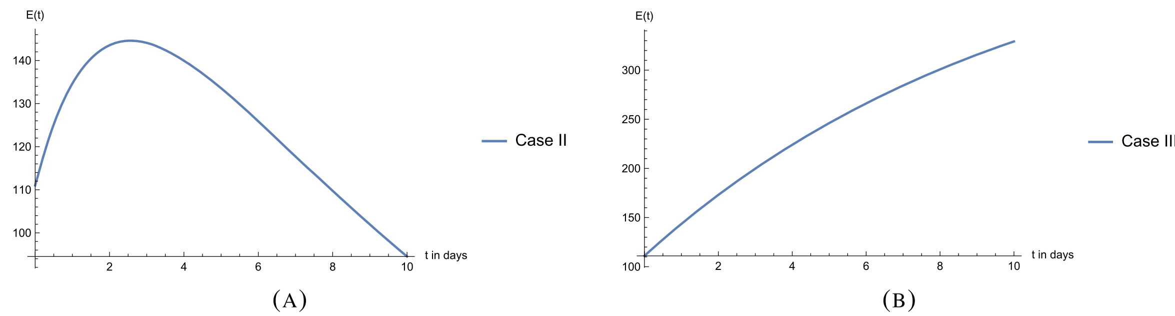


Fig. 14 Choice behavior of $E(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$, case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$

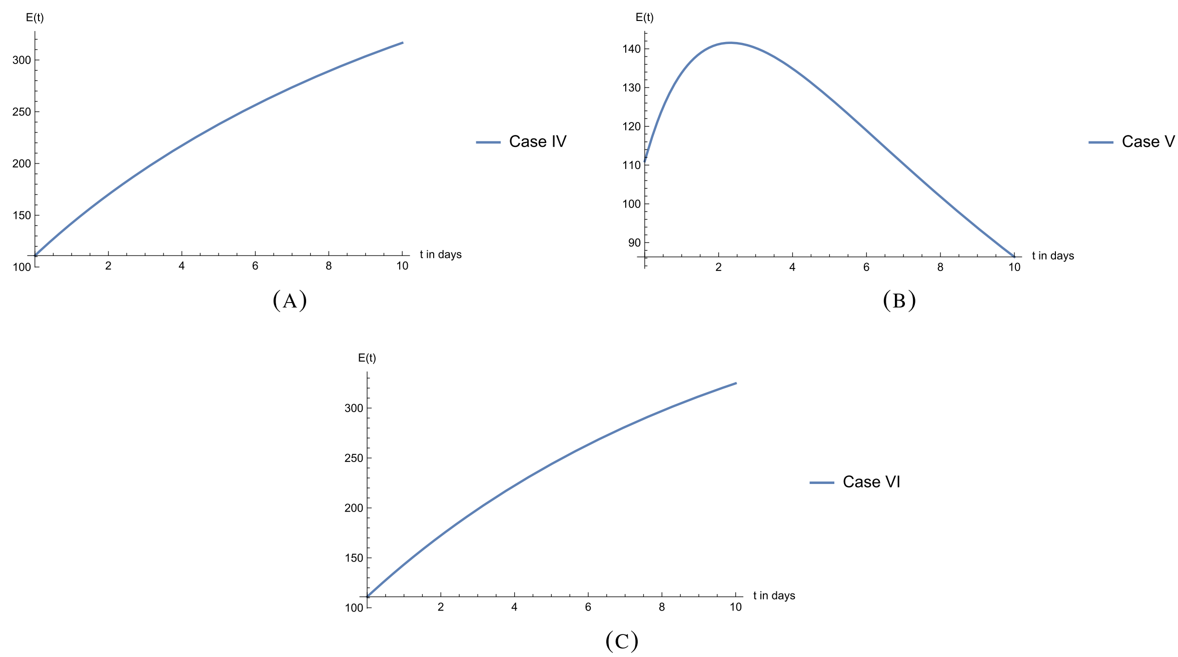
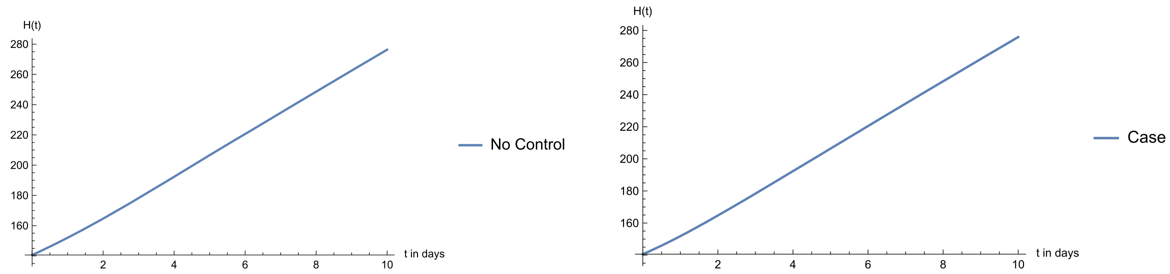
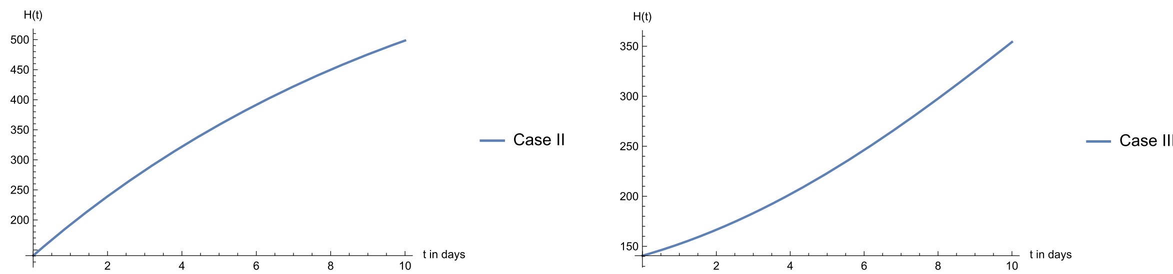


Fig. 15 Choice behavior of $E(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$, Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$



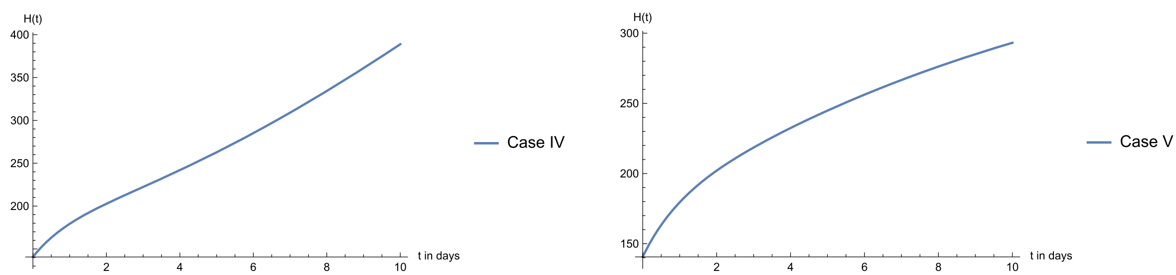
(A) (B)

Fig. 16 Choice behavior of $H(t)$ for no control i.e. $u_1 = u_2 = u_3 = u_4 = 0$ and case I i.e. $u_1 \neq 0, u_2 = u_3 = u_4 = 0$

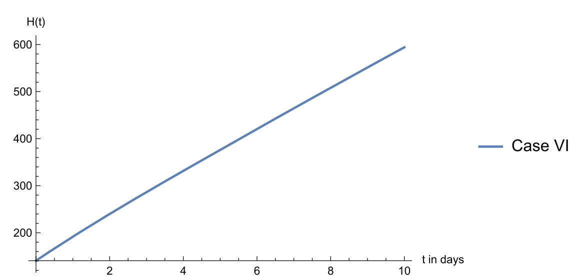


(A) (B)

Fig. 17 Choice behavior of $H(t)$ for the case II i.e. $u_1 = u_4 \neq 0, u_2 = u_3 = 0$, case III i.e. $u_1 = u_2 = u_3 \neq 0, u_4 = 0$



(A) (B)



(C)

Fig. 18 Choice behavior of $H(t)$ for three different control parameters. Case IV i.e. $u_1 = u_2 = u_3 = u_4 \neq 0$, Case V i.e. $u_1 = u_2 = u_4 \neq 0, u_3 = 0$ and Case VI i.e. $u_1 = u_3 = u_4 \neq 0, u_2 = 0$

will remain constant, whereas the infected population is decreasing exponentially. The infected population, as compared to Case III, is decreasing rapidly in Case IV, while $H(t)$ and $E(t)$ exhibit the same behavior as in Case III, i.e., increasing.

In Case V, the control parameters are considered as $u_1 = u_2 = u_4 \neq 0, u_3 = 0$, which means that awareness is focused solely on the minimally exposed population with medical therapy and isolation. Only the highly exposed population is not going to be educated; as a consequence, in Fig., 6, the $R_L(t)$ population remains constant, but $R_H(t)$ decreases drastically. In Fig. 12, the $I(t)$ decreases rapidly along with $E(t)$, but $H(t)$ in Fig. 18 keeps increasing.

The last case, where $u_1 = u_3 = u_4 \neq 0$ and $u_2 = 0$, is similar to Case IV. With $u_2 = 0$, this means that the minimally exposed population is not being educated about the addiction to TikTok, whereas all other control parameters are being applied, i.e., isolation, medical treatment, and educating the highly exposed population. The susceptible population in Fig. 3 remains the same, and since $u_2 = 0$, $R_L(t)$ decreases in Fig. 6. In Fig. 9, the behavior of $R_H(t)$ remains constant, whereas $I(t)$ in this case decreases, but $E(t)$ increases, and so does $H(t)$.

Therefore, the best strategy among these control variables is described in Case III, Case IV, and Case VI-either isolation along with educating people about this app can reduce addicted users or providing medical therapy along with all these strategies can help in reducing addicted TikTok users.

6 Conclusion

In this paper, we present a new mathematical model that describes the addictive behavior of TikTok users and proposes control strategies to reduce addiction rates. The model includes a discussion of TikTok user behavior as well as four optimal control strategies, including isolation, education of highly or lightly addicted populations, and medical therapy for those addicted. To demonstrate its validity, we examine the reproduction number and perform stability analysis of trivial and non-trivial equilibrium points.

Six types of populations (i.e., susceptible, minimally exposed, highly exposed, infected, educators, and healers) and four control parameters were selected for optimal control theory. u_1 represents isolation, u_2 represents education, u_3 represents awareness, and u_4 represents medical therapy. These parameters suggest six choices for the user to adapt to overcome this addiction. In the first case, it was considered that only taking away the devices for a certain period could reduce addiction, but the TikTok mathematical model identified three better options. These three best options are illustrated in Figs. 3, 6, 9, 12, 15, and 18 as Case III, Case IV, and Case VI. The remaining three are considered the worst choices, according to the TikTok model. Our findings suggest that the most effective strategy for reducing addiction rates is to isolate and educate the population about the harmful effects of this social media application, thereby contributing to the well-being of humanity. In the future, more sub-classes can be formed to study the behavior of the human population towards TikTok addiction, and this can also improve the control parameters to control this addiction.

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Data Availability The data underlying the results presented in the study are available within the manuscript.

Declarations

Author contributions All authors contributed to the study conception and design. All authors read and approved the final manuscript.

Conflict of interest The authors declare that they have no competing interests.

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