Alexandria Engineering Journal 107 (2024) 245–253

Contents lists available at ScienceDirect

Alexandria Engineering Journal

journal homepage: www.elsevier.com/locate/aej

Original article

Bifurcation, chaotic analysis and soliton solutions to the (3+1)-dimensional p-type model

Muhammad Nadeem [a], Omar Abu Arqub [b], Ali Hasan Ali [c,d,e,*], Husam A. Neamah [f]

[a] School of Mathematics and Statistics, Qujing Normal University, Qujing 655011, China
[b] Department of Mathematics, Faculty of Science, Al-Balqa Applied University, Salt 19117, Jordan
[c] Department of Mathematics, College of Education for Pure Sciences, University of Basrah, 61001 Basrah, Iraq
[d] Institute of Mathematics, University of Debrecen, Pf. 400, H-4002 Debrecen, Hungary
[e] Department of Business Management, Al-imam University College, 34011 Balad, Iraq
[f] Department of Electrical Engineering and Mechatronics, Faculty of Engineering, University of Debrecen, Debrecen, 4028, Ótemető, u. 4-5, Hungary

ARTICLE INFO

Keywords:
(3+1)-dimensional p-type model
Phase portrait analysis
Bifurcation analysis
Chaotic analysis
Wave solutions

ABSTRACT

This study examines the modified Sardar sub-equation method (MSSEM) for deriving the novel solutions of the (3+1)-dimensional p-type model. This framework is commonly employed to explain the behavior of optical solitons in nonlinear media. The applications of MSSEM allows us to acquire the precise analytical solutions, which incorporate a diverse array of optical soliton solutions. We discuss the dynamical structure of the solitons, bifurcation and chaos theory to develop the multiple soliton solutions, including rational, hyperbolic, exponential, and trigonometric functions and depending on the principle of balancing equation. Moreover, by using bifurcation and chaos theory, we examine the governing model with and without the perturbation term and provide the three-dimensional, two-dimensional, and density profiles to improve the clarity of obtained results. The different aspects of the solutions are evident in our visual representations. These solutions are applicable to a wide range of domains, including fluid physics, oceanography, physics, engineering, and nonlinear optics.

1. Introduction

Nonlinear partial differential equations (NLPDEs) are fundamental tools for modeling complex physical events in many scientific disciplines. These equations usually describe the quantities that depend on several spatial and temporal dimensions [1–3]. These equations are widely used in many different fields, including fluid dynamics, mathematical biology, and quantum physics [4–8]. These formulas are essential to comprehending fluid instabilities and turbulence, among other phenomena [9]. The (3+1)-dimensional p-type model is one particular subtype of NLPDEs [10].

$$\mathcal{A}_{xxxy} + a_1\mathcal{A}_{yt} + a_2(\mathcal{A}\mathcal{A}_x)_y + a_3\mathcal{A}_{xx} + a_4\mathcal{A}_{zz} = 0, \tag{1}$$

where $a_i's$ are the real components. This particular model contains one temporal dimension of t and three spatial dimensions of x, y, z, which is highly valuable for analyzing dynamic processes. The governing model is used to represent the instabilities and plasma waves in various fields, including plasma physics. The (3+1)-dimensional p-type model is a theoretical construct used to investigate a variety of physical processes within the context of quantum field theory and other domains. This model explains specific properties of materials and spontaneous processes in solid-state structures, which include magnetism and the conventional theory of particle physics. Mohan et al. [11] developed a Painleve integrable generalized (3+1)-dimensional evolution model and they used an auxiliary function to find rogue wave and dispersive-soliton solutions created by applying the Cole–Hopf transform through symbolic computation.

Numerous strong and effective analytical mathematical techniques have been developed, including traveling wave transformation [12], Darboux transformations [13], inverse-scattering technique [14], the generalized Kudryashov approach [15], Sardar sub-equation technique [16], $\frac{G}{G'}$-expansion technique [17], auxiliary equation technique [18], exp-function method [19], Unified direct method [20], Adomian technique [21], Lie symmetry technique [22], bilinear neural network technique [23], new extended direct algebraic [24], modified F-expansion approach [25] and so on [26–28]. These various methods offer a variety of techniques for deriving exact analytical solutions and give researchers a rich arsenal for addressing the difficulties posed by nonlinear evolution equations.

* Corresponding author.
E-mail address: ali.hasan@science.unideb.hu (A.H. Ali).

https://doi.org/10.1016/j.aej.2024.07.032
Received 12 May 2024; Received in revised form 29 June 2024; Accepted 7 July 2024

In this work, we propose the concept of MSSEM for the soliton solutions of p-type model [29]. Using the proposed method, various new wave solutions have been generated, such as combo solutions, hyperbolic, exponential and trigonometric solutions. Additionally, use bifurcation and chaotic analysis to analyze the dynamical behavior of the model for perturbed and non-perturbed systems [30]. These results have significant applications in various discipline of mathematical physics. Furthermore, the aforementioned results demonstrate the efficacy of the proposed methodology, which finds application in a variety of nonlinear physical models encountered in the applied sciences. The presence of a bright soliton is crucial in the field of nonlinear optics as it enables the generation of compact and dependable light waves for transmission systems. Periodic solitons are employed to characterize periodic waves and oscillations in a variety of fields, including fluid dynamics, plasma physics, and geophysical phenomena. Singular solutions are indispensable for understanding wave behaviors in a diverse array of systems to elucidate complex problems, including motion of liquids, plasmas, and nonlinear optical phenomena.

These are the main sections of the study. In Section 2 covers the description for the MSSEM. Applications of MSSEM to the (3+1)-dimensional p-type model are given in 3. Section 4 are organized to investigate the dynamical system of the governing model by using bifurcation and chaos theory. Section 5 covers the numerical discussions of obtained solutions. The conclusion is organized in Section 6.

2. Mathematical analysis of MSSEM

By including additional terms and instances in the solution ansatz, the MSSE strategy expands the capabilities of the traditional Sardar sub-equation technique for the creation of nonlinear equations. This approach solves NLPDEs effectively in a variety of circumstances. Consider the general form of NLPDEs

$$N\left(\mathcal{A}, \mathcal{A}_x, \mathcal{A}_t, \mathcal{A}_{xx}, \mathcal{A}_{xz}, \mathcal{A}_{xt}, \dots\right) = 0. \tag{2}$$

Step-1: Consider the wave transformation

$$\mathcal{A} = \mathcal{V}(\phi), \qquad \phi = -\beta_4 t + \beta_1 x + \beta_2 y + \beta_3 z. \tag{3}$$

Inserting Eq. (3), into Eq. (2), acquired the nonlinear ordinary differential equations (NLODEs).

$$\widetilde{N}(\mathcal{V},\ \mathcal{V}',\ \mathcal{V}'', \dots) = 0. \tag{4}$$

Step-2: Assume the general solution for Eq. (4) through MSSEM

$$\mathcal{V}(\phi) = \gamma_0 + \sum_{j=1}^{K} \gamma_j \mathcal{Z}^j(\phi), \qquad \gamma_j \neq 0, \tag{5}$$

where $\mathcal{V} = \mathcal{V}(\phi)$ assures

$$\mathcal{Z}'(\phi)^2 = \alpha_2 \mathcal{Z}(\phi)^4 + \alpha_1 \mathcal{Z}(\phi)^2 + \alpha_0, \tag{6}$$

as $\alpha_0 \neq 1$, α_1, and $\alpha_2 \neq 0$ are integers. Assume that the invertible γ_j is zero and compute γ_0 and γ_1. To get K, the balancing principle is applied. Various clusters for Eq. (6) are:

Cluster-1:

• If $\alpha_0 = 0$, $\alpha_1 > 0$ and $\alpha_2 \neq 0$, we acquired

$$\mathcal{Z}_1(\phi) = \sqrt{-\frac{\alpha_1}{\alpha_2}} \operatorname{sech}\left(\sqrt{\alpha_1}(\phi + L)\right), \tag{7}$$

$$\mathcal{Z}_2(\phi) = \sqrt{-\frac{\alpha_1}{\alpha_2}} \operatorname{csch}\left(\sqrt{\alpha_1}(\phi + L)\right). \tag{8}$$

Cluster-2:

• The constants are M_1 and M_2 and $\alpha_0 = 0$, $\alpha_1 > 0$ and $\alpha_2 = +4M_1 M_2$, we acquired

$$\mathcal{Z}_3(\phi) = \frac{4M_1\sqrt{\alpha_1}}{\left(4M_1^2 - \alpha_2\right)\sinh\left(\sqrt{\alpha_1}(\phi + L)\right) + \left(4M_1^2 - \alpha_2\right)\cosh\left(\sqrt{\alpha_1}(\phi + L)\right)}. \tag{9}$$

Case-3:

• The constants are W_1 and W_2, and $\alpha_0 = \frac{\alpha_1^2}{4\alpha_2}$, $\alpha_1 < 0$ and $\alpha_2 > 0$, we acquired

$$\mathcal{Z}_4(\phi) = \sqrt{-\frac{\alpha_1}{2\alpha_2}} \tanh\left(\sqrt{-\frac{\alpha_1}{2}}(\phi + L)\right), \tag{10}$$

$$\mathcal{Z}_5(\phi) = \sqrt{-\frac{\alpha_1}{2\alpha_2}} \coth\left(\sqrt{-\frac{\alpha_1}{2}}(\phi + L)\right), \tag{11}$$

$$\mathcal{Z}_6(\phi) = \sqrt{-\frac{\alpha_1}{2\alpha_2}} \left(\tanh\left(\sqrt{-\frac{\alpha_1}{2}}(\phi + L)\right) + i\operatorname{sech}\left(\sqrt{-2\alpha_1}(\phi + L)\right)\right), \tag{12}$$

$$\mathcal{Z}_7(\phi) = \sqrt{-\frac{\alpha_1}{8\alpha_2}} \left(\tanh\left(\sqrt{-\frac{\alpha_1}{8}}(\phi + L)\right) + \coth\left(\sqrt{-\frac{\alpha_1}{8}}(\phi + L)\right)\right), \tag{13}$$

$$\mathcal{Z}_8(\phi) = \frac{\sqrt{-\frac{\alpha_1}{2\alpha_2}}\left(\sqrt{W_1^2 + W_2^2} - W_1\cosh\left(\sqrt{-2\alpha_1}(\phi + L)\right)\right)}{W_1\sinh\left(\sqrt{-2\alpha_1}(\phi + L)\right) + W_2}, \tag{14}$$

$$\mathcal{Z}_9(\phi) = \frac{\sqrt{-\frac{\alpha_1}{2\alpha_2}}\cosh\left(\sqrt{-2\alpha_1}(\phi + L)\right)}{\sinh\left(\sqrt{-2\alpha_1}(\phi + L)\right) + i}. \tag{15}$$

Cluster-4:

• If $\alpha_0 = 0$, $h_1 < 0$ and $h_2 \neq 0$, we acquired

$$\mathcal{Z}_{10}(\phi) = \sqrt{-\frac{\alpha_1}{\alpha_2}} \sec\left(\sqrt{-\alpha_1}(\phi + L)\right), \tag{16}$$

$$\mathcal{Z}_{11}(\phi) = \sqrt{-\frac{\alpha_1}{\alpha_2}} \csc\left(\sqrt{-\alpha_1}(\phi + L)\right). \tag{17}$$

Case-5:

• If $\alpha_0 = \frac{\alpha_1^2}{4\alpha_2}$, $\alpha_1 > 0$ and $\alpha_2 > 0$ and $W_1^2 - W_2^2 > 0$, we acquired

$$\mathcal{Z}_{12}(\phi) = \sqrt{-\frac{\alpha_1}{2\alpha_2}} \tan\left(\sqrt{\frac{\alpha_1}{2}}(\phi + L)\right), \tag{18}$$

$$\mathcal{Z}_{13}(\phi) = -\sqrt{-\frac{\alpha_1}{2\alpha_2}} \cot\left(\sqrt{\frac{\alpha_1}{2}}(\phi + L)\right), \tag{19}$$

$$\mathcal{Z}_{14}(\phi) = -\sqrt{-\frac{\alpha_1}{2\alpha_2}} \left(\tan\left(\sqrt{2\alpha_1}(\phi + L)\right) - \sec\left(\sqrt{2\alpha_1}(\phi + L)\right)\right), \tag{20}$$

$$\mathcal{Z}_{15}(\phi) = \sqrt{-\frac{\alpha_1}{8\alpha_2}} \left(\tan\left(\sqrt{\frac{\alpha_1}{8}}(\phi + L)\right) - \cot\left(\sqrt{\frac{\alpha_1}{8}}(\phi + L)\right)\right), \tag{21}$$

$$\mathcal{Z}_{16}(\phi) = \frac{\sqrt{-\frac{\alpha_1}{2\alpha_2}}\left(\sqrt{W_1^2 - W_2^2} - W_1\cos\left(\sqrt{2\alpha_1}(\phi + L)\right)\right)}{W_2 + W_1\sin\left(\sqrt{2\alpha_1}(\phi + L)\right)}, \tag{22}$$

$$\mathcal{Z}_{17}(\phi) = \frac{\sqrt{-\frac{\alpha_1}{2\alpha_2}}\cos\left(\sqrt{2\alpha_1}(\phi + L)\right)}{\sin\left(\sqrt{2\alpha_1}(\phi + L)\right) - 1}. \tag{23}$$

Cluster-6:

• If $\alpha_0 = 0$, $\alpha_1 > 0$, we acquired

$$\mathcal{Z}_{18}(\phi) = \frac{4\alpha_1 e^{\sqrt{\alpha_1}(\phi + L)}}{e^{2\sqrt{\alpha_1}(\phi + L)} - 4\alpha_1\alpha_2}, \tag{24}$$

$$\mathcal{Z}_{19}(\phi) = \frac{4\alpha_1 e^{\sqrt{\alpha_1}(\phi + L)}}{1 - 4\alpha_1\alpha_2 e^{2\sqrt{\alpha_1}(\phi + L)}}. \tag{25}$$

Cluster-7:

• If $\alpha_0 = 0,\ \alpha_1 = 0$ and $\alpha_2 > 0$, we acquired

$$\mathcal{Z}_{20}(\phi) = \frac{1}{\sqrt{\alpha_2}(\phi + L)}, \tag{26}$$

$$\mathcal{Z}_{21}(\phi) = \frac{i}{\sqrt{\alpha_2}(\phi + L)}. \tag{27}$$

Step-3: After applying Eq. (6) to Eq. (4) and the second-order required derivatives of Eq. (5), the polynomial that results is a power of $\mathcal{Z}(\phi)$. Also, gather coefficients representing similar powers of $\mathcal{Z}(\phi)$, and then set all of the parameters to zero to obtain the algebraic systems for γ_0, γ_j $(j = 1, 2, 3, \ldots, K)$.

Step-4: The solution to Eq. (1) can be obtained by computing algebraic systems using Wolfram Mathematica.

3. Application of MSSEM to (3+1)-dimensional p-type model

The MSSEM facilitates the procedure of finding exact solutions by converting complicated nonlinear PDEs to streamlined sub-equations. This can frequently make the procedure more simple and efficient than other techniques. The MSSEM is useful when creating NLPDE solutions, such as the p-type model. This technique involves an algebraic system to obtain unknown constants, then taking an ansatz for a solution using additional variables and a unique function. The results show that our recommended technique is reliable, accurate and efficient for the soliton solution for the p-type model. Now, using the wave transformation presented in Eq. (3) to Eq. (1), we obtain the NLODE as follows

$$a_3\beta_1^2\mathcal{V}''(\phi) + a_4\beta_3^2\mathcal{V}''(\phi) - a_1\beta_2\beta_4\mathcal{V}''(\phi) + a_2\left(\beta_1\beta_2\mathcal{V}(\phi)\mathcal{V}''(\phi) + \beta_1\beta_2\mathcal{V}'(\phi)^2\right) + \beta_2\beta_1^3\mathcal{V}^{(4)}(\phi), \tag{28}$$

Integrate twice Eq. (28) to acquire the required NLODEs of the second order.

$$a_3\beta_1^2\mathcal{V}(\phi) + \frac{1}{2}a_2\beta_2\beta_1\mathcal{V}(\phi)^2 + a_4\beta_3^2\mathcal{V}(\phi) - a_1\beta_2\beta_4\mathcal{V}(\phi) + \beta_2\beta_1^3\mathcal{V}''(\phi). \tag{29}$$

Using the balance principle in Eq. (29), we obtain $j = 2$. Using $j = 2$, the general solution becomes.

$$\mathcal{V}(\phi) = \gamma_0 + \gamma_2\mathcal{Z}(\phi)^2 + \gamma_1\mathcal{Z}(\phi). \tag{30}$$

Findings the coefficient by equating it with $j = 0, 1, 2, 3, \ldots$, to the power of $\mathcal{Z}(\phi)^j$. Inserting Eq. (30) in Eq. (29), to obtain an algebraic equation systems.

$$a_3\beta_1^2\gamma_0 + a_4\beta_3^2\gamma_0 - a_1\beta_2\beta_4\gamma_0 = 0 \wedge 2\alpha_0\beta_2\beta_1^3\gamma_2 + \frac{1}{2}a_2\beta_2\beta_1\gamma_0^2 = 0,$$
$$+\left(\alpha_1\beta_2\beta_1^3\gamma_1 + a_3\beta_1^2\gamma_1 + a_2\beta_2\beta_1\gamma_0\gamma_1 + a_4\beta_3^2\gamma_1 - a_1\beta_2\beta_4\gamma_1\right) = 0,$$
$$+\left(4\alpha_1\beta_2\beta_1^3\gamma_2 + a_3\beta_1^2\gamma_2 + \frac{1}{2}a_2\beta_2\beta_1\gamma_1^2 + a_2\beta_2\beta_1\gamma_0\gamma_2 + a_4\beta_3^2\gamma_2 - a_1\beta_2\beta_4\gamma_2\right) = 0,$$
$$\left(6\alpha_2\beta_2\beta_1^3\gamma_2 + \frac{1}{2}a_2\beta_2\beta_1\gamma_2^2\right) + \left(2\alpha_2\beta_2\beta_1^3\gamma_1 + a_2\beta_2\beta_1\gamma_1\gamma_2\right) = 0.$$

The equation system above is resolved by symbolic computation.

Family 1:

$$\left\{a_2 \to \frac{4\sqrt{3}a_4\sqrt{\alpha_0}\sqrt{\alpha_2}\beta_3^2}{a_3\gamma_0},\quad \beta_1 \to \frac{i\sqrt{a_4}\beta_3}{\sqrt{a_3}},\quad \beta_2 \to 0, \gamma_1 \to 0,\quad \gamma_2 \to \frac{\sqrt{3}\sqrt{\alpha_2}\gamma_0}{\sqrt{\alpha_0}}\right\}. \tag{31}$$

The solutions for Family 1 are the expressions that follow:

• Bright soliton solution.

$$\mathcal{A}_{1,1} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\operatorname{sech}^2\left(\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{32}$$

• Singular soliton solution is given in Box I.

$$\mathcal{A}_{1,2} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\operatorname{csch}^2\left(\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{33}$$

• Hyperbolic soliton solution. • Dark soliton solution.

$$\mathcal{A}_{1,4} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\tanh^2\left(\frac{\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{\sqrt{2}}\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{35}$$

• Singular soliton solution.

$$\mathcal{A}_{1,5} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\coth^2\left(\frac{\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{\sqrt{2}}\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{36}$$

• Combo of bright and dark soliton solution is given in Box II.

• Combo of dark and singular soliton solution is given in Box III.

• Hyperbolic soliton solution is given in Box IV.

$$\mathcal{A}_{1,9} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\cosh^2\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}\left(\sinh\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right) + i\right)^2}, \tag{40}$$

• Periodic soliton solution is given in Box V.

$$\mathcal{A}_{1,10} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\sec^2\left(\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{\sqrt{\alpha_0}\sqrt{\alpha_2}}, \tag{41}$$

$$\mathcal{A}_{1,11} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\csc^2\left(\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{\sqrt{\alpha_0}\sqrt{\alpha_2}}, \tag{42}$$

$$\mathcal{A}_{1,12} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\tan^2\left(\frac{\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{\sqrt{2}}\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}} \tag{43}$$

$$\mathcal{A}_{1,13} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\cot^2\left(\frac{\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{\sqrt{2}}\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}, \tag{44}$$

$$\mathcal{A}_{1,15} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\left(\tan\left(\frac{\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{2\sqrt{2}}\right) - \cot\left(\frac{\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)}{2\sqrt{2}}\right)\right)^2}{8\sqrt{\alpha_0}\sqrt{\alpha_2}}, \tag{46}$$

$$\mathcal{A}_{1,16} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\left(\sqrt{M_1^2 - M_2^2} - M_1\cos\left(\sqrt{2}\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)\right)^2}{2\sqrt{\alpha_0}\sqrt{\alpha_2}\left(M_2 + M_1\sin\left(\sqrt{2}\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)\right)^2}, \tag{47}$$

$$\mathcal{A}_{1,17} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\cot^2\left(\sqrt{2}\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{48}$$

• Exponential soliton solution.

$$\mathcal{A}_{1,18} = \gamma_0 + \frac{16\sqrt{3}\sqrt{\alpha_2}\alpha_1^2\gamma_0\exp\left(2\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)}{\sqrt{\alpha_0}\left(-4\alpha_1\alpha_2 + \exp\left(2\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}} + L - \beta_4 t + \beta_3 z\right)\right)\right)^2},$$

$$\mathcal{A}_{1,3} = \gamma_0 + \frac{16\sqrt{3}\sqrt{\alpha_2}\alpha_1\gamma_0 M_1^2}{\sqrt{\alpha_0}\left(\left(4M_1^2-\alpha_2\right)\sinh\left(\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)+\left(4M_1^2-\alpha_2\right)\cosh\left(\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}. \tag{34}$$

Box I.

$$\mathcal{A}_{1,6} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\left(\tanh\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)+i\text{sech}\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{37}$$

Box II.

$$\mathcal{A}_{1,7} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\left(\tanh\left(\frac{\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)}{2\sqrt{2}}\right)+i\coth\left(\frac{\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)}{2\sqrt{2}}\right)\right)^2}{8\sqrt{\alpha_0}\sqrt{\alpha_2}}. \tag{38}$$

Box III.

$$\mathcal{A}_{1,8} = \gamma_0 - \frac{\sqrt{3}\alpha_1\gamma_0\left(\sqrt{M_1^2+M_2^2}-M_1\cosh\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}{2\sqrt{\alpha_0}\sqrt{\alpha_2}\left(M_2+M_1\sinh\left(\sqrt{2}\sqrt{-\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}, \tag{39}$$

Box IV.

$$\mathcal{A}_{1,14} = \gamma_0 + \frac{\sqrt{3}\alpha_1\gamma_0\left(\tan\left(\sqrt{2}\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)-\sec\left(\sqrt{2}\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}{2\sqrt{\alpha_0}\sqrt{\alpha_2}}, \tag{45}$$

Box V.

(49)

$$\mathcal{A}_{1,19} = \gamma_0 + \frac{16\sqrt{3}\sqrt{\alpha_2}\alpha_1^2\gamma_0\exp\left(2\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)}{\sqrt{\alpha_0}\left(1-4\alpha_1\alpha_2\exp\left(2\sqrt{\alpha_1}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)\right)\right)^2}. \tag{50}$$

• Plane wave soliton solution.

$$\mathcal{A}_{1,20} = \gamma_0 + \frac{\sqrt{3}\gamma_0}{\sqrt{\alpha_0}\sqrt{\alpha_2}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)^2}. \tag{51}$$

• rational soliton solution.

$$\mathcal{A}_{1,21} = \gamma_0 + \frac{\sqrt{3}\gamma_0}{\sqrt{\alpha_0}\sqrt{\alpha_2}\left(\frac{i\sqrt{a_4}\beta_3 x}{\sqrt{a_3}}+L-\beta_4 t+\beta_3 z\right)^2}. \tag{52}$$

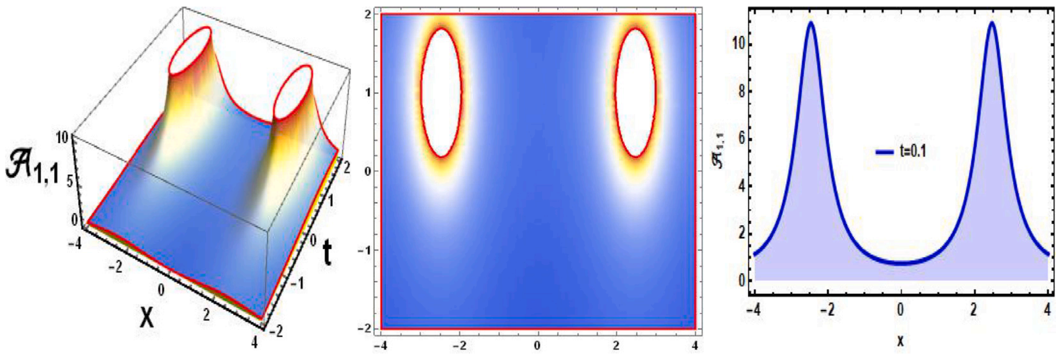
4. Dynamical system

An equation that represents how a system changes over time is called a dynamical system (DS). The differential equations that make up this system explain how the system's state varies over time. The behavior of complex systems is studied using dynamic systems in a variety of disciplines, including physics, engineering, biology, economics, and ecology.

A fundamental DS can be illustrated by the movement of planets in a solar system, the behavior of the stock market, the population dynamics of a species, or the motion of a pendulum. Analysis of long-term behavior, periodicity, and stability are frequently employed in the investigation of dynamical systems.

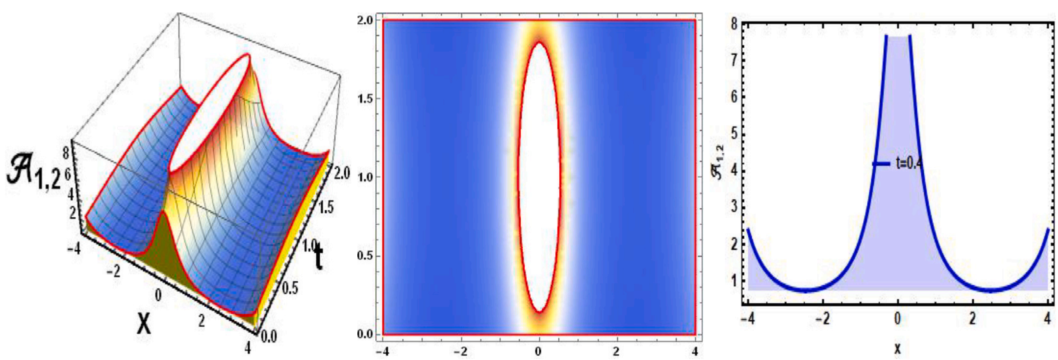
4.1. Bifurcation analysis

The theory of bifurcation analysis is an essential concept for the dynamical system. If one or more system parameters are changed, then the dynamical system describes a qualitative shift in the behavior of



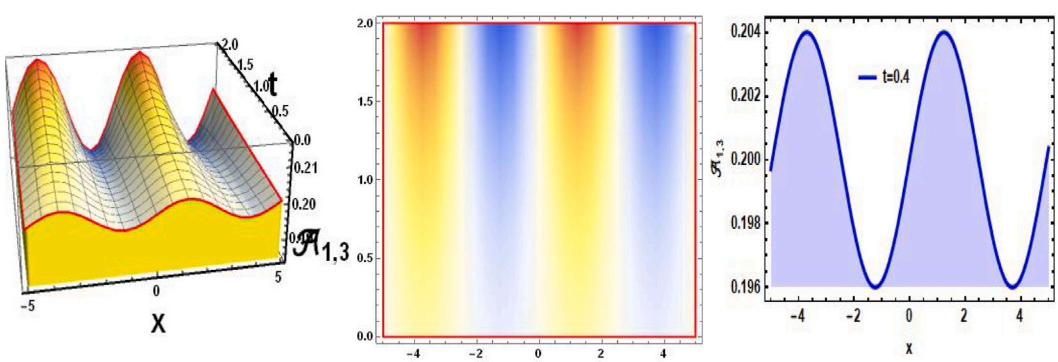
(a) 3-dimensional (b) Density plot (c) 2-dimensional

Fig. 1. Physical structure of 2-soliton of bright solution of $\mathcal{A}_{1,1}$ by choosing appropriate parametric values.



(a) 3-dimensional (b) Density plot (c) 2-dimensional

Fig. 2. Physical structure of 1-soliton of singular solution of $\mathcal{A}_{1,2}$ by choosing appropriate parametric values.



(a) 3-dimensional (b) Density plot (c) 2-dimensional

Fig. 3. Physical structure of hyperbolic solitons of $\mathcal{A}_{1,3}$ by choosing appropriate parametric values.

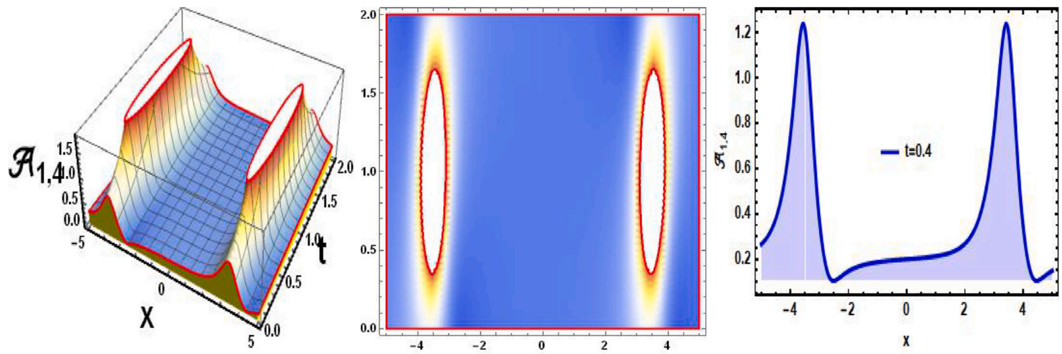
a distributed system. A bifurcation can occur when the behavior of a dynamical system change suddenly. When two unstable and one stable equilibriums clash and destroy each others, then the stability of equilibrium points can exchange such that the production of one unstable equilibrium point and the destruction of two stable equilibrium points. Bifurcation theory is useful in many domains, including as physics, biology, economics, and engineering, and it is crucial for comprehending the behavior of nonlinear dynamical systems. Consider Eq. (29), and apply the Galilean transformation to acquire the first order DS such as:

$$\begin{cases} \mathcal{V}'(\phi) = W, \\ \mathcal{V}''(\phi) = W' = B_1 \mathcal{V}(\phi) + B_2 \mathcal{V}^2(\phi), \end{cases} \tag{53}$$

where $B_1 = \frac{-1}{\beta_1^3 \beta_2}(a_3\beta_1^2 + a_4\beta_3^2 - a_1\beta_2\beta_4)$ and $B_2 = \frac{-a_2}{2}\frac{1}{\beta_1^2}$. The following are the possible cases for the DS without perturbation term at the fixed parametric values are $\beta_1 = 0.2$, $\beta_2 = 0.5$, $\beta_3 = 0.21$, & $\beta_4 = 0.9$.

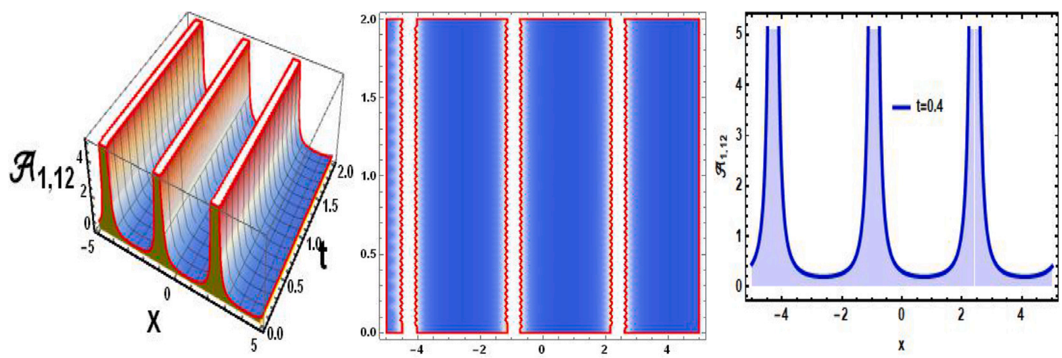
Case-1: When $B_1 > 0$, $B_2 > 0$ and $B_1 > 0$, $B_2 < 0$, Eq. (53), gives the behaviors of saddle and center point by choosing the values of parameters are $a_3 = 1$, & $a_2 = 0.7$, as depicted in part-(a) of Fig. 6.

Case-2: When $B_1 < 0$, $B_2 < 0$ and $B_1 < 0$, $B_2 > 0$, Eq. (53), gives the behaviors of Cuspidal and center point by choosing the values of parameters are $a_3 = 0.3$, & $a_2 = -0.7$, as depicted in part-(b) of Fig. 6.



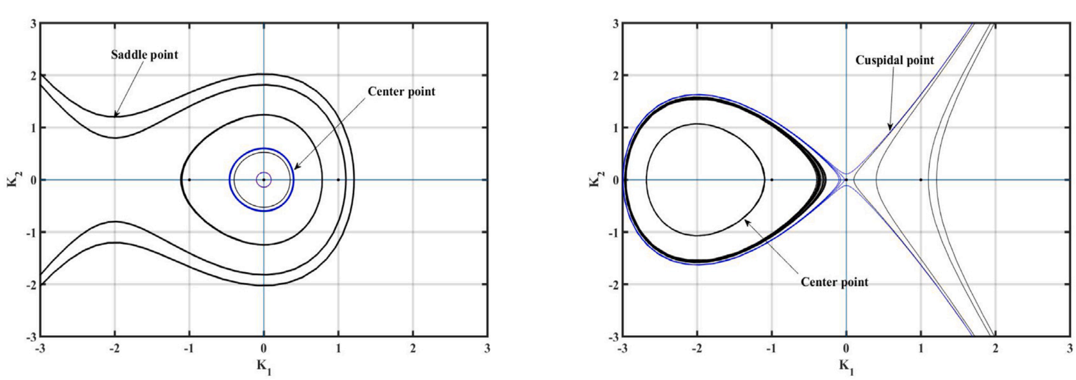
(a) 3-dimensional3 (b) Density plot (c) 2-dimensional

Fig. 4. Physical structure of 2-solitons of dark solution of $\mathcal{A}_{1,4}$ by choosing appropriate parametric values.



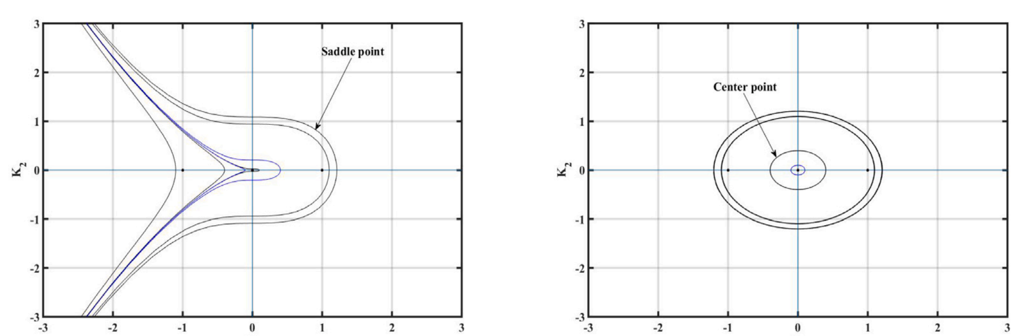
(a) 3-dimensional (b) Density plot (c) 2-dimensional

Fig. 5. Physical structure of periodic solitons of $\mathcal{A}_{1,12}$ by choosing appropriate parametric values.



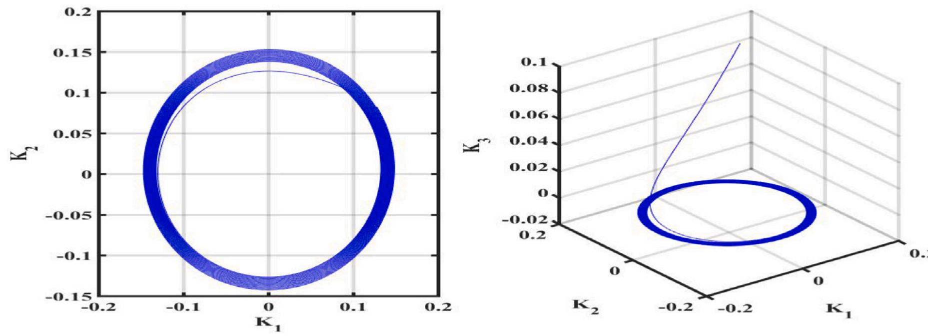
(a) Saddle and Center points (b) Cuspidal and Center points

Fig. 6. Illustration of bifurcation analysis of Eq. (53) under values of parameters are $a_3 = 1,\ \beta_1 = 0.2,\ \beta_2 = 0.5,\ \beta_3 = 0.21,\ \beta_4 = 0.9,\ \&\ a_2 = 0.7$.

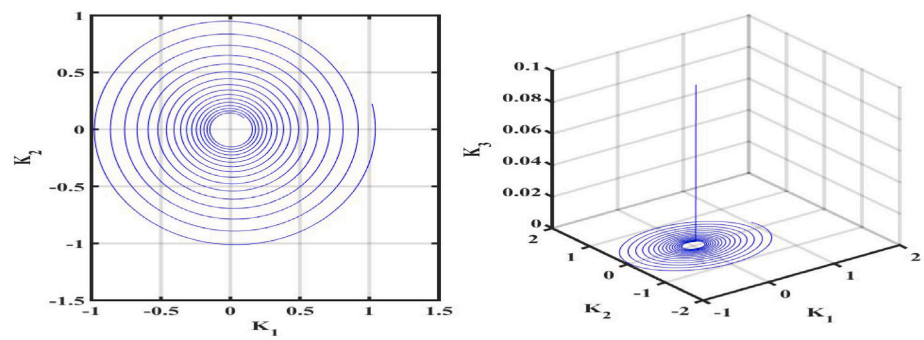


(a) Saddle point (b) Center pint

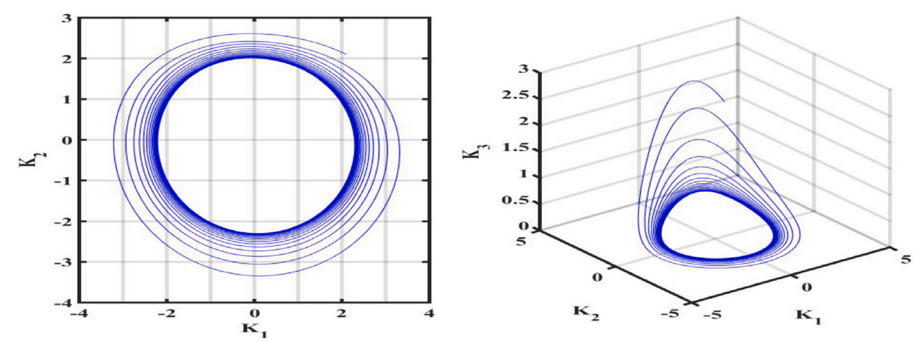
Fig. 7. Illustration of bifurcation analysis of Eq. (53) under values of parameters are $a_3 = 1,\ \beta_1 = 0.2,\ \beta_2 = 0.5,\ \beta_3 = 0.21,\ \beta_4 = 0.9,\ \&\ a_2 = 0.7$.



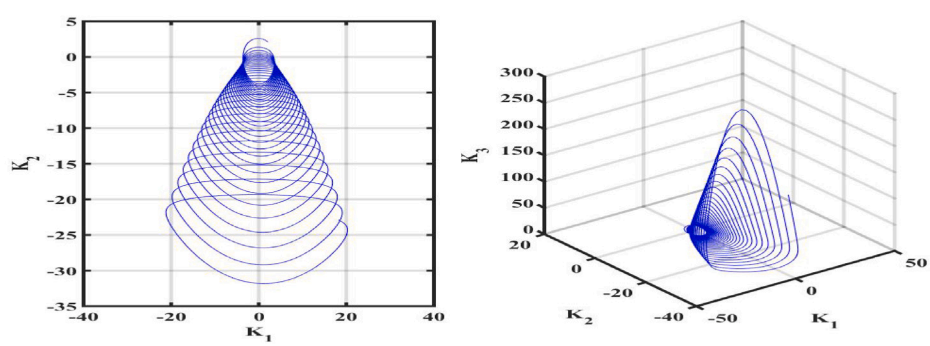
(a) $\delta = 0.1$ & $\Psi = -2.1$.



(b) $\delta = -0.1$ & $\Psi = 2.1$.



(c) $\delta = 2.1$ & $\Psi = 0.1$.



(d) $\delta = 1.1$ & $\Psi = -1.1$.

Fig. 8. Visualization of chaotic analysis with perturbation at different amplitudes and frequencies of Eq. (54).

Case-3: When $B_1 = 0$, $B_2 > 0$, Eq. (53), gives the behaviors of saddle point by choosing the values of parameters are $a_3 = 1.2$ & $a_2 = 1.7$, as depicted in part-(a) of Fig. 7.

Case-4: When $B_1 > 0$, $B_2 = 0$, Eq. (53), gives the behaviors of center point by choosing the values of parameters are $a_3 = 1.4$ & $a_2 = 1.27$, as depicted in part-(b) of Fig. 7.

4.2. Chaotic analysis

Chaos theory examines deterministic systems that exhibit complex and unexpected behavior despite the fact that they are governed by fundamental mathematical equations. The slightest variations in the initial conditions can lead to results that are significantly different. Ultimately, the system's phase space is formed by the trajectories of adjacent points. The phase space of the system is tightly packed with an endless number of periodic orbits. The weather, turbulent fluid movement, the double pendulum, and the logistic map are a few well-known instances of chaotic systems. Chaos theory has been employed in a variety of fields, such as economics, engineering, biology, physics, and the social sciences. It has profoundly altered our perceptions of randomness and predictability in nature as well as our understanding of deterministic systems. Eq. (53) exhibits chaotic behavior when a perturbation term is included. such as

$$\begin{cases} \mathcal{V}'(\phi) = W, \\ \mathcal{V}''(\phi) = W' = B_1 \mathcal{V}(\phi) + B_2 \mathcal{V}^2(\phi) + \delta \cos(\Psi \phi), \\ W'' = \delta, \end{cases} \tag{54}$$

where δ is amplitude and Ψ is frequency. Changing the values of Ψ and δ to get the different structures of chaos with perturbation.

5. Numerical results

The results obtained from the proposed method for solving the (3+1)-dimensional p-type model differ slightly from those obtained by other investigators with existing methods. Our acquired results offer a fresh and simplified understanding of the intricate physical processes associated with these advanced models. Eq. (1) in particular provides a range of special solutions, including bright and dark solitons, periodic waves, and other wave solutions. We have computed our results, which include several previously unreported discoveries, are not just extraordinary but also unique after comparing them to the existing literature [10]. The proposed method has been used to generate a large number of wave solutions in the form of trigonometric and hyperbolic functions. These solutions are unique and have significant implications in mathematical physics. In order to provide a physical explanation for the results, the numerous unique structures of the acquired solutions, including dark, luminous, singular, and numerous periodic solutions, are plotted. Figs. 1–5 show the solitons solutions. Figs. 6 and 7 illustrates the dynamical structures for bifurcation analysis without perturbation term. Fig. 8 illustrates the dynamical structures for chaotic analysis with perturbation terms at different values of frequencies and amplitudes. The density plots depict the distribution of energy. These plots are an effective medium to examine convoluted data and variations in (3+1)-dimensional p-type models. They help with the study, interpretation, and sharing of theory and experimental results in many areas of physics.

6. Conclusion

In our current work, we successfully utilized the concept of MSSEM for novel wave solutions of the (3+1)-dimensional p-type model. These wave results provide rational, trigonometric, exponential, and hyperbolic functions, which have significant applications in a variety of physical and applied domains. Furthermore, graphs show various structural patterns of solutions, such as singular solutions, periodic waves, bright and dark solitons. Using MATLAB, we can effectively visualize these solutions in multiple dimensions, which allowed us to reveal a clear understanding of the physical properties of the proposed model. The dynamical structures of the perturbed and nonperturbed models were studied through bifurcation and chaos. The wide range of wave solutions indicates that the proposed technique is feasible and successful. This approach frequently necessitates extensive changes and transformations, which can be extremely rigorous and challenging to perform accurately. The MSSEM may not be applicable to all varieties of nonlinear PDEs, particularly those with more complex dynamical problem. By investigating and using these results, we can find even more ways to solve complex problems and encourage new ideas in a variety of fields, such as the Lyapunov exponent and bifurcation diagram for our future work.

CRediT authorship contribution statement

Muhammad Nadeem: Writing – original draft, Methodology, Investigation. **Omar Abu Arqub:** Supervision, Formal analysis, Funding acquisition. **Ali Hasan Ali:** Project administration, Software, Resources, Writing – review & editing. **Husam A. Neamah:** Visualization, Software, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] S. Malik, M.S. Hashemi, S. Kumar, H. Rezazadeh, W. Mahmoud, M. Osman, Application of new kudryashov method to various nonlinear partial differential equations, Opt. Quantum Electron. 55 (1) (2023) 8.

[2] S. Tang, X. Feng, W. Wu, H. Xu, Physics-informed neural networks combined with polynomial interpolation to solve nonlinear partial differential equations, Comput. Math. Appl. 132 (2023) 48–62.

[3] N. Rahman, M.A. Akbar, Traveling waves solutions of nonlinear klein Gordon equation by extended (G'/G)-expansion method, Ann. Pure Appl. Math. 3 (1) (2013) 10–16.

[4] S. Jin, N. Liu, Y. Yu, Time complexity analysis of quantum algorithms via linear representations for nonlinear ordinary and partial differential equations, J. Comput. Phys. 487 (2023) 112149.

[5] M.J. Simpson, R.J. Murphy, O.J. Maclaren, Modelling count data with partial differential equation models in biology, J. Theoret. Biol. 580 (2024) 111732.

[6] N. Ullah, M.I. Asjad, A. Hussanan, A. Akgül, W.R. Alharbi, H. Algarni, I. Yahia, Novel waves structures for two nonlinear partial differential equations arising in the nonlinear optics via sardar-subequation method, Alex. Eng. J. 71 (2023) 105–113.

[7] J. Wu, Z. Yang, Global existence and boundedness of chemotaxis-fluid equations to the coupled solow-swan model, AIMS Math. 8 (8) (2023) 17914–17942.

[8] J. Ahmad, Z. Mustafa, Multi soliton solutions and their wave propagation insights to the nonlinear Schrödinger equation via two expansion methods, Quantum Stud. Math. Found. (2024) 1–17.

[9] U. Younas, H.F. Ismael, T. Sulaiman, M.A.S. Murad, N.A. Shah, M. Sharifpur, A diversity of patterns to new (3+ 1)-dimensional Hirota bilinear equation that models dynamics of waves in fluids, Results Phys. 54 (2023) 107124.

[10] S.K. Dhiman, S. Kumar, Analyzing specific waves and various dynamics of multi-peakons in (3+ 1)-dimensional p-type equation using a newly created methodology, Nonlinear Dynam. (2024) 1–14.

[11] B. Mohan, S. Kumar, R. Kumar, Higher-order rogue waves and dispersive solitons of a novel P-type (3+ 1)-D evolution equation in soliton theory and nonlinear waves, Nonlinear Dynam. 111 (21) (2023) 20275–20288.

[12] C. Liu, Z. Li, The dynamical behavior analysis and the traveling wave solutions of the stochastic Sasa–Satsuma equation, Qual. Theory Dyn. Syst. 23 (4) (2024) 157.

[13] S.-S. Chen, B. Tian, L. Liu, Y.-Q. Yuan, C.-R. Zhang, Conservation laws, binary darboux transformations and solitons for a higher-order nonlinear Schrödinger system, Chaos Solitons Fractals 118 (2019) 337–346.

[14] Y.-D. Hu, X.-H.W.H. Zhou, L. Wang, B.-Z. Wang, A more general electromagnetic inverse scattering method based on physics-informed neural network, IEEE Trans. Geosci. Remote Sens. (2023).

[15] H.K. Barman, R. Roy, F. Mahmud, M.A. Akbar, M. Osman, Harmonizing wave solutions to the fokas-lenells model through the generalized kudryashov method, Optik 229 (2021) 166294.

[16] H.U. Rehman, R. Akber, A.-M. Wazwaz, H.M. Alshehri, M. Osman, Analysis of Brownian motion in stochastic Schrödinger wave equation using sardar sub-equation method, Optik 289 (2023) 171305.

[17] M.N. Alam, M.A. Akbar, M.F. Hoque, Exact travelling wave solutions of the (3+ 1)-dimensional mkdV-ZK equation and the (1+ 1)-dimensional compound KdVB equation using the new approach of generalized G'/G \left(g^'/g\right)-expansion method, Pramana 83 (2014) 317–329.

[18] J. Sabi'u, M.T. Shaayesteh, A. Taheri, H. Rezazadeh, M. Inc, A. Akgül, New exact solitary wave solutions of the generalized (3+ 1)-dimensional nonlinear wave equation in liquid with gas bubbles via extended auxiliary equation method, Opt. Quantum Electron. 55 (7) (2023) 586.

[19] M. Shakeel, Attaullah, N.A. Shah, J.D. Chung, Modified exp-function method to find exact solutions of microtubules nonlinear dynamics models, Symmetry 15 (2) (2023) 360.

[20] S. Javed, A. Ali, J. Ahmad, R. Hussain, Study the dynamic behavior of bifurcation, chaos, time series analysis and soliton solutions to a Hirota model, Opt. Quantum Electron. 55 (12) (2023) 1114.

[21] O. González-Gaxiola, A. Biswas, J. Ruiz de Chavez, A. Asiri, Bright and dark optical solitons for the concatenation model by the Laplace-adomian decomposition scheme., Ukrainian J. Phys. Opt. 24 (3) (2023).

[22] A. Hussain, M. Usman, F. Zaman, T. Ibrahim, A. Dawood, Symmetry analysis, closed-form invariant solutions and dynamical wave structures of the benney-luke equation using optimal system of Lie subalgebras, Chinese J. Phys. 84 (2023) 66–88.

[23] S. Hossain, S. Umer, R.K. Rout, M. Tanveer, Fine-grained image analysis for facial expression recognition using deep convolutional neural networks with bilinear pooling, Appl. Soft Comput. 134 (2023) 109997.

[24] T. Shahzad, M.Z. Baber, M.O. Ahmad, N. Ahmed, A. Akgül, S.M. Ali, M. Ali, S.M. El Din, On the analytical study of predator–prey model with holling-II by using the new modified extended direct algebraic technique and its stability analysis, Results Phys. 51 (2023) 106677.

[25] M.S. Islam, K. Khan, M.A. Akbar, A. Mastroberardino, A note on improved F-expansion method combined with riccati equation applied to nonlinear evolution equations, Royal Soc. Open Sci. 1 (2) (2014) 140038.

[26] A. Ali, S. Javed, M. Nadeem, L.F. Iambor, S. Mureşan, A soliton solution for the kadomtsev–petviashvili model using two novel schemes, Symmetry 15 (7) (2023) 1364.

[27] M. Gu, C. Peng, Z. Li, Traveling wave solution of (3+ 1)-dimensional negative-order kdV-Calogero-bogoyavlenskii-schiff equation, Aims Math. 9 (3) (2024) 6699–6708.

[28] M.A. Akbar, N.H.M. Ali, M.T. Islam, Multiple closed form solutions to some fractional order nonlinear evolution equations in physics and plasma physics, AIMS Math. 4 (3) (2019) 397–411.

[29] M.I. Khan, A. Farooq, K.S. Nisar, N.A. Shah, Unveiling new exact solutions of the unstable nonlinear Schrödinger equation using the improved modified sardar sub-equation method, Results Phys. 59 (2024) 107593.

[30] Y. Chahlaoui, A. Ali, J. Ahmad, S. Javed, Dynamical behavior of chaos, bifurcation analysis and soliton solutions to a konno-onno model, PLoS One 18 (9) (2023) e0291197.