



Exact solutions for strongly
interacting
electron systems in $D > 1$ dimensions

Ph.D. thesis

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In my work I have presented exact results related to relevant and up-to-date problems of many-body physics. Important feature of these results are that they apply to higher than one dimensional systems. I emphasized the importance of the exact results, but I also underlined that in this field there are no standard well-tried methods.

My results can be summarized in the following points.

- I generalized Brandt and Gieseke's approach [?] to extended Hubbard model which contains next-nearest neighbour interaction terms and I deduced a ground state phase diagram of the system. My results are valid in all $D > 1$ spatial dimensions at half filling.
 - A completely saturated ferromagnetic phase, different types of commensurate spin density waves, commensurate charge-density waves and a region with phase separation were found, the last one being in our knowledge for the first time signaled at the level of approximation free results in $D > 1$ dimensions.
 - The stability domains of the fully saturated ferromagnetic phase, spin and charge density waves were extended in comparison with previous publications.
 - The presented phase diagram is complete in the localized limit $t_l = X_l \rightarrow 0$ in all dimensions $D > 1$ for arbitrary value of fully anisotropic or non-negative isotropic Heisenberg couplings.
 - The obtained results clearly emphasize that the next-nearest neighbour couplings have their role in building up the subtle balance between the stability domains of different ordered phases in the parameter space, introducing completely new phases or modifying considerably the emergence regions for other phases within the phase diagram.
- Under the conditions in which even the exact solution for the one dimensional periodic Anderson model is not known at finite U , I have presented exact solutions for two dimensional periodic Anderson

model in the interacting case. The described solutions are present on a surfaces of the $T = 0$ parameter space of the model. The physical properties of the solutions depend on the values of the coupling constants and the filling. Two types of solution emerge: a paramagnetic Mott insulator and a non-Fermi liquid type phase. Both solutions describe the interacting $U > 0$ model, and the deduced ground-state wave functions cannot be obtained perturbatively from the non-interacting case.

- The first solution emerges at $3/4$ filling and represents a paramagnetic Mott insulator, the ground-state being completely localized.
 - The second solution represents in two dimensions a new non-Fermi liquid normal (non-symmetry broken) phase and emerges at $N/N_\Lambda \geq 3/4$ filling. In this phase the $n_{\vec{k}}$ momentum distribution function is continuous together with its derivatives of any order; it is a well defined Fermi energy, but the Fermi momentum is not definable thus the system has no Fermi surface. In the parameter space this phase emerges in the vicinity of the above mentioned Mott insulating phase. The ground-state is paramagnetic with large spin degeneracy. The reason of this interesting behaviour is due to the fact, that the flat-band feature of the model — which is given by the diagonalization of the formally non-interacting Hamiltonian $\hat{H}_{eff} = \hat{H}_0 + U\hat{N}_f$ — remains valid also in the interacting case. This partially filled band is situated above a normal band with dispersion. Low lying excitations increases the number of particles at the Fermi energy. In this process particles are removed from the lower band and come up into the upper flat band.
- I presented a technique to deal with the simple Hubbard model in the case of infinitely repulsive interaction. The formalism treats separately the spin and charge degrees of freedom at the level of description,

showing explicitly the effect of the spin background in the movement of the hole.

- The Hamiltonian can be reduced into invariant subspaces with given value of the total spin and its z -component. I explicitly showed that the matrix element of the Hamiltonian reduced into such a subspace can be given using matrix elements of an irreducible representation of the symmetric group of degree N . (N is the number of particles.) It can be useful for numerical diagonalization for small clusters.
- I developed a high-temperature expansion technique in which the sum over the spin degrees of freedom can be expressed exactly in terms of characters of the symmetric group. Based on this technique the efficiency of the world line Monte Carlo algorithm can be increased. By the limited computer capacity available for me I attempted to study the magnetic properties of the model near half filling.
- Within the frame of the presented formalism the Nagaoka mechanism of ferromagnetism becomes clear, and also the reason why it cannot survive the generalization for more than one hole: the fermionic feature of particles does not play any role. The situation is similar for one dimensional system with open boundary condition (the number of particles is arbitrary), or in case of odd number of particles with periodic boundary conditions, or in case of even number of particles with anti-periodic boundary conditions.

Publications

- [1] *Exact phase diagram for extended Hubbard model in $D > 1$ dimensions with next-nearest-neighbour interaction terms*, P. Gurin and Zs. Gulácsi, Low Temp. Phys. **21**, 2643 (1996).

- [2] *Hubbard model with next-nearest-neighbour interaction terms in higher dimensions: New exactly solvable cases*, P. Gurin and Zs. Gulácsi, Phil. Mag. B **76**, 827 (1997).
- [3] *Exact results related to the extended Hubbard model with increased interaction range in $D > 1$ dimensions*, P. Gurin and Zs. Gulácsi, Phil. Mag. B **78**, 315 (1998).
- [4] *$T \geq 0$ properties of the infinitely repulsive Hubbard model for arbitrary number of holes*, P. Gurin and Zs. Gulácsi, Phil. Mag. B **81**, 321 (2001).
- [5] *Magnetic properties of the infinitely repulsive Hubbard model near half filling*, P. Gurin and Zs. Gulácsi, Phil. Mag. B, in press
- [6] *Exact solutions for the periodic Anderson model in 2D: A Non-Fermi liquid state in normal phase*, P. Gurin and Zs. Gulácsi, Phys. Rev. B, in press

Conference presentations, posters

- [7] *Exact results related to the phase diagram of the extended Hubbard model containing next-nearest-neighbour terms*, P. Gurin and Zs. Gulácsi, Adriatica Research Conference on the Electron Quantum Liquid in Systems of Reduced Dimensions, Book of Abstract p.71, ICTP, Trieste, Italy, July 2-5, 1996
- [8] *Exact results for extended Hubbard model with next-nearest-neighbour interaction terms in $d = 2, 3$ dimensions*, P. Gurin, 16th General Conference of the EPS Condensed Matter Division, Book of Abstract p.145, Leuven, Belgium, August 25-28, 1997
- [9] *Search for exact solutions for the Hubbard model in higher dimensions*, P. Gurin, III Trieste Conference on Statistical Field Theory, Book of Abstract p.112, ICTP, Trieste, Italy, June 22-26, 1998

- [10] *The two hole problem in the $U = \infty$ Hubbard model: Ferromagnetism vs. singlet state*, P. Gurin, The 20th IUPAP International Conference on Statistical Physics, Book of Abstract PO03/36, Paris, France, July 20-24, 1998