

# Towards Equations for Brain Dynamics and the Concept of Extended Connectome

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## Abstract

The brain is a system at the edge of chaos equipped with nonlinear dynamics and functional energetic landscapes. However, still doubts exist concerning the type of attractors or the trajectories followed by particles in the nervous phase space. Starting from a system governed by differential equations in which a dissipative strange attractor coexists with an invariant conservative torus, we developed a 3D model of brain phase space which has the potential to be operationalized and assessed empirically. We achieved a system displaying both a torus and a strange attractor, depending just on the initial conditions. Further, the system generates a funnel-like attractor equipped with a fractal structure. Changes in three brain phase parameters lead to modifications in funnel's breadth or in torus/attractor superimposition. We have found that the higher frequencies of evoked activities are more deterministic due to the greater funnel breadth with decreasing degrees of freedom. In contrast, the resting state is formed by lower frequencies represents greater degrees of freedom. Thus, our model explains a large repertoire of brain functions and activities, such as sensations/perceptions, memory and self-generated thoughts.

## Keywords

Brain Dynamics; Strange Attractor; Invariant Torus; Nonlinear Systems

## Introduction

The functional neocortex is the connectome, a synoptically interconnected network of neurons. The energy need of synaptic activation depends of the frequency of use, which decreases the resistance over time. Thus, synaptic transmission is not a proportional with the energy of activation, but dependent on many factors, such as the synaptic conditions and state, neurotransmitter presence, chemical environment and the ATP/ADP ratio. These conditions may result in less, equal, or more energy being transferred than arrived at the bouton. Part of the energy transmitted from one neuron to the next creates a memory trace in the synapse, which enhances the efficiency of future transmissions. Therefore, the brain is a complex, non-linear system, formed by a large number of interacting

and inter-dependent components which exhibit emergent properties, spontaneous self-organization, avalanches [1, 2, 3] and sensitivity to initial small changes. Synaptic transmission is susceptible to unpredictable behavior via the hierarchy of physical organization of the brain. Furthermore, it seems likely that the neural states might

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be unique to specific mental states. The brain might be viewed as a single interconnected feedback system that can be described by mathematical phase space analysis, which can be flexibly and speedily reconfigured: many brain areas are capable of carrying out a variety of functions and are able to switch between those functions in a context-sensitive, dynamic fashion [4].

It has been proposed that the brain operates at the edge of chaos, near a critical regime, where the maximum information function lies between randomness and regularity [5, 6, 7]. The concept of nonlinear brain needs to be framed into the energy landscape theory, originally built for a statistical description of protein's potential surfaces [8, 9]. Such landscape is characterised not just by low-energy valleys - stationary points where the gradient vanishes -, but also by high-energy peaks and transition states. When nervous activity takes place, the energy tends to converge towards a stable state equipped with a minimum energy level, so that the interplay between neuronal structure and activity at many different spatiotemporal scales gives rise to functional "attractors" [10, 11] that emphasize the importance of random walks [12], metastability [13] and self-organized criticality [14]. Although the simplest candidate in which local minima might occur is a simple "fixed-point attractor", which is a funnel located in a functional phase space where trajectories converge as time progresses, other models have also been proposed: for example, resting state networks might emerge as structured noise fluctuations around a stable low firing activity equilibrium state in the presence of latent "ghost" multistable attractors. [15]. It has been also suggested that brain function does not exhibit erratic brain dynamics nor attractors, rather a stable sequence, called transient heteroclinic channel [13]. A foremost issue in such a context is the scale-free dynamics – also called  $1/f^\alpha$  behavior, power law, self-similarity, fractal-like distribution [16, 17, 18, 19].

Conversely, it has been recently proposed that mental operations follow constrained, topological donut-like trajectories along preferential functional brain railways [20, 21, 22, 23, 24]. Experimental and theoretical clues allow us to conjecture that some brain activities are shaped in guise of a multidimensional torus. The theoretical claims of brain multidimensionality are widespread [3, 25, 26] as an example, models characterized by dimensionality reduction have been used in the study of human central nervous system [27]. It has been demonstrated that high dimensionality brain spontaneous activity structures occur,

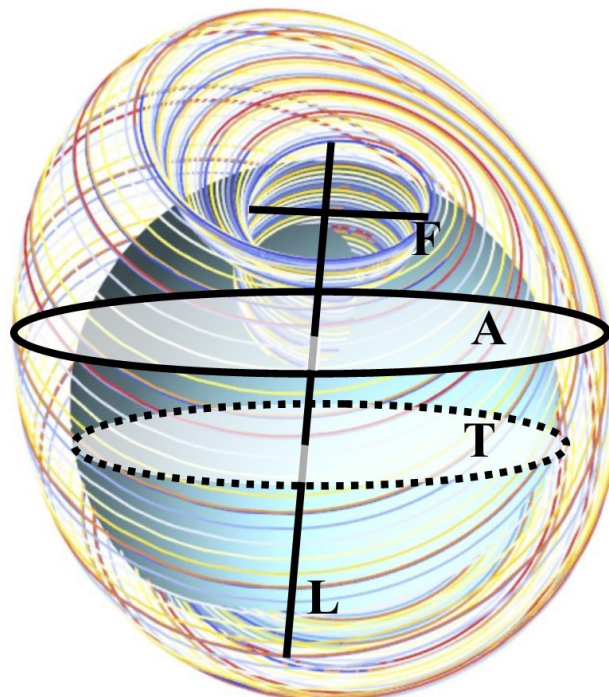
consisting of multiple, reproducible temporal sequences [28]. It has also been shown that the exceedingly diverse nonlinear selectivity in single-neuron activity in the prefrontal cortex is a signature of high-dimensional neural representations: crucially, this dimensionality is predictive of animal behaviour as it collapses in error trials [29]. In addition, recent findings suggest that nervous structures process information through topological as well as spatial mechanisms: for example, hippocampal place cells might produce topological templates in order to represent spatial information [30]. A torus displays a donut-like shape: it means that the trajectory followed by a particle inside it, is closed. To make an example, if one walks along one of the curves of a torus, she perceives in traversing in a straight trajectory, even though her environment is embedded in a higher dimension. The torus may be compared with a video game with biplanes in aerial combat: when a biplane flies off one edge of gaming display, it does not crash but rather it comes back from the opposite edge of the screen [31]. Mathematically speaking, the display edges have been "glued" together. Our brain exhibits similar behaviour, i.e., the unique ability to connect past, present and future events in a single, coherent picture [24, 32, 33], as if we were allowed to watch the three screens of past-present-future glued together in a mental kaleidoscope. The same occurs during other brain functions, e.g., memory retrieval, recursive imagination and mind wandering [34], in which concepts flow from one state to another. The torus is naturally visualized intrinsically, by ignoring any extrinsic properties a surface may have: in the same way, we humans perceive our thoughts intrinsically and naturally adopt "private", subjective or personal standpoints. The concept of personality corresponds very closely with the concept of the strange attractor [35]. If the brain is considered a chaotic system, accompanied by a quasi-attractor pattern in phase space, then a personality can be seen as a logical and necessary consequence. The brain, like other extremely complex systems, is unpredictable in the level of detail but roughly predictable on the level of structure. This means that the dynamics of its physical variables display a strange attractor with a complex structure. Thus, consciousness can be viewed as a phenomenon in Physics [36] and the complex neural dynamics of the brain can be studied in the framework of state space dynamics and criticality. In Physics, concepts of phase space transitions and the Renormalization Group are powerful tools for interpreting phenomena involving many scales of length and time in complex systems. The significance of these concepts lies

in their accounting for the emergence of different levels of new collective behaviors in complex systems, each level with its distinct ontology, organization and laws, as a new pattern of reality.

Earlier we have shown that the brain forms an energy structure, which is insulated from the environment. Thus, sensory interaction is carefully regulated based on energy/information exchange, which engenders self-organization and criticality [22]. Via via self-regulation the brain formulates low entropy, so called resting state.

In this multifaceted framework, a recently proposed three-dimensional, continuous, time-reversible system - governed by a set of first-order ordinary differential equations (ODEs) -comes into play. This system has an unusual property: it exhibits conservative behavior for some initial conditions, and dissipative for others [37]. The conservative regime has quasi-periodic orbits whose amplitude depends on the initial conditions, while the dissipative regime is chaotic. It means that, in the state space, a strange attractor coexists with an invariant torus. Our aim was to build a neural model, starting from the Sprott's ODEs, which might explain brain dynamics during different mental activities.

**Figure 1:** Depicts four measurable system's features. In this case, as an example, we displayed the system originating from the initial conditions 0.5002, -0.5, -0.0791. The solid circle A stands for the disk embedded into the strange attractor's maximum transversal diameter, while the solid line F for the diameter of the funnel lying into the strange attractor. The solid line L stands for the length of the funnel lying into the strange attractor, while the dotted circle T for the disk embedded into the torus's circumference



## Materials and Methods

### Sprott's Equations

We built a 3-dimensional phase space based on the unusual Sprott's system, equipped with no equilibrium points (stable or unstable) and only bounded orbits for all initial conditions. The system is described by the following ODEs:

$$\begin{aligned} \dot{x} &= y + 2xy + xz, \\ \dot{y} &= 1 - 2x^2 + yz, \\ \dot{z} &= x - x^2 - y^2. \end{aligned} \quad (1)$$

The only possible solutions are (quasi)-periodic or chaotic, depending just on the starting point of the initial conditions. Notice that Sprott's images stand for the 4D case (x, y, z, t), with time as a (hidden) parameter on the derivatives.

We achieved the evaluable and measurable features described in Figure 1.

## A Model of Brain Phase Space: The Extended Connectome

The next step was to evaluate whether it is possible to assess brain dynamics through our Sprott's system implementation. The natural candidate to the role of nervous phase space is the human connectome. According to the widely accepted description of the connectome's networks, the cortical and sub cortical structures form a hierarchical network, equipped with preferential pathways for fast communication and winner-takes-all mechanisms [38, 39]. However, recent papers unveil the role of a connectome that is more extended than previously believed, investigating network connections in different species: the *Caenorhabditis elegans* globally integrative rich club of neurons [40], the mouse micro scale cortico-thalamic pathway [41], the rodent peripheral nervous system connectivity and cortical midline structures [42], the human sub cortical connectome [43], thalamus [44] and basal ganglia - including striatum and putamen [45]. Taken together, such observations suggest that the nodes and the hubs of the connectome are not confined to the "higher" and phylogenetically recent structures, but are also diffused along the entire neural pathway; including the central and peripheral nervous system. It is possible that, in the years to come, the concept of connectome will be also extended to the peripheral receptors. Indeed, most peripheral structures play a crucial role in information processing: receptors perform complex neural computations that were thought to be carried out by sensory cortex and have a prominent role in sensory processing. As an example, the brain has outsourced some aspects of touch processing to the nerve endings in the fingertips [46]. In sum, we can state that the electric oscillatory activity of the extended connectome – shaped in guise of a phase space - gives rise to a free-energy landscape.

Our hypothesis is that external (or internal) inputs activate just a point on a 3D brain phase space (the connectome), and the activation of such a point leads to a trajectory displaying the shape of either a torus, or a strange attractor. The particle trajectories depend just on its initial location in the phase space.

We evaluated the system behavior during increases or decreases of every one of the three brain parameters, in order to detect which type of trajectory is achieved when a single change in  $x$ ,  $y$  or  $z$  occurs. The values of some of the three coordinates were extracted from real-data plots of studies based on EEGs and fMRI neurotechniques (18, 29, 47, 48)

## Results

We achieved a system displaying both a torus and a strange attractor, depending on the starting conditions, i.e., the initial points where the trajectories started their movements. Figure 2 shows the results of our implementation of Sprott's attractors with different initial conditions ( $x, y, z$ ). A noteworthy result is that the strange attractors display funnel-shape morphology, with the appearance of a twisting funnel that dips in and out of the sphere: it means that the funnel is embedded in a fractal structure. The funnel is more or less narrow, depending on the initial conditions: to make an example, when  $x$  increases, the funnel is much narrower, while, when  $y$  increases, the funnel is larger. Depending from initial conditions, the strange attractor and the torus are more or less superimposed: i.e., when  $y$  changes, the strange attractor and the torus are closely intertwined, making difficult to evaluate whether a starting point leads to a fractal or non-fractal trajectory; vice versa, when  $z$  increases, the strange attractor surrounds the torus in guise of a cloud; furthermore, when  $x$  increases, the two structures are neatly separated.

In particular, we achieved the following results:

### When the initial conditions are 0, 0, 0, the following system is displayed

L and T are partially superimposed and intertwined.  
T diameter = A diameter  
F = high values (a large funnel)

### When the initial conditions are 1, 0, 0

L and T are sharply separated  
 $T < A$   
F = the minimum values (very narrow funnel's diameter)

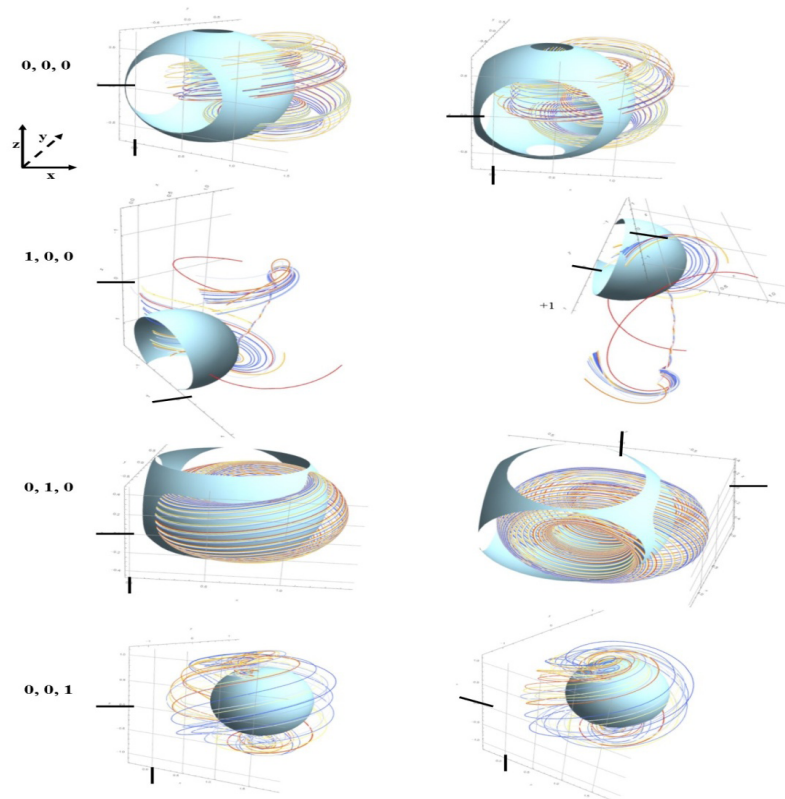
### When the initial conditions are 0, 1, 0

T (almost completely) includes and surrounds L.  
 $T > A$   
A is partially embedded into T  
F = the highest values (the widest funnel)

### When the initial conditions are 0, 0, 1

A includes and surrounds T  
 $A > T$   
T is embedded into A  
F = low values (narrow funnel)

**Figure 2:** Implementation of Sprott’s attractors for four different values of  $x, y$  and  $z$ . The black lines show the points zero on the respective axis. See the main text for further details



## Conclusions

Self-organized criticality (SoC), a spontaneous dynamic state established and maintained in complex networks, is a universal characteristic of neural systems [49]. We investigated how the energy/information flow via synaptic transmission in the brain is related to psychological experiences, such as emotion and qualia. In the brain blood glucose levels are maintained by careful endocrine control, with the primary hormones insulin and glucagon acting in a complementary manner. The brain’s high metabolism is ensured by the disproportionate consumption of oxygen and glucose. For example, the brain’s local activity can be inferred by energetic changes registered by fMRI, PET, and EEG, because sensory stimulation is associated with a sharp increase in neuronal 2-deoxyglucose uptake [50]. In neurons, most ATP is dedicated to supporting synaptic transmission [51] and energy restrictions are often implicated in the brain’s vulnerability to hypoxic-ischemic damage, and the reduction of consciousness. Thus, it is expected that the brain’s available energy supply shows close relationship to the frequencies and amplitude of electric activation. Namely, the ATP/ADP

ratio places an upper limit on the neurons energy use, which encourages the formation of frequencies away from their energetic mean value, leading to a power law of oscillation frequencies. In addition, the memories formed by the network of synaptic connections function as preferred paths that act as attractors in the phase space. Attractor networks naturally stabilize by seeking energy minima, and the relative positions of basins of attraction define the geometry of an energy landscape [4, 52]. As a result, the transition into an active attractor state occurs as a transition into an information/energy maximum.

Starting from the peculiar Sprott’s system of ODEs, we built a neural testable model equipped with both a conservative torus and a dissipative strange attractor. When a moving particle starts its trajectory from a given position  $x, y, z$  in the 3D nervous phase space, we may predict whether it will fall in the torus or into the strange attractor. The funnel shape is fractal, and not just a simple fixed-point attractor. A narrower funnel means that the trajectory is constrained towards a small zone of the phase space. When the two structures are closely superimposed, we might hypothesize a state of phase transition at the

edge of the chaos, equipped with high symmetry, in which it is difficult to evaluate every single initial position: a slightly change in the starting point could indeed lead to completely different outcomes. When the torus and the strange attractor are clearly split, a single starting point gives rise to a sharply defined outcome. It means that in the latter case, the two conformations are neatly separated, as if the system went out of phase transition and a symmetry breaking occurred.

Looking for possible psychological correlates of our system, we hypothesize that the proposed scheme might stand for a large repertoire of brain functions and activities, such as sensations/perceptions, emotions, mood-state/affect, memory, abstraction, sequencing/planning, choice, judgment, creativity, self-generated thoughts [34]. For example, when the particle falls into the conservative torus conformation (where two antipodal particles cannot never meet or become closer, due to their recurring rotations) the correlated psychical activities could be either repetitive patterns of movements or thoughts, or preserved memories, or mind wandering, or unsure perceptions, in which different ideas are not properly melted together. In turn, when the particle falls into the dissipative strange attractor conformation (equipped with steep and narrow funnels), the correlated psychical activities could be driven by strong emotional impetus of survival, or motivation, focus, such as goals, pride, aggravation and others.

We might hypothesize that, due to external or internal inputs, changes take place in one or more of the three parameters. Such changes activate a single point of the 3D functional brain lattice, where a “particle 1” starts its trajectory into one of the two described dynamical conformations. When another input occurs, the corresponding variation in the parameters’ value puts in motion a “particle 2” from a different starting point in the phase space. We might speculate that: a) either the particle 2’ trajectory prevails and cancel the particle 1’s; 2) or the two particles move at the same time, following each one its own orbit; 3) or the particle 1, following the activation of the starting point of the trajectory 2, switches from its trajectory to the trajectory 2, or to another one.

Our model also satisfies the brain free-energy constraints required by the most influential current brain theories [20, 53, 54, 55]. Indeed, spontaneous activity displays a lower energetic level than the evoked one. The energy need and information carrying capacity of brain waves is proportional to their frequencies [56] and all positive emotions and neutral mental states have smaller

energy requirement than negative emotional states [57], showing the relationship between the energy need of brain oscillations to the physiological of mental, emotional experience. As stated above, we know that each spike has a certain consumption of ATP [58] and that each spike is formed by an oscillation, equipped with both amplitude and a frequency. During spontaneous activities, the frequency is very low, but the amplitude is substantial. In contrast, the high frequencies that emerge during evoked activities manifest only small amplitudes. For the Ohm’s law, in the first case, the AMPLITUDE is determinant in energy consumption, whereas in the second case the energy consumption due to the amplitude of the oscillation is negligible, compared with the energy consumption due to its FREQUENCY. This means that the evoked brain activity, equipped with a mean spike frequency higher than the spontaneous activity, spends more energy. Due to their great energy need, great energy requiring activation, such as powerful stimulus or negative emotions represent a sharper funnel that can dramatically decrease the degrees of freedom, because the decrease in temporal dimensionality [20, 21]. However, the exponentially increasing energy gradient taxes the ATP production abilities of the brain and leads to mental, behavioral consequences manifested as lack of patience, aggravation and emotional breakdown. This occurs, because the ATP cannot keep up with the metabolic requirement of neural activation [55, 59]. Therefore, neurodegenerative pathways are thought to converge on energy failure, and synaptic terminals degenerate early in many neurodegenerative diseases associated with impaired energy metabolism [60].

This is in contrast to spontaneous activities, which are the physiological counterpart of the default mode network. This low-energy rewiring activity is characterized by low-frequency, high amplitude oscillations equipped with power laws, infinite correlation length and (probably) falling into strange attractors and phase transitions. Since the low frequencies extinguish information carrying capacity of the brain’s electric activities, it leads to recurring, wayward thoughts with its psychological correlates (such as mind wandering or self-generated thoughts) [34]. During resting sensations, non-dissipative torus is restored. However, we have shown in our earlier work that the resting state is not immune from increasing frequencies [20, 21]. Accumulated emotional energies can interfere with the resting state, leading to more deterministic thought patters. Thus, ongoing brain activity is capable for the dynamic reconfiguration of

neural function [61] depending on the energy landscape.

Of course, ours is just an in-progress, provisional scheme. As suggested by Sprott himself (personal communication), we might also not to stop at three dimensions plus time, and we might take into account no integer dimensions too. For example, two interacting particles can be described as the motion of a single particle in a 6-dimensional space, where three of the dimensions characterize the particle position, and the other three characterize the components of velocity in each of the three spatial directions. A dynamical phase space's landscape (such as the brain) that consists of  $N$  interacting micro areas could be described by a single point moving in a  $2N$ -dimensional phase space, although it will typically be the case that the dynamics is confined to a lower-dimensional subset of that very high-dimension space. It must also be kept into account that the torus, being structurally unstable in dynamical systems with dimensions greater than three, tends to evolve into a strange attractor. It means that, in long timescales, the trajectory inexorably falls into a funnel-like attractor equipped with the lowest possible energetic level, as proposed by protein-folding models based on the principle of minimum frustration [9].

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