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tmcs@inf.unideb.hu

<http://tmcs.math.klte.hu>

Teaching
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The background of students’ performance

RITA NAGY-KONDOR

Abstract. The question to which we were seeking was: how can we reveal the students’ strategies and mental process by following their work precisely and by finding out what correlation these have with their efficiency. Our aim was to understand the factors behind of students’ achievement. We tried to follow up the process of problem solving by looking at the number of wrong turnings.

Key words and phrases: descriptive geometry, dynamic geometry systems.

ZDM Subject Classification: G80, B40.

1. Introduction

Several studies have demonstrated the opportunities computers offer in teaching mathematics. The appearance of computer geometry software packages seems to be spurring a new interest in geometrical constructions and supports the possibility of using construction tasks in Dynamic Geometry Environments [8]. In one of our previous experiments, eighth-grade students and their teachers both welcomed and found computers and dynamic geometric systems to be useful elements in Mathematics lessons [6]. It was an obvious extension for us to extend this experiment to the university level. Our aim was to understand the factors behind of students’ achievement. We tried to follow up the process of problem solving by looking at the number of Undos in dynamic geometric system Cinderella.

2. Antecedents

The feedback provided by dynamic geometry software permits interaction between the visual and the theoretical aspects of geometry. If a constructed figure in the drag mode does not keep the shape that was expected, it means that the construction process must be wrong. The drag mode can also invalidate a conjectured property and thus lead the students to abandon it [5].

Two criteria of validation for solutions of construction problems in dynamic geometry environments:

- Drag Test Criterion: A solution of a construction problem carried out in a dynamic geometry environment is a valid if and only if the final construction retains its geometrical properties under dragging.
- Compatibility Criterion: A solution of a construction problem carried out in a dynamic geometry environment is a valid if and only if the final construction retains its geometrical properties under dragging and its construction process does not violate the dynamic geometry environment construction restrictions, that is, constraints in the use of software operations equivalent to those imposed on straightedge-and-compass constructions (see [8, p. 36]).

Hölzl distinguishes two ways of using the mediating functions of the drag mode: a test mode and a search mode. Hölzl conclude that “The analyses and supplementary observations from the overall research project indicate that such an advanced appreciation of the meaning of the drag mode is not a short term affair but develops in mutual dependence with the ability to grasp a geometric situation – a learning process that is characterised by different layers of conceptions.” (see [3, p. 83]).

Arzarello et al. also monitored how students use the mouse of the computer while solving a problem. They found the following modalities: wandering dragging, bound dragging, guided dragging, dummy locus dragging, line dragging, linked dragging, dragging test. Dragging test: moving draggable or semi-draggable points in order to see whether the drawing keeps the initial properties. If so, then the figure passes the test; if not, the drawing was not constructed according to the geometric properties you wanted it to have. Dragging test was used to test a conjecture. Therefore looking at how students used dragging provided an insight into their cognitive processes (see [2, p. 67]).

Drag mode in dynamic geometric systems accordingly reinforces in students the logical sequence of events. Dragging supports the production of conjectures. The steps taken in a construction may be retraced and the construction can thus

be analysed and logical mistakes and incorrect assumptions can be revealed. With these facilities the program helps to make concepts clear; mistakes, uncertainties hidden in students' train of thought emerge for both teachers and students, helping to uncover weak points.

The students had been using paper and pencil methods during their descriptive geometry studies in our college in recent years. It often happened that students' performance was good in the paper and pencil test, but their performance in the oral exam at the end of the semester was much worse. Only then did mistakes in their thinking processes and defects in their knowledge emerge. It motivated us to try dynamic geometry software in teaching descriptive geometry.

One of the programs that supports computer-aided descriptive geometrical design is a Czech development but it is not a dynamic geometric software [9]. Using this program it is possible to make constructions for the Monge-projection, axonometry and perspective. Another program was developed by a Hungarian expert and helps the teacher to explain the theory and practice of the Monge-projection, the reconstruction of the spatial objects in the mind and, with the help of interactive feature, to understand spatial relationships [4]. Designs can be saved in BMP format with both programs.

3. Settings, methods, methodology

We carried out our educational experiments in groups of 23 first year students at the Technical College Faculty of Debrecen University. There were fourteen computers, some of them were used by two students. The pre-test took place on the first week to check the students' previous knowledge. We examined the students' spatial outlook and fundamental knowledge of descriptive geometry. Here we skip the tasks of pre-test. On average the tasks were solved with 80% success or more. The last two tasks were the least successful (41% and 21%). Here either the projection or the object was missing. In this paper we analyse three students who stood out from among the other students. In the experiment described here our aim was to reveal the background of students' achievement. We tried to reconstruct the process of solving a problem with reference to the number of wrong turnings, which is not an easy job. After all it is difficult to get insight into someone's mental processes.

Descriptive geometrical construction is taught only in one semester in two lectures and two seminars weekly. During this period the students must learn the

elements of the Monge-projection as far as the interpenetration of plane figures and interpenetration of curved surfaces is concerned.

To help practise in Cinderella, the dynamic geometric program that we selected, each design can also be saved as interactive website. We made the constructions using interactive worksheets. Students could work from the Internet during seminars. If the solution was correct, the program indicated that. The program accepted any kind of solution that led to the correct result. The layout of computed-aided design programs is familiar to students, so they did not mind learning how to use this program. We made more progress with the group using computers than we did on another occasion with a group making constructions with pencil and paper, since the computer program was capable of producing the basic constructions instantly.

In the class the computer and the notebook were typically used simultaneously. Although the icon used for visibility is present in the program, it cannot be transferred to the worksheet. Therefore we constructed on paper several of the exercises concerning visibility, and in the case of computer-aided exercises we discussed visibility, as in the following exercise too.

Task: construct the line of concurrence for planes ABC and DEF .

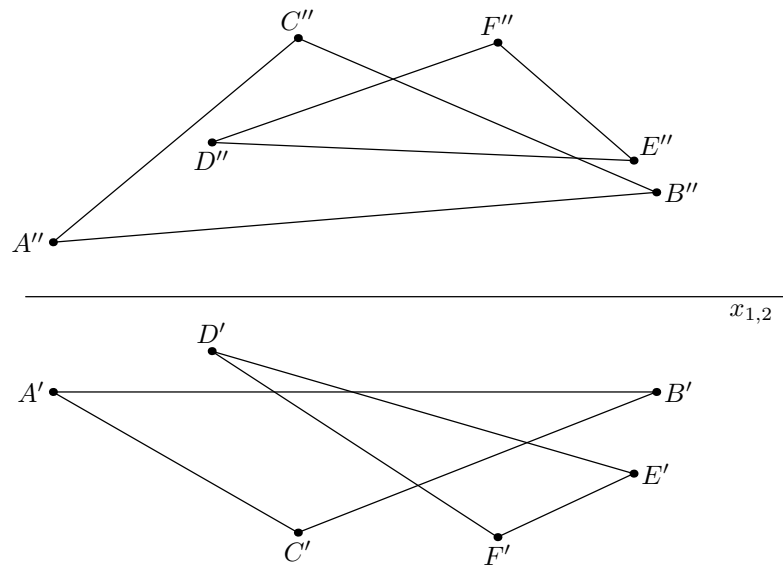


Figure 1. The task and its solution

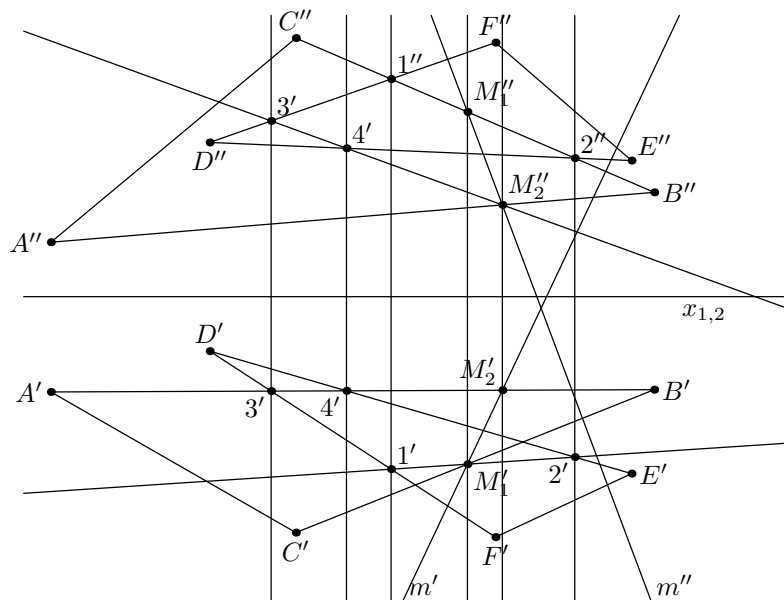


Figure 2. The task and its solution

Solving a problem means to reach the target from the initial state by determining the intermediate elements [1]:

Initial state:

$x_{1,2}$ axis,
two projections of ABC
triangle and DEF triangle

Intermediate elements:

K_1 : meet for $C''B''$ and $D''F''$: $1''$
 K_2 : meet for $C''B''$ and $D''E''$: $2''$
 K_3 : meet for $A'B'$ and $D'F'$: $3'$
 K_4 : meet for $A'B'$ and $D'E'$: $4'$
 K_5 : $1'$
 K_6 : $2'$
 K_7 : $3''$
 K_8 : $4''$
 K_9 : meet for $1'2'$ and $B'C'$: M_1'
 K_{10} : meet for $3''4''$ and $A''B''$: M_2''
 K_{11} : M_1''
 K_{12} : M_2'

Target:

two projections
for m line of
concurrence

Means of descriptive geometry:

A_1 : method of congruent straight lines (to construct intersection point)
 A_2 : figures of joint elements also fit
 A_3 : order lines are vertical to $x_{1,2}$

One of the possible sequences of solution:

$$\text{Initial state} \rightarrow \left\{ \begin{array}{l} K_1 \rightarrow K_5 \\ K_2 \rightarrow K_6 \\ K_3 \rightarrow K_7 \\ K_4 \rightarrow K_8 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} K_9 \rightarrow K_{11} \\ K_{10} \rightarrow K_{12} \end{array} \right\} \rightarrow \text{Target}$$

4. Results

In this experiment we observed and analysed three students' work during the lesson (Student *A*, Student *B*, Student *C*). All of the three students' performances in the first – paper and pencil – test were better than the average of their group, so we looked into the significant differences in their methods of working, the time needed for solution and the use of computer-aided design facilities in dynamic geometry environment. The students worked independently.

Apart from changing the order of the K_1 , K_2 and K_3 , K_4 solution steps, all three of them followed the same way of solution. It is advisable to construct the second intersection point (M_2) starting from the first diagram; whereas it does not matter in the case of the first one (M_1) which diagram you start from. They did not notice this at first, only when they reached the end of constructing M_2 . That is why a lot of time was spent between finding the two intersection points.

Drag modalities mentioned in the article by Arzarello et al. are useful for solving this exercise as well. Especially at the end of the task students can check their assumptions by using the dragging test. The dragging test is used as a means of validating a conjecture, in particular conjectures that are originated by a visual or a construction. However, only the Student *A* used the drag mode. So the use of drag mode was hardly experienced among our students. To check their assumptions they used rather the Undo and the eraser, since incorrect components can be cancelled with a single mouse click. It was observed that at the beginning the students did not use dragging very much. This is a behaviour which has been observed in many experiments with students at different school levels [2].

According to our observations the types of Undo used by students may be the following:

- Undo was used after an experimental step. This type serves to check the students' assumptions and ideas. The computer provides more facilities to make experiments, since incorrect elements can be cancelled quickly and other lines cannot accidentally be erased, as is the case with design on paper. However,

Table 1. How many times he clicked on the icon

Icons available in the exercise	Student A	Student B	Student C
 Add a Point	8	13	0
 Moving an Element	1	0	0
 Compass	1	0	0
 Join	5	6	3
 Meet	0	0	9
 Define a Parallel	2	1	0
 Define a Perpendicular	6	5	6
 Undo	7	6	1
 New	0	0	0

Table 2. How many points, lines he drew

	Student A		Student B		Student C		Necessary for construction
	drew	used Undo	drew	used Undo	drew	used Undo	
Point	18	6	15	3	13	1	12
Line	11	1	13	3	10	0	10

this does not mean that the task be solved without any prior knowledge just by experimentation.

- Undo was used after a logical mistake. The students would make the same mistake on paper as well.
- Undo was used after improper use of the software. In this case the mistake would not be made on paper.

Table 3. Time used for construction

	Student A	Student B	Student C
Time passed between constructing projection 1 and 2 of intersection point M_1	06:07	01:31	02:53
Time passed between constructing projection 1 and 2 of intersection point M_2	10:52	05:46	06:29
Total time	11:10	06:12	07:02

Lines that students drew incorrectly were undone at once. This sort of mistake was noticed immediately because it was usually caused by the incorrect use of icons drawing vertical and parallel lines. The students selected a line other than the one to which they wanted to draw a vertical and a parallel. This is an example of third type of Undo.

Table 4. The types of Undo

	Student A	Student B	Student C
Experimental	4	4	0
Logical mistake	2	1	0
Improper use of the software	1	1	1

Student C used the Undo least frequently, he undid only one step. He took the least time between finding the two intersection points (less than 4 minutes). His work was the most thoughtful and he achieved the best score among the three of them in the first test too. Although he took more time on the construction than did Student B , this suggests that whereas B was rushing his work, drawing lots of elements without thinking and then undoing them, C was thinking before every step. Student B had altogether six and Student A seven Undos, due either to careless moves or because of bewilderment or while using trial-and-error methods. In the case of Student B thoughtlessness and rushing the work was more typical, while for Student A the problem was perplexity, since B solved the exercise in almost half the time. The high number of Undos may advert to the behaviour and mentality of the person in question, as well as being characteristic of his approach to problem solving.

In drawing the point of concurrence the Students A and B used exclusively the point drawing icon whereas Student C used the icon determining the common point of two straight lines.

Student *C*'s higher degree of awareness and more secure background knowledge are also demonstrated by the fact that, while the others variously used the icon for constructing parallels and verticals, *C* worked only with the vertical construction icon, using the property of the order lines that they are perpendicular to the $x_{1,2}$ axis.

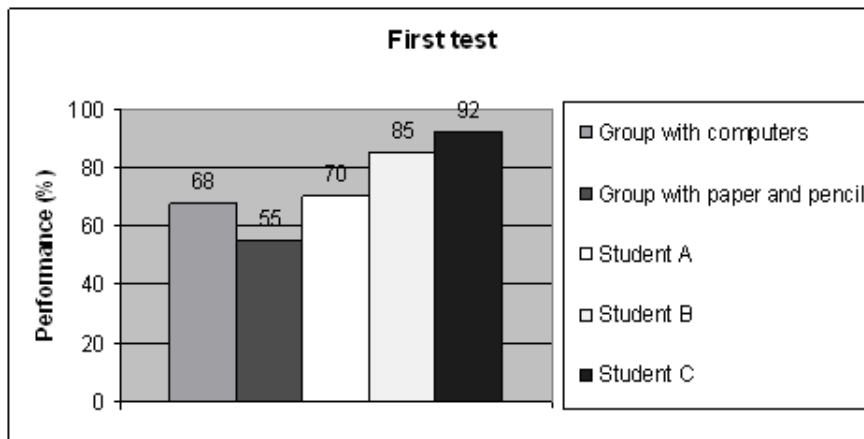


Figure 3

The performance of Student *A* in the first – paper and pencil – test was 70%, Student *B* 85% and Student *C* 92%. Student *A* made a lot of logical mistakes and there were signs of erasures on his answer paper. In the case of Student *B* several Undos were visible (arising from his experimentations) and his drawings were pretty inaccurate. Mistakes due to inaccuracy of construction do not arise in the case of computer work, not does the problem that there may not be enough space on the paper for the construction, which often occurred in the group using traditional methods. Student *B* found it easier to perform constructions with the computer: the program helped him to do the constructions. Student *C* made no logical mistakes and there were no signs of erasure. It made no difference to him whether he worked with paper and pencil or on the computer. He does constructions well in either medium. Thus Students *A* and *B* are experimental types and *C* is theoretical type.

5. Summary

Our aim was to reveal details hidden in the students’ mental processes. We tried to retrace the process of problem solving on the basis of the number of wrong turnings. The interactive worksheets helped us to improve their problem solving abilities making it easier to follow the students’ thoughts and to find logical mistakes.

We can draw a conclusion from our results that the program is a useful aid in improving students’ approach to geometry in primary school [6], as well as in higher education. Interactive worksheets prepared by computer can be used to advantage for teaching descriptive geometry in parallel with traditional construction, while avoiding the use of the computer for personal purposes. Students can check the validity of their solutions to problems of construction by using the drag test criterion. The results also show this. Even though the drag test criterion in dynamic geometry environment does not constitute an analogue of the straightedge-and-compass criterion in the paper-and-pencil Euclidean geometry environment, the compatibility criterion of validation Stylianides introduced and aimed to make the establishment a firm correspondence between geometrical constructions in dynamic geometry environment and the paper-and-pencil Euclidean geometry environment [8]. In our experiment the drag test criterion is equivalent to the compatibility criterion, because the menu on the interactive worksheets ensures that the available software tools cannot violate the construction restrictions in the dynamic geometry environment. Unfortunately among our students hardly any of them were able to gain experience of the drag test. So the teacher’s task is to put more emphasis on the use of the drag test in the teaching session. The goal is to use fewer Undos and more drag tests.

It is remarkable that while the performance of all of the three students in the first test was better than the average of their group not using computers and all of them correctly solved the problem analysed above, nonetheless in the case of the supervised task there were significant difference in their methods of working, the time needed to solve the problem and how they exploited the opportunities for computer-aided construction. Our experiment also shows the difference in pace at which students solved a problem. It suggests a conclusion that has already been emphasised several times: the importance of differentiated education. In our experiment by using the computer program we revealed the student’s difficulties in problem solving. Undos, experimentation, trial-and-error methods and perplexity attest to the insecure background knowledge behind the

good performance. A reason for a lot of the Undos is the automatic and careless execution of well-practised steps (a lot of time was taken between finding the two intersection points). The goal in every construction problem is to increase the awareness on the part of the students, as well as the working out of a problem solving sequence and deliberate execution. The teacher's task is to motivate the students to explain their problem solving steps when doing paper and pencil exercises too.

If the software is simply made available, it does not mean that people will more or less automatically take advantage of the opportunities that it affords. The program might become an obstacle to the transition from empirical to theoretical thinking, as it allows the validating of a proposition without the need to use a theory [2]. Therefore direction by the teacher is very important.

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RITA NAGY-KONDOR
FACULTY OF TECHNICAL ENGINEERING
UNIVERSITY OF DEBRECEN
HUNGARY

E-mail: rita@mfk.unideb.hu

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