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Testing the Lee-Carter model on Hungarian mortality data

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ABSTRACT

This article tests the popular Lee-Carter model's performance for Hungarian mortality rate forecasting. Hungary passed through a mortality crisis which makes the task particularly difficult. Previous forecasts and model choices are validated, and updated forecasts are produced. We find that the behaviour of mortality rates is normalizing, and so the basic Lee-Carter model is becoming applicable.

KEYWORDS

mortality rate forecasting, Lee-Carter model, Hungary

JEL CLASSIFICATION INDICES

G22, J11

1. INTRODUCTION

The number of births across Europe has been steadily declining in the past decades, as a result of numerous socio-cultural, economic, educational and medical factors (Ágh et al. 2018). In addition to birth, it is increasingly important to forecast the long-term mortality rate. However, these

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are hard to produce and validate mortality rates, since annual mortality rate recording does not have a long history.

Assessing Hungarian mortality rates are especially difficult, because male life expectancy decreased from the mid-1960s during the socialist regime (Bálint – Kovács 2015). It was peculiar, since in the developed countries life expectancy increased and mortality rates decreased in this period. The Hungarian mortality crisis lasted until the mid-90, so the recovery happened (and is happening) just recently. Scheiring et al. (2018) produced a systematic review of the case of Hungary, and found it puzzling.

Understanding the behaviour of mortality rates implied by the regime change is a challenge. Large political changes (even if positive) can have huge social and health costs – the mortality crisis of the former communist countries produced 10 million excess deaths between 1990 and 2000 (Cornia – Paniccià 2000). While several important explanatory variables have been identified, the cited authors claim that much research remains to be done.

Baran et al. (2007) produced mortality rate forecasts by dropping most of the available data – all observation before 1989. It seems rational to omit the abnormal mortality rates, however, it also seems to be a great sacrifice, since very little data remains. The aim of this article is to reevaluate this choice of ignoring the communist era, and to produce new forecasts using an updated mortality database.

2. ECONOMIC IMPORTANCE OF THE MORTALITY RATE

Mortality rate forecasting methods, and the Lee-Carter model (presented in the next section) are widely applied. Mortality predictions can help planning healthcare and pension systems, and they also have actuarial applications, such as the pricing of annuities (Renshaw – Haberman 2006). The forecast of pricing of life insurance products depends on the ability of three things (Yan et al. 2019):

- mortality rates by age groups,
- interest rate dynamics, and
- the relationship between mortality events and interest rate dynamics.

Previously, simple and subjective methods were used in mortality forecasting, more sophisticated methods have only been applied recently (Booth – Tickle 2008). There has been an increase in research activity in the recent decades, assisted by freely available databases and software. Mortality forecasting gained importance in recent times due to the unexpected growth in life expectancy. Mortality improved substantially in the past century, and it seemed (and seems) difficult to forecast, this phenomenon is called “mortality risk”. Life expectancy increased steadily for over 150 years, and there is no sign of a slowdown, in a few decades, there may be a country where average life expectancy passes 100 years, if the trend continues (Oeppen – Vaupel 2002). They emphasize that the official forecasts haven’t considered the continuous increase in life expectancy, and this may allow politicians to postpone painful changes in the pension- and healthcare systems. Underestimating mortality improvements is a problem for annuity providers (Blake – Burrows 2001).

Tuljapurkar (2005) claims that the old-age-dependency ratio (the number of old people divided by the number of working age people) underlies pension costs: the more retirees per worker, the more stress on the pension system. Stochastic forecasts of mortality (together with forecasts of fertility) might be used to adjust the retirement age cut-off while keeping the old-age-dependency ratio at a desired level. Lee – Miller (2001) draws attention to the fact that government forecasts are often too pessimistic about the future. Mortality forecasts are often based on subjective judgments and expert opinions, and they can (and, historically, did) lead to a systematic underprediction of the gain in life expectancy. Statistical methods, such as the extrapolative approach of the Lee-Carter model may generate smaller errors. Booth – Tickle (2003) used the Lee-Carter model to produce population projections. They claim that elderly population will be larger in the future than official forecasts suggest, and it is especially true for females and ages of 85+.

The Lee-Carter model may also be applied in macroeconomic studies, since mortality rates often correlate with macroeconomic variables. Reichmuth – Sarferaz (2008) found that mortality rates of young adults differ from the other age groups. The 25 years olds show increased mortality in recessions, while older age groups not. Hanewald (2009) found significant correlations between real GDP and the mortality index (k_t) with the Lee-Carter model in various OECD countries.

3. LEE-CARTER MODEL

The Lee-Carter model is a popular algorithm for mortality rate forecasting. It was originally developed for the United States, but now applied to many other countries. The model uses singular value decomposition (SVD) to find a time series vector k_t that captures most of the overall trend of mortality. (The model might be extended to include multiple such vectors.) The original Lee-Carter model estimates the logarithm of the mortality rate $m_{x,t}$ of age group x at time t (1).

$$ln\left(m_{x,t}\right) = a_x + b_x k_t + e_{x,t} \tag{1}$$

The b_x values measure the changes in log-mortality rates with respect to the changes in the mortality trend k_t.

The estimates of the parameters can be derived using the least squares method, i.e., minimizing (2). However, this problem does not have a unique solution. We need the conditions $\sum_x b_x = 1$ and $\sum_t k_t = 0$ in order to have unique least squares solution.

$$\sum_x \sum_t \left[ln\left(m_{x,t}\right) - a_x - b_x \cdot k_t\right]^2 \tag{2}$$

The above-mentioned conditions imply that the least squares estimates of a_x are the average age-specific log-mortality rates (3). The estimates of b_x (4) and k_t (5) can be obtained from the SVD of the centered log-mortality matrix $M_{x,t} = ln(m_{x,t}) - \widehat{a}$, in the form of $M = UDV^T$. The formulas below already normalize the estimates of b_x to sum to 1. $D_{1,1}$ is the largest singular value – the original Lee-Carter model uses only the first singular value to produce the estimates.[1] The estimates of future mortality rates are then produced by substituting forecasted values of k_t (6).

[1]A more detailed description of the parameter estimation of the Lee-Carter model can be found in Baran et al. (2007).

$$\widehat{a}_x = \frac{1}{T}\sum_t ln\left(m_{x,t}\right) \tag{3}$$

$$\widehat{b}_x = \frac{U_{x,1}}{\sum_x U_{x,1}} \tag{4}$$

$$\widehat{k}_t = V_{t,1}D_{1,1}\sum_x U_{x,1} \tag{5}$$

$$\widehat{m}_{x,T+t} = \exp(\widehat{a}_x + \widehat{b}_x \cdot \widehat{k}_{T+t}), t = 1, 2, \ldots \tag{6}$$

Some generalizations of the above-described original Lee-Carter model naturally arise. One such extension is keeping multiple singular vales ($D_{i,i}$) from the SVD (Booth et al. 2001; Baran et al. 2007). This means keeping more information, which was deemed unnecessary for the original model that was proposed for US mortality rates, but it may be useful and may provide better approximations for certain countries, especially for the countries with anomalous mortality patterns due to non-trivial political reasons. In such cases, the multi-factor Lee-Carter model may provide a better estimate.

Forecasts might be produced by predicting future k_t values using time series models. Autoregressive models are a reasonable and popular choice. We are using the Box-Jenkins method (Box et al. 2015) to find the best fitting ARIMA models. We have implemented the Lee-Carter model in *R* (Team et al. 2013). The *forecast* package R (Hyndman and Khandakar, 2007) was used to produce ARIMA forecasts.[2]

4. DATA

Our dataset was obtained from mortality.org (University of California, Berkeley (USA), the Max Planck Institute for Demographic Research (Germany), and from the Hungarian Central Statistical Office. A detailed description of the data sources is available at the website.[3] We have

Table 1. Evaluation metrics for 2004–2017, averaged over all ages

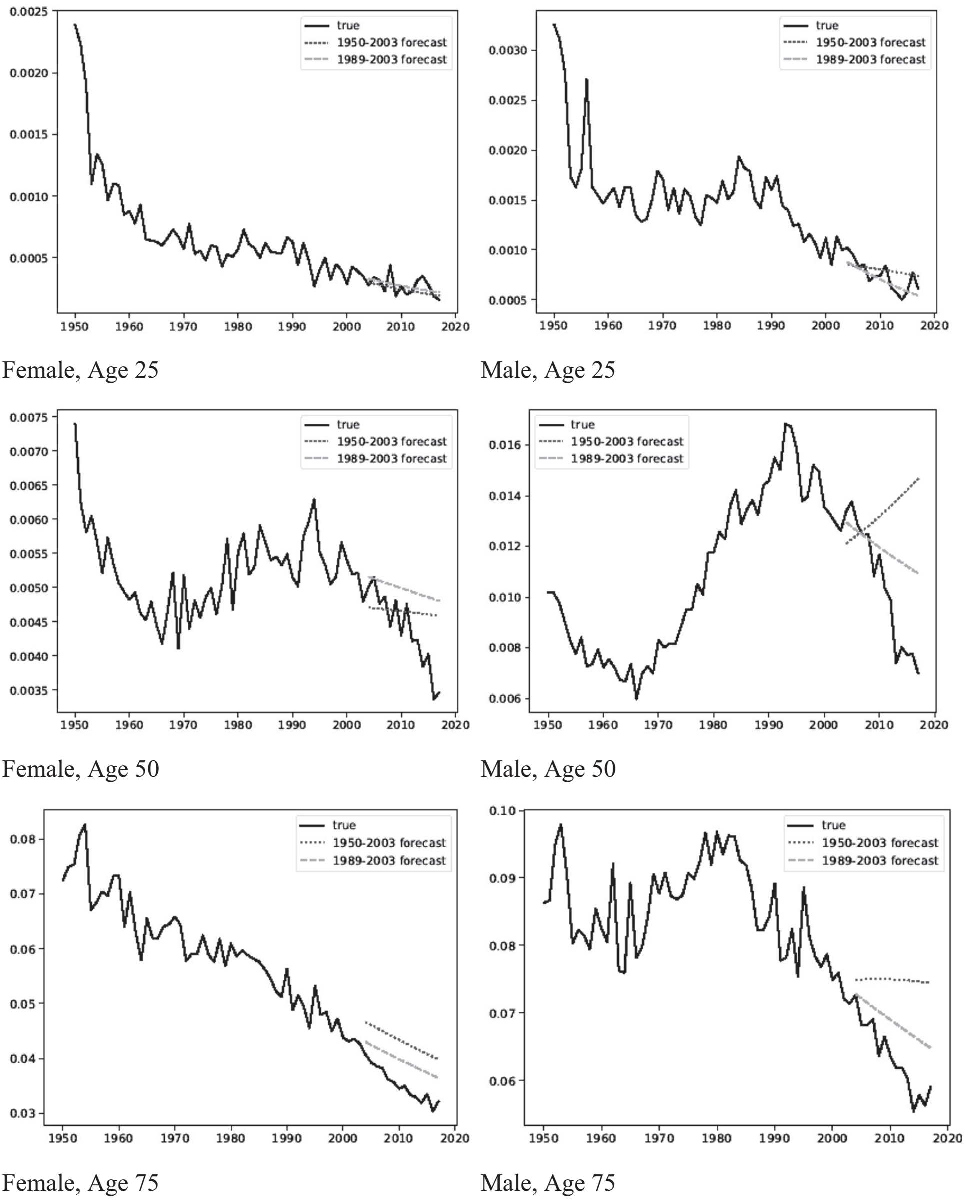
Metric	1950–2003 model	1989–2003 model
RMSE	0.0071	0.0040
SMAPE	24.35	17.47
MedAE	0.0058	0.0028

Note: RMSE: Root mean squared error; SMAPE: Symmetric mean absolute percentage error; MedAE: Median absolute error.

[2]The R package forecast provides methods and tools for displaying and analyzing univariate time series forecasts.

[3]https://www.mortality.org/hmd/HUN/DOCS/ref.pdf.

downloaded the latest available life tables of Hungary, separately for females and males. The life-tables contain observations from 1950 to 2017, for each age from 0 to 109 and 110+ years (though we dropped ages above 100 years).



Female, Age 25 Male, Age 25

Female, Age 50 Male, Age 50

Female, Age 75 Male, Age 75

Fig. 1. Forecasts for the 2004–2017 period

5. FORECASTS

We reproduce and test the mortality forecasts of Baran et al. (2007), who used a variant of the Lee-Carter method to forecast Hungarian mortality rates from 2004. Following Booth et al. (2001), they used multiple terms of the SVD to approximate mortality rates (7). We described the intuition for this model choice in the section titled Lee-Carter model.

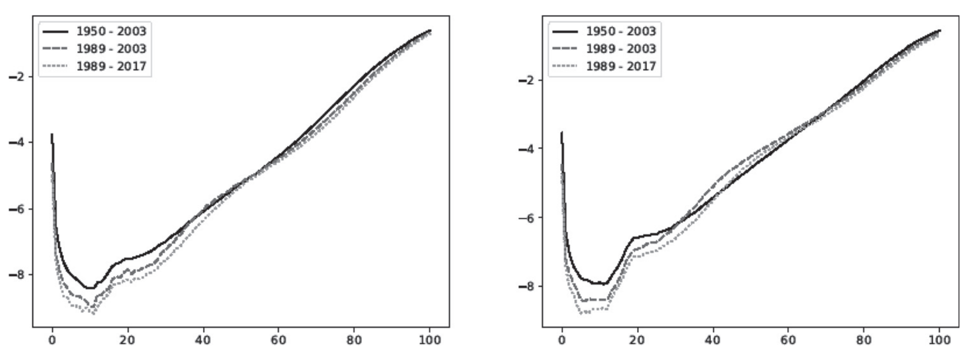
$$ln\left(m_{x,t}\right) = a_x + b_x^{(1)}k_t^{(1)} + b_x^{(2)}k_t^{(2)} + b_x^{(3)}k_t^{(3)} + e_{x,t} \quad (7)$$

The original (one-factor) Lee-Carter model typically works well in the developed democracies, but not in all countries. Hungarian mortality rates are a particularly anomalous case (see, for example, Józan 2008), which was intensively studied in the literature.

The model fitted by Baran et al. (2007) to all available data resulted in increasing mortality rate estimates for middle-aged men. This did not seem credible, and it was the result of peculiar changes in mortality, due to the aforementioned historical reasons. Hence, they used a smaller dataset containing mortality rates for only 15 years (1989–2003). This latter model produced more reasonable estimates, yet the authors expressed their concerns regarding the goodness of predictions.

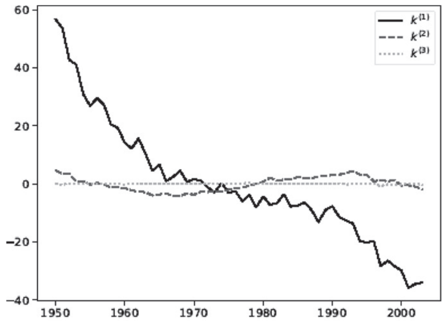
One and a half decades have passed, and now we have data to validate the proposed models. Also, we can use the (nearly) doubled data volume to produce new, and probably more reliable long-term estimates. We applied the same higher order Lee-Carter models, and the same ARIMA models as Baran et al. (2007). The 1950–2003 and 1989–2003 periods were used to fit the models, while the now available 2004–2017 period was used for evaluating the forecasts. We got similar, but not identical results. Our dataset may differ slightly, and it is missing the first year (1949) data. We found that the 1989 –2003 model did, indeed, produce more accurate forecasts for the subsequent 14 years. The results of the evaluation are displayed in Table 1. The forecasts (for 3 arbitrarily chosen ages) are shown in Fig. 1.

We used the 1989–2017 period to refit the model and make forecasts for the next years. The general patterns of mortality across age (a_x) are displayed in Fig. 2. Our updated database produced lower estimated mortality rates in most age groups. The mortality changes (k_t) are shown in Fig. 3. The Box-Jenkins method was used to find the most appropriate time series models for forecasting k_t.

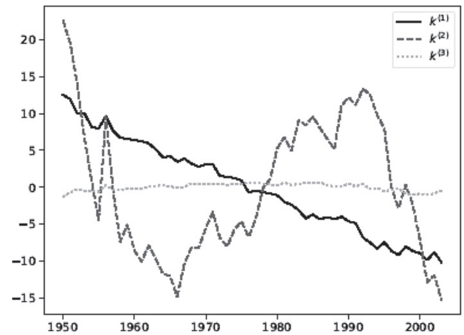


Female Male

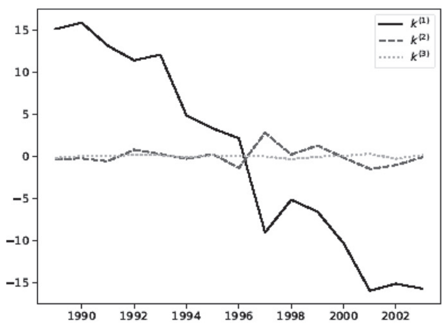
Fig. 2. Age specific mortality, *a*



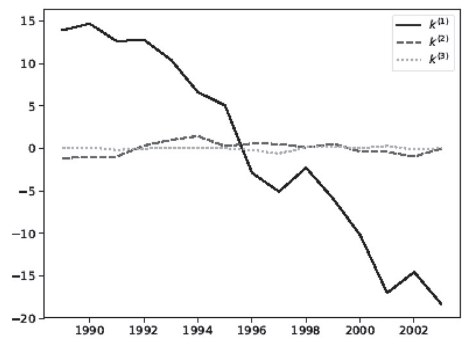
Female, 1950–2003



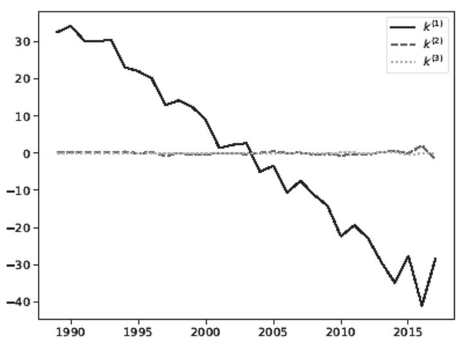
Male, 1950–2003



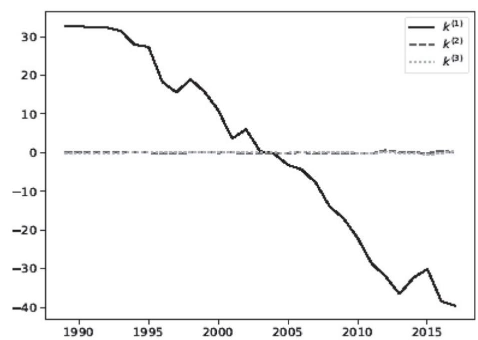
Female, 1989–2003



Male, 1989–2003



Female, 1989–2017



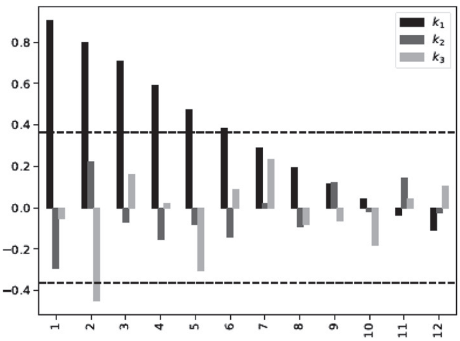
Male, 1989–2017

Fig. 3. Mortality changes over time, ***k***

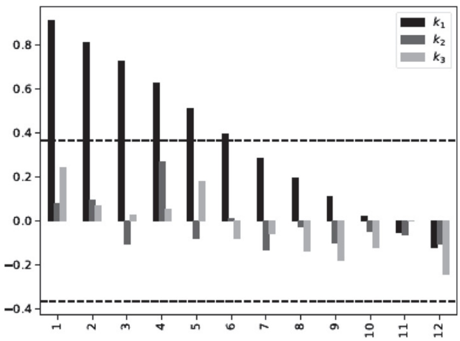
We found that the $k^{(2)}$ and $k^{(3)}$ values can be considered as white noises for either male or female mortality rates. The results of the Ljung-Box test (1978) with 10 lags are available in Table 2. The autocorrelation and partial autocorrelation plots are displayed in Fig. 4. Thus, the use of a higher order Lee-Carter model does no longer seem justified. It suggests that Hungary's

Table 2. Ljung-Box test results

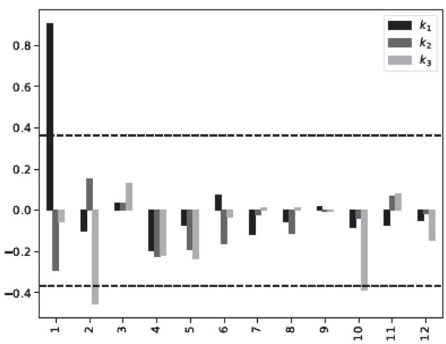
	$k^{(1)}_{\text{male}}$	$k^{(2)}_{\text{male}}$	$k^{(3)}_{\text{male}}$	$k^{(1)}_{\text{female}}$	$k^{(2)}_{\text{female}}$	$k^{(3)}_{\text{female}}$
Q	97.224	7.4648	15.72	102.49	5.072	6.7178
$P-$value	$2.22e^{-16}$	0.681	0.1079	$<2.2e^{-16}$	0.8863	0.7518



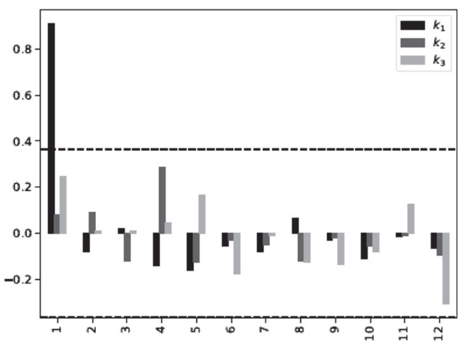
ACF, Female



ACF, Male



PACF, Female

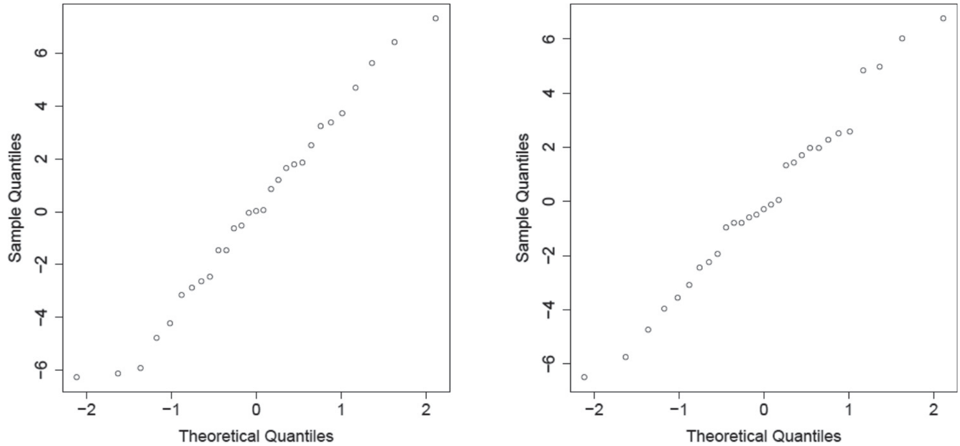


PACF, Male

Fig. 4. ACF and PACF plots

mortality rates are normalizing, and so the original Lee-Carter model is becoming more applicable.

The $k^{(1)}$ series of male mortality rates can be described by an ARIMA (0,1,0) with drift, that is, the same model as the one proposed in the original Lee-Carter method (Lee – Carter 1992). In case of female mortality rates, an ARIMA (1,1,0) with drift model seemed the most suitable. The residuals of both models can be considered white noise. The P-values of the Ljung-Box test for independence with 10 lags are 0.5403 for k^{female} and 0.9634 for k^{male}. The Shapiro-Wilk test's P-values are 0.7921 for k^{female}, and 0.8916 for k^{male}, thus the null hypothesis of



$k^{(1)}_{female}$ $k^{(1)}_{male}$

Fig. 5. Normal QQ plots of residuals

normally distributed residuals cannot be rejected. The residuals' normal QQ plots are displayed in Fig. 5.

The fitted models for $k^{(1)}$ are the following (standard errors in parentheses):

- $k_t^{\text{female}} = -2.4842\ (0.4004) + k_{t-1}^{\text{female}} - 0.7975\ (0.1421) * (k_{t-1}^{\text{female}} - k_{t-2}^{\text{female}}) + \delta_t^{\text{female}}$
- $k_t^{\text{male}} = -2.5842\ (0.6348) + k_{t-1}^{\text{male}} + \delta_t^{\text{male}}$

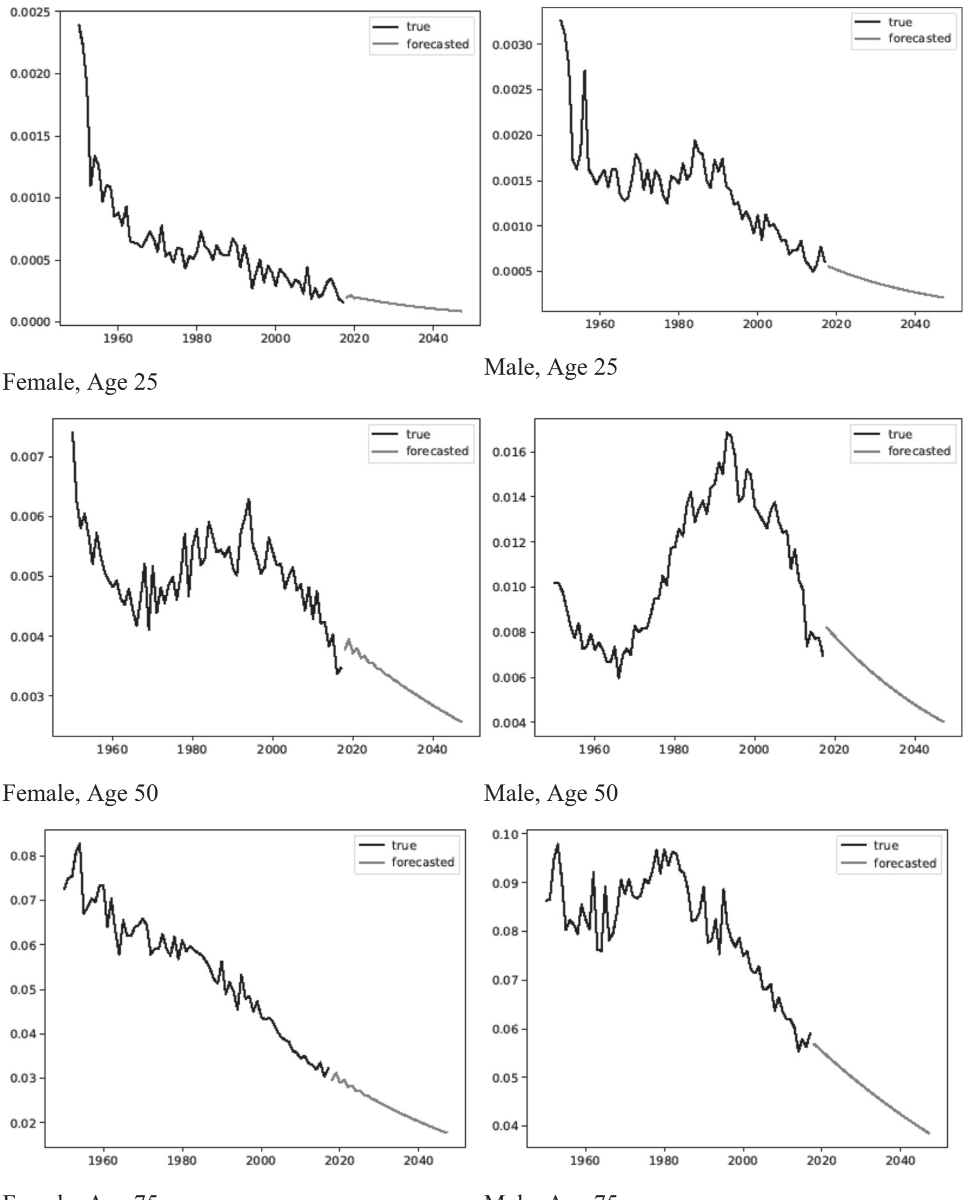
Some sample mortality rate forecasts are displayed in Fig. 6 for the 2018–2047 period.

6. CONCLUSION

We knew from the very beginning, that the popular Lee-Carter model does not work satisfactorily well for the mortality rate in the case of certain countries. The well-known irregularities of Hungarian mortality rates cause difficulties in applying the original model, as well. It does not necessarily mean that the Lee-Carter model is not applicable, the extended models (such as a multi-factor estimate) may produce better forecasts.

In our research, we used Hungary's latest mortality tables to evaluate the earlier forecasts of Baran et al. (2007), and we showed new forecasts for the future. Our results justify that better forecasts can be produced by ignoring the mortality rates from the socialist era. It seems that the past 15 years made the original Lee-Carter model more applicable. It does not mean that a multi-factor Lee-Carter model could not produce better fitting estimates, rather that there is not much difference, and so that it is not reasonable to use a higher order model. We used twice as much data as Baran et al. (2007), which allowed us to produce forecasts with lower standard errors.

Our findings may have economic applications, for example, in macroeconomics or life insurance, since both fields rely on long-term forecasts of mortality. We found that the Lee-Carter



Female, Age 25 | Male, Age 25

Female, Age 50 | Male, Age 50

Female, Age 75 | Male, Age 75

Fig. 6. Forecasts for the future

model is becoming easier to apply as convenient softwares are increasingly available. For example, the automatic ARIMA procedure in the forecast package of *R* can make the forecasting part much easier for those with less knowledge of time series analysis. While we think that it is not currently necessary to apply a higher order Lee-Carter model, it may change in the future. There may be economic or social irregularities that may cause the original model to malfunction. For example, it is possible for the COVID-19 pandemic to produce previously unseen patterns in mortality, which could (once again) disprove the model.

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