

# $\alpha_s$ from energy-energy correlations and jet rates in $e^+e^-$ collisions

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We present a comparison of the computation of energy-energy correlations and Durham algorithm jet rates in  $e^+e^-$  collisions at next-to-next-to-leading logarithmic accuracy matched with the  $\mathcal{O}(\alpha_s^3)$  perturbative prediction to LEP, PEP, PETRA, SLC, and TRISTAN data. With these predictions we perform extractions of the strong coupling constant taking into account non-perturbative effects modelled with modern Monte Carlo event generators that simulate NLO QCD corrections.

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## Introduction

The strong interaction in the Standard Model (SM) is described by Quantum Chromodynamics (QCD), see Ref. [1] for a review. The theory successfully describes the interactions between quarks and gluons and is a source of numerous predictions. One of the precise QCD predictions that depends strongly on the only theory parameter, the coupling constant of the strong interaction  $\alpha_s$ , is the topology of the  $e^+e^- \rightarrow$  hadrons events. In these events at high energies, hadrons predominantly appear in collimated bunches, called *jets*. The topologies of  $e^+e^- \rightarrow$  partons events can be predicted with high precision in perturbation theory and the observables of the final hadronic state observed in the experiments are closely related to them.

The state of the art predictions for QCD for such observables currently includes exact fixed-order next-to-next-to-leading order (NNLO) corrections for the three-jet event shapes and jet rates. The specialized numerical matrix element integration codes allow a straightforward computation of any suitable, i.e. collinear and infrared safe, event shape or jet observable.

In this paper we describe two analyses that utilise the NNLO predictions matched to next-to-next-leading-log (NNLL) resummed calculations for the region with  $e^+e^- \rightarrow 2$ -partons topology.

The first analysis considers the energy-energy correlation (EEC). EEC is the normalised energy-weighted cross section defined in terms of the angle between two particles  $i$  and  $j$  in an event [2]:

$$\frac{1}{\sigma_t} \frac{d\Sigma(\chi)}{d\cos\chi} \equiv \frac{1}{\sigma_t} \int \sum_{i,j} \frac{E_i E_j}{Q^2} d\sigma_{e^+e^- \rightarrow ij+X} \delta(\cos\chi - \cos\theta_{ij}),$$

where  $E_i$  and  $E_j$  are the particle energies,  $Q$  is the centre-of-mass energy,  $\theta_{ij} = \chi$  is the angle between the two particles, and  $\sigma_{tot}$  is the total hadronic cross section. EEC was the first event shape for which a complete NNLL resummation was performed [3] while the fixed-order NNLO corrections to this observable were computed only recently [4].

The second analysis considers the 2- and 3-jet rates obtained with the Durham jet algorithm [5]. The algorithm is described in detail elsewhere [5], only a brief description is given below. As every jet clustering algorithm, the Durham jet algorithm combines the energy and the momenta of particles (partons or hadrons) into jet objects. This is done using a measure in phase space between pairs of particles  $i$  and  $j$  with corresponding energies  $E_i$  and  $E_j$  as

$$d_{ij} = 2 \min(E_i^2, E_j^2) (1 - \cos\theta_{ij}),$$

where  $\theta_{ij}$  is the angle between the momenta of particles. At a given stage of the combination procedure a pair of objects  $i$  and  $j$  with minimal  $d_{ij}$  is found. The object  $i$  is merged (e.g. by adding 4-vectors) with object  $j$ . Therefore, at every given stage, the number of objects (jets) can be related to the parameter  $y = \min\{d_{ij}\}/Q^2$ . Consequently, the jet rates are defined as  $R_n(y) = \frac{\sigma_{n-jets}(y)}{\sigma_{tot}}$ , where  $\sigma_{n-jet}(y)$  is the cross-section of  $n$ -jet events. In this analysis we used the implementations of the algorithm from the `FastJet 3.1` [6] package. The NNLL resummation for the 2-jet rates is described in Ref. [7].

## Extraction procedure

The  $\alpha_s$  extraction procedure is based on the comparison of data to the perturbative QCD pre-

diction combined with non-perturbative (hadronization) corrections, and contains ingredients described below.

### Fixed-order and resummed calculations

In NNLO perturbative QCD at the default renormalization scale of  $\mu = Q$ , the fixed-order predictions for observable  $O$ , vanishing in the 2-jet limit, reads

$$O_{\text{f.o.}} = \frac{\alpha_s(Q)}{2\pi} A + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C + \mathcal{O}(\alpha_s^4),$$

where  $A$ ,  $B$  and  $C$  are the perturbative coefficients at LO, NLO and NNLO, normalised to the LO cross section for  $e^+e^- \rightarrow \text{hadrons}$ ,  $\sigma_0$ . In the presented analyses the coefficients  $A$ ,  $B$ ,  $C$  were calculated using the CoLoRFulNNLO method [8, 4] as function of angle  $\chi$  (for EEC) or  $y$  (for jet rates). The NNLL resummed predictions and matching procedures were used as described in Ref. [9] and Ref. [10] (for EEC) and in Ref. [7] (for 2-jet rates). For the three jet rate  $R_3$  the resummed prediction has a much lower logarithmic accuracy [5] and does not guarantee a good theoretical control in the region where logarithms are large. Therefore, for the three jet rate  $R_3$  in this analysis only fixed order predictions were used.

### Finite $b$ -quark mass corrections

The theoretical predictions described above are computed in massless QCD. In order to take into account finite  $b$ -quark mass effects, we subtract the fraction of  $b$ -quark events,  $r_b(Q)$  from the massless result and add back the corresponding massive contribution. Hence, we include mass effects directly at the level of matched distributions of corresponding observables  $O$ ,

$$O = (1 - r_b(Q))O_{\text{massless}} + r_b(Q)O_{\text{massive}}^{\text{NNLO}*}.$$

Here  $O_{\text{massless}}$  is the matched distribution, computed in massless QCD as outlined above, while  $O_{\text{massive}}^{\text{NNLO}*}$  is the fixed-order massive distribution. The complete massive NNLO corrections are currently unknown, so we model them by supplementing the massive NLO prediction of the parton level Monte Carlo generator Zbb4 [11], with the NNLO coefficient of the massless fixed-order result. The fraction of  $b$ -quark events  $r_b(Q)$  is defined as

$$r_b(Q) \equiv \sigma_{\text{massive}}(e^+e^- \rightarrow b\bar{b}) / \sigma_{\text{massive}}(e^+e^- \rightarrow \text{hadrons}),$$

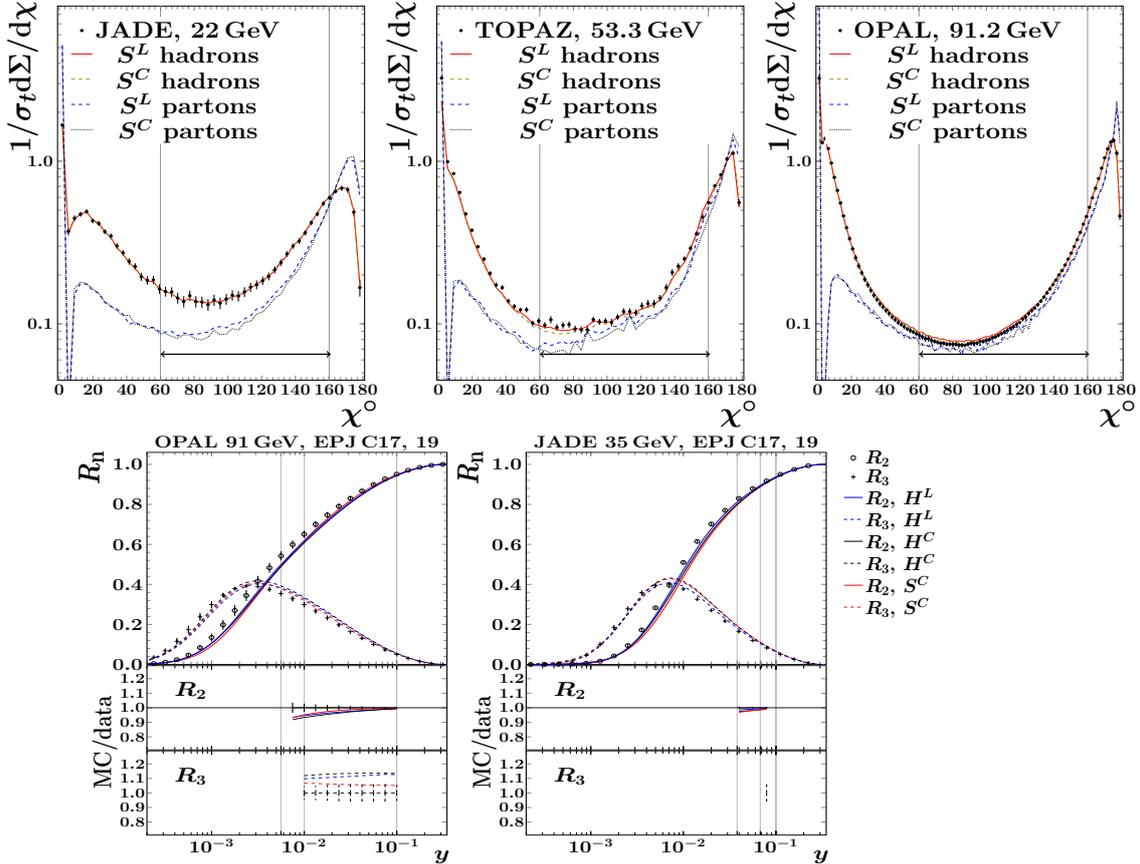
where all quantities are calculated up to  $\mathcal{O}(\alpha_s^3)$ .

### Data sets

To extract the strong coupling the predictions described above were confronted with the available data sets. The criteria to include the data were high precision measurements obtained with charged and neutral final state particles, presence of corrections for detector effects, correction for initial state photon radiation and sufficient amount of supplementary information. Namely, for the EEC analysis the data obtained in SLD, L3, DELPHI, OPAL, TOPAZ, TASSO, JADE, MAC, MARKII, CELLO, and PLUTO experiments were included, see details in Ref. [10]. The corresponding centre-of-mass energy range is  $\sqrt{s} = 14 - 91.2 \text{ GeV}$ . For the jet rates analysis, the data obtained in the OPAL, JADE, DELPHI, L3, and ALEPH experiments were included, see details in Ref. [12]. The corresponding centre-of-mass energy range is  $\sqrt{s} = 35 - 207 \text{ GeV}$ .

### Monte Carlo generation setup

In both analyses, the non-perturbative effects in the  $e^+e^- \rightarrow$  hadrons process are modelled using state-of-the-art particle-level Monte Carlo (MC) generators `SHERPA` [13] and `Herwig 7` [14]. The MC generated event samples describe the data relatively well, see Fig. 1.



**Figure 1:** Selected data and predictions with different Monte Carlo setups for EEC (top) and jet rates (bottom) analyses. For the EEC analysis the hadron level distributions are accompanied with corresponding parton level distributions.

The full description of the MC event generator setups is given in Ref. [12] and [10], only a brief overview is given below. The `SHERPA` samples were generated using the matrix element generators `AMEGIC` and `COMIX`. The `Herwig 7` samples were generated using the matrix element generator `MadGraph5`. To simulate one-loop QCD correction the `GoSam` one-loop library for the EEC analysis and the `OpenLoops` one-loop library for the jet rates analysis are employed. In all cases the 2-parton final state processes had NLO accuracy in perturbative QCD and the matrix elements were calculated assuming massive  $b$ -quarks.

To test the fragmentation and hadronization model dependence, the parton level events were hadronized with different hadronization setups. Here and below the results of the  $\alpha_s$  extraction are labelled according to these hadronization setups.

### MC event samples for EEC analysis

The events generated by `SHERPA` were hadronized with a native implementation of the cluster model (label  $S^C$ ) and the Lund string fragmentation model as implemented in `Pythia 6` (label  $S^L$ ). The events generated by the `Herwig 7` were hadronized by the native implementation of the cluster model (label  $H^M$ ).  $S^L$  was chosen to be the default setup. The hadronization corrections were used multiplicatively, i.e.  $\text{EEC}^{(\text{hadrons})}(\chi) = k(\chi, s) \times \text{EEC}^{(\text{partons})}(\chi)$ , where the coefficients  $k(\chi, s)$  are extracted for every bin from the MC simulated samples. Before the extraction, the MC simulated samples were re-weighted on an event-by-event basis so the energy-energy correlation distributions on hadron level coincide with data, see Ref. [10] for details.

### MC event samples for jet rates analysis

The events generated by `Herwig` were hadronized with the native implementation of the cluster model (label  $H^C$ ) and the Lund string fragmentation model as implemented in `Pythia 8` (label  $H^L$ ). The events generated by `SHERPA` were hadronized by the native implementation of the cluster model (label  $S^C$ ).  $H^L$  was chosen to be the default setup. The hadronization correction procedure is designed to take into account that the jet rates add up to unity, see Ref. [12] for details.

### Fit procedure and estimation of uncertainties

The perturbative part of the predictions was calculated for every data point as described in previous sections. To find the optimal value of  $\alpha_s$ , the `MINUIT2` program was used to minimise the value of

$$\chi^2(\alpha_s) = \sum_{\text{data sets}} \chi^2(\alpha_s)_{\text{data set}},$$

where  $\chi^2(\alpha_s)$  was calculated for each data set as

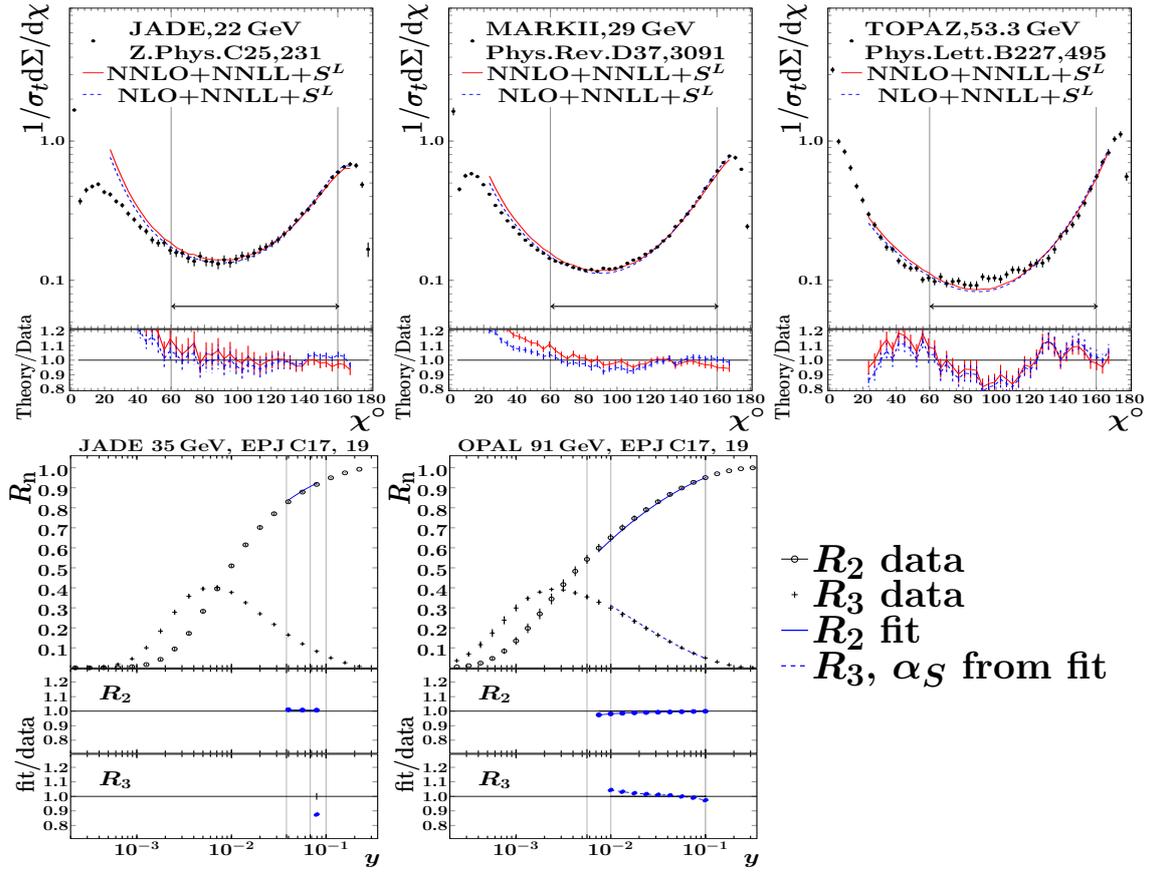
$$\chi^2(\alpha_s) = (\vec{D} - \vec{P}(\alpha_s))V^{-1}(\vec{D} - \vec{P}(\alpha_s))^T,$$

with  $\vec{D}$  standing for the vector of data points,  $\vec{P}(\alpha_s)$  for the vector of calculated predictions and  $V$  for the covariance matrix for  $\vec{D}$ . The default scale used in the fit procedure was  $\mu = Q = \sqrt{s}$ .

The fit ranges were chosen to avoid regions where resummed predictions or hadronization correction calculations are not reliable. The uncertainty on the fit result ('exp.') was estimated with the  $\chi^2 + 1$  criterion as implemented in the `MINUIT2` program. For both analyses the fits were performed taking into account the correlations between measurements within each data set, that were estimated from Monte Carlo simulations. The distributions obtained in the reference fits are shown in Fig. 2.

### Systematic uncertainties and validity checks of the results

The full description of validity checks performed in both analyses is given in Refs. [12, 10]. Below we describe briefly the way of estimation of the main systematic uncertainties. The systematic uncertainties were estimated with procedures used in previous studies [15]. To estimate the effects caused by the absence of higher-order terms in the perturbative predictions, the scale variation procedures were performed. The fits were repeated, with variation of the resummation



**Figure 2:** Selected data and fit results in the EEC (top) and jet rates (bottom) analyses.

$\mu_{\text{res.}} = x_R \times Q$  scale and renormalization  $\mu_{\text{ren.}} = x_L \times Q$  scale by a factor  $2^{\pm 1}$ , see results in Fig. 3. The corresponding estimations are labelled below as ('ren.') for renormalization and as ('res.') for resummation scale variation.

The bias of hadronization model selection ('hadr.') is studied with alternative setups for MC hadronization corrections described above, i.e. from  $S^C$  for the EEC analysis and  $H^C$  for the jet rates analysis.

## Summary

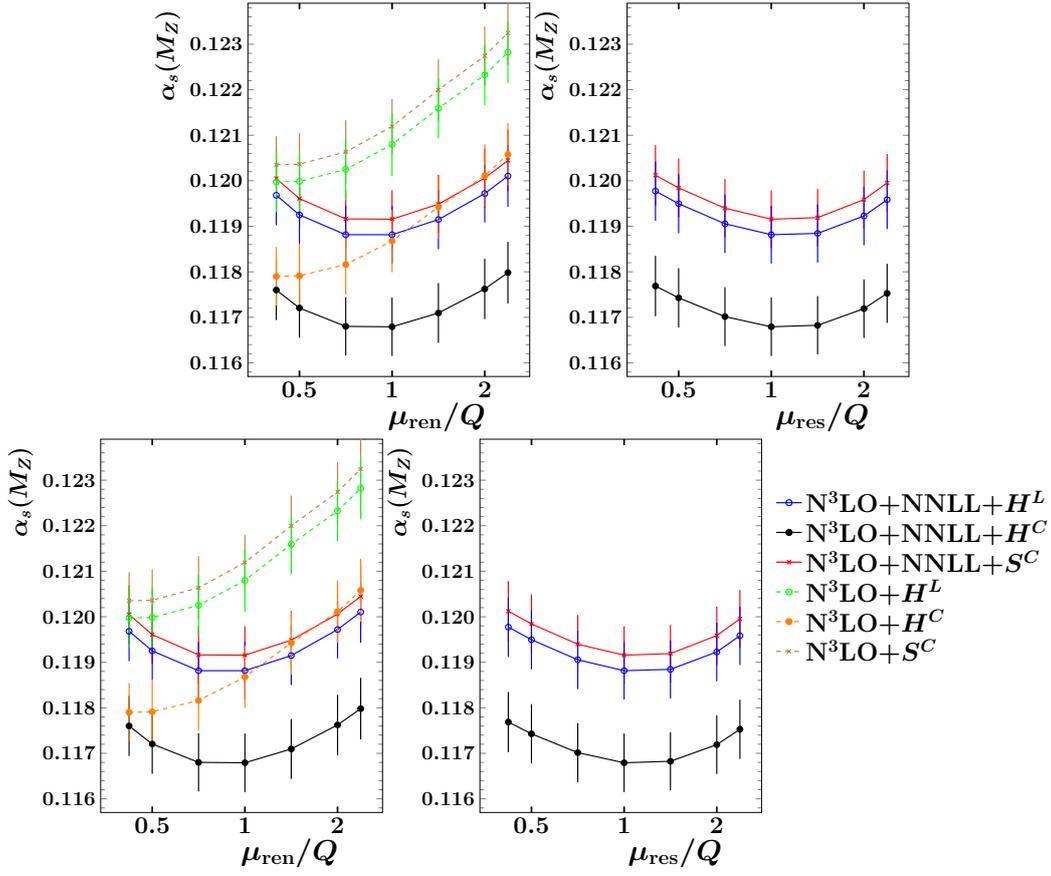
For the central value of the final result for EEC (jet rates) analysis, we quote the results obtained from the fits with  $S^L$  ( $H^L$ ) hadronization model with uncertainties and estimations of biases obtained as described above. The final result of the EEC analysis is

$$\alpha_s(m_Z) = 0.11750 \pm 0.00018 (exp.) \pm 0.00102 (hadr.) \pm 0.00257 (ren.) \pm 0.00078 (res.),$$

and for the jet rates analysis it is

$$\alpha_s(m_Z) = 0.11881 \pm 0.00063 (exp.) \pm 0.00101 (hadr.) \pm 0.00045 (ren.) \pm 0.00034 (res.).$$

Both results are in agreement with the latest world average  $\alpha_s(m_Z) = 0.1181 \pm 0.0011$  [16].



**Figure 3:** Dependence of fit results in the EEC (top) and jet rates (bottom) analyses on the renormalization and resummation scales.

Both analyses provide determinations of  $\alpha_s(m_Z)$  determination which have one of the highest numerical and theory precisions ever obtained from the corresponding observables. In addition to that, in the case of the jet rates analysis, for the first time the hadronization-related uncertainty is much larger than other uncertainties.

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