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ORIGINAL RESEARCH  
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# Manufacturing of spur gears having normal teeth on different pressure angles by module disc milling cutter

Sándor Bodzás\*

Department of Mechanical Engineering, University of Debrecen, 4028 Debrecen, Ótmető str. 2-4., Hungary

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## ABSTRACT

The aim of this study is the manufacturing analysis of five spur gear pairs where the initial geometric parameters are the same only the pressure angle is different. Firstly, the gears must be designed and modelled. After that, I analyse the modification of this geometric parameter for the manufacturing parameters of the pinion and the gear in the case of gear cutting by module disc milling cutter. Using this technology the one tooth cutting can repeat from tooth to tooth in the function of the number of teeth. I would like to find correlations between the pressure angle and the manufacturing parameters. For this purpose, I define the initial technological parameters and calculate necessary technological parameters for the manufacturing process in a general way. I also define the manufacturing parameters for the given gear geometries. This analysis is practical and theoretical at the same time since the results and the process can help the manufacturing engineers to develop the gear manufacturing processes and applying my results for similar manufacturing problems.

## KEYWORDS

spur gear, pressure angle, manufacturing, analysis, gear, pinion

## 1. INTRODUCTION

The involute profile is selected based on experience of geometric, tooth connection and load transmission. It can be generated by constructive and mathematical way. The involute curve is always generated from the base circle of the gear [10, 11, 19, 21].

The parametric equation of the involute curve is (Fig. 1) [10, 21]

$$\left. \begin{aligned} x &= r_b \cdot \sin \varphi - r_b \cdot \varphi \cdot \cos \varphi \\ y &= r_b \cdot \cos \varphi + r_b \cdot \varphi \cdot \sin \varphi \end{aligned} \right\} \quad (1)$$

The polar angle is the  $\text{inv} \alpha$ . The  $\varphi$  angle is

$$\varphi = \text{inv} \alpha + \alpha \quad (2)$$

The following two equations can also be derived on Fig. 1. [10, 21]:

$$\rho = r_b \cdot \varphi \quad (3)$$

$$\rho = r_b \cdot \tan \alpha \quad (4)$$

Based on (3) and (4)

$$\varphi = \tan \alpha \quad (5)$$

Substituting (5) into (2) and expressing  $\text{inv} \alpha$

$$\text{inv} \alpha = \tan \alpha - \alpha \quad (6)$$

The common normal line, which is the common tangent line of the base circles, has to go

\*Corresponding author.

E-mail: bodzassandor@eng.unideb.hu



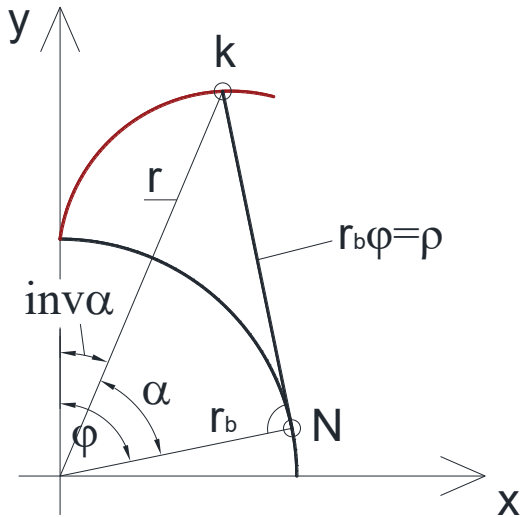


Fig. 1. The generation of the involute curve by mathematical way

through on the *C* main point (Fig. 2) [10, 11, 19, 21]. The connection always takes place on this line. This line is called line of action. This line and the common tangent line of the rolling circles ( $r_{w1}$ ,  $r_{w2}$ ) always include an  $\alpha_w$  angle [21]. This angle is called pressure angle. If the centre distance is modified from  $a$  to  $a'$  the  $\alpha_w$  will be also modified to  $\alpha'_w$ . The pressure angle that belongs to the pitch circle radius ( $r_p$ ) is called base profile angle ( $\alpha_p$ ).

Starting from the pitch and the rolling circle radiuses, the radius of the base circle from which the arc can be generated is [10, 11, 19, 21]

$$r_b = r_p \cdot \cos \alpha_p = r_w \cdot \cos \alpha_w \tag{7}$$

$$r_w = r_p \cdot \frac{\cos \alpha_p}{\cos \alpha_w} \tag{8}$$

The centre distance is

$$a = r_{w1} + r_{w2} = r_{p1} \cdot \frac{\cos \alpha_p}{\cos \alpha_w} + r_{p2} \cdot \frac{\cos \alpha_p}{\cos \alpha_w} = a_0 \cdot \frac{\cos \alpha_p}{\cos \alpha_w} \tag{9}$$

The elementary centre distance is [10, 11, 19, 21]

$$a_0 = r_{p1} + r_{p2} = \frac{d_{p1} + d_{p2}}{2} \tag{10}$$

### 1.1. The properties of spur gear having normal teeth

The basic rack gear tooth profile contains the base parameters of the normal section (circular pitch, whole depth, basic rack gear tooth profile angle and clearance). This profile has infinite number of teeth along a line. The basic rack gear tooth profile of an involute gear is standardized (Fig. 3) [10, 11, 19, 21].

During the manufacturing process, the pitch circle of the gear is rolled down on the tool centre line without slip [4, 5, 7, 8, 12, 15, 16]. The tool centre line and the tool reference line can be different because the pitch circle of the gear can be rolled down any parallel lines of the tool centre line [1, 2, 5, 11, 12, 21]. The phenomenon when the tool centre line and the tool reference line are not same is called gear having addendum modification [10, 11, 21]. This process is also called addendum modification. This parameter can be calculated by the following formula [10, 11, 21]

$$x_1 \cdot m_{ax} \tag{11}$$

The  $x$  is positive when the basic profile is moved from the gear axis (Fig. 4b). The  $x$  is negative when the basic profile is moved to the gear axis (Fig. 4a). If  $x=0$ , the tool centre line and the tool reference line are the same. This type of gear pair is called  $x$ -zero gear drive [10, 11, 21].

### 1.2. Determination of the tooth thickness in general way

Based on Fig. 5a the tooth thickness is [10, 11, 21]

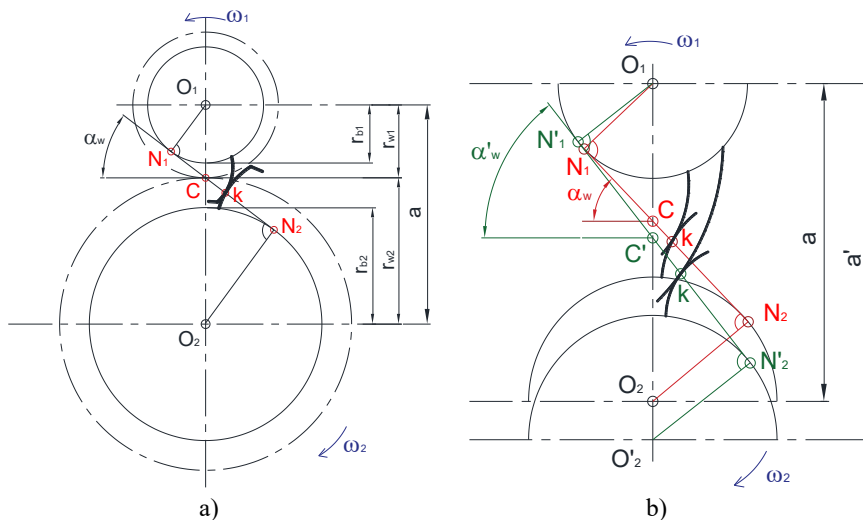


Fig. 2. Connection of the involute curves in the case of different centre distances



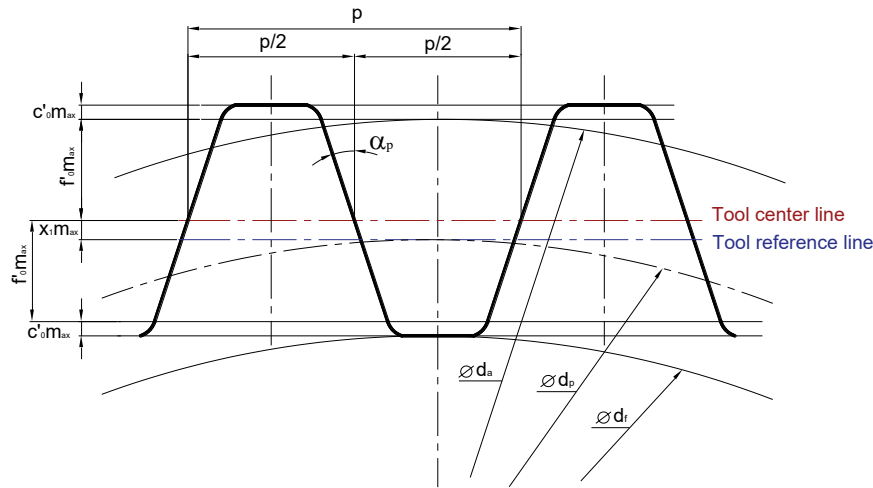


Fig. 3. Tool basic rack gear tooth profile in the case of involute gear having normal teeth

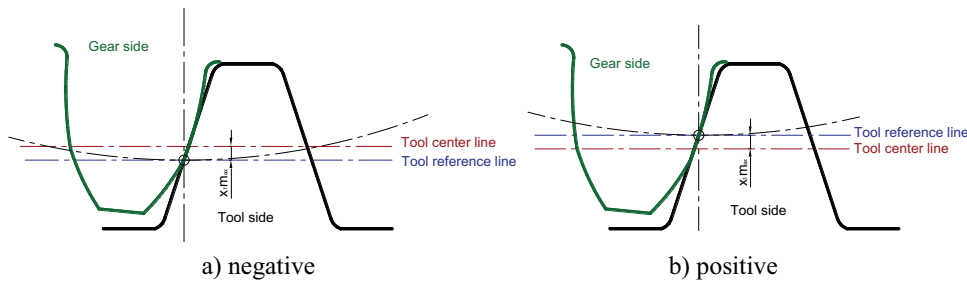


Fig. 4. The connection possibilities of the tool basic rack gear tooth profile and the gear profile

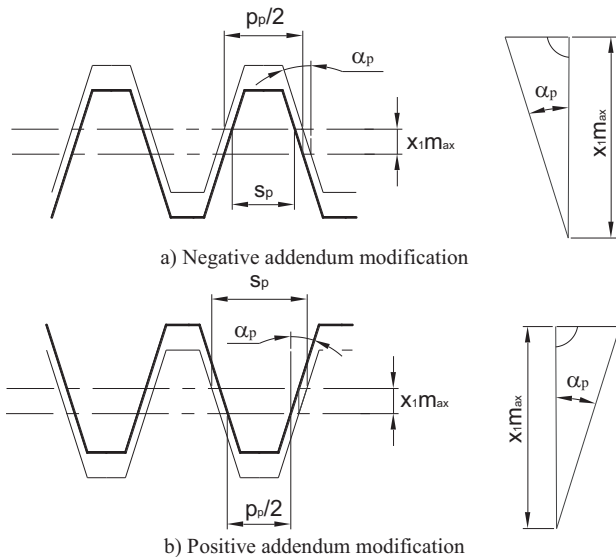


Fig. 5. The correlation between the addendum modification and the tooth thickness on the pitch circle on the basic rack gear tooth profile

$$s_p = \frac{p_p}{2} - 2 \cdot x \cdot m_{ax} \cdot \tan \alpha_p = m_{ax} \cdot \left( \frac{\pi}{2} - 2 \cdot x \cdot \tan \alpha_p \right) \quad (12)$$

Based on Fig. 5b the tooth thickness is [10, 11, 21]

$$s_p = \frac{p_p}{2} + 2 \cdot x \cdot m_{ax} \cdot \tan \alpha_p = m_{ax} \cdot \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha_p \right) \quad (13)$$

The  $\sigma$  angle is (Fig. 6) [10, 11, 21]

$$\sigma = \text{inv} \alpha_p + \frac{s_p}{2 \cdot r_p} = \text{inv} \alpha + \frac{s}{2 \cdot r} \quad (14)$$

The  $s$  tooth thickness is on an arbitrary circle is

$$s = 2 \cdot r \cdot \left( \frac{s_p}{2 \cdot r_p} + \text{inv} \alpha_p - \text{inv} \alpha \right) \quad (15)$$

### 1.3. Manufacturing of spur gear by module disc milling cutter

Spur and helical gears could be manufactured by plain milling technology on a horizontal knee type milling machine (Figs 8 and 9) or a CNC milling machine (Fig. 9) [1, 3–5, 8, 9, 12, 15, 16]. The profile of the module disc milling cutter is the same as that of the tooth space [1, 2, 4, 7–9, 11, 12]. The tool is doing rotation ( $\vec{v}_c$ ) and linear ( $\vec{v}_f$ ) motions at the same time. This linear motion could also be provided by the workpiece (Fig. 7). After one tooth is ready the division form tooth to tooth could be made possible by a dividing head (Fig. 8) in a classical way. The milling process can restart again [1–4, 9, 11, 12, 15, 16].

The geometric shape of the module disc milling cutter can be seen in Fig. 10. The geometry of the tool depends on the number of teeth and the module of the gear [1, 2, 9, 12, 15, 16].



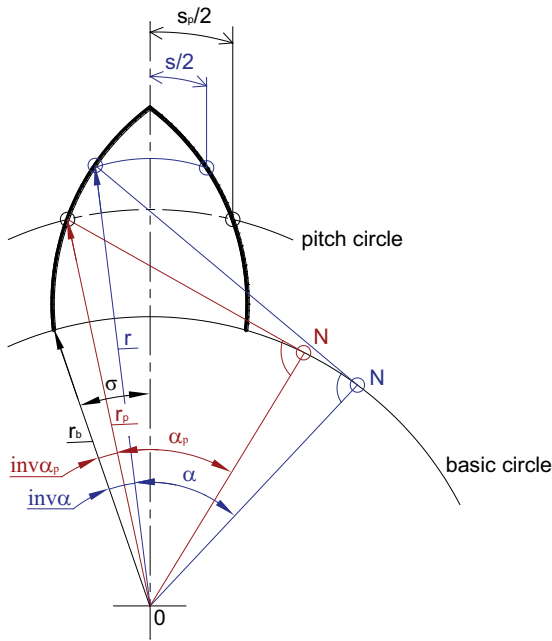


Fig. 6. The correlations of the tooth thicknesses on the pitch circle ( $r_p$ ) and an arbitrary ( $r$ ) circle

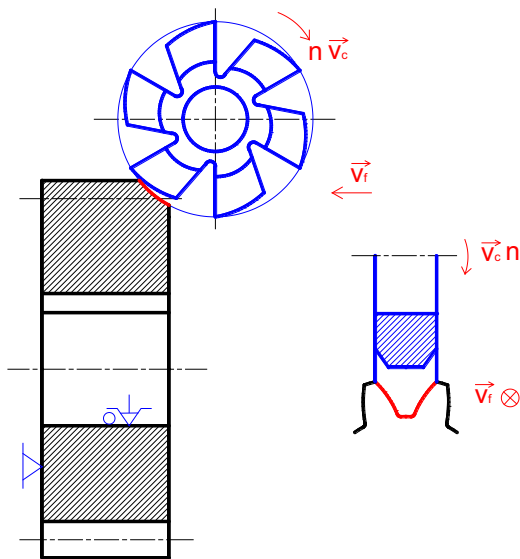


Fig. 7. Manufacturing of spur gear by module disc milling cutter [3, with permission from Debrecen University Press]

## 2. GEOMETRIC DESIGN AND MODELLING OF SPUR GEARS HAVING NORMAL TEETH

Knowing the references' recommendations [10, 11, 13, 19, 20, 21] and the initial gear parameters all of the other geometric parameters can be calculated by MATLAB software, which was created by me. The formulas for the gear design were programmed into this software. The output parameters of this program are the calculated geometric parameters, the profile curves of the elements and a *txt* file that contains the point coordinates of the profile points. The involute profile

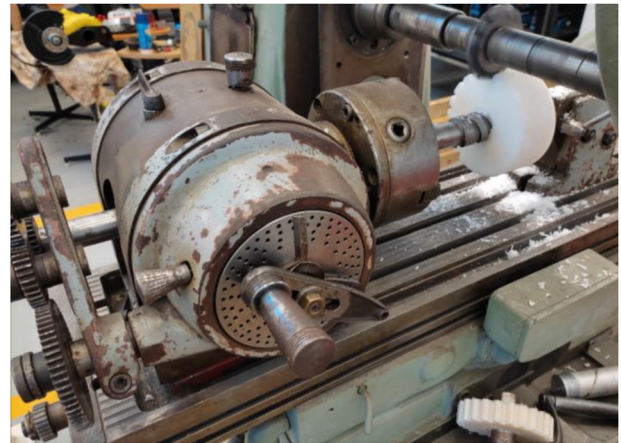


Fig. 8. The structure of the dividing head



Fig. 9. Spur gear manufacturing by a CNC milling machine

curves on the pinion and the gear in the case of  $\alpha_w = 23^\circ$  as an example can be seen in Fig. 11.

Geometrically, the shape of the involute curves is similar to both gear pairs since the base circle diameters are the same for each gear pair. The differences are the arc length between the root and the outside circles. The tooth connections take place on different  $d_w$  rolling circle diameters [10, 11, 21]. The calculated geometric parameters of the designed gear pairs can be seen in Table 1.

Knowing the geometric parameters of the gear pairs the CAD models and the assembly can be done by SolidWorks software. These models are important for the manufacturing simulations (CAM) [3, 17, 18] and the tooth contact analysis (TCA) [10]. The CAD models of a gear pair ( $\alpha_w = 23^\circ$ ) can be seen in Fig. 12.

The effect of the pressure angle on the geometric parameters can be seen in the diagrams of Fig. 13.





Fig. 10. The geometry of the module disc milling cutter

### 3. ANALYSIS OF THE MANUFACTURING PARAMETERS

#### 3.1. Determination of the technological parameters by general way

The gear cutting process by module disc milling cutter is applicable in a conventional way (gear cutting on horizontal knee type milling machine) or in a modern way (application of CNC machine) too [1, 3–5, 8, 9, 12, 15, 16]. Since the basis of the modern way is the conventional way, I analyse this gear cutting technology in the conventional way. The gear parameters are changing (tooth thickness, diameters, pressure angle, etc.), consequently the tool geometry must be also changed [1, 2, 7–9, 11, 12, 15, 16]. Gears having different geometries need different cutting tools in geometric aspects.

The cutting process for one tooth can be seen in Fig. 14. This process must be repeated in the function of the number of teeth. The gear is fixed into a clamping device. After the cutting of one tooth the gear has to be divided according to the circular pitch. The tool has two motions: rotation ( $\vec{v}_c$ ) and linear ( $\vec{v}_f$ ) motions at the same time [1, 2, 4–9, 11, 12,

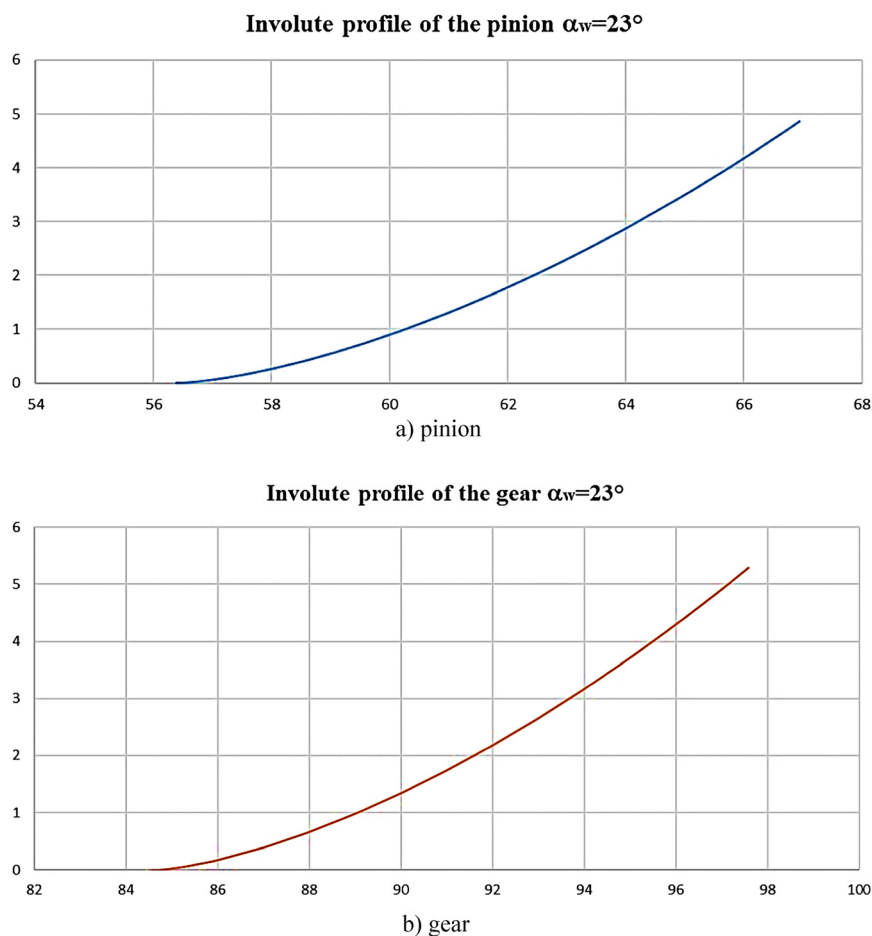


Fig. 11. The profiles of the gear pairs ( $\alpha_w = 23^\circ$ )

Table 1. The calculated geometric parameters of the gear pairs

Geometric parameters	Gear drive I	Gear drive II	Gear drive III	Gear drive IV	Gear drive V
$m_{ax}$ [mm]	6				
$z_1$	20				
$z_2$	30				
$\alpha_p$ [°]	20				
$c'_0$	0.2				
$u$	1.5				
$l_m$ [mm]	50				
$\alpha_w$ [°]	20	21	22	23	24
$d_{p1}$ [mm]	120				
$d_{p2}$ [mm]	180				
$d_{b1}$ [mm]	112.763				
$d_{b2}$ [mm]	169.144				
$d_{w1}$ [mm]	120	120.785	121.618	122.501	123.434
$d_{w2}$ [mm]	180	181.178	182.428	183.752	185.151
$\sum x$	0	0.167	0.353	0.559	0.786
$a$ [mm]	150	150.982	152.023	153.126	154.293
$a_0$ [mm]	150				
$y$	0	0.163	0.337	0.521	0.715
$h'$ [mm]	12	11.976	11.901	11.770	11.576
$h_a$ [mm]	6	5.988	5.950	5.885	5.788
$c$ [mm]	1.2				
$p_p$ [mm]	18.849				
$p_w$ [mm]	18.849	18.973	19.103	19.242	19.389
$h_f$ [mm]	7.2	7.188	7.150	7.085	6.988
$h$ [mm]	13.2	13.176	13.101	12.970	12.776
$d_{a1}$ [mm]	132	132.761	133.520	134.271	135.011
$d_{a2}$ [mm]	192	193.154	194.329	195.522	196.728
$d_{f1}$ [mm]	105.6	106.409	107.317	108.331	109.458
$d_{f2}$ [mm]	165.6	166.802	168.126	169.582	171.175
$j_s$ [mm]	0.942	0.948	0.955	0.962	0.969
$x_1$	0	0.065	0.134	0.208	0.286
$x_2$	0	0.102	0.218	0.351	0.499
$s_{p1}$ [mm]	9.424	9.710	10.014	10.335	10.674
$s_{p2}$ [mm]	9.424	9.871	10.380	10.957	11.608
$s_{w1}$ [mm]	8.953	9.005	9.045	9.071	9.082
$s_{w2}$ [mm]	8.953	9.019	9.103	9.208	9.336

15, 16]. The 'E' middle point of the tool is controlled [17, 18]. We need to provide safety distance before and after the cutting process. These distances are called overruns ( $x_1$  and

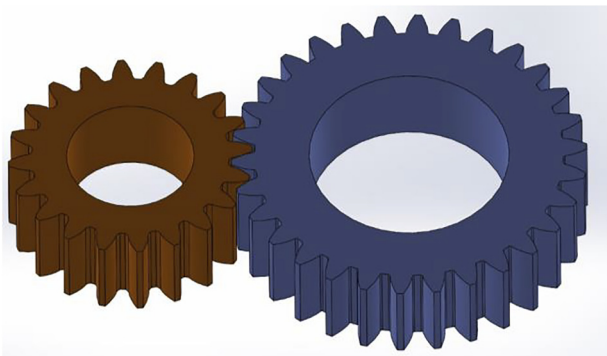


Fig. 12. The CAD model of a gear pair ( $z_1=20$ ,  $z_2=30$ ,  $max=6$  mm,  $\alpha_w=23^\circ$ , Gear drive IV)

$x_2$ ). The initial parameters of the execution of the technology are the following: the tool and workpiece material, the workpiece (pinion and gear) geometry, the tool geometry, the feed for one edge ( $f_z$ ), the adjustable number of revolution ( $n$ ) and the overruns ( $x_1$ ,  $x_2$ ).

Knowing the  $h$  whole depth and the  $D$  tool diameter the  $m$  distance is (Fig. 14a EBD triangle)

$$m = \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{D}{2} - h\right)^2} = \sqrt{D \cdot h - h^2} \quad (16)$$

The  $\varphi_c$  angle of contact is (Fig. 14a EBD triangle)

$$\varphi_c^\circ = \text{atan}\left(\frac{m}{\frac{D}{2} - h}\right) \quad (17)$$

The  $i$  arc of contact is (Fig. 14a)

$$i = \varphi_c^\circ \cdot \frac{D \cdot \pi}{360^\circ} \quad (18)$$

The separated chip volume can be seen in Fig. 15. The widest distance is the  $w_a$  tooth space on the outside circle of the gear.

The chip thickness is continuously changing along the  $i$  arc of contact, that is why this parameter will be considered by average value, which is called  $h_m$  medium chip thickness [1, 2, 3, 6, 7, 12, 14]. The  $V_h$  volume, which has a complex shape, can be approximated by the  $V_1$  and  $V_2$  prism volumes (Fig. 15) because of the simplification of the calculation and design process:

$$V_h = V_1 = V_2 \quad (19)$$

It means

$$h \cdot w_a \cdot f_z = i \cdot w_a \cdot h_m \quad (20)$$

$$h_m = \frac{h \cdot f_z}{i} \quad (21)$$

Substituting (18) into (21) the  $h_m$  medium chip thickness is

$$h_m = h \cdot f_z \cdot \frac{360^\circ}{\varphi_c^\circ \cdot D \cdot \pi} \quad (22)$$

Considering Fig. 6 and formulae (12)–(15)  $w_a$  tooth space on the outside circle can be determined. A side view for the manufacturing process can be seen in Fig. 16. The blue milling cutter is milling one tooth. The  $\alpha_a$  angle can be calculated from the OFG triangle:

$$\cos \alpha_a = \frac{d_b}{d_a} \rightarrow \alpha_a = \dots \quad (23)$$

Based on (6)

$$\text{inv } \alpha_a = \tan \alpha_a - \alpha_a \quad (24)$$

$$\text{inv } \alpha_p = \tan \alpha_p - \alpha_p \quad (25)$$

Based on (15) and considering the  $j_s$  backlash that we have to provide between the teeth [10, 11, 12, 19, 20, 21] the  $s_a$  tooth thickness on the  $d_a$  outside circle is (Fig. 16)

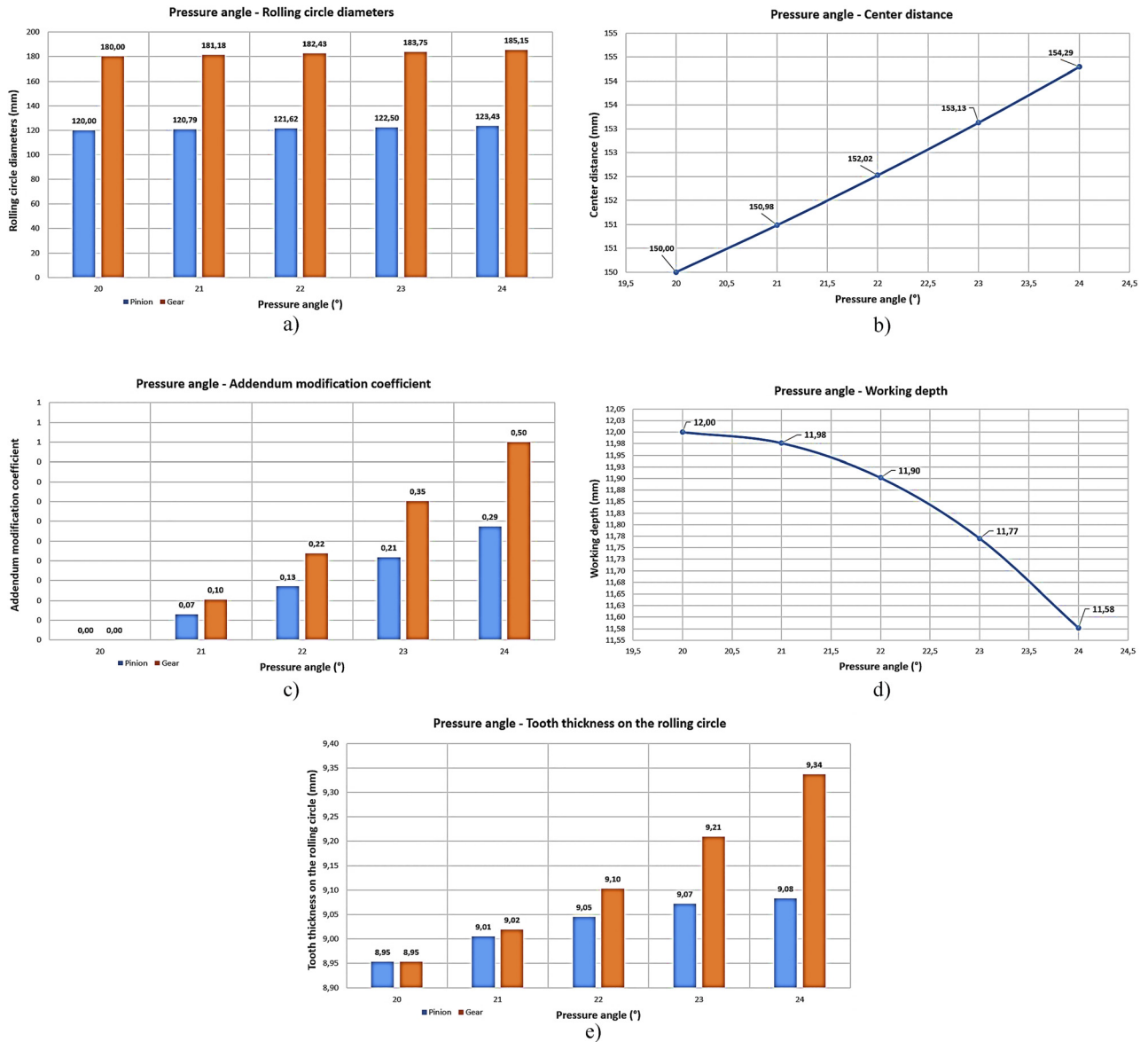


Fig. 13. The effect of the modification of the pressure angle on the geometric parameters

$$s_a = d_a \cdot \left( \frac{s_p}{d_p} + \text{inv } \alpha_p - \text{inv } \alpha_a \right) \cdot d_a - \frac{j_s}{2} \quad (26)$$

The perimeter of the given circle has to be equal with the multiplication of the number of teeth and the given pitch, which is interpreted on the given circle of the gear [10, 11, 21]:

$$d_p \cdot \pi = z \cdot t_p \rightarrow \frac{d_p}{t_p} = \frac{z}{\pi} \quad (27)$$

$$d_a \cdot \pi = z \cdot t_a \rightarrow \frac{d_a}{t_a} = \frac{z}{\pi} \quad (28)$$

Based on (27) and (28) the circular pitch on the outside circle is

$$t_a = \frac{d_a \cdot t_p}{d_p} \quad (29)$$

Based on (26) and (29) the  $w_a$  tooth thickness on the outside diameter is (Fig. 16)

$$w_a = t_a - s_a \quad (30)$$

Knowing the specific cutting force ( $k_c$ ) from the material property, the  $h_m$  and the  $w_a$  the cutting force for one edge of the tool can be determined:

$$F_{c1} = k_c \cdot h_m \cdot w_a \quad (31)$$

The  $t$  tooth pitch means the peripheral distance between two neighbouring teeth on the tool [1-9, 12, 14-16]:

$$t = \frac{D \cdot \pi}{z_t} \quad (32)$$

The  $\psi$  switch number means the number of the working teeth along the  $i$  arc of contact (Fig. 14) [1-9, 12, 14-16]:



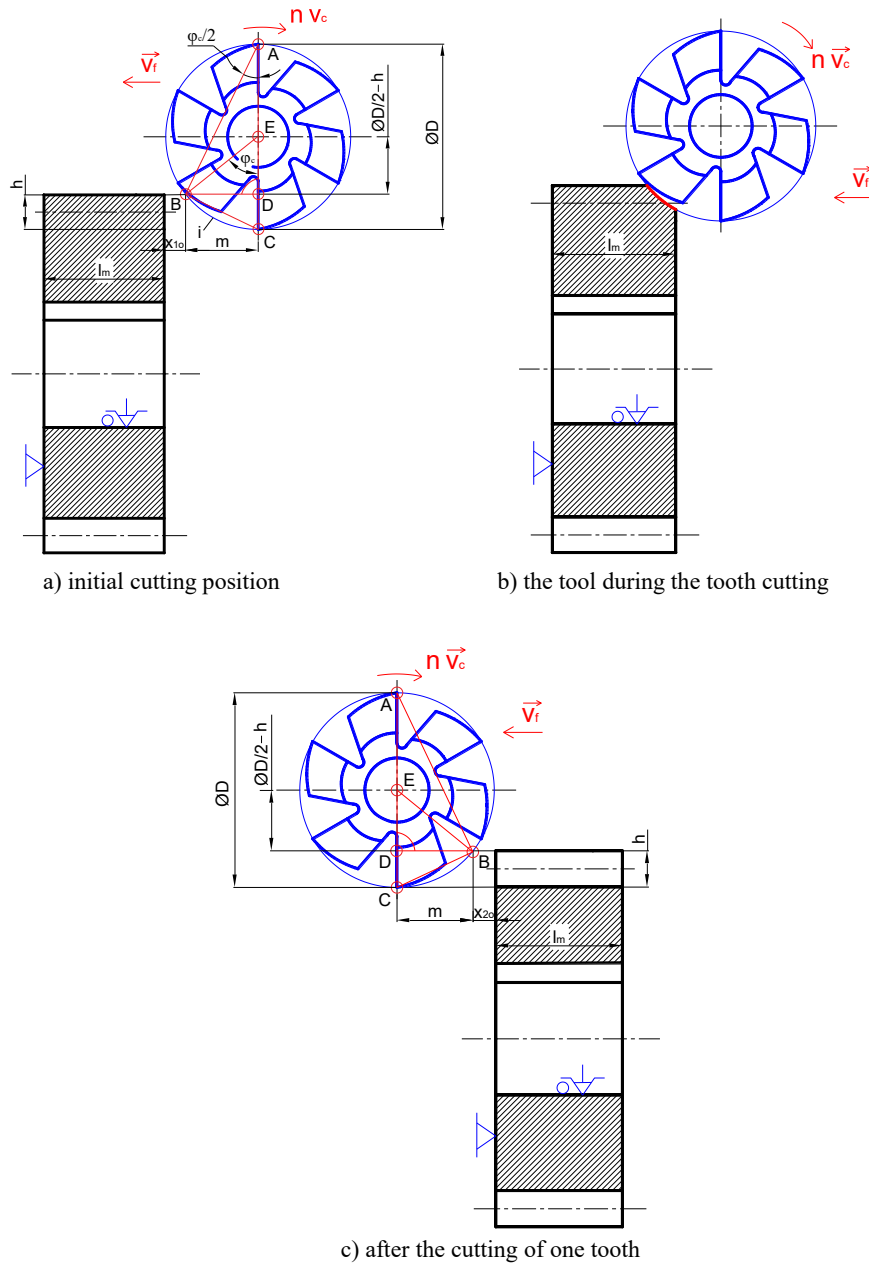


Fig. 14. The cutting process for one tooth

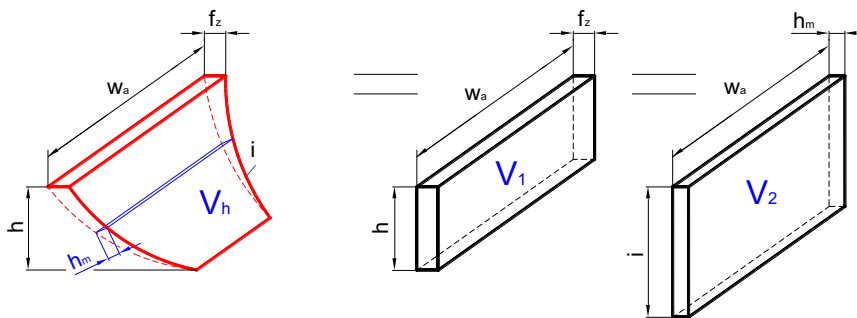


Fig. 15. The approximation by volume constancy



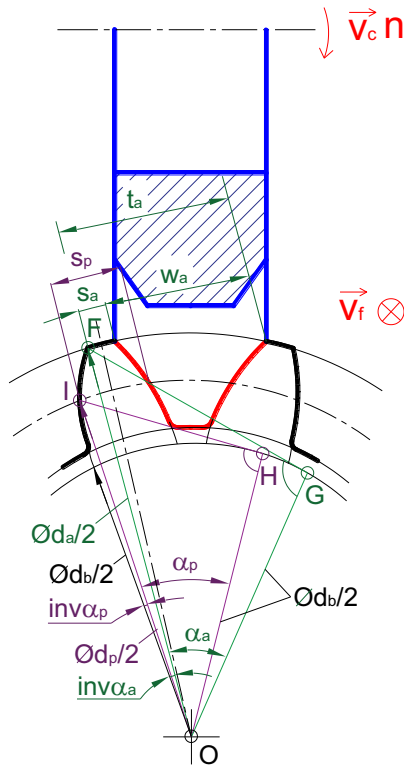


Fig. 16. Determination of the  $w_a$  tooth space on the outside circle

Table 2. The initial manufacturing parameters

Manufacturing parameters	Value
$D$ [mm]	120
$z_t$	18
$f_z$ [mm]	0.06
$l_m$ [mm]	50
$k_c$ [ $N\ mm^{-2}$ ]	5,000
$x_{1o}, x_{2o}$ [mm]	3

$$\psi = \frac{i}{t} = \frac{\varphi}{360} \cdot z_t \quad (33)$$

Considering the  $\psi$  switch number the total cutting force along the  $i$  arc of contact is

$$F_c = \psi \cdot F_{c1} \quad (34)$$

Substituting (22), (31) and (33) into (34)

$$F_c = k_c \cdot w_a \cdot h \cdot f_z \cdot \frac{z_t}{D \cdot \pi} \quad (35)$$

The rotational cutting speed is [1-9, 12, 14-16]

$$v_c = D \cdot \pi \cdot n \quad (36)$$

The feed speed is [1-9, 12, 14-16]

$$v_f = f_z \cdot z \cdot n \quad (37)$$

Knowing of the cutting force (35) and the cutting speed (36) the cutting power is

$$P_c = F_c \cdot v_c = k_c \cdot w_a \cdot h \cdot f_z \cdot z_t \cdot n \quad (38)$$

Based on Fig. 14, the overall machining time that is needed for the manufacturing of all of the teeth is

$$T = \frac{L}{v_f} \cdot z = \frac{m + x_{1o} + l_m + x_{2o} + m}{v_f} \cdot z \quad (39)$$

### 3.2. Manufacturing design and analysis for the designed gear pairs

Knowing the geometric parameters of the gear pairs, the geometric and manufacturing formulas I tried to find correlations between the geometric formulas and the manufacturing parameters for the concrete cases and analyse the results. The geometric parameters are found in Table 1. The initial manufacturing parameters are found in Table 2. According to the subchapter 3.1., I made an Excel table to determine the manufacturing parameters for each gear pair. The results can be seen in Tables 3 and 4 for the pinions and the gears.

The correlation between the pressure angle ( $\alpha_w$ ) and the angle of contact ( $\varphi_c$ ) can be seen in Fig. 17. The diagram is the same for the cohesive connecting pinions and gears. The shape of the diagram is parabola. The angle of contact is exponentially decreasing while the pressure angle is increasing.

The correlation between the pressure angle ( $\alpha_w$ ) and the arc of contact ( $i$ ) can be seen in Fig. 18. The diagram is the same for the cohesive connecting pinions and gears. The

Table 3. Manufacturing parameters for the pinions

Manufacturing parameters	Pressure angles ( $\alpha_w$ ) [°]				
	20	21	22	23	24
$v_c$ [ $m\ min^{-1}$ ]	45.216				
$m$ [mm]	37.546	37.516	37.423	37.258	37.012
$\varphi_c$ [°]	38.759	38.722	38.608	38.407	38.107
$i$ [mm]	40.568	40.529	40.409	40.199	39.885
$h_m$ [mm]	0.019522	0.019505	0.019452	0.019358	0.019219
$w_a$ [mm]	17.037	17.288	17.527	17.750	17.951
$F_{c1}$ [N]	1663.068	1686.138	1704.787	1718.105	1725.060
$\psi$	2.028	2.026	2.020	2.010	1.994
$F_c$ [N]	3373.353	3416.913	3444.468	3453.295	3440.237
$P_c$ [W]	2542.159	2574.985	2595.751	2602.403	2592.562
$v_f$ [ $mm\ min^{-1}$ ]	129.6				
$T$ [min]	20.230	20.221	20.192	20.141	20.065

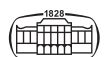


Table 4. Manufacturing parameters for the gears

Manufacturing parameters	Pressure angles ( $\alpha_w$ ) [°]				
	20	21	22	23	24
$v_c$ [m min <sup>-1</sup> ]	45.216				
$m$ [mm]	37.546	37.516	37.423	37.258	37.012
$\varphi_c$ [°]	38.759	38.722	38.608	38.407	38.107
$i$ [mm]	40.568	40.529	40.409	40.199	39.885
$h_m$ [mm]	0.019522	0.019505	0.019452	0.019358	0.019219
$w_a$ [mm]	16.144	16.392	16.601	16.762	16.864
$F_{c1}$ [N]	1575.952	1598.717	1614.679	1622.500	1620.632
$\psi$	2.028	2.026	2.020	2.010	1.994
$F_c$ [N]	3196.647	3239.757	3262.407	3261.133	3231.980
$P_c$ [W]	2408.993	2441.481	2458.550	2457.590	2435.620
$v_f$ [mm min <sup>-1</sup> ]	129.6				
$T$ [min]	30.346	30.332	30.288	30.212	30.098

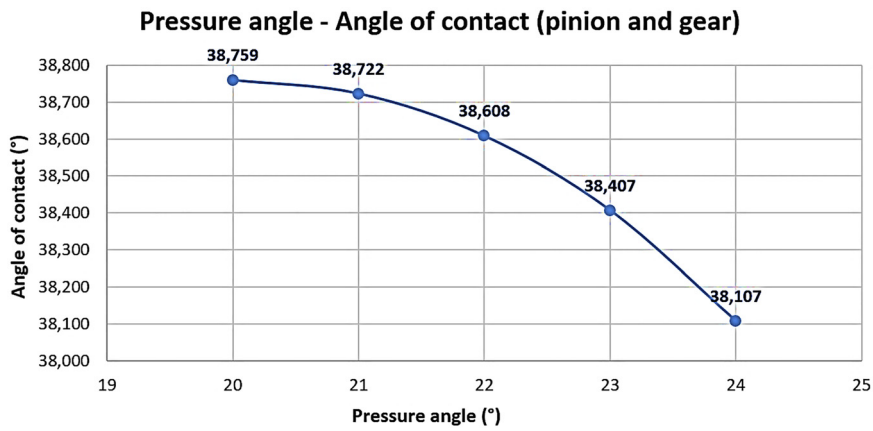


Fig. 17. The correlation between the pressure angle and the angle of contact

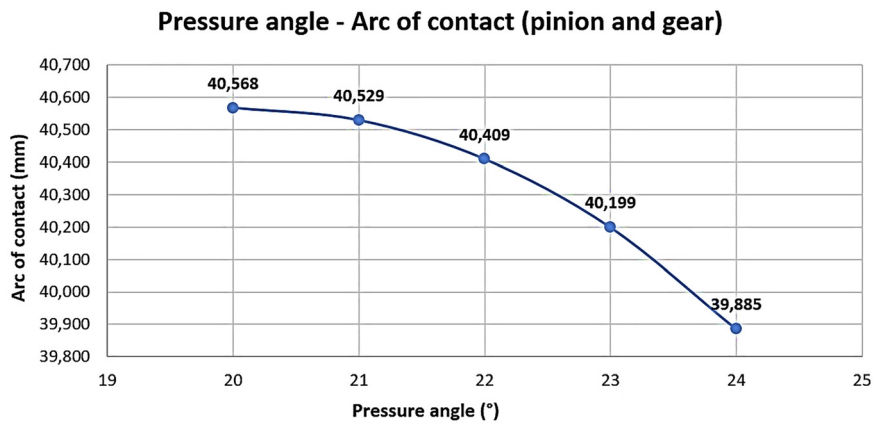


Fig. 18. The correlation between the pressure angle and the arc of contact

shape of the diagram is parabola. The arc of contact is exponentially decreasing while the pressure angle is increasing. The highest result is in the case of  $\alpha_w = 20^\circ$ . The lowest result is in the case of  $\alpha_w = 24^\circ$ .

The correlation between the pressure angle ( $\alpha_w$ ) and the cutting force for one edge ( $F_{c1}$ ) can be seen in Fig. 19. I got higher results for the pinion than for the gear. The results

are increasing in the function of the pressure angle in the case of the pinion. The highest result is received in the case of  $\alpha_w = 24^\circ$ . The shape of the diagram is parabola. The higher the pressure angle, the higher the cutting force for one edge on the tool tooth.

The highest result is received in the case of  $\alpha_w = 23^\circ$  on the gear. The results are continuously increasing in the



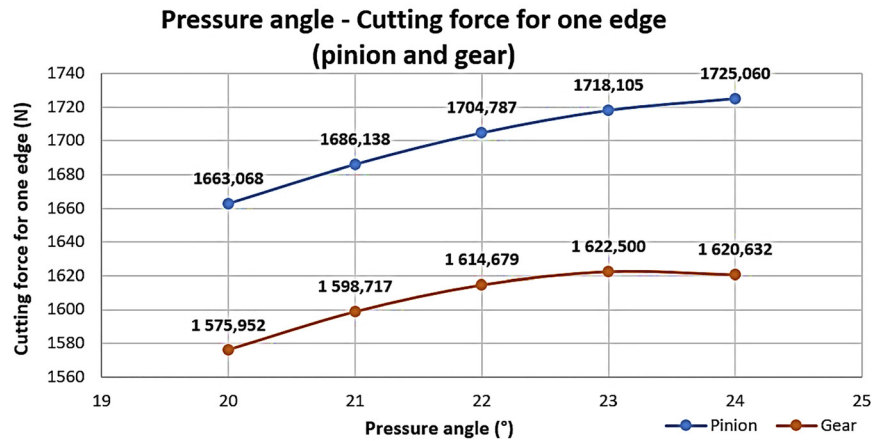


Fig. 19. The correlation between the pressure angle and the cutting force for one edge on the tool

function of the increasing pressure angle until  $\alpha_w = 23^\circ$ . The shape of the diagram is a parabola until this main point. The lowest result is in the case of  $\alpha_w = 20^\circ$ .

The correlation between the pressure angle ( $\alpha_w$ ) and the total cutting force ( $F_c$ ) can be seen in Fig. 20. I got higher results for the pinion than for the gear. The results are continuously increasing in the function of the

increasing pressure angle until  $\alpha_w = 23^\circ$  in both cases. The shapes of the diagrams are a parabola until these main points. The highest results are in the case of  $\alpha_w = 23^\circ$  in both cases. The lowest results are in the case of  $\alpha_w = 20^\circ$  in both cases.

The correlation between the pressure angle ( $\alpha_w$ ) and the switch number ( $\psi$ ) can be seen in Fig. 21. I got the same

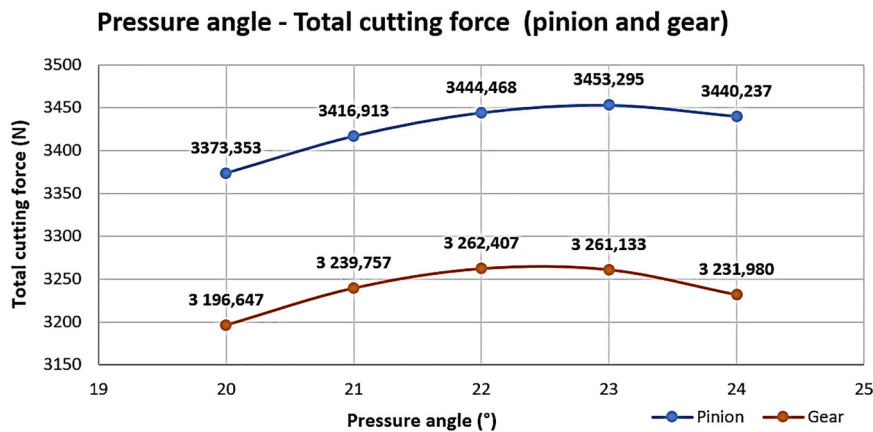


Fig. 20. The correlation between the pressure angle and the total cutting force on the tool

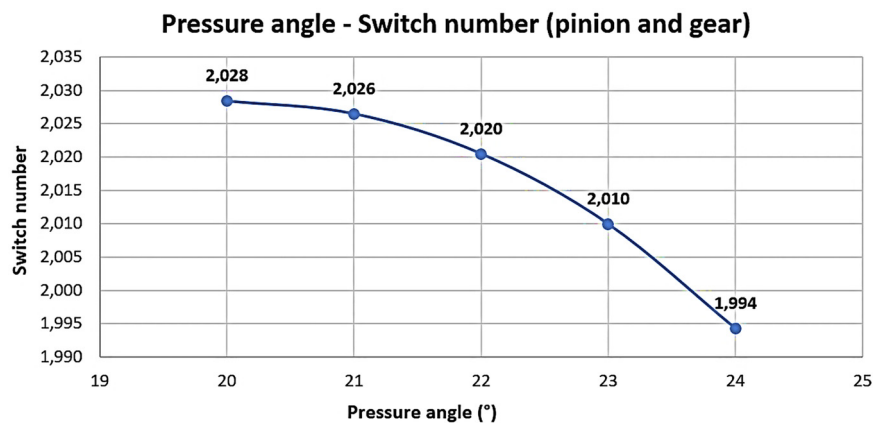
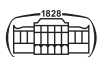


Fig. 21. The correlation between the pressure angle and the switch number



results for the cohesive, connecting pinion and gear. The shape of the diagram is a parabola. The higher the pressure angle, the lower the switch number. The highest result is received in the case of  $\alpha_w = 20^\circ$ . The lowest result is received in the case of  $\alpha_w = 24^\circ$ .

The correlation between the pressure angle ( $\alpha_w$ ) and the cutting power ( $P_c$ ) can be seen in Fig. 22. I got higher results for the pinion than for the gear. The shapes of the diagrams are a parabola until  $\alpha_w = 23^\circ$  in both cases. The results are continuously increasing in the function of the increasing pressure angle until this main point. The highest results are

in the case of  $\alpha_w = 23^\circ$  in both cases. The lowest results are in the case of  $\alpha_w = 20^\circ$  in both cases.

The correlation between the pressure angle ( $\alpha_w$ ) and the machining time ( $T$ ) can be seen in Fig. 23. The shapes of the diagrams are a parabola in both cases. I got higher results for the gear than for the pinion. The main reason is the higher number of teeth around the perimeter of the gear (Table 1). The higher the pressure angle, the less the machining time in both cases. I got the highest results in the case of  $\alpha_w = 20^\circ$  for the cohesive, connecting pairs. I got the lowest results in the case of  $\alpha_w = 24^\circ$  for the cohesive, connecting pairs.

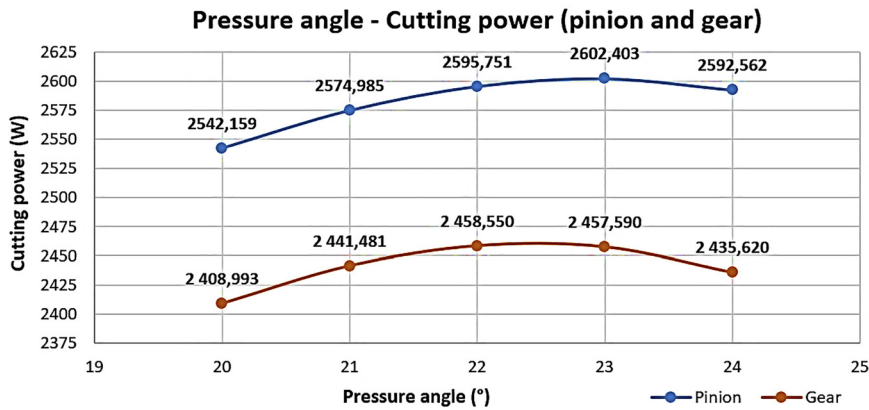
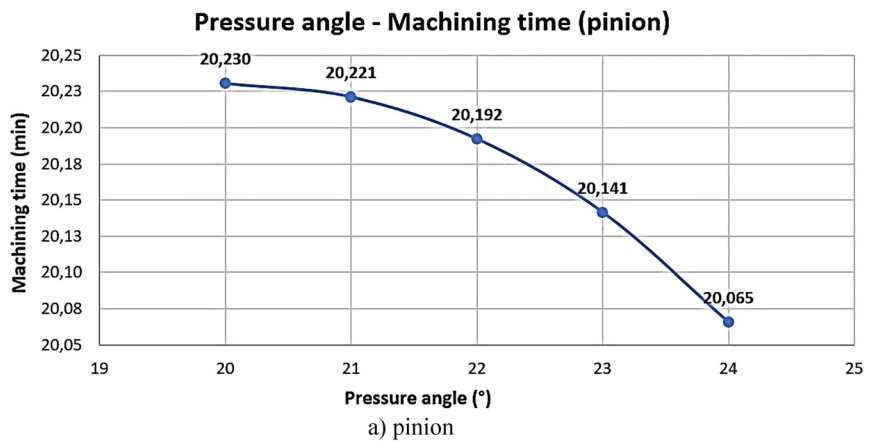
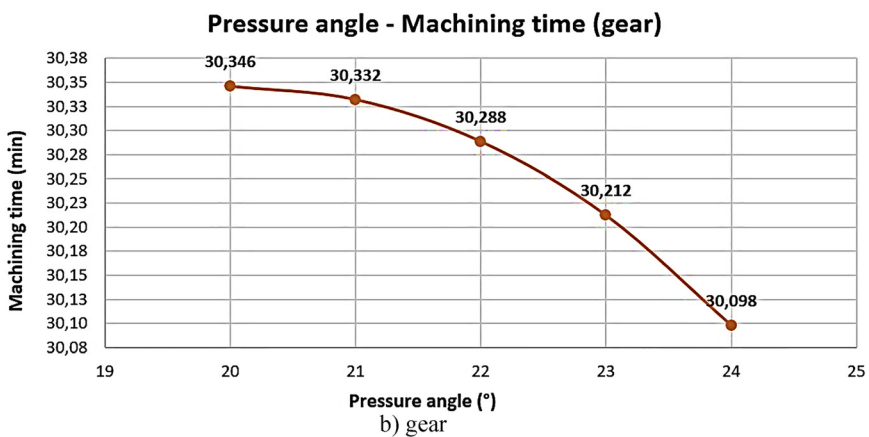


Fig. 22. The correlation between the pressure angle and the cutting power



a) pinion



b) gear

Fig. 23. The correlation between the pressure angle and the machining time



## 4. CONCLUSION

The aim of this study is to find correlations between the modified geometric parameter, that is the pressure angle and the manufacturing parameters in the case of gear cutting by module disc milling cutter for the pinion and the gear. This technology can be executed in a conventional way (using a horizontal knee type milling cutter) or a computer numerical controlled way (using of a CNC milling machine). In this work, I determined the necessary technological parameters for both cases in a general way.

Firstly, I designed five types of connecting gear pairs where the difference between the initial parameters was the pressure angle beside the constancy of the other initial geometric parameters. I made a computer program in MATLAB language to enhance the design time and process for the output geometric parameters and the involute profile points. The received geometric results can be imported into the SolidWorks designer software where the CAD models can be generated for the tooth contact analysis (TCA) and the computer aided manufacturing (CAM) analysis. The CAM analysis is important for making CNC programs for CNC machines among other things if we choose CNC manufacturing for the gears.

I selected the conventional manufacturing process for the gears since this is the oldest way for which the newest methods are built up. I determined all of the necessary technological parameters for the manufacturing design in a general way. After that, I chose initial parameters with concrete values for the manufacturing design. I made an Excel table to determine the manufacturing parameters for the pinion and the gear. Considering the results, I made diagrams for the possible correlations of the analysed technological parameters and the pressure angle. I determined the consequences.

This study is theoretical and practical at the same time. In a theoretical way, there are a lot of ways to continue this research. In a practical way, this study can help the manufacturing engineers to design such manufacturing technologies for spur gears since my developed process is general. The received formulas are useable for the manufacturing of different types of spur gears by module disc milling cutter in a conventional or CNC way.

Obviously, changing the tooth space geometrically a different type of module disc type milling cutter is needed. It is another field how it is possible to design the tool for these manufacturing problems.

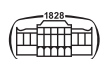
## ACKNOWLEDGEMENT

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**NOMINATIONS**

Symbol	Unit	Parameter
$\varphi$	[°]	The sum of the involute angle and the angle between the arbitrary radius and the base circle radius
$\alpha$	[°]	Angle between the arbitrary radius and the base circle radius
$\rho$	[mm]	Curvature radius of the involute curve
$\sigma$	[°]	Angle between the basic circle radius and the tooth centre line
$\psi$		Switch number
$\alpha_a$	[°]	Angle between the outside circle radius and the base circle radius
$\varphi_c$	[°]	Angle of contact
$\vec{v}_c$	[m min <sup>-1</sup> ]	Real cutting speed
$\vec{v}_f$	[mm min <sup>-1</sup> ]	Feed speed
$c'_0$		Clearance factor ( $c'_0 = 0.25$ )
<b>a</b>	[mm]	Normal centre distance
<b>a<sub>0</sub></b>	[mm]	Elementary centre distance
<b>c</b>	[mm]	Clearance
<b>C</b>		Main point
<b>CAD</b>		Computer Aided Design
<b>CAM</b>		Computer Aided Manufacturing
<b>CNC</b>		Computer Numerical Control
<b>D</b>	[mm]	Diameter of the module disc milling cutter
<b>d<sub>a1</sub></b>	[mm]	Outside circle diameter of the pinion
<b>d<sub>a2</sub></b>	[mm]	Outside circle diameter of the gear
<b>d<sub>b1</sub></b>	[mm]	Base circle diameter of the pinion
<b>d<sub>b2</sub></b>	[mm]	Base circle diameter of the gear
<b>d<sub>f1</sub></b>	[mm]	Root circle diameter of the pinion
<b>d<sub>f2</sub></b>	[mm]	Root circle diameter of the gear
<b>d<sub>p1</sub></b>	[mm]	Pitch circle diameter of the pinion
<b>d<sub>p2</sub></b>	[mm]	Pitch circle diameter of the gear
<b>d<sub>w1</sub></b>	[mm]	Rolling circle diameter of the pinion
<b>d<sub>w2</sub></b>	[mm]	Rolling circle diameter of the gear
<b>F<sub>c</sub></b>	[N]	Total cutting force
<b>F<sub>c1</sub></b>	[N]	Cutting force for one edge
<b>f<sub>z</sub></b>	[mm]	Feed for one edge
<b>h</b>	[mm]	Whole depth
<b>h'</b>	[mm]	Working depth
<b>h<sub>a</sub></b>	[mm]	Addendum
<b>h<sub>f</sub></b>	[mm]	Dedendum
<b>h<sub>m</sub></b>	[mm]	Medium chip thickness
<b>i</b>	[mm]	Arc of contact
<b>inv α</b>	[°]	Arbitrary involute angle (polar angle)
<b>inv α<sub>a</sub></b>	[°]	Involute angle of the outside circle
<b>inv α<sub>p</sub></b>	[°]	Involute angle of the pitch circle
<b>inv α<sub>w</sub></b>	[°]	Involute angle of the rolling circle
<b>j<sub>s</sub></b>	[mm]	Backlash

(continued)

Continued

Symbol	Unit	Parameter
<b>k<sub>c</sub></b>	[N mm <sup>-2</sup> ]	Specific cutting force
<b>l<sub>m</sub></b>	[mm]	Tooth length
<b>m<sub>ax</sub></b>	[mm]	Transverse module
<b>m</b>	[mm]	Distance between the tool centre line and the corner point of the workpiece
<b>n</b>	[min <sup>-1</sup> ]	Adjustable number of revolution
<b>O<sub>1</sub>, O<sub>2</sub></b>		Middle points of the pinion and the gear
<b>P<sub>c</sub></b>	[W]	Cutting power
<b>P<sub>p</sub></b>	[mm]	Circular pitch on the pitch circle
<b>P<sub>w</sub></b>	[mm]	Circular pitch on the rolling circle
<b>r</b>	[mm]	Arbitrary radius of the involute curve
<b>r<sub>b</sub></b>	[mm]	Base circle radius
<b>r<sub>p1</sub></b>	[mm]	Pitch circle radius of the pinion
<b>r<sub>p2</sub></b>	[mm]	Pitch circle radius of the gear
<b>r<sub>w1</sub></b>	[mm]	Rolling circle radius of the pinion
<b>r<sub>w2</sub></b>	[mm]	Rolling circle radius of the gear
<b>s<sub>a</sub></b>	[mm]	Tooth (arc) thickness on the outside circle
<b>s<sub>p1</sub></b>	[mm]	Tooth (arc) thickness of the pinion on the pitch circle
<b>s<sub>p2</sub></b>	[mm]	Tooth (arc) thickness of the gear on the pitch circle
<b>s<sub>w1</sub></b>	[mm]	Tooth (arc) thickness of the pinion on the rolling circle
<b>s<sub>w2</sub></b>	[mm]	Tooth (arc) thickness of the gear on the rolling circle
<b>t</b>	[mm]	Tooth pitch on the module disc milling cutter
<b>T</b>	[min]	Machining time
<b>t<sub>a</sub></b>	[mm]	Tooth pitch on the outside circle
<b>TCA</b>		Tooth Contact Analysis
<b>t<sub>p</sub></b>	[mm]	Tooth pitch on the pitch circle
<b>u</b>		Tooth ratio
<b>V<sub>1</sub>, V<sub>2</sub></b>	[mm <sup>3</sup> ]	Approximate chip volumes
<b>V<sub>h</sub></b>	[mm <sup>3</sup> ]	Separated chip volume
<b>w<sub>a</sub></b>	[mm]	Tooth space on the outside circle
<b>x<sub>c</sub></b>		Addendum modification coefficient
<b>x, y</b>		Coordinates of the given point on the involute curve
<b>x<sub>1</sub>, x<sub>2</sub></b>		Addendum modification coefficient of the pinion and the gear
<b>x<sub>1o</sub>, x<sub>2o</sub></b>	[mm]	Overruns
<b>y</b>		Centre distance increment
<b>z<sub>1</sub></b>		Number of teeth of the pinion
<b>z<sub>2</sub></b>		Number of teeth of the gear
<b>z<sub>t</sub></b>		Number of teeth around the perimeter of the module disc milling cutter
<b>α<sub>p</sub></b>	[°]	Base profile angle
<b>α<sub>w</sub></b>	[°]	Pressure angle
<b>∑ x</b>		Sum of the addendum modification coefficients

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