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Cite this article: Danku Z, Batool A, Pál G, Kun F. 2026 Effect of the range of interaction on the time evolution of failure cascades. *Proc. R. Soc. A* **482**: 20260050.
<https://doi.org/10.1098/rspa.2026.0050>

Received: 20 January 2026

Accepted: 2 June 2026

Subject Areas:

complexity, statistical physics

Keywords:

cascading failure, avalanche profile, creep rupture, acoustic emission

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Electronic supplementary material is available online at <https://doi.org/10.6084/m9.figshare.c.8582663>.

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Effect of the range of interaction on the time evolution of failure cascades

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We investigate how the range of interaction between system elements influences the statistical and temporal characteristics of failure cascades in driven disordered systems. Focusing on creep-induced failure of heterogeneous materials under constant external load, we employ a fibre bundle model in which elements of random strength are arranged on a square lattice and the load released by failed fibres is redistributed according to a distance-dependent power law. This formulation allows the effective interaction range to be tuned continuously between equal-load-sharing and localized-load-sharing limits. We show that both avalanche size and duration distributions exhibit power-law behaviour, with exponents that vary systematically with the interaction range, signalling a broad crossover from mean-field-like global load sharing to stress-concentration-dominated localized load sharing. Beyond avalanche statistics, we analyse the temporal evolution of individual events and demonstrate that their average activity profiles are generically parabolic. Crucially, the symmetry of these profiles is highly sensitive to the interaction range: long-range redistribution produces symmetric, mean-field-like avalanches, whereas localized interactions lead to strongly right-skewed profiles. These results provide a unified statistical and dynamical characterization of failure cascades and highlight

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1. Introduction

A wide range of complex systems exhibit avalanche dynamics when driven by a slowly varying or constant external load [1,2]. In such systems, the response to the driving force is not smooth but proceeds through intermittent bursts of activity—avalanches—that span a broad range of sizes and durations. This behaviour has been observed in systems as diverse as magnetic domain wall motion [2], plastic deformation [3], seismic activity [4] and fracture processes in heterogeneous materials [5]. Despite the diversity of microscopic mechanisms, many of these systems display universal statistical features in their avalanche dynamics, suggesting the presence of common underlying principles governing intermittent failure processes [1,6–8].

Avalanche dynamics provides a unifying statistical framework for understanding intermittent activity across a broad class of physical systems, linking microscopic failure events to macroscopic response. In particular, the temporal organization of avalanches, their size and duration distributions, and the evolution of activity during individual events have emerged as key descriptors of the collective dynamics. These features are not only of fundamental interest, but also of practical relevance, as they often encode information about the internal state of the system and the nature of interactions governing failure propagation.

Beyond material failure, cascade phenomena have been extensively studied in the context of complex networks, where local breakdown events may trigger system-wide avalanches through load redistribution and connectivity constraints. Early studies demonstrated that heterogeneous network topologies are particularly susceptible to cascading failures, such that the removal or failure of a single node can initiate an avalanche of subsequent breakdown events [9–11]. These works highlighted strong conceptual analogies between network-induced cascades and failure processes in physical systems, despite differences in the interpretation of load, stress or flow.

Creep rupture, in which a material fails over time under a constant subcritical load, provides a particularly relevant physical realization of avalanche dynamics [12–15]. During creep rupture, progressive damage accumulation leads to intermittent cracking events that emit acoustic signals. These signals can be detected as crackling noise in acoustic emission (AE) measurements, offering a powerful experimental probe into the microscopic dynamics of fracture and damage evolution [13–15].

To model the failure of disordered materials, the fibre bundle model (FBM) has emerged as a paradigmatic theoretical framework that captures essential aspects of cascade dynamics. In FBMs, material elements are represented as parallel fibres with randomly distributed strength thresholds [16–21]. As fibres fail, their load is redistributed to the remaining intact fibres, potentially triggering further failures and giving rise to avalanches. Owing to the generality of this redistribution-induced failure mechanism, FBMs have been successfully applied beyond fracture mechanics to describe cascading processes in traffic systems [22], flow networks [23] and power grid failures [24,25].

More recently, attention has turned to how the structure of the underlying load-sharing network influences the statistical and dynamical properties of failure cascades [26–28]. Using network-based extensions of FBMs, including topologies generated by the Watts–Strogatz rewiring technique, previous studies revealed a transition from localized failure to mean-field behaviour as structural randomness increases. In this transition, the system evolves from abrupt global failure to a regime characterized by numerous failure cascades with scale-free size distributions [27,28].

In this work, we address a complementary and largely unexplored aspect of cascade dynamics: how the *range* of load redistribution, rather than the topology of the interaction network, affects both the statistical properties and the temporal evolution of failure cascades. We focus on

creep rupture of heterogeneous materials under constant subcritical load, where progressive damage accumulation leads to sudden local failure events [15,29,30]. Using a fibre bundle model defined on a square lattice, we redistribute the load released by a failed fibre among intact ones according to a power-law decay with distance [31]. By tuning the power-law exponent, the effective interaction range can be continuously varied, interpolating between global load sharing and strongly localized redistribution. We show that avalanche size and duration distributions exhibit power-law behaviour with exponents that vary systematically with the interaction range, indicating a broad crossover between equal-load-sharing and localized-load-sharing universality classes. Beyond statistical properties, we analyse the temporal evolution of individual avalanches and demonstrate that their average profiles are generically parabolic in shape. Crucially, the symmetry of these profiles is highly sensitive to the interaction range: while long-range redistribution yields symmetric, mean-field-like profiles, localized interactions give rise to strongly right-skewed avalanche shapes. We provide a quantitative characterization of this transition and show that the degree of profile asymmetry serves as a robust indicator of the effective load-sharing range, suggesting potential applications in non-destructive testing methods aimed at inferring internal stress redistribution properties from AE data.

2. Damage enhanced creep in a fibre bundle model of variable range of interaction

To study the temporal evolution of cracking avalanches accompanying the creep rupture of heterogeneous materials, we use an FBM. In our FBM, the sample is discretized into parallel fibres arranged on a square lattice of side length L . The fibres are assumed to behave as linearly elastic elements with identical Young modulus $E = 1$. The bundle is subjected to a constant external load σ_0 along the fibre direction, chosen below the fracture strength σ_c of the system (subcritical loading). To capture the key mechanisms of time-dependent fracture while flexibly controlling the interaction range between material elements, the model construction has three main components:

(a) Breaking of fibres

Fibres in the model fail through two distinct mechanisms. *Immediate breaking* occurs when the local load σ_i on fibre i exceeds its strength threshold σ_{th}^i ($i = 1, \dots, N$, with $N = L^2$ being the total number of fibres). Under constant subcritical load $\sigma_0 < \sigma_c$, this mechanism alone leads to partial failure, after which the system stabilizes. Time dependence is introduced by allowing intact fibres under load $\sigma_i > 0$ to *accumulate damage* $c_i(t)$, which causes failure once it exceeds a threshold value c_{th}^i [32–34]. The rate of damage accumulation Δc_i follows a power-law dependence on the local load,

$$\Delta c_i = a \sigma_i(t)^\beta dt, \quad (2.1)$$

so that the total accumulated damage at time t is given by

$$c_i(t) = \int_0^t a \sigma(t')^\beta dt', \quad (2.2)$$

where the integration is carried out over the loading history of fibres ($i = 1, \dots, N$). The exponent β governs how sensitive damage accumulation is to the heterogeneity of the stress field and may take any value in the range $0 \leq \beta < \infty$. This mechanism provides a simplified representation of material ageing due to processes such as thermally activated cracking or environmental degradation, which can lead to failure over time even under subcritical loads [32–34].

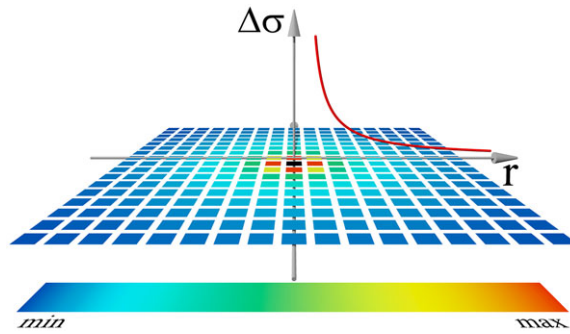


Figure 1. Illustration of the model construction. Fibres are arranged on a square lattice and oriented perpendicular to the lattice plane. Upon fibre failure, the load carried by the broken fibre is redistributed among the intact fibres according to a power-law decay with distance from the failure point. The example shows a lattice of linear size $L = 17$, with a single broken fibre highlighted in black at the centre. The curve represents the load-sharing function of equation (2.4) for exponent $\gamma = 1$, while the colour scale indicates the fraction of load $\Delta\sigma/\sigma$ transferred to intact fibres after the failure of a fibre carrying load σ . The scale ranges from deep blue, corresponding to $\Delta\sigma/\sigma = 0.0015$ for fibres far from the failure point, to red, corresponding to the largest load fraction $\Delta\sigma/\sigma = 0.016$ received by the nearest neighbours.

(b) Controlling the range of load sharing

Under stress-controlled loading, the load released by a failed fibre must be redistributed among the remaining intact fibres,

$$\sigma_j \rightarrow \sigma_j + \Delta\sigma_j. \quad (2.3)$$

Following [31], we assume that all intact fibres participate in load sharing, but the excess load $\Delta\sigma_j$ received by fibre j decreases with the distance from the failed fibre i , following a power-law

$$\Delta\sigma_j = \frac{1}{Z} \frac{\sigma_i}{r_{ij}^\gamma}. \quad (2.4)$$

Here, σ_i is the load of the failed fibre, r_{ij} is the distance between fibres i and j , and γ controls the spatial decay of load transfer. The distance r_{ij} between fibres i and j , located at positions $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively, is defined as their physical distance, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, with periodic boundary conditions taken into account in both directions of the square lattice. The normalization factor Z ensures load conservation and is computed as

$$Z = \sum_j r_{ij}^{-\gamma}. \quad (2.5)$$

The exponent γ can vary in the range of $0 \leq \gamma < +\infty$. The limiting cases $\gamma \rightarrow 0$ and $\gamma \rightarrow +\infty$ correspond to equal load sharing (ELS), where all fibres receive nearly equal load, and localized load sharing (LLS), where only nearest neighbours absorb the excess load [31,35].

Redistributed load may cause additional fibres to exceed their strength and fail, resulting in further redistribution. These successive steps of breaking and load transfer generate avalanches of failure events, which either terminate (stabilizing the system) or evolve into catastrophic breakdown. The effective range of load sharing, controlled by γ , plays a central role in determining both the statistical and dynamical properties of these avalanches (figure 1).

The local load $\sigma_i(t)$ used in the breaking and damage rules already contains the stress enhancement generated by previously failed fibres. Hence, broken clusters act as local stress concentrators: neighbouring intact fibres receive an increased fraction of the redistributed load and, consequently, have a higher probability of failure and accumulate damage faster. This provides the fibre-bundle analogue of a notch- or crack-tip effect without introducing any additional damage factor. Related effects of pre-existing cracks in fibre bundle models were analysed in [36].

(c) Random strength of fibres

Fibres fail irreversibly if either their local load σ_i or accumulated damage $c_i(t)$ exceeds the respective thresholds σ_{th}^i and c_{th}^i ($i = 1, \dots, N$). To capture material heterogeneity, these strength parameters are sampled from probability distributions $p(\sigma_{th})$ and $p(c_{th})$, respectively. Following our earlier work [32,34,37,38], both quantities are uniformly distributed: σ_{th}^i takes values in the interval $[0, 1]$, and c_{th}^i in the interval $[1 - \delta, 1 + \delta]$, where δ quantifies the disorder in damage thresholds.

Based on our previous studies on FBMs with two failure mechanisms [38], we use $\delta = 0.2$ and $\beta = 5$ in the simulations. This choice ensures that fibres in highly loaded regions—typically around cracks—accumulate damage more rapidly, promoting spatial localization of damage and facilitating crack growth. The time-dependent damage model and the tunable load-sharing range have proven successful in capturing various features of fracture in disordered materials [32,34,37,38]. Here, we apply these approaches to investigate how the range of load redistribution influences the temporal structure of individual failure cascades.

To this end, we performed computer simulations on a square lattice of side length $L = 401$, systematically varying the load-sharing exponent in the range of $0 \leq \gamma \leq 100$, which spans from perfectly global to highly localized redistribution. Periodic boundary conditions are applied in both directions of the lattice. At fixed disorder δ and damage exponent β , the value of the external load σ_0 also influences avalanche activity. Earlier work [38] showed that under localized load sharing, increasing σ_0 initially enhances avalanche activity by enabling more frequent large bursts. However, near the system's fracture strength, avalanche activity diminishes as events more easily become critical and destroy the entire system. Based on these findings [38], we fixed the load $\sigma_0/\sigma_c = 0.1$ to ensure strong avalanche activity. At each parameter set, simulations were repeated $K = 3000$ times with different random realizations of fibre strength to obtain statistically reliable results. Note that the value of the critical load σ_c depends on the load-sharing exponent γ , which had to be determined by independent simulations [31].

3. Failure avalanches triggered by damage sequences

Under a constant external load σ_0 , an initial fraction of fibres fail immediately. If σ_0 exceeds the ultimate strength of the system, catastrophic failure occurs right away. However, for subcritical load $\sigma_0 < \sigma_c$, the bundle stabilizes after this initial damage, and time evolution proceeds from this partially broken state. Local failure events are followed by load redistribution over the remaining intact fibres according to the load-sharing function equation (2.4). Already in this initial state, the stress field becomes heterogeneous: higher loads emerge near failed fibres, while intact regions experience lower loads. Without the time-dependent damage mechanism, this partially failed configuration would remain stable indefinitely. However, as time elapses, intact fibres under load gradually accumulate damage $c_i(t)$ and fail when their damage threshold c_{th}^i is exceeded [32,38].

The timescale of this process is governed by the damage exponent β , and the dynamics is essentially slow: fibres break one by one in a sequence of damage-induced events. Each damage breaking redistributes load to the intact fibres, and after a sufficient number of such events, the accumulated stress may provoke the immediate failure of another fibre. This initiates a cascade of immediate breakings, as load redistribution may trigger further failures. Since load redistribution and immediate breaking occur on a significantly shorter timescale than damage accumulation, no damage is accumulated during an avalanche. As a result, the creep rupture process consists of avalanches of immediate failure—responsible for acoustic bursts in experiments—triggered by slow damage sequences.

The spatial structure of fibre breakings is illustrated in figure 2 for several values of the load-sharing exponent γ , with the damage exponent fixed at $\beta = 5.0$ and the external load was set to the value $\sigma_0/\sigma_c = 0.1$. For $\gamma = 0$, the stress field remains homogeneous throughout the loading process, since all intact fibres receive equal load from failed ones. Consequently, the failure dynamics is fully governed by the random strength disorder of fibres. Our earlier studies have

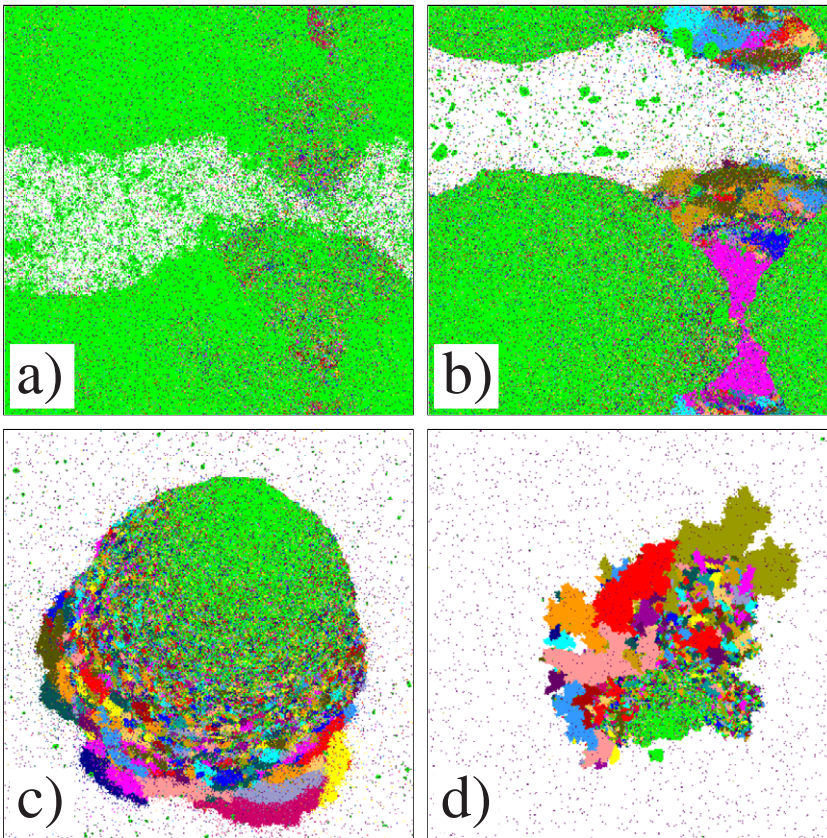


Figure 2. Final states of the time evolution of the bundle, where broken fibres are coloured according to the breaking mechanism: intact fibres are white, green indicates fibres which broke due to damage accumulation, while all other colours stand for fibres failing due to immediate breaking. Extended areas forming a connected set of points of the same colour are breaking avalanches. Results are presented for four different values of load-sharing exponent γ : 2 (a), 2.5 (b), 3 (c) and 6 (d).

shown that this mean-field-like behaviour, obtained exactly for $\gamma = 0$, persists as long as γ remains sufficiently small.

Figure 2a, corresponding to $\gamma = 2.0$, shows that a large fraction of fibres fail due to damage accumulation (indicated in green). This reflects the fact that many damage-induced failures are needed to trigger avalanches of immediate breaking, which are shown in colours other than green. Another consequence of the long-range interaction of fibres and of the nearly homogeneous stress field is that fibres breaking in avalanches do not form spatially connected clusters on the lattice which further reduces stress concentration.

At high γ exponents, however, the redistributed load becomes increasingly localized around failed fibres. According to the damage law equation (2.2), fibres under higher load accumulate damage faster, which results in damage breakings also becoming spatially localized. This, in turn, enhances local stress concentration and facilitates the triggering of larger avalanches that form compact, connected clusters (cracks in the model). Figure 2d illustrates this behaviour for $\gamma = 6$, where the coloured spots (excluding green) correspond to distinct avalanche events. It can also be observed that after some initial breakings, randomly scattered over the bundle, the failure process quickly localizes and both damage breakings and avalanches of immediate breakings occur along the front of a growing crack.

As γ increases from zero to large values, the system undergoes a transition from equal load sharing to fully localized behaviour, where load redistribution occurs solely over the nearest neighbours of failed fibres. This crossover is clearly visible in figure 2b,c: for intermediate values

of γ , fewer damage breakings (smaller and smaller green-coloured regions) are sufficient to trigger immediate-failure avalanches, which become more spatially correlated as we approach the localized regime. In figure 2c already for $\gamma = 3$, the failure process is dominated by the growth of a single crack.

This transition from mean-field to localized behaviour has been previously studied in [31] under quasi-static loading, where the external load is slowly increased and no damage mechanism is included. Those studies identified a critical value of the load-sharing exponent around $\gamma_c \approx 2.1$, where the ELS to LLS transition sets on. Later, this value was made more precise: $\gamma_c \approx 2.15$ in [39].

In the present work, we focus on creep rupture under constant external load, where time dependence arises from the internal damage mechanism of fibres. Our aim is to analyse how the range of load sharing influences both the statistics and temporal dynamics of failure avalanches in this setting.

4. Statistics and dynamics of avalanches

The size Δ of an avalanche is defined as the number of fibres breaking in the cascade of immediate failure events, which is proportional to the newly created crack surface in the specimen. Failure avalanches always begin with a single fibre breaking due to a load increment arising from a preceding damage event, and then evolve through consecutive steps of load redistribution and immediate breaking. The avalanche duration W is defined as the number of breaking-load redistribution steps until the avalanche terminates. The number of fibres Δ_s breaking in step u defines a sub-avalanche of size $\Delta_s(u)$. These quantities are related by

$$\Delta = \sum_{u=1}^W \Delta_s(u), \quad (4.1)$$

where u serves as a discrete time index within the avalanche. The physical time-scale T of avalanche duration is assumed proportional to W , which reflects the finite propagation time of elastic interactions. Since the avalanche duration is much shorter than the timescale of damage accumulation, we assume that no damage events occur during avalanches.

We emphasize that the above definition of an avalanche does not involve any temporal binning or waiting-time threshold. The creep dynamics is implemented with a strict separation of timescales between slow damage accumulation and fast immediate breaking. During the damage-controlled part of the evolution, fibres fail one by one due to the accumulation of damage. If the load redistribution following such a damage event triggers immediate breaking, the damage evolution is stopped and the resulting avalanche is followed through successive redistribution-breaking steps. The avalanche terminates only when all remaining intact fibres satisfy $\sigma_i < \sigma_{th}^i$. Only after this relaxation process has ended is the damage clock advanced again to generate the next damage-induced failure. Consequently, avalanches triggered by different damage events are always separated in the simulations, and no artificial merging of temporally close events can occur.

Both the size Δ and duration W of avalanches are stochastic quantities, characterized by the probability distributions $p(\Delta)$ and $p(W)$. As shown in figure 3, the shape of these distributions strongly depends on the load-sharing exponent γ . In the limiting cases of low and high γ , corresponding to the ELS and LLS regimes, the distributions exhibit scale-free behaviour,

$$p(x) \sim x^{-\tau} \exp\left(-\frac{x}{x_0}\right), \quad (4.2)$$

i.e. a power law with exponent τ , followed by an exponential cut-off governed by x_0 . The exponent τ depends on the effective range of load sharing. In the ELS limit $\gamma \lesssim \gamma_c$, the avalanche size and duration exponents are $\tau_\Delta = 2.5 \pm 0.04$ and $\tau_W = 3.5 \pm 0.05$, respectively, in

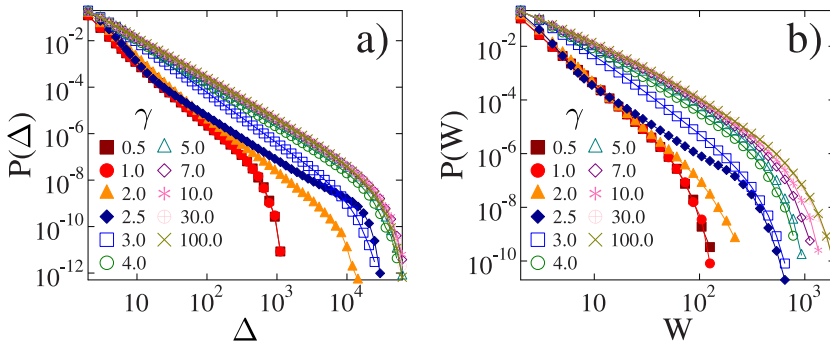


Figure 3. Probability distributions of (a) avalanche size $p(\Delta)$ and (b) avalanche duration $p(W)$ for several values of the load-sharing exponent γ . In both panels, scale-free behaviour is observed in the limiting cases of small and large γ , corresponding to the ELS and LLS regimes, respectively. A crossover between two power-law regimes emerges for intermediate values of γ .

good agreement with their mean-field counterparts $\tau_{\Delta}^{\text{ELS}} = 5/2$ and $\tau_W^{\text{ELS}} = 3.75$ known from quasi-static FBM [40].

As γ increases above γ_c , the load redistribution becomes more localized, enhancing stress concentration near cracks. As a result, larger avalanches can grow in the bundle, and hence, the cut-offs Δ_{max} and W_{max} increase, while the exponents τ_{Δ} and τ_W gradually decrease, converging to $\tau_{\Delta} = 1.8 \pm 0.05$ and $\tau_W = 2.3 \pm 0.06$ in the LLS limit.

In the intermediate regime, around $\gamma \approx 2$ – 5 , the distributions $p(\Delta)$ and $p(W)$ exhibit crossover behaviour between two power-law regimes with different exponents (see figure 3). To quantify how load sharing affects avalanche statistics, the extraction of avalanche exponents was carried out as follows. The distributions $p(\Delta)$ and $p(W)$ were logarithmically binned, and the slopes were determined by linear regression in the log–log representation over the scaling interval. In the ELS and LLS limiting regimes, this interval is well defined between the small-event region and the large-event cut-off. In the crossover regime, however, the distributions are visibly curved and display two apparent regimes. We, therefore, did not fit the full distributions by a single power law. Instead, the fits were restricted to the large-avalanche sector, excluding both the small-event regime, where the curves are steeper, and the far tail affected by the finite-size cut-off. This choice is also consistent with the analysis of avalanche profiles, where scaling is expected for large durations.

As a robustness check, we repeated the exponent estimation using a maximum-likelihood (MLE) method for discrete power-law distributed variables. The MLE estimates were found to be consistent, within statistical uncertainty, with the regression-based values obtained from the same scaling intervals. In the crossover regime, the fitted values should nevertheless be interpreted as effective exponents characterizing the large-event sector, rather than as asymptotic universal exponents of a separate critical state.

Figure 4a presents the fitted exponents τ_{Δ} and τ_W over the full range of $0 \leq \gamma \leq 100$ considered in our study. For $\gamma \lesssim \gamma_c$, both exponents remain approximately constant, confirming the mean-field behaviour. Around $\gamma \approx \gamma_c$, the avalanche size exponent τ_{Δ} rapidly decreases, indicating the onset of the transition to LLS-like behaviour. For $\gamma \gtrsim 5$, the exponents appear to saturate, although further analysis revealed that the crossover region is in fact broader, extending up to $\gamma \approx 8$ – 10 (see later the behaviour of avalanche shape parameters).

The behaviour of the duration exponent τ_W is qualitatively similar but exhibits a more complex, non-monotonic trend: as γ increases, τ_W initially rises, then drops and finally settles near the LLS value. We carefully analysed the impact of fitting range and statistical fluctuations to ensure the robustness of the results in figure 4a.

We note that the exponents τ_{Δ} and τ_W in the ELS limit are universal for a broad class of strength distributions $p(\sigma_{\text{th}})$, their numerical values we obtained are consistent with $\tau_{\Delta}^{\text{ELS}}$ and τ_W^{ELS}

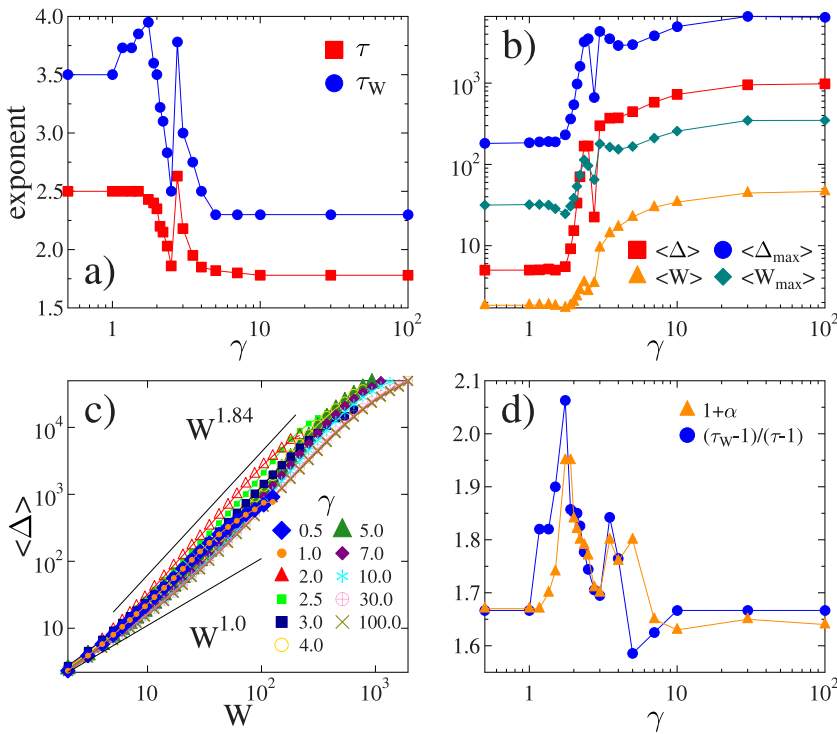


Figure 4. (a) Avalanche exponents τ_Δ and τ_W for size and duration distributions as functions of the load-sharing exponent γ . The onset of deviation from mean-field behaviour occurs near $\gamma \approx \gamma_c$ for τ_Δ and somewhat earlier for τ_W . The exponents gradually approach their LLS values beyond $\gamma \gtrsim 5$. The crossover region is broad and exhibits enhanced uncertainty. (b) Average avalanche size $\langle \Delta \rangle$ and duration $\langle W \rangle$, and their average maxima $\langle \Delta_{\max} \rangle$ and $\langle W_{\max} \rangle$ as functions of γ . All quantities have an overall increasing trend with stronger stress localization, and the curves display non-monotonic behaviour in the crossover regime. (c) Average size of avalanches of fixed duration for various γ values. The power-law relation equation (4.3) holds across all γ . (d) Test of the scaling relation equation (4.4). Both sides of the relation are plotted as functions of γ , showing consistent agreement across regimes.

[16,41–43]. By contrast, the LLS limit and the intermediate regime depend more strongly on the disorder in fibre strength [27,28,34].

To further explore how the broad crossover from the equal to the localized load-sharing regime occurs, in figure 3 we show that the cut-offs of the distributions of avalanche characteristics undergo also a complex evolution when the range of load sharing γ is varied. To estimate the characteristic avalanche size and duration and the value of the cut-offs, we determined the average size $\langle \Delta \rangle$ and duration $\langle W \rangle$ of avalanches and the average value of the largest burst size $\langle \Delta_{\max} \rangle$ and duration $\langle W_{\max} \rangle$. Figure 4b shows that as γ increases, the average avalanche size $\langle \Delta \rangle$ and duration $\langle W \rangle$, along with their maximum values $\langle \Delta_{\max} \rangle$ and $\langle W_{\max} \rangle$, all have an overall increasing trend due to enhanced stress localization which gives rise to larger avalanches. Notably, $\langle \Delta \rangle > \langle W \rangle$ and likewise for their maxima, indicating that multiple fibres typically break in each sub-avalanche. Importantly, these curves are not monotonic in the crossover regime, where $\langle \Delta_{\max} \rangle$ and $\langle W_{\max} \rangle$ show complex, non-smooth behaviour consistent with that of τ_Δ and τ_W .

Avalanche size and duration are not independent, i.e. longer avalanches tend to have larger size. To quantify this effect, we measured the average avalanche size $\langle \Delta \rangle$ for fixed duration W . As shown in figure 4c, the relation follows a power law,

$$\langle \Delta \rangle \sim W^{1+\alpha}, \quad (4.3)$$

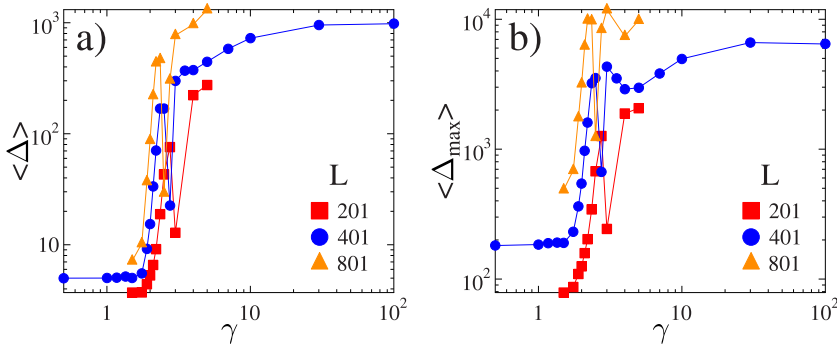


Figure 5. Finite-size analysis of the crossover regime. The average avalanche size $\langle \Delta \rangle$ (a) and the average of the largest avalanche $\langle \Delta_{\max} \rangle$ (b) are shown as functions of the load-sharing exponent γ for three system sizes, $L = 200, 401$ and 800 . For each system size and each value of γ , the critical load $\sigma_c(\gamma, L)$ was determined independently and the creep simulations were performed at fixed relative load $\sigma_0/\sigma_c(\gamma, L) = 0.1$. For the two additional system sizes $L = 200$ and $L = 800$, simulations were performed only in the crossover regime. The data show that increasing L sharpens the crossover and shifts its upper boundary slightly towards smaller γ , while the non-monotonic behaviour in the crossover regime persists.

where the exponent α varies with γ . In figure 4d, we observe that in both the ELS and LLS limits the curve of $1 + \alpha$ tends to the constant $1 + \alpha \approx 1.66 \pm 0.02$, with some erratic variations in the crossover region.

The three exponents τ_Δ , τ_W and α are related by the scaling law [6,44],

$$1 + \alpha = \frac{\tau_W - 1}{\tau_\Delta - 1}. \quad (4.4)$$

We tested this relation in figure 4d by plotting both sides as a function of γ , using independently measured exponent values. The scaling relation equation (4.4) is based on the assumption that avalanche size and duration are governed by a single joint scaling form. As shown in figure 4d, this relation is well satisfied in the limiting ELS and LLS regimes, where the avalanche dynamics is controlled by a single dominant mechanism. In the crossover regime, however, significant deviations occur. We interpret these deviations not simply as statistical fluctuations, but as a signature of mixed dynamics: disorder-controlled breaking, localized crack propagation and crack–crack interaction all contribute to the avalanche ensemble. Consequently, the power-law exponents extracted in this interval should be regarded as effective exponents characterizing the large-event sector, rather than as asymptotic critical exponents of a separate universality class.

To test whether the anomalous behaviour observed in the crossover regime is affected by finite-size effects, we carried out additional simulations for two further system sizes, $L = 200$ and $L = 800$, focusing on the vicinity of the ELS–LLS transition. For each pair (γ, L) , the critical load $\sigma_c(\gamma, L)$ was determined independently, and the creep simulations were performed at the same relative load $\sigma_0/\sigma_c(\gamma, L) = 0.1$ as for the main system size $L = 401$. The results, shown in figure 5, demonstrate that finite-size effects are indeed present in the crossover regime. As the system size increases, the transition becomes somewhat sharper and the interval of γ values where anomalous behaviour occurs shifts slightly towards smaller γ . Nevertheless, the non-monotonic variation of the avalanche characteristics survives for all system sizes considered. In particular, the average avalanche size $\langle \Delta \rangle$ and the average of the largest avalanche $\langle \Delta_{\max} \rangle$ in figure 5 still show enhanced fluctuations and non-smooth behaviour for $L = 800$. This confirms that the anomalies are not artifacts of the finite lattice size $L = 401$, but rather originate from the mixed dynamics of the crossover regime.

The persistence of this behaviour with increasing system size supports the interpretation that additional microscopic mechanisms become relevant when the load redistribution is neither fully global nor fully localized. To identify the microscopic origin of this mixed dynamics, we examined

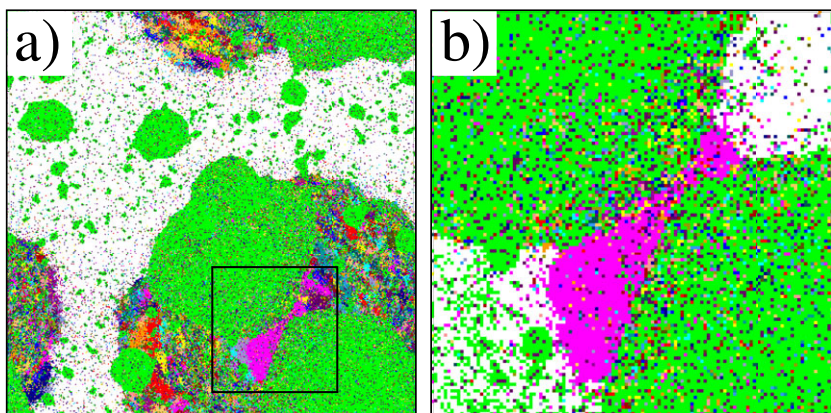


Figure 6. (a) Final state damage structure of an FBM at $\gamma = 2.5$, where fibres breaking due to damage are indicated by green colour, while avalanches of immediate breakings are drawn in colours different from green. The black square highlights a region where avalanches are generated between two nearby cracks where the highly loaded regions along crack perimeters overlap. (b) Magnified view of the square shaped region of (a) where an anomalously large avalanche emerged due to the high load concentration in the region spatially constrained by the two nearby cracks.

in detail the spatial evolution of damage and its coupling to avalanche propagation in the intermediate range of γ , where the load redistribution is neither fully global nor fully localized. In this crossover regime, the stress field becomes increasingly heterogeneous, prompting us to analyse how interacting cracks influence avalanche dynamics. Clusters of broken fibres form cracks whose perimeters experience strong load concentration: the load released at each breaking event accumulates along the boundary of the growing broken region. These perimeter fibres, repeatedly subjected to high overload, act as stress-focusing sites for subsequent avalanches.

When an evolving avalanche encounters such a crack, the overloaded region at its front overlaps with the high-load crack perimeter. This overlap amplifies local stress concentration and forces the avalanche to extend over a larger area before the system can stabilize. Figure 6a provides a clear example: an avalanche triggered between two nearby cracks grows anomalously large because the stress-enhanced zones of the cracks overlap; the magnified region in figure 6b shows the confined geometry responsible for this amplification.

This crack-mediated mechanism increases the weight of large avalanches, thereby reducing the apparent exponents τ_Δ and τ_W . In the ELS regime, stress remains nearly homogeneous and this amplification cannot occur, while in the LLS limit the growth of a single dominant crack prevents merging between extended damage zones. Consequently, the effect appears only at intermediate values of γ , near the ELS–LLS transition γ_c , where multiple cracks coexist and interact. This explains the strong fluctuations in both avalanche averages and exponents seen in figure 4a,b.

The same mechanism also explains why the tails of the avalanche distributions are especially sensitive in the crossover regime. The anomalously large avalanches generated by crack interaction have a disproportionate influence on the tails of the distributions. Although such events are less frequent than ordinary avalanches, their large size and duration strongly affect the fitted large-event exponents and the averages of the maximal avalanche size and duration. Therefore, the crossover regime should not be viewed as a clean critical regime with a single set of universal exponents. Instead, it is a mixed-dynamics region in which two limiting failure modes coexist: disorder-controlled diffuse failure characteristic of ELS, and stress-concentration-driven crack propagation characteristic of LLS. The crack-merging events illustrated in figure 6 are a characteristic manifestation of this coexistence.

Having clarified the microscopic origin of the broad crossover, we now turn to the temporal structure of avalanches. As shown below, not only the statistics but also the average shape of avalanches reflects the underlying interaction range and exhibits similarly rich behaviour.

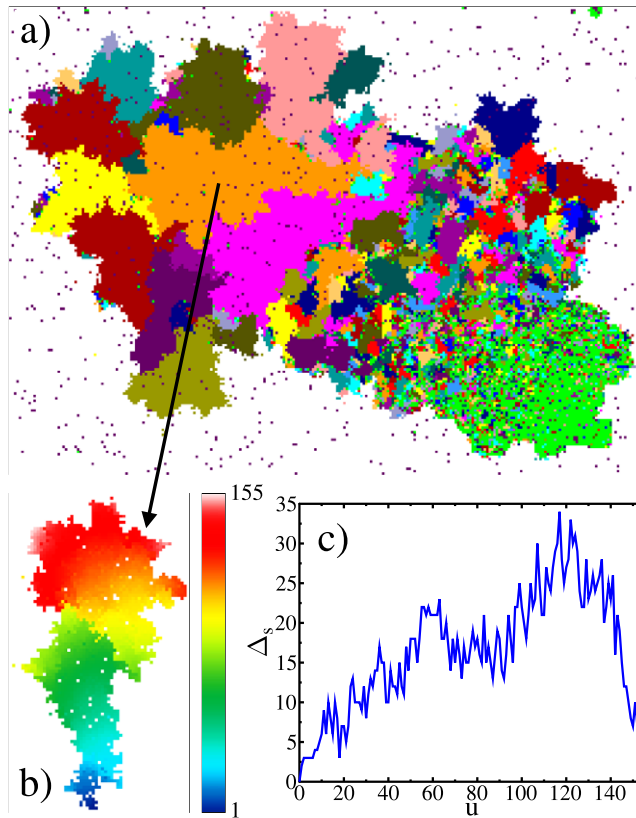


Figure 7. (a) Avalanches in a fibre bundle at the load-sharing exponent $\gamma = 5$. Due to stress localization, after some initial random breakings, a crack started to grow from the lower right corner driven by damage-induced failures. Once sufficient stress had accumulated at the crack front, bursts of immediate breakings—indicated by distinct colours—erupted with increasing size. (b) Time evolution of a single avalanche of duration $W = 155$, marked by the arrow in (a). Different colours highlight the sub-avalanches in consecutive steps. (c) The size of sub-avalanches Δ_s as a function of the internal time u .

5. Temporal profile of avalanches

Single avalanches have a non-trivial dynamics, which is illustrated in figure 7 for the case of the load-sharing exponent $\gamma = 5$, where avalanches occur along the propagating front of a growing crack. Figure 7a shows all avalanches occurring before global failure, while figure 7b zooms in on the temporal structure of a specific event. Here, fibres that break in each time-step u form sub-avalanches of size Δ_s , colour-coded to reflect their sequential development as the avalanche spreads. The fluctuating nature of the avalanche profile $\Delta_s(u)$, shown in figure 7c, reflects the underlying randomness in fibre strength σ_{th}^i, c_{th}^i ($i = 1, \dots, N$).

To extract the typical time evolution of avalanches, we average the size of sub-avalanches $\Delta_s(u)$ over all events of a fixed duration W [7,28,34,40,44]. The resulting average temporal profiles $\langle \Delta_s(u) \rangle$ are presented in figure 8 for several values of the load-sharing exponent γ , keeping the duration $W = 30$ constant. It can be seen in figure 8a that all curves display a characteristic parabolic shape, while their amplitude and symmetry vary markedly with γ . As load sharing becomes increasingly localized with increasing γ , the peak height first increases, indicating that avalanches of the same duration become larger in size. At larger γ , however, the profile height decreases and approaches a limiting curve characteristic of the LLS regime. Perhaps the most striking feature is the transition from symmetric profiles in the ELS regime ($\gamma \lesssim \gamma_c$) to right-skewed profiles for $\gamma > 5$, where load sharing is strongly localized. Interestingly, in the crossover regime, the symmetry varies non-monotonically, and even left-skewed shapes can

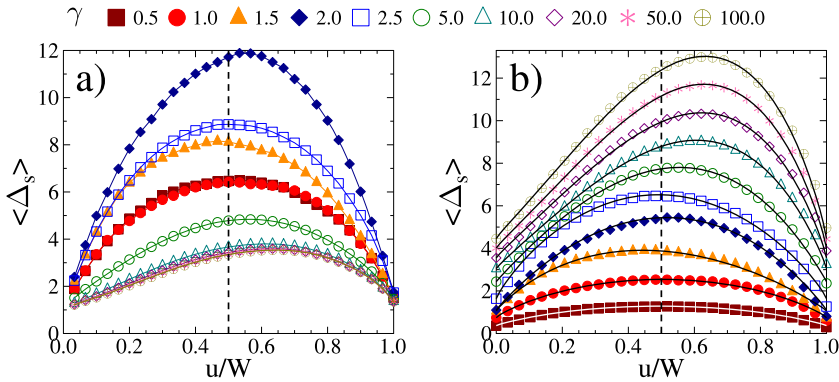


Figure 8. (a) Average temporal profiles of avalanches for several values of the load-sharing exponent γ at fixed duration $W = 30$. The horizontal axis is rescaled by the avalanche duration W , such that $u/W \in [0, 1]$. The vertical dashed line marks the midpoint $u/W = 0.5$. (b) Demonstration of the fitting of profiles with the functional form equation (5.2). For clarity, the curves of different γ values are vertically offset.

emerge. This apparently anomalous behaviour is associated with the interaction of nearby cracks in the crossover regime. In this range of γ , the stress field is already sufficiently heterogeneous for localized cracks to form, but the load redistribution is still sufficiently long-ranged that several cracks may coexist and interact. When an avalanche is triggered in the stress-enhanced region between nearby cracks, or when its propagating front reaches the overloaded perimeter of a pre-existing crack, many fibres are already close to their immediate breaking threshold. This may produce a rapid initial increase of activity, followed by a slower decay as the cascade exhausts the locally overloaded region and stabilizes. Such events shift the maximum of the average profile towards earlier reduced times $u/W < 1/2$, giving rise to left-skewed profiles.

The asymmetry of the temporal profile reflects how the cascade unfolds in time: right-skewed profiles suggest a slow build-up followed by a sudden termination, while left-skewed ones indicate a rapid onset followed by a more gradual decay of the breaking activity. Figure 9 explores this further by showing how profiles evolve with W for fixed γ values covering the range of durations $10 \leq W \leq 100$. It can be seen that longer avalanches grow in amplitude, as expected from the scaling relation equation (4.3). Remarkably, the symmetry pattern remains consistent across durations—each value of γ imprints a characteristic profile shape so that the maxima of profiles fall to the same u/W values apart from fluctuations. Note that for longer durations the $\langle \Delta_s \rangle$ curves get ramified due to the decreasing number of avalanches (see also the distributions $p(\Delta)$ and $p(W)$ in figure 3).

Figure 10 confirms the robustness of these shapes via a scaling collapse. By appropriately rescaling the vertical axis with W^α , where α is tuned for best overlap near the peak, profiles of different durations W collapse onto a universal curve. While some deviations appear near the boundaries ($u/W \rightarrow 0$ or 1), the scaling form,

$$\langle \Delta_s(u, W) \rangle = W^\alpha f\left(\frac{u}{W}\right), \quad (5.1)$$

holds remarkably well, reinforcing that both α and the shape of the scaling function $f(x)$ are sensitive to the load redistribution range.

To quantify the shape and asymmetry more precisely, we fit the profiles using the analytic form proposed in [5] for fracture avalanches,

$$f(x) \sim [x(1-x)]^\alpha \left[1 - a \left(x - \frac{1}{2} \right) \right], \quad (5.2)$$

where α captures the overall curvature and a quantifies the degree and direction of asymmetry. Note that due to consistency, the parameter α is the same in equations (4.3), (5.1), (5.2) [6,44]. Since deviations of scaling are observed in figure 10, we apply the functional form equation (5.2) to

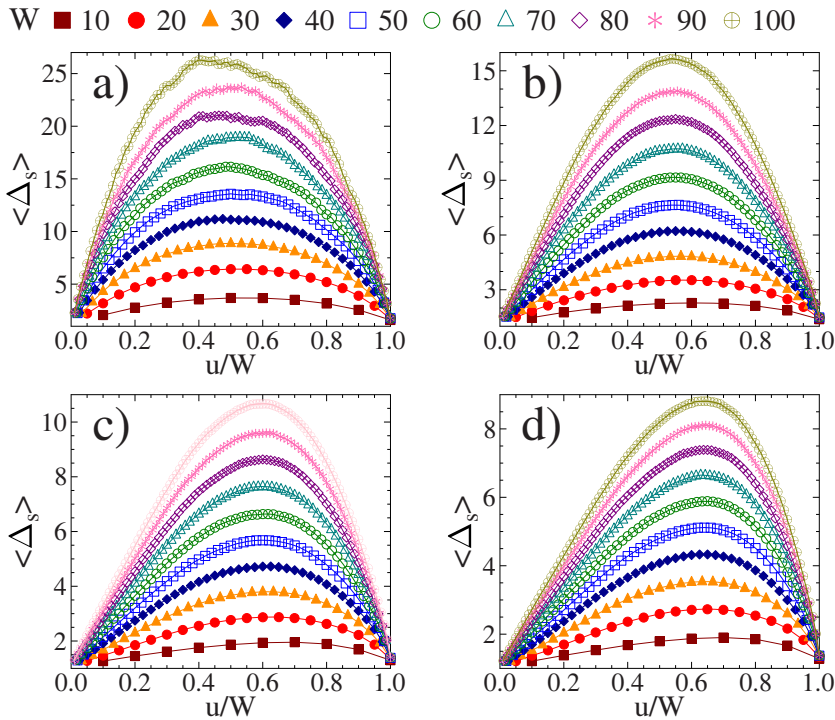


Figure 9. Average temporal profiles of avalanches for different durations W at four selected values of the load-sharing exponent γ : (a) 2.5, (b) 5.0, (c) 10.0 and (d) 100.0. The time axis is scaled with W . Fluctuations increase with duration due to the decreasing number of events.

individual profiles and study how the parameters α and a depend on the duration W at different values of the load-sharing exponent γ . Figure 8b demonstrates that equation (5.2) provides a high quality description of the profiles at any value of γ .

The fitted parameters are summarized in figure 11. In the ELS regime ($\gamma < \gamma_c$), the shape exponent α is practically constant at all durations apart from fluctuations at high W values, however it has a decreasing tendency with increasing W , ranging from $\alpha \approx 1$ down to $\alpha \approx 0.55$ – 0.6 . By contrast, when the ELS to LLS transition is completed for $\gamma \gtrsim 10$, α stabilizes around 0.8–0.9 being independent of the duration above $W = 20$, in agreement with quasi-static LLS studies [34]. Figure 11b demonstrates that the asymmetry parameter a practically does not depend on the avalanche duration for $W > 10$. In the ELS regime, it remains close to zero indicating the high degree of symmetry, while it becomes increasingly negative as γ grows—highlighting the onset of pronounced right-skewness. The reason is that in the presence of stress concentrations an avalanche front emerges along which a large amount of stress gets accumulated. As a consequence, the front gradually accelerates and stops suddenly when entering a material region with high local strength. The systematic positive excursion of a in the crossover regime shows that the left-skewness is a robust statistical feature rather than a fluctuation of individual avalanche profiles. Positive values of a correspond to profiles whose maximum is shifted towards earlier reduced times $u/W < 1/2$, while negative values indicate the right-skewed profiles characteristic of localized load sharing. The positive values of a occur for several avalanche durations in the crossover regime, whereas a remains close to zero in the ELS regime and becomes negative in the LLS limit. Thus, the left-skewed profiles are confined to the crossover region, where multiple interacting cracks coexist.

These results reveal that the average shape and asymmetry of avalanche profiles serve as sensitive indicators of the range of load sharing in the system, offering potential for non-destructive material diagnostics.

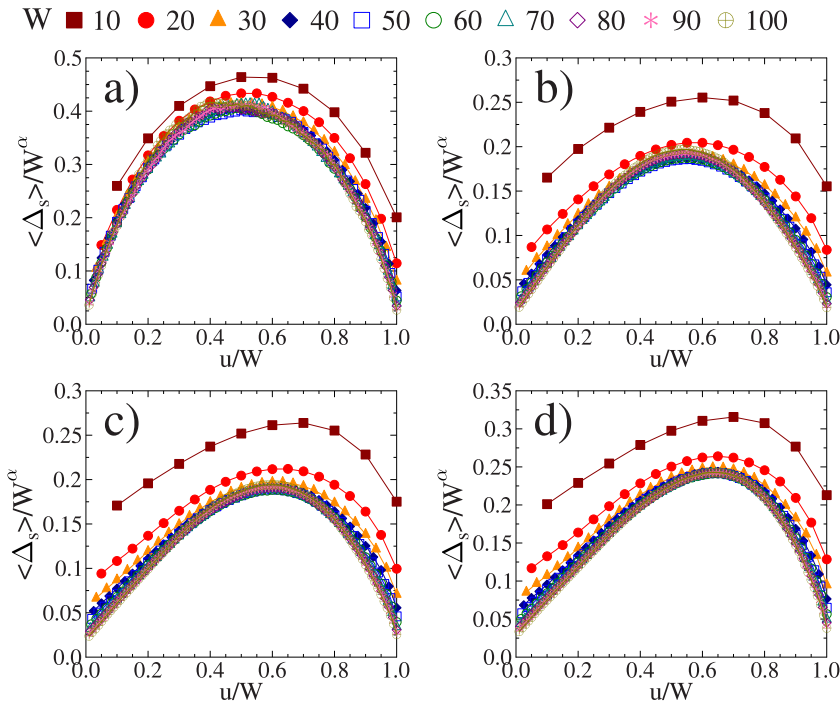


Figure 10. Rescaled average avalanche profiles $\langle \Delta_s \rangle(u, W)$ for various durations W at fixed load-sharing exponent γ . A good collapse is obtained by rescaling the vertical axis with W^α , with the scaling exponent α depending on γ : (a) $\alpha = 0.9$ for $\gamma = 2.5$, (b) $\alpha = 0.95$ for $\gamma = 5.0$, (c) $\alpha = 0.87$ for $\gamma = 10.0$, (d) $\alpha = 0.78$ for $\gamma = 100.0$.

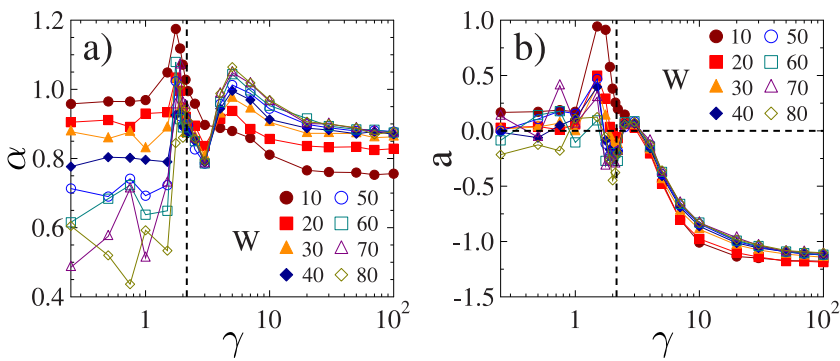


Figure 11. Parameters extracted from fitting the average avalanche profiles to equation (5.2): (a) shape exponent α and (b) asymmetry parameter a as a function of γ for various durations W . The exponent α reflects the steepness and sharpness of profiles, while a captures their temporal skewness. The vertical dashed line in both figures indicates the value of $\gamma_c = 2.15$.

6. Discussion

Fracture of heterogeneous materials typically proceeds through intermittent bursts of activity, triggered by the sudden nucleation and propagation of cracks that arrest upon encountering locally stronger regions. These cracking events are accompanied by the release of elastic energy in the form of waves, detectable as acoustic emissions. Crackling noise measurements therefore provide a powerful window into the microscopic dynamics of fracture. Among various fracture scenarios, creep rupture—failure under a constant subcritical external load—poses a

particularly challenging problem, both fundamentally and practically. Since the applied load remains below the nominal strength of the material, the observed time dependence must originate from underlying microscopic processes such as viscoelastic relaxation, frictional sliding or damage accumulation driven by thermal activation. Extracting meaningful information from acoustic bursts thus requires understanding not only avalanche statistics, but also the temporal organization of individual failure events.

In this work, we investigated the statistics and dynamics of failure cascades in a fibre bundle model designed to capture damage-enhanced creep rupture in disordered materials. The model incorporates two essential ingredients: (i) fibres may fail either instantaneously when their local load exceeds a strength threshold, or gradually through damage accumulation under sustained load; and (ii) after failure, the released load is redistributed among intact fibres according to a power-law decay with distance. This formulation allows the effective interaction range to be tuned continuously via the exponent γ . Owing to the separation of timescales between slow damage accumulation and rapid immediate breaking, the rupture process proceeds as a sequence of damage-triggered avalanches, which we characterized in terms of their size, duration and temporal profile.

Building on earlier studies [32,34,38,45], we fixed the disorder and damage parameters and systematically varied only the interaction-range exponent γ . This approach allowed us to isolate the role of load redistribution range in controlling avalanche behaviour. We found that increasing γ drives the system from the ELS regime to the LLS limit. In the ELS case ($\gamma = 0$), avalanche statistics follow mean-field predictions: disorder dominates the failure process, and the size and duration distributions exhibit power-law behaviour with exponents τ_{Δ}^{ELS} and τ_W^{ELS} . As γ increases, stress becomes increasingly concentrated around failed regions, promoting the formation of larger and longer avalanches. Consequently, both the average and maximal avalanche size and duration grow with γ , while the corresponding power-law exponents decrease.

Between the ELS and LLS limits, we identified a broad crossover regime. Avalanche size statistics proved relatively robust: the exponent τ_{Δ} and the averages $\langle \Delta \rangle$ and $\langle \Delta_{\max} \rangle$ remain nearly constant up to $\gamma \approx \gamma_c$, consistent with earlier quasi-static FBM results [31,39,46]. By contrast, avalanche duration is more sensitive to the interaction range: the exponent τ_W begins to deviate from its ELS value already slightly above $\gamma = 1$. Within the crossover region, both exponents and averages exhibit non-monotonic behaviour, including local extrema, before the transition completes near $\gamma \approx 10$.

A central result of this work concerns the temporal evolution of individual avalanches. By averaging the activity over events of fixed duration, we demonstrated that avalanche profiles are generically parabolic. However, the symmetry of these profiles provides a highly sensitive fingerprint of the interaction range. In the ELS regime, profiles are symmetric and closely resemble the excursions of unbiased random walkers [6]. In the LLS limit, profiles become strongly right-skewed. This asymmetry reflects the underlying stress redistribution: in ELS, load is redistributed homogeneously and avalanche dynamics is governed primarily by disorder, whereas in LLS, stress accumulates at the advancing avalanche front, leading to a slow build-up of activity followed by an abrupt termination upon encountering a strong region. Interestingly, in the intermediate regime we also observed left-skewed profiles, indicating non-trivial dynamics during the crossover between global and localized load sharing.

It is worth noting that the temporal asymmetry of avalanche profiles may also carry information on the proximity to macroscopic failure. Recently, Bodaballa *et al.* studied this question in the fibre bundle model and showed that, under quasi-static loading, the asymmetry of the avalanche profile changes systematically as the critical load is approached [47]. In the present creep model the external load is kept fixed and failure is approached through progressive damage accumulation; therefore, the distance from failure is not controlled by the external load in the same way as in quasi-static FBM simulations. Nevertheless, our results indicate that the interaction range strongly affects the symmetry of the avalanche profiles, and a more detailed analysis of the last avalanches before global failure may provide further insight into possible precursors of creep rupture.

An important distinction emerges when comparing creep rupture with quasi-static fracture. In the creep process studied here, the avalanche exponents τ_{Δ} and τ_W are larger in the ELS regime and smaller in the LLS limit, opposite to the trends observed under quasi-static loading [31,40]. This difference is not solely caused by the slow damage accumulation controlled by β . The exponent β mainly sets the timescale on which fibres approach failure, whereas the avalanche itself evolves through load redistribution at fixed external load. Thus, during creep, the background load remains constant and subcritical over the entire failure process. This allows even relatively large avalanches to arrest, because no additional external load is supplied between burst. In the ELS regime, spatial stress concentrations cannot build up, and avalanche growth is limited by the globally redistributed load. By contrast, in the LLS regime, broken clusters generate enhanced local stresses at their boundaries, which facilitates the failure of neighbouring fibres and supports avalanche propagation. Under quasi-static loading, however, the external load is increased between avalanches, and large bursts rapidly become catastrophic as the increasing background stress destabilizes the remaining fibres. This difference underscores the crucial role of the loading protocol in determining cascade dynamics.

Finally, it is instructive to place the present results in the broader context of cascading failure phenomena studied in complex networks. Beyond fibre bundle models, cascade dynamics driven by load redistribution and threshold-triggered breakdown have been extensively investigated in networked systems, where the interplay between interaction range, heterogeneity and topology plays a central role [9–11]. These studies demonstrated that localized overloads or node failures can propagate through networks in the form of avalanches, giving rise to scale-free event statistics and critical transitions between robust and fragile regimes. While the physical interpretation of load and failure differs across systems, the underlying cascade mechanisms exhibit strong conceptual similarities.

In recent work on network-based fibre bundle models [27,28,48], the focus was placed on how the topology of load-sharing connections influences avalanche dynamics, revealing a transition from localized to mean-field behaviour as network randomness increases. By contrast, the present study isolates a complementary and more general control parameter—the spatial range of interactions—while keeping the underlying lattice structure fixed. From a network perspective, tuning the load redistribution exponent effectively interpolates between globally coupled and locally constrained interaction kernels, a mechanism closely related to cascade processes studied in heterogeneous networks.

Importantly, while network cascade studies often emphasize global robustness, percolation thresholds or final system collapse, our results highlight the diagnostic value of the temporal structure of individual cascades. The sensitivity of avalanche profile asymmetry to interaction range provides a dynamical fingerprint that is largely independent of microscopic details. This suggests that temporal avalanche profiles may serve as a unifying observable across material failure, network-induced cascades and other driven threshold systems, offering new opportunities for interpreting crackling noise signals and identifying underlying interaction mechanisms in complex systems.

Data accessibility. The source files of the simulation programs used and the data generated in the study are available in the electronic supplementary material [49].

Declaration of AI use. We have not used AI-assisted technologies in creating this article.

Authors' contributions. Z.D.: investigation, methodology, software, visualization; A.B.: investigation, methodology; G.P.: investigation, methodology; F.K.: conceptualization, data curation, formal analysis, funding acquisition, investigation, methodology, project administration, resources, supervision, validation, writing—original draft.

All authors gave final approval for publication and agreed to be held accountable for the work performed therein.

Conflict of interest declaration. We declare we have no competing interests.

Funding. This work was supported by the University of Debrecen Program for Scientific Publication. Project no. TKP2021-NKTA-34 has been implemented with the support provided from the National Research, Development and Innovation Fund of Hungary, financed under the TKP2021-NKTA funding scheme.

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