

Slip avalanches in a fiber bundle model

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Abstract. - We study slip avalanches in disordered materials under an increasing external load in the framework of a fiber bundle model. Over-stressed fibers of the model do not break, instead they relax in a stick-slip event which may trigger an entire slip avalanche. Slip avalanches are characterized by the number slipping fibers, by the slip length, and by the load increment, which triggers the avalanche. Our calculations revealed that all three quantities are characterized by power law distributions with universal exponents. We show by analytical calculations and computer simulations that varying the amount of disorder of slip thresholds and the number of allowed slips of fibers, the system exhibits a disorder induced phase transition from a phase where only small avalanches are formed to another one where a macroscopic slip appears.

There is a large variety of non-equilibrium systems which exhibit crackling noise, i.e. they respond to a slow continuous external driving in the form of bursts of local events [1]. Examples can be mentioned from earthquakes and fracture of disordered materials [2,3], through Barkhausen noise in ferromagnets [4], to martensitic shape memory alloys and plastic deformation of solids [5,6]. During the last decade experimental and theoretical investigations revealed that the probability distributions of the characteristic quantities of bursts have scale free behavior with universal exponents [1, 2, 6, 7]. An intense research has been initiated to understand the underlying mechanism of the observed universality. The investigation of simple models which grasp the crucial features of systems exhibiting crackling noise proved to be essential. Along this line, based on the analogy of the plastic deformation and fracture of heterogeneous materials, recently, a micromechanical model was introduced in Ref. [7] which can reproduce the main features of crackling noise in these types of systems with only one tuning parameter.

In the present Letter we study the emergence of crackling noise in heterogeneous materials which respond to an increasing external load by local rearrangements with stick-slip mechanism. We consider a fiber bundle model [10–14] where over-stressed fibers do not break, instead they increase their relaxed length in a slip event until they can sustain the load. The system is driven by small load

increments giving rise to the slip of a single fiber which may then trigger an entire avalanche of slip events due to load redistribution in the bundle. We show by analytic calculations and computer simulations that the load increment triggering the slip bursts, furthermore, the number of slipping fibers and the total slip length of the bundle are all characterized by power law distributions. We demonstrate that the amount of disorder and the total number of allowed slips play a crucial role in the system: a disorder induced phase transition [1, 6, 8, 9] is obtained from a low disorder phase where the system snaps with macroscopic bursts to the high disorder one where only small avalanches pop up. Our model provides an adequate description of the micromechanics of disordered systems which store hidden length [15], and it can also be considered as the fiber bundle analogue of the Burridge-Knopoff model of earthquakes with an infinite range of interaction [16].

Our model consists of N fibers assembled in parallel. Under an increasing external load σ the fibers exhibit a linearly elastic behavior characterized by the same Young modulus E . The important novel element of the model is that when the deformation ε of a fiber reaches a threshold value ε_{th} the fiber does not break. Instead, its relaxed length increases until the load reduces to zero on the fiber. The mechanism of relaxation is the slip of the fiber end, or it can also be interpreted as the unfolding of

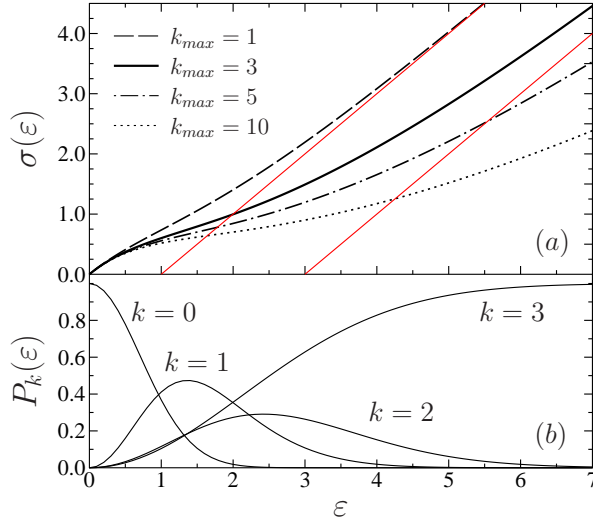


Fig. 1: (Color online) (a) Constitutive behavior of the bundle with exponentially distributed quenched failure thresholds. $\sigma(\varepsilon)$ tends to an asymptotic linear response preceded by a longer horizontal plateau when k_{max} increases. The intersection of the asymptotic straight lines with the horizontal axis indicate the remaining deformation when unloading the system. (b) Breaking probabilities $P_k(\varepsilon)$ for $k_{max} = 3$.

subunits of fibers which provide some stored length [15]. The slip thresholds ε_{th}^i , $i = 1, \dots, N$ are random variables with a probability density $p(\varepsilon_{th})$ and distribution function $P(\varepsilon_{th})$. After the slip event the fiber gets stucked again so that it can support load and can suffer further slips. The load kept by fiber i at a deformation ε after slipping reads as $\sigma_i = E(\varepsilon - \varepsilon_{th}^i)$ so that no hardening or softening is assumed in the system. When a fiber slips again either the same slip threshold is retained (quenched disorder) or new threshold values can be drawn from the same probability distribution $p(\varepsilon_{th})$ (annealed disorder). The total number of slip events k_{max} a fiber can suffer is a very important parameter of the model which can vary in the range $1 \leq k_{max} < +\infty$. For the load redistribution following slip events we assume an infinite range of interaction, i.e. equal load sharing, which is ensured by the condition that the strain ε is the same for all fibers. In the present paper our analysis is restricted to the case of quenched disorder so that the load of fiber i in the bundle after $k_i \leq k_{max}$ slips takes the form $\sigma_i = E(\varepsilon - k_i \varepsilon_{th}^i)$. Further details of the model construction can be found in Ref. [15] including also the case of annealed slip thresholds.

Based on the assumption of equal load sharing the constitutive equation of the parallel bundle can be obtained analytically by integrating the load kept by the subsets of fibers with different slip indices k [15]

$$\sigma(\varepsilon) = E\varepsilon [1 - P(E\varepsilon)] + \sum_{k=1}^{k_{max}-1} \int_{\varepsilon/(k+1)}^{\varepsilon/k} p(E\varepsilon_1) E(\varepsilon - k\varepsilon_1) d\varepsilon_1$$

$$+ \int_0^{\varepsilon/k_{max}} p(E\varepsilon_1) E(\varepsilon - k_{max}\varepsilon_1) d\varepsilon_1. \quad (1)$$

Note that the integrals have to be performed over the entire loading history of the bundle. For very large deformations $\varepsilon \rightarrow \infty$, practically all fibers have suffered k_{max} slips so that Eq. (1) can be rewritten as $\sigma(\varepsilon) \sim E\varepsilon - k_{max}E \int_0^{\varepsilon/k_{max}} p(\varepsilon_1) \varepsilon_1 d\varepsilon_1$ where the integral provides the average value of the slip thresholds $\langle \varepsilon_{th} \rangle$. It means that the bundle has an asymptotic linear behavior with the initial value of the Young modulus, however, when unloading the system $\sigma \rightarrow 0$ an irreversible permanent deformation remains whose maximum ε_r^{max} value is proportional to the average slip length $\langle \varepsilon_{th} \rangle$ and the number of slip events k_{max} allowed $\varepsilon_r^{max} = k_{max} \langle \varepsilon_{th} \rangle$. We note that for a finite bundle of N fibers with quenched slip thresholds the constitutive equation Eq. (1) can be written in a discrete form

$$\sigma(\varepsilon) = E\varepsilon - (E/N) \sum_{i=1}^N k_i(\varepsilon) \varepsilon_{th}^i, \quad (2)$$

where $k_i(\varepsilon)$ denotes the number of slips suffered by fiber i up to deformation ε .

In the explicit calculations we use Weibull distributed threshold values ε_{th} with the probability density function

$$p(\varepsilon_{th}) = m \frac{\varepsilon_{th}^{m-1}}{\lambda^{m-1}} e^{-(\varepsilon_{th}/\lambda)^m}, \quad (3)$$

where the parameter λ setting the scale of the thresholds is fixed to $\lambda = 1$ in our entire study. The Weibull exponent $1 \leq m < +\infty$ is a very important characteristics of the system, which controls the amount of disorder in the slip thresholds. Increasing the value of m from 1 to infinity the probability density Eq. (3) varies from the exponential distribution to the delta function of zero width. Figure 1 illustrates the constitutive curve $\sigma(\varepsilon)$ of the model with exponentially distributed quenched slip thresholds ($m = 1$) for several different values of k_{max} . It can be seen that increasing the maximum number of breakings allowed a plastic plateau develops, i.e. the final asymptotic linear part of the constitutive curve is preceded by a longer and longer horizontal plateau. The slope of $\sigma(\varepsilon)$ in the asymptotic regime is equal to the Young modulus $E = 1$ of fibers. Note that the simple form of $\sigma(\varepsilon)$ in Fig. 1(a) is the consequence of the monotonous behavior of the exponential distribution, i.e. varying the value of m and k_{max} along the plastic plateau $\sigma(\varepsilon)$ can have a more complex functional form which will be explored below. Macroscopic failure of the system can be captured in the model by assuming that the fibers break after having suffered k_{max} slips, which has been studied in Ref. [15]. In the present paper we focus on the microscopic stick-slip process of the fiber bundle with quenched slip thresholds retaining the fibers' stiffness after k_{max} slips (no breaking).

Quasi-static stress controlled loading of the fiber bundle can be performed by incrementing the external load

with a small amount $\delta\sigma$ just to provoke the slip of a single fiber. Since the external load σ is kept constant during the slip, the load dropped by the slipping fiber must be overtaken by the other ones which can give rise to further slip events. This way a single slip induced by the load increment $\delta\sigma$, can trigger an entire avalanche of slips, which increases the macroscopic strain ε of the system by the amount $\delta\varepsilon$. This jerky microscopic dynamics has the consequence that the deformation of the bundle has a step-wise increase under a quasi-statically increasing external load σ . We characterize the slip avalanches by their size Δ defined as the number of fibers slipping in the avalanche, and by the emerging slip length $\delta\varepsilon$ which is the increment of the strain ε of the bundle. All the three quantities, the load increment $\delta\sigma$ which triggers the avalanche, the avalanche size Δ , and the slip length $\delta\varepsilon$ are random variables so that the stick-slip process on the micro-scale can be characterized by their probability distributions $P(\delta\sigma)$, $P(\Delta)$, and $P(\delta\varepsilon)$, respectively. Note that under strain controlled loading no slip avalanches can arise.

For simple fiber bundles where fibers break irreversibly when the local load surpasses their threshold value, it has recently been shown [10,11] for the case of equal load sharing that the size distribution of avalanches $P(\Delta)$ can be obtained in a closed analytical form as

$$P(\Delta) \approx \frac{e^\Delta}{\sqrt{2\pi}\Delta^{3/2}} \int_0^{\varepsilon_c} p(\varepsilon) \frac{1-a(\varepsilon)}{a(\varepsilon)} e^{\Delta[a(\varepsilon)-\ln a(\varepsilon)]} d\varepsilon. \quad (4)$$

Here $a(\varepsilon)$ denotes the average number of fibers which break as a consequence of a single fiber failure induced by the external load increment at the deformation ε . The integration over ε is carried out up to the critical point ε_c of the system where catastrophic collapse occurs. The dominating contribution to the integral is provided by the vicinity of the maximum of the exponent of the integrand $\psi(\varepsilon) = a(\varepsilon) - \ln a(\varepsilon)$, which is obtained at $a = 1$. After Taylor expansion of $a(\varepsilon)$ and of the exponent $\psi(\varepsilon)$ about the maximum the asymptotics of the size distribution $P(\Delta)$ reduces to the power law form

$$P(\Delta) \sim \Delta^{-\tau}. \quad (5)$$

The exponent $\tau = 5/2$ proved to be universal for a broad class of disorder distributions where the macroscopic constitutive curve $\sigma(\varepsilon)$ of the system has a single quadratic maximum [10,11,14].

In order to understand the dynamics of slip avalanches in our model, first the sequence of slipping events has to be analyzed. The probability $P_k(\varepsilon)$ that a randomly selected fiber in the bundle has suffered exactly k slips up to the deformation ε can be obtained analytically as

$$\begin{aligned} P_0(\varepsilon) &= 1 - P(\varepsilon), \\ P_k(\varepsilon) &= P\left(\frac{E\varepsilon}{k}\right) - P\left(\frac{E\varepsilon}{k+1}\right), \quad 1 \leq k < k_{max}, \\ P_{k_{max}} &= P\left(\frac{E\varepsilon}{k_{max}}\right), \end{aligned} \quad (6)$$

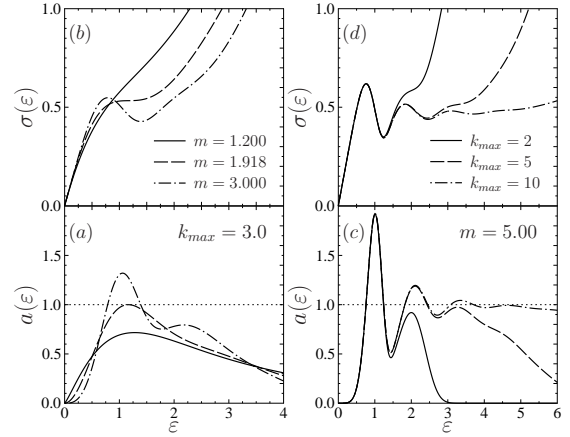


Fig. 2: Constitutive curves $\sigma(\varepsilon)$ and average number of induced slips $a(\varepsilon)$ for a fixed $k_{max} = 3$ varying the value of m (a,b), and for a fixed $m = 5.0$ varying the value of k_{max} (c,d). Note that $m_3^c \approx 1.918$.

where P denotes the cumulative distribution of the slip thresholds. The functional form of $P_k(\varepsilon)$ is presented in Fig. 1(b) for $m = 1$ and $k_{max} = 3$. From the above equations one can determine the probability density $p_k^{k+1}(\varepsilon)$ of events that a fiber which has suffered k slips until the deformation ε was reached, will slip again due to the strain increment $d\varepsilon$

$$p_k^{k+1}(\varepsilon) = \frac{1}{k+1} p\left(\frac{\varepsilon}{k+1}\right), \quad 0 \leq k < k_{max}, \quad (7)$$

where p is the original probability density of the slip thresholds. When the external load is increased by the amount $\delta\sigma$ at the deformation ε to provoke the slip of a single fiber i which has already slipped k times, the strain increment $\delta\varepsilon = \delta\varepsilon_k$ arising due to load redistribution after the slip can be obtained from the constitutive equation of finite bundles Eq. (2). Keeping the load σ fixed during the slip, the strain increment reads as $\delta\varepsilon_k = \varepsilon_{th}^i/N = \varepsilon/(kN)$. It follows that the average number of fibers $a(\varepsilon)$ which slip as a consequence of a single slip can be determined as $a(\varepsilon) = N \sum_{k=0}^{k_{max}-1} \delta\varepsilon_k p_k^{k+1}(\varepsilon)$ which leads to the form

$$a(\varepsilon) = \varepsilon \sum_{k=1}^{k_{max}} \frac{1}{k^2} p\left(\frac{E\varepsilon}{k}\right). \quad (8)$$

It is important to emphasize that the derivative of the constitutive equation $\sigma(\varepsilon)$ of Eq. (1) can be expressed in terms of $a(\varepsilon)$ as

$$\frac{d\sigma}{d\varepsilon} = E \left[1 - \sum_{k=1}^{k_{max}} \frac{\varepsilon}{k^2} p\left(\frac{E\varepsilon}{k}\right) \right] = E [1 - a(\varepsilon)], \quad (9)$$

which show that the constitutive curve $\sigma(\varepsilon)$ has extrema at locations ε_c where the average number of induced slips becomes unity $a(\varepsilon_c) = 1$. It also follows from Eq. (9) that

at the extremal points of $a(\varepsilon)$ the constitutive curve $\sigma(\varepsilon)$ has an inflexion point $d^2\sigma/d\varepsilon^2 = 0$.

Our calculations revealed that varying the amount of disorder m and the number of allowed slip events k_{max} the statistics of avalanches exhibits a very complex behavior. Starting from Eqs. (1,8,9) we can determine analytically the phase diagram of the system on the $m - k_{max}$ plane which classifies all possible functional forms of the constitutive curves $\sigma(\varepsilon)$ and of avalanche size distributions $P(\Delta)$. Writing $a(\varepsilon)$ in the form $a(\varepsilon) = \sum_{k=1}^{k_{max}} a_k(\varepsilon)$, it can be seen that each term $a_k(\varepsilon) = (\varepsilon/k^2)p(E\varepsilon/k)$ has a single maximum at the strain ε_k^c where $\varepsilon_k^c p(E\varepsilon_k^c/k) = -1$ holds. It follows that if a_1 has a maximum at ε_1^c with the value a_1^c , then the maxima of the other terms $a_k(\varepsilon)$ are placed equidistantly as $\varepsilon_k^c = k\varepsilon_1^c$ with decreasing values $a_k^c = a_1^c/k$. Due to the overlap of the functions $a_k(\varepsilon)$, the consecutive maxima of $a(\varepsilon)$ do not coincide with that of $a_k(\varepsilon)$, however, the equidistant spacing and the decreasing sequence survive. For the case of Weibull distributions the above analysis results in $\varepsilon_k^c = k\lambda$ and $a_k^c = m/(ke)$, where e is the base of natural logarithm. It can be seen that for $k_{max} = 1$, when only a single slip is allowed, at the critical Weibull exponent $m_1^c = e$ the constitutive curve $\sigma(\varepsilon)$ has an inflexion point at the position ε_1^c where $a(\varepsilon)$ has a maximum with the value $a(\varepsilon_1^c) = 1$. Similarly, for any $k_{max} \geq 1$ one can find an $m_{k_{max}}^c$ value of the Weibull exponent, where the constitutive curve has an inflexion point with the properties $d\sigma/d\varepsilon|_{\varepsilon_{k_{max}}^c} = 0$ and $d^2\sigma/d\varepsilon^2|_{\varepsilon_{k_{max}}^c} = 0$, where at the same time $a(\varepsilon_{k_{max}}^c) = 1$ and $da/d\varepsilon|_{\varepsilon_{k_{max}}^c} = 0$ hold.

The phase diagram of the system is presented in Fig. 3, where the decreasing line represents the $m_{k_{max}}^c$ curve which was determined numerically. Note that for $k_{max} = 1$ the critical Weibull exponent is $m_1^c = e$ and $m_{k_{max}}^c \rightarrow 1$ holds for $k_{max} \rightarrow +\infty$. In order to obtain the asymptotics of the size distribution of slip avalanches $P(\Delta)$ analytically from Eq. (4) for parameters along the $m_{k_{max}}^c$ curve, the Taylor expansion of $a(\varepsilon)$ and $\psi(\varepsilon)$ about $\varepsilon_{k_{max}}^c$ has to be continued beyond the first order terms. Following the derivations of Ref. [14] the first non-vanishing terms are $a(\varepsilon) \simeq 1 + C_1(\varepsilon - \varepsilon_{k_{max}}^c)^2$ and $\psi(\varepsilon) \simeq 1 + C_2(\varepsilon - \varepsilon_{k_{max}}^c)^4$, which result in a power law asymptotics $P(\Delta) \sim \Delta^{-\tau}$ with the exponent $\tau = 9/4$. A similar behavior was found in Ref. [14], where a different physical mechanism led to a similar constitutive curve of the system.

The parameter regime of m and k_{max} below the $m_{k_{max}}^c$ curve of Fig. 3 defines the high disorder phase of the model, where $\sigma(\varepsilon)$ is monotonically increasing $d\sigma/d\varepsilon > 0$ and the maximum of $a(\varepsilon)$ is always smaller than 1. Figure 2(a, b) illustrates the constitutive behavior $\sigma(\varepsilon)$ and the average number of induced slips $a(\varepsilon)$ for $k_{max} = 3$ varying the value of m , where the critical disorder parameter is $m_3^c \approx 1.918$. Since the minimum value of the derivative $d\sigma/d\varepsilon$ Eq. (9) is positive in the high disorder phase, the avalanche size distribution $P(\Delta)$ behaves as in simple fiber bundles when the loading process was stopped at a deformation ε_m

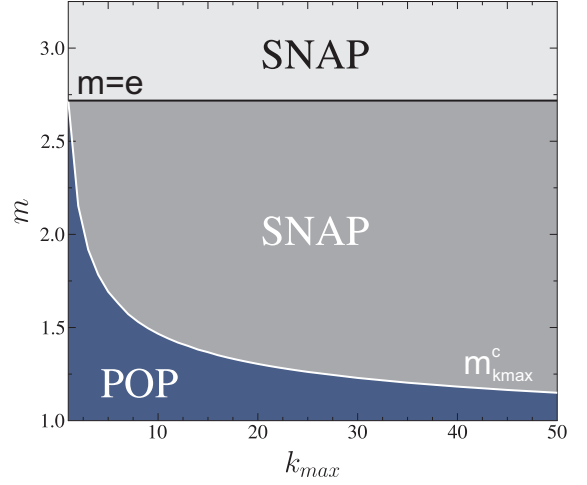


Fig. 3: (Color online) Phase diagram of the system. The decreasing line indicates the $m_{k_{max}}^c$ curve which separates the *POP* and *SNAP* regimes.

before the critical point of macroscopic failure [10]. Hence, for $m < m_{k_{max}}^c$ from Eq. (4) the size distribution of bursts takes the form

$$P(\Delta) \sim \Delta^{-\tau} e^{-[a(\varepsilon_m) - 1 - \ln a(\varepsilon_m)]\Delta}, \quad (10)$$

i.e. the power law regime of exponent $\tau = 9/4$ is followed by an exponential cutoff, where in our case ε_m is the position of the inflexion point of the constitutive curve. Since in the high disorder phase of the model only relatively small avalanches pop up away from the phase boundary, we call this phase as *POP* phase, following the terminology of Ref. [6]. Figure 4(a) presents the size distribution of slip avalanches $P(\Delta)$ obtained by computer simulations for $k_{max} = 7$ at different m values in the range $m \leq m_7^c$. A high quality power law behavior can be observed with a diverging cutoff as approaching the critical point $m \rightarrow m_7^c$ in agreement with the above derivation.

In the low disorder regime, above the $m_{k_{max}}^c$ curve of Fig. 3, the constitutive curve can have local maxima along the plateau regime. It follows from the above derivation that at a given value of k_{max} the number of maxima of $\sigma(\varepsilon)$ is one if the value of m falls in the interval $m_{k_{max}}^c < m < e$. Under stress controlled loading a macroscopic avalanche appears resulting in a horizontal jump when the maximum of $\sigma(\varepsilon)$ is reached. Consequently, this phase of the model is called *SNAP* phase [6]. At very low disorder $m > e$ the constitutive curve has a local maximum already at $k_{max} = 1$ and further maxima occur with decreasing height accompanied by a similar oscillating behavior of $a(\varepsilon)$ as k_{max} increases. This feature can be observed in Figures 2(c, d), which present the behavior of $\sigma(\varepsilon)$ and $a(\varepsilon)$ for $m = 5$ varying the value of k_{max} . In the *SNAP* phase the distribution of avalanche sizes $P(\Delta)$ is determined by the first maximum of $\sigma(\varepsilon)$ which has a quadratic shape. Consequently, similarly to the case of simple fiber bundles,

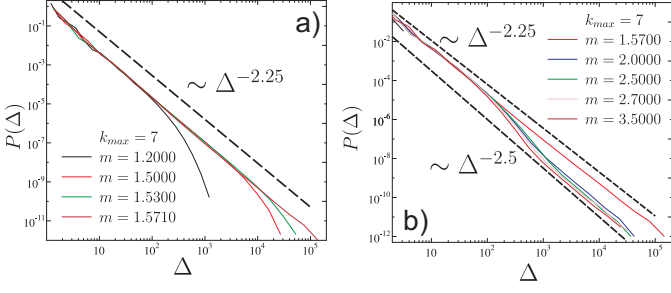


Fig. 4: (Color online) (a) Size distribution of slip avalanches $P(\Delta)$ in the *POP* phase for $k_{max} = 7$ varying the amount of disorder m . A high quality power law behaviour is obtained with a diverging cutoff as the phase boundary is approached $m \rightarrow m_7^c \approx 1.572$. (b) Comparison of avalanche size distributions in the *POP* and *SNAP* phases. Excellent agreement is obtained with the analytic predictions.

$P(\Delta)$ has a power law functional form $P(\Delta) \sim \Delta^{-\tau}$ without cutoff regime but with an exponent $\tau = 5/2$ higher than in the *POP* phase [10, 11, 14]. Burst size distributions of the *POP* and *SNAP* phases are compared in Fig. 4(b), where nice agreement can be observed with the analytic predictions.

In order to understand the transition from the *POP* to the *SNAP* phase when the amount of disorder m is varied, we further analyze the burst size distribution Eq. (10). For the specific case of $k_{max} = 1$ we have $a(\varepsilon_m) = a_1^c$, where $a_1^c = m/e$ converging to 1 when the amount of disorder is decreased in the *POP* phase $m \rightarrow m_1^c = e$. After Taylor expanding the terms in the exponential function of Eq. (10) about m_1^c , we obtain the form

$$P(\Delta) \sim \Delta^{-\tau} e^{-\Delta/\Delta_0}, \quad (11)$$

where the characteristic burst size has a power law divergence

$$\Delta_0 \sim (m_{k_{max}}^c - m)^{-\nu} \quad (12)$$

with the cutoff exponent $\nu = 2$. The exact proof is for $k_{max} = 1$ but our numerical calculations revealed that the analytic results Eqs. (11,12) hold for all values of k_{max} in the *POP* phase. In order to numerically verify the above analytic predictions, we assume that the cutoff avalanche size Δ_0 is proportional to the average size of the largest avalanche $\Delta_0 \sim \langle \Delta^{max} \rangle$. Figure 5(a) presents $\langle \Delta^{max} \rangle$ obtained by computer simulations of a bundle of $N = 10^7$ fibers with $k_{max} = 7$ varying the Weibull exponent m in a broad range. It can be seen that approaching the phase boundary m_7^c from the *POP* phase $\langle \Delta^{max} \rangle$ diverges, i.e. it exhibits a sharp maximum in the finite system. Figure 5(b) presents the same data as a function of the distance from the critical point m_7^c , where a power law behavior is evidenced with an exponent $\nu = 2.05 \pm 0.05$ in a good agreement with Eq. (12). The results imply that varying

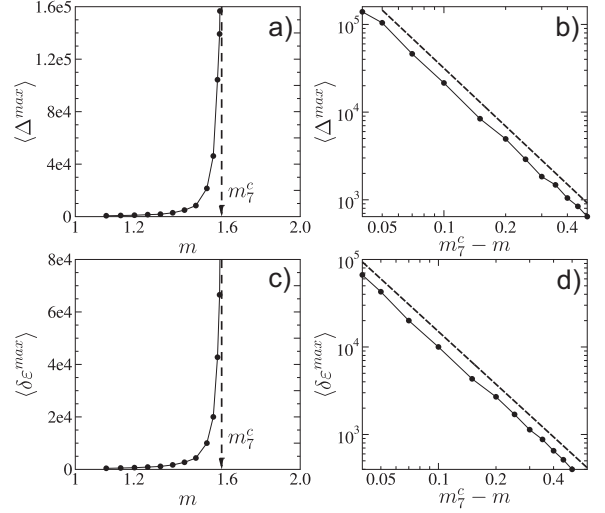


Fig. 5: The average size of the largest avalanche $\langle \Delta^{max} \rangle$ (a) and the average largest slip length $\langle \delta \varepsilon^{max} \rangle$ (c) as a function of the Weibull exponent m below the critical point m_7^c . The same quantities are presented in (b,d) as a function of the distance from the critical point $(m_7^c - m)$.

the amount of threshold disorder the bundle of stick-slip fibers undergoes a disorder induced phase transition from the high disorder phase where small avalanches pop up to the low disorder one where macroscopic avalanches snap the system [1, 6, 8].

In spite of the complexity of the behavior of avalanche sizes, computer simulations revealed a universal functional form for both the distribution of the slip length $P(\delta \varepsilon)$ and for the load increments $P(\delta \sigma)$. The total slip length, i.e. the strain increment $\delta \varepsilon$ occurred during an avalanche of size Δ reads as $\delta \varepsilon = (1/N)(\varepsilon_{th}^{i_1} + \varepsilon_{th}^{i_2} + \dots + \varepsilon_{th}^{i_\Delta})$, where $\varepsilon_{th}^{i_j}$ denotes the slip threshold of fibers taking part in the avalanche. The distribution of slip length $P(\delta \varepsilon)$ is presented in Fig. 6(a) for several values of m . It can be observed that $P(\delta \varepsilon)$ exhibits a universal power law behavior

$$P(\delta \varepsilon) \sim \delta \varepsilon^{-\phi}, \quad (13)$$

where the value of the exponent $\phi = 2.25 \pm 0.05$ was obtained numerically independently of m and k_{max} . Similarly to the avalanche size Δ , the characteristic slip length defined as the average value of the largest slip length $\langle \delta \varepsilon^{max} \rangle$ is sensitive to the precise shape of $\sigma(\varepsilon)$ in the vicinity of the extremal points. It can be observed in Fig. 5(c) that $\langle \delta \varepsilon^{max} \rangle$ has a sharp maximum approaching the phase boundary from the *POP* phase and a power law divergence of the type of Eq. (12) is evidenced in Fig. 5(d). The critical exponent $\nu \approx 2$ proved to be the same as for the characteristic burst size. Figure 6(b) presents that the distribution of load increments has a power law decay

$$P(\delta \sigma) \sim \delta \sigma^{-\alpha}, \quad (14)$$

where the value of the exponent is universal $\alpha = 2$, it does

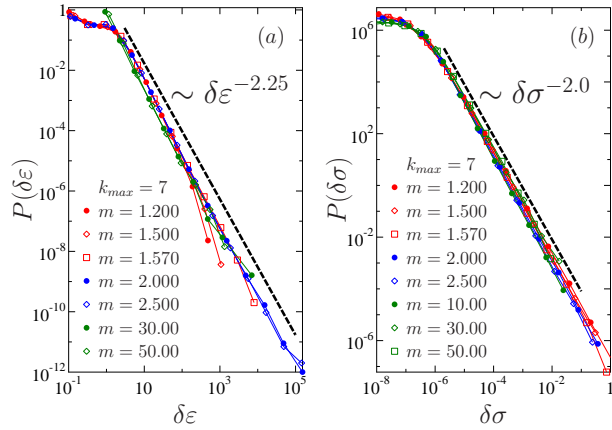


Fig. 6: (Color online) Probability distribution of slip length $P(\delta\epsilon)$ of avalanches (a) and the distribution of load increments $P(\delta\sigma)$ which trigger the avalanches (b) for different values of the Weibull exponent m . Universal power law behavior is obtained in both cases.

not depend neither on m nor on k_{max} . It has to be emphasized that the asymptotics of the distribution $P(\delta\sigma)$ is determined by the beginning of the loading process where large load increments are required to drive the system. The reason of universality is that the low stress regime of the loaded system is insensitive to the parameters m and k_{max} .

In summary, we studied the statistics of slip avalanches in a fiber bundle model where overstressed fibers can relax in a series of stick slip events. We showed that on the macro-scale the stick-slip mechanism leads to plastic behavior with a permanent deformation remaining after the load is released. On the micro-scale single slips induced by external load increments trigger bursts which give rise to a step-wise strain increase. The distribution of load increments and of slip length exhibit a universal power law behavior with exponents independent of the model's parameters. The size distribution of bursts proved to be sensitive to the amount of disorder and to the number of fibers' slips. Our calculations revealed that at high enough disorder only small avalanches pop up, while at low disorder macroscopic avalanches can snap the system. We set up the phase diagram of the model and showed that the transition between the *POP* and *SNAP* phases is analogous to disorder induced phase transitions. Besides the theoretical interest, our calculations provide insight into the statistics of restructurings of systems with hidden length such as biomaterials. Our study was restricted to the case of Weibull distributions where the amount of disorder can be represented by the exponent m . Generalization to other distributions defined over an infinite domain is straightforward using the standard deviation as a measure of disorder. The value of the critical exponents τ, ν, ϕ , and α do not have any dependence on the functional form of the disorder distribution.

It has been shown in Ref. [6] that the mode of external driving has a crucial effect on the critical non-equilibrium steady states in slowly driven bistable heterogeneous systems with controllable disorder: changing the driving from soft to hard a crossover is obtained from the classical order-disorder universality class to the quenched Edwards-Wilkinson class of SOC type. In our investigations of the avalanche statistics only stress controlled loading was considered, which corresponds to the perfectly soft driving of Ref. [6]. Our phase diagram of Fig. 3 can be considered as an extension of the zero softness part of the phase diagram of Ref. [6] with the additional degree of freedom of varying the number of allowed slip events under an infinite range of interaction. It is very interesting to extend our study to vary the mode of driving which is currently in progress.

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