PhD Thesis

GeoGebra in Teaching and Learning Mathematics in Albanian Secondary Schools

Pellumb Kllogjeri

Supervisor: **Dr. Péter Körtesi**
Institute of Mathematics, University of Miskolc

The Doctoral School in Mathematics and Computer Science,
Program in Didactics and Statistics of Mathematics,
The University of Debrecen
Hungary

Debrecen, 2015

PhD Thesis

GeoGebra in Teaching and Learning Mathematics in Albanian Secondary Schools

Pellumb Kllogjeri

Supervisor: **Dr. Péter Körtesi**
Institute of Mathematics, University of Miskolc

The Doctoral School in Mathematics and Computer Science,
Program in Didactics and Statistics of Mathematics,
The University of Debrecen
Hungary

Debrecen, 2015

Abstract

•••• This didactical research investigated the effectiveness of GeoGebra software in teaching and learning mathematics in the secondary school in Albania by experimenting simultaneously the students and the teachers. There are many computer programs used for mathematics but we have noticed that Geogebra software is the most fitted to different age groups, it is useful for elementary mathematics teachers and even at the BSC level university teaching, it is useful for the students in learning mathematics in any level of the school, it is fun and entertainment experiencing GeoGebra - stimulating this way the students to move deeper and wider in mathematics. The objectives of this research were to: 1) Determine the school capacities (like computer laboratories) for using technology in teaching and learning mathematics. 2) Determine the computer and internet capacities and abilities of the teachers of mathematics (are they able to teach by using computer programs). 3) Investigate the influence of GeoGebra software in increasing the level of the students in mathematics by comparing two groups where, one is taught in traditional way and the other using GeoGebra.

The strategy used to investigate the influence of GeoGebra software in increasing the level of the students in mathematics and decide whether the mathematical course taught by using GeoGebra software is as effective as more traditional methods of instruction, was:

- Were selected two classes of the same secondary school and of the same grade. One class (the control group) received traditional teaching and instruction in the chapter of Derivatives (text of Analysis 3), while the other (the experimental group) took the same course by using GeoGebra software.
- were collected data about the level of the two groups in mathematics based in the previous chapter only
- at the end of the course, each group took the same comprehensive exam
- the experimental group had two additional tests: one in the beginning of the chapter and one regarding its comprehensions and skills in GeoGebra software(the second one was to study the relation between GeoGebra and mathematics)
- comparisons of the results were done between the two groups and within the experimental group with the purpose of investigating and determining if the treatment with GeoGebra software in teaching and learning process caused a change in the individuals' math knowledge and skills.

- the intervention taking place between the two measures was the use of GeoGebra software in teaching and learning mathematics
- a paired t-test was performed and the observed difference between the groups was summarized in a p-value. The formulation of the testing hypotheses was related to the means of the two methods of instruction (Do two methods have the same mean?), also having into consideration that the same subjects were observed twice (those of the experimental group).

The result was that the difference was sufficiently great , meaning there is evidence that the treatment (the new teaching and learning method) causes change in the observed variable that is, in the level of mathematics.

On the other side, by comparing the two tests results (the end chapter results and GeoGebra results), is revealed that the increased level of knowledge and skills in GeoGebra is accompanied with an increase of the level of knowledge and skills in mathematics. The value calculated showed that there is correlation between GeoGebra and mathematics.

Acknowledgements

First special thanks to my scientific supervisor: Dr. Péter Körtesi (Institute of Mathematics, University of Miskolc). He gave excellent support, advice and encouragement throughout my studies, regarding the participation in conferences and the published papers. I also thank Dr. Kovács András (University of Debrecen) who contributed so much by helping me with many didactical sources and creating comfortable facilities for studying and staying while I have been visiting the University of Debrecen many times and, by organizing and arranging several meetings with doctors and professors of mathematics to have talks and consultations about my studies and didactical experiment in Albania.
I am so thankful to the key persons of my University in Elbasan, Albania: Prof. Dr. Liman Varoshi (rector of the university "Aleksander Xhuvani", Elbasan), Prof. As. Peci Naqellari (Dean of the Faculty of Natural Sciences), Prof. Dr. Agron Tato (chief of the Department of Mathematics and Informatics) who have continually supported by creating for me facilities through the inter-universities program of CEEPUS to visit and cooperate with professors and doctors of the other universities of Europe in the field of mathematics and continue my doctoral studies.

I am grateful to the CEEPUS program through which I have been supported by several grants within its framework of mobilities for teachers and students – an exchange program between universities, and I have been able to carry out the experiment.

Particular thanks I extend to the teachers of mathematics in the secondary schools where the experiment is performed, who every time have been willing to help and offer me unconditional support in regard with the provision of all things I needed**: ready-made computer laboratory, maintaining the communication and relations with the students, text-books and necessary information about the students of the two classes etc.**

Finally, I thank my wife, Lindita Kllogjeri, for her lovely devotion in time and energies by releasing me from many family commitments and encouraging and creating for me good commodities for studies.

Contents

Chapter 3: GeoGebra Experiment for Teaching and Learning Mathematics (2009-2010)

Chapter 4: Statistical Comparison of the Two Data Sets of the Experimental Group

Chapter 5: REPETITION OF GEOGEBRA EXPERIMENT FOR TEACHING AND LEARNING MATHEMATICS (2010 – 2011)

Chapter 6: GeoGebra Surveys –Qualitative Variables

Chapter 1

Introduction

1.1 Short History

I am working in the University of Elbasan, Albania, partner of the CEEPUS network CEEPUS II- 0028-09-10, which includes, as partner University, the University of Debrecen and the University of Miskolc.

Having the opportunity of participating in the existing CEEPUS cooperation and network activities, like that of one month mobility for teachers in February 2007 in the University of Miskolc, one year later, during the CEEPUS Summer University in Miskolc, Hungary, 2008 August 8-20, I heard for the first time about GeoGebra software and program, presented by Zsolt Lavicza, who introduced GeoGebra program for the participants.

I was very much impressed by this program and immediately I became an active part of the teaching by helping the students, present there, to use the software. There I learned that this program was linked with the Miskolc GeoGebra Institute in Hungary, so I shared my concern about this program and asked associated Prof. Dr. Péter Körtesi to act as scientific advisor in my planned doctoral studies of how to make this software available for the teachers and students in Albania.

Our University has been visited by prof.Peter Körtesi and prof. Imre Juhasz, both members of the Miskolc GeoGebra Institute who have offered us direct consultation for my research. Also, I have been able to visit the University of Debrecen and the University of Miskolc several times.

- Through the International GeoGebra Institute I have found direct connections to the didactical research in GeoGebra done in other universities, especially with those lead by Zsolt Lavicza in Cambridge University, Markus Hohenwarter in Florida State University and Ágnes Tuska in Fresno University in California. Due to the existing research interest in Elbasan University I have received a supporting letter by them to plan and act in making up a large cooperation in Albania, in order to create the Albanian GeoGebra. The work was started and I have translated the five (5) properties files of GeoGebra using Attesoro software for the translation. The translation was reviewed after the remarks and suggestions of Judith Hohenwarter (codesigner of

GeoGebra with **Dr. Markus Hohenwarter)** and, now we have GeoGebra version in Albanian.

- Also, are translated some basics of GeoGebra based on the Introductory Book of Markus and Judith Hohenwarter.

1.2 Training on GeoGebra.

I led the teacher training with GeoGebra - the first in Albania. The training schedule was consisted of two hours teaching and practice every month. There were 20 teachers participating in this first training and there will be others in the future. The initiated teacher training program in Albania served as an impetus for other teachers' trainings and for the dissemination of the program for secondary and elementary mathematics teachers, and it will create a large community of users of the program, cooperating with the international user community.

*** In the beginning there was a talk and exchange of thoughts. The conclusions of the talk were:

- none of the teachers had heard about GeoGebra
- watching some simple applications of GeoGebra in constructing geometrical figures they appreciated it very much and were moved to immediately start the training
- they considered GeoGebra as a mean to be involved and used in a program for <u>further qualification of the math teachers of the secondary schools .</u>

After this proposal of teachers, I discussed my research project on GeoGebra software with the Head of the Department of Mathematics and Informatics, prof. Agron Tato, and with the chief inspector of the mathematics in the Education Directorate of our District and I got full support by them and other mathematics specialists to firstly start the math teachers training and later with students of secondary schools. They were very positive and considered this program of great benefit in improving the teaching and learning methods and, is agreed to build such a program including GeoGebra and Maple, Analysis and Algebra. The future plan is to make up a control research, in what extent people like to use, or find useful the program, e.g. questionnaires, internet forum, feedback, follow up the extension of the use of Geogebra via an Albanian Wiki page, and display there comments, feedback or rating function, etc. All this is due to easy access, free download available for students and teachers, large international community and so on. The global experience accumulated is good to be shared in Albania too, it will be easy to create the Albanian language version and related internet pages.

……….

1.3 Pedagogical Experiment and Dissertations with theme on GeoGebra

I have finished an experiment with a class(third year) of a secondary school where I taught the first chapter on Derivatives using GeoGebra and geometrically and visually demonstrating the concepts and properties of monotonous functions, extreme values, the mean value theorem etc. For comparing the results there was another testing class where this chapter is taught in the traditional way. To draw right conclusions were kept detailed notes. The other fact regarding GeoGebra is that, in summer (June) 2009 there was a dissertation from a student in the last year of mathematics branch whose subject was on GeoGebra and under my tutorship (What is GeoGebra and how does it work). His presentation was very much appreciated from the commission of the diploma and graded with top mark. The commission asked him how to help in introducing GeoGebra and making available for them.

- Another dissertation of a student with theme from GeoGebra was led by me in May 2010 (Use of GeoGebra for geometrical illustrations) and another one in June 2011(GEOGEBRA and STATISTICS).

I plan to use the program to produce teaching materials for all levels of teaching for Albanian high schools and universities as well.

Why GeoGebra?

Taking into consideration:

- the situation of using the computer programs and potentialities in the education system in Albania, that is far away so far
- the lack of financial potentialities to get and use computer programs in the teaching and learning process but GeoGebra is a free software which is a great advantage for our education system
- the fact that in many secondary and 9-year schools there are computer laboratories but very little used
- the low level of capabilities of math teachers(including others as well) in using computer programs
- the willingness of prof. Imre Juhasz and prof. Körtesi, both members of the Miskolc GeoGebra Institute, to offer me direct consultation for my planned research.

I understood and valued that GeoGebra program is very important for secondary school mathematics teachers especially, for elementary mathematics teachers and even at the BSC level university teaching in Albania.

1.4 Background of Albanian Education and Mathematics

Albanian Academy of Sciences, founded in 1972, is the most important scientific institution in the Republic of Albania. It includes scientists from the uppermost-academic institutions and centers and organizations within and outside the country. The Academy has 28 academic members and 26 associate and honorary members. Academy has two sections: Section of Humanities and Albanology and the Section of Natural Sciences and Technology. In its structure includes research projects and Innovation Technological Development, Division of Public Relations, Science and Publishing Library.
”The Albanian Academy of Sciences has its inception in 1750, when it was created in Voskopoja(close to Korca), the so-called ”New Academy”. It was both high school and cultural center, known in Albania and abroad also. Although, the”New Academy” had neither the structure nor the duties of the Academies in other countries of Europe, it was associated with heritage and cultural developments of the Albanian nation. Albanians have been for centuries under foreign oppression and administering, however they could survive by maintaining and developing their language and original culture. Cultural and scientific contributions of the Albanians can be found not only in Albania but in many European countries and beyond, as well. They have left traces in three mail areas:

1. In the humanitarian field (phylosophy, history, literature, theology, folklore) through: Demetrio Frengu (1443-1525), John Buzuku (16th century), Marin Barleti(1460-1512), Pjeter Budi(1566-1623), Franc Bardhi(1606-1643), Pjeter Bogdani (1625-1689), etc.
2. In the field of art (musicians, painters and sculptors): Jan Kukuzeli(1010-1075)-the most known figure in music, the painter Onufri(16th century), David Selenica and Constantine Shpataraku (18th century).
3. In the field of science by scholars like John Gazulli (1400-1465)-astronomer and mathematician, Leonik Tomeu (1456-1531)-astronomer, philosopher and professor of Nicolas Copernicus in University of Padua, etc.” ([1], translated by the author, see the original in Annexe 1.)

Successive occupations damaged and hindered the progressive and cultural development of the country, but failed to stop and vanish the Albanian cultural heritage and the authenticity of the Albanian art and culture. The first important evidences of the Albanian language and literature go back to 16th century culture (with Buzuku-1555 and Matrenga-1592), though the Albanian language is one of the most ancient indo-european languages. The National Renaissance (the 2nd half of 19th century) has given a great impetus to the progress of scientific opinion on historical issues and for the spread of the education in the native language. Its fight for freedom and indipendence was to affirm the Albanian nation and culture and, to unify the Albanian language alphabet and process the knowledge in various branches of science. Many of the outstanding figures of the nation like, Sami Frasheri, Hasan Tahsini, Refat Frasheri, etc., even operating outside our country, made efforts not only for freedom and independence, but also for the development of the Albanian education, culture and science. The historical efforts of many freedom-loving and education-loving Albanians led to the opening of the first elementary school in Albania (in Korca: March 7, 1887) and, of the first secondary school in Elbasan: December 1909. The Normal school of Elbasan is the foundation of the first university in Albania because its purpose was to make teachers for the Albanian schools.

This foundation was laid when the Albanian club of Thessaloniki (Greece) called for a congress be held in the heartland of Albania, in Elbasan: 20 – 27 August 1909. This eight-day conference was designed to foster an educational movement throughout the country. ”There it was agreed to found a normal school at Elbasan, with a six-year course to train young man as teachers. Man educated in European universities were located to form the faculty...The Normal School of Elbasan was opened that very December with an enrollment of 143 students” [2].
After declaring independence in November 1912, through the efforts of the Albanian intellectuals and scholars were created some cultural centers and clubs that carried out research functions in some of the main cities of Albania. The laying of the bedrock of science became more organized with the consolidation of the Albanian state. After the First World War were done researches in the fields of language, ethnology and the history of the Albanians and, in geology and natural resources, vegetation and archaeological excavations. In these researches and studies were involved Albanian specialists who were prepared in foreign universities. The scientific activity began to fully take its shape and was further developed during the years 30’s-40’s of 20th century, when high intellectuals who were abroad put the foundations of the Albanian studies like, Alexander Xhuvani, Kostaq Cipo, Eqrem Cabej, Bilal Golemi, Gjovalin Gjadri. Hasan Ceka etc. After the liberation of the country (1944) was established the Institute of Studies (1946), which was organized as the Institute of Sciences (1948) and, it became the first center for researches and scientific activities. In 1957 was founded the University of Tirana, which also included the Institute of Science, putting this way the foundations for scientific work in Albania. Starting in late 60s and after was created a network of specialized scientific institutions in specific fields. Until 1972 were operating 25 scientific research insitutions, so became imperative the need of establishing the Academy of Sciences as a national research institutions, **10 October 1972**.

In the beginning, the Academy had 17 members and five correspondent members and operated in a number of institutions of social and albanological sciences, in natural and technical sciences as well. Until the end of the 80s, its scientific activity suffered a number of limitations in its scientific activity, mainly of political and ideological nature. The international links of the Albanian scientists and their participation in international activities, including academic exchanges, were very limited as result of the isolation of our country and of the political climate. The scientific studies carried out by the institutes of the Academy of Sciences have solved a number of important problems related to the study of the history, language and culture of the Albanian people, the study of the nature and the natural resources of the country by introducing and using methods of advanced technologies in industrial and agriculture production, in the improvement of environment and health of the population etc. Some of the institutions are: Institute of Pedagogical Studies, Institute of History and Archeology, Institute of Health Care, Institute of Agriculture Researches, Institute of Geological Researches, Institute of Veterinarian Researches, Breed Improvement Institute, etc. In early 2008, following a reform in the research, the scientific institutions started to function independent of the Academy of Sciences. Today, the Academy of Sciences has joint relations with the national scientific academies of the region and other academies of the world: Austria, Great Britain, Bulgaria, Egypt, Italy, China, Croatia, Montenegro, Macedonia, Poland, Romania, Russia, Slovenia and Turkey.

The Department of Mathematics of Tirana has its beginning in year 1946, when was found the first Pedagogical Institute in Albania. The cathedra of Mathematics and Physics were part of this Institute. When the State University of Tirana was founded (1957), this cathedra was gradually transformed in a dignitous universitary cathedra which has given to the country many scientists in many fields. As result of the serious efforts done by prominent teachers working in a long span of time, in this cathedra is faced a work of extraordinary dimensions for meeting the great needs of our country for teachers of mathematics and carrying out research work as well in different sectors of the Albanian economy. In year 1971 this cathedra was supported by the government to have a more professional direction and was created the Center of Computerized Mathematics followed by the creation of the scientific profiled groups in 1972. This event led to the replacement of Cathedra of Mathematics by more specialized cathedras. Very soon was created the Department of Mathematics consisted of the sections: Analysis and Differential Equations, Algebra, Geometry, Probability and Statistics, Numerical Analysis and Operational Researches. Following the year 1980, other state universities were opened in other cities of Albania, accompanied by other departments of mathematics. Today, in addition to the Department of Mathematics in Tirana, there are departments of mathematics (including

Informatics in some of them) in the universities of Elbasan, Korca, Vlora, Durres, Gjirokastra and Shkodra. These are indication and fruits of the voluminous work done by the first disciplined and full of passion mathematicians of Albania. All the new mathematicians and all the teachers of mathematics working in Albania are very proud of them.

Mathematicians of 19th century

Hasan Tahsini, born in 1811 in Saranda, Albania(Shqipëri), died in 1881. He was a reminiscent, philosoph, mathematician and psychologist.

Bios: Hasan Tahsini was son of a peasant from Ninati of Saranda(the southest town of Albania). Since his childhood he was committed to the knowledge, finished the elementary school in his village and after it was taken to Instambul(Stamboll) for further studies up to university. Because of his variety and deepness in knowledge and intelectual capabilities, he was appointed rector of the University of Instambul, just opened that time. He has done researches in different fields of natyral sciences and presented them in many articles and scientific books in the Turkish language. Among them are "Psychology" and "The Basis of Astronomy". In the last one are presented his mathematical abilities and works. The data and knowledge presented in these books are the most progressive of that time and consequently, he was given the title of a promonant knower in the Ottoman Empire.
Edwin E. Jackues writes, "As economists, the "only great names in Turkey" were two, both Albanians, including Kotchi Bey of Korcha(ibid.). Turkey's outstanding astronomer, Hasan Tahsini(1811-1881), originated in Albania's southern village of Ninati, near Saranda. He became famous for his works in mathematics, physics and psychology, but especially in astronomy and for his invention of astronomical instruments..., his book on astronomy was unique(Nalb 1987, 6:25). Tahsini was named the first rector of the University of Constantinople(Nalb 1984, 5:27). He collaborated with other Albanian patriots at Constantinople in the developing of the famous "Stambul Alphabet", even suffering persecution for his patriotism(FESH 1985, 1073)". [3]

Hysni Babameto, born in Gjirokastra, 16 October, 1888, died in 20 May, 1970 in Tirana. He was teacher of mathematics and educator in Albania, and the first pioneer of the Albanian education in 20th century. He is the veteran of the Albanian national education. After finishing the elementary school in Gjirokastra, he was sent by his family to continue the secondary school in Instambul which finished with excellent results in 1908. After it, he studied in the University of Instambul but, because of his health reasons was obliged to interrupt his studies and start the mission of educator in his birth country. He started his teacher occupation in 1910 and worked for more than 50 years in Albania, in the beginning in Gjirokastra, later in Shkodër, and Korçë and definitevily from 1929, in Tirana. Hysni Babameto is distinguished as a prominent methodist in teaching mathematics and has left un-erasable prints to many generations. His career traverses the borders of many political systems of Albania in the 20th century. Professor Babameto is author of several mathematical textbooks.

Mathematicians of communist regime and after the changes(1990)

*** There is a pleiad of mathematicians, contributing in mathematical education and scientific work in Albania during the communist regime, but unfortunately, very little or nothing is written and known about their contributions in Albania. The publicity has been open and arena of the political actors only, and continues to be so. Qazim Turdiu, Petraq Pilika, Shaban Baxhaku, Osman Kraja, Aleko Minga, Kujtim Dedej, Agim Karcanaj, Mina Naqo etc. There are of those involved in common studies and research work with mathematicians of the world like Alfred Kume, Fioralba Cakoni etc.

1.5 The Research Plan for doctoral studies

I. INTRODUCTORY STUDY

Goals:

- Know if the planned GeoGebra is known in an extent or not
- Know the level of computer knowledge among the math teachers of different schools
- Know the computer and internet facilities in every secondary and 9-year school in Elbasan
- Know the situation of using computer programs for math in different schools of Elbasan
- Prepare the training plan and schedule for math teachers

Revised study-- **31 December 2009**

Goals: - Summarize and get conclusions about the results achieved during the first stage

- Present the results with facts and analysis on different topics of the research

- Improve the methods and techniques used in the first stage

- See the possibility of its extension in other groups that have not been involved in the first stage (for instance with talented students of 9-year schools), etc.

II. RESEARCH AND DOCTORAL PREPARATION

1. Building the Albanian translation of the software: as I preliminary contacted the research people who manage the GeoGebra program and I was informed about the translation in different languages, I have got their support that the same thing be done in Albanian language. It is easy to access, free download available for students and teachers as well, large international community, the experience they accumulated is good to be shared in Albania too, it will be easy to create the Albanian language version and related internet pages.

2. Plan to take part in the activities of the doctoral school, to have consultations with my scientific advisors and to consult other professors of the didactical programme. Using CEEPUS partnership I will use the possibility to cooperate with other partners: Russe, Plovdiv (Bulgaria), Bratislava, Ruzomberok (Slovakia), and NoviSad, Subotica (Serbia) where they have teacher training and are introducing the GeoGebra as well in teaching, in order to get and share their experience

3. Special attention I will show to the application of GeoGebra in teaching probability and statistics, which is very close to my previous research interests.

4. I plan to participate in different GeoGebra conferences in the coming years: the first one is the one organized in July 13 – 15, 2009 in Hagenberg, Austria, the GeoGebra Conference. The purpose is to get and share experience and to contribute. Hope be able to take part in ICME 2010.

Goals: - Build up relations with other partners and share ideas and cooperate in the future for the implementation of GeoGebra.

- Present my research plan on GeoGebra for Albania.

- Meet and discuss more details about my PhD research with Zsolt Lavicza, Markus Hohenwarter, Sárváry and Körtesi.

5. Plan to participate in different International Conferences on Technology Enhanced Learning, Quality of Teaching and Reforming of Education to present GeoGebra software in other fields of teaching and learning technology and, increase the number of publications.

6. Research visits in the University of Debrecen and the University of Miskolc (1-2 visits- up to one month to each institute planned for the next academic year), using the CEEPUS partnership, and other international cooperation grants.

Goals: - consultations with my scientific advisors and other professors of the didactical programme

- report about my research work done in Albania(facts on GeoGebra training, the translation work done, risen problems and challenges)

- gain further experience from these two universities and get advises from my professors and make plans for the future

6. I plan to make a study of introduction of GeoGebra in the University of Elbasan and other universities, as part of math teaching methods program for Math Branch (Acamedic years 2009 – 2010 - 2011).

III. PUBLICATIONS

1. I plan to have 2-3 publications: publish the results of my research submitting the research papers to the journal of Teaching Mathematics and Computer Science of the University of Debrecen, in Springer, Communications in Computer and Information Science or, to the journals of the above mentioned partnership, like the North University of Baia Mare which has a didactics of mathematics journal, and other journals as well - like Octagon or the didactical journal of the University of Ruse. Of course my results will be published in Albanian ones.

2. Make up a detailed didactical manual - which will be published not only for the use in Albania, but to be included in the international Geogebra research, as materials in this domain, due to the very recent new features.

3. Study the possibility of creating a local or national magazine for teachers and students to publish creative works on GeoGebra program

Chapter 2

Use of Technology in Mathematics Education

2.1 Technology Resources for the Classroom

In present age we are witnesses and practioners of computer-based education which is highly speed progressing.The computer-based education allowes educators and students to use educational programing language and e-tutors to teach and learn, to interact with one another and share together the results of their work. The computer-based education is done possible by special electronic tools among which the most important are the mathematical programmes. There are many resources used in the classroom. The teacher remains the most important technology resource in the classroom while technology serves as an accessory to it. Today, after the teacher, the most important accessory is e-technology which is a modern one. The role of technology in mathematics education can be summarized by the two questions addressed by Nemirowsky et al in "Manipulatives, Limit Objects, and Mathematics Learning": "1) If mathematics education aims at familiarizing students with abstractions, and abstractions cannot be directly touched, seen, heard, etc., why would bodily activity be relevant to learning about mathematical abstractions? And 2) How and why should the use of tools which engage eyes and hands in drawing, writing, manipulating, or touching be relevant to learning about mathematical abstractions?" (*10thInternational Congress on Mathematical Education, Pg.390*).

Teachers who wish to make better use of modern technology must first make themselves familiar and proficient with what is available. They should make use of all opportunities to further their technology skills, including help from colleagues of technology, education conference workshops, summer and distance-delivery courses, education network offerings, international or national or local trainings, inter-universities project opportunities. Teachers should involve themselves with site or district technology planning efforts, also develop a personal technology plan for themselves and their classrooms. They must be willing to effectively use (thoughtful planning and implementation) the existing technology in their classrooms. The integration of technology into content area learning requires from teachers to constantly balance the mastery of technology with content area mastery.

The implication of this requirement is: greater the mastery of any technology of the classroom teacher be less effort is needed for classroom use and student mastery during the teaching and learning mathematics process. The most useful types of computer-based applications are: Computer simulation games that offer opportunities at nearly all grade levels for teachers to involve individual students or groups of students in activities directly related to content of the lesson; data bases which allow users to sort, change, and update data, search for specific information, delete and add information, and publish the data in a variety of formats; new opportunities for global communication via computer and in other ways(effective network use); word processing programs which can virtually be tweaked for classroom publishing and allow for increasingly more sophisticated work; presentation software and hardware has become more attractive for use by students and teachers in presentations with computer graphics as well as digitized sound and video and so on. Communications technology is having a profound effect on individual participation in public affairs. It may already be the case that those with access via technology have disproportionately more influence on the processes of social life. If this is true today, then the unforeseen technologies and their effects tomorrow make it even more requisite for educators and educational institutions to ensure technological literacy in their students, and to prepare them to encounter both the effects and the implications of communications technology on the working places. Technology affects both the content of the social studies and how the social studies are taught. Almost any creative learning experience may be enhanced by the use of technology, low- or high-tech. The differences in technology found between schools and in classrooms within schools, compounded by the differences in technology skills among teachers, affect the degree of technology integration and the way technology is integrated. Individual teachers are the key to successful integration of technology into the learning opportunities of students. Teachers need the support of good technology planning and staff development opportunities to stimulate and enhance the use of technology in their classrooms and to build a foundation for successful mastery of technology for themselves and their students.

2.2 Technology in the Albanian school: facts and challenges

After the changes in 1990 done in Albania and supported by the international community and institutions a lot of changes are done in the education system of Albania. The changes have involved the qualification of the teachers (mainly in secondary schools and universities) and improvement of education accessories. Many schools and universities are helpt with computer laboratories and internet access. The state authorities have given much consideration to the national strategy on ICTs. An

important role in this strategy, especially after 2005, plays the issue of ICTs application in all levels of the educational process. The application has started since 2006 and was monitored by the Ministry of Science and Education in order to build the necessary infrastructure, its acknowledgment and application in schools.
The usage of ICTs during the recent years has marked an obvious progress in Albania. The number of users of ICTs, the internet users, the computer applications, the usage of ICTs from the managing bodies of institutions etc. has increased. The spreading of ICTs in the everyday life and activity of the Albanian society set the conditions and also was considered necessary for the Albanian educational system to meet the demands of the contemporary development in the relevant field.

According to the official data, at the end of **2008,** the schools were supplied with computer laboratories as well as the internet connection. There were installed over 1500 labs, comprising about 1800 PCs. Over 500 000 learners are by now using the computer and internet at their schools. The new set up computer labs were supplied with a certain number of computers (15 or 17 according to the number of learners). Big schools established 2 or more labs. Each school was supplied with virtual laboratories: video projectors and laptops (almost 2000 laptops and 2000 video projectors).This was due to the national program realization of ICT in education.

ICTs are seen as important tools to enable and support the move from traditional " teacher- centric" teaching styles to more " learner – centric" methods.Research consensus and the experience holds that the most effective uses of ICTs are those in which the teacher, aided by ICTs can challenge pupils' understanding and thinking, either through whole-class discussions and individual/small group work using ICTs.
Naturally, the installation of information technology laboratories and the school provision with the technical elements of ITC is a primary condition, in order to make use of these technologies during the teaching process.

ICTs is included as an obligatory subject matter in the academic curricula starting from 7 -th grade of elementary schools. Textbooks for this subject are prepared in conformity with the level of education and the school ages of children. ICTs program in elementary school aim to develop the basic competencies in the use of ICTs.
The ICTs program in secondary school aim to develop the student competencies in the use of ICTs up to qualification in certification program as *ECDL (European Computer Driving License) .*

*** Irrespective of these achievements there is a necessity to increase the level of the teacher’s qualification and competences in the use of ICTs in teaching. Teacher’s training and professional development is seen as the key driver for the successful usage of ICTs in Albanian schools.

A study done last in the recent years regarding the use of computer laboratories shows that:

- the mentioned laboratories are found mainly in the cities and towns, rarely are found in country or villages
- they are not used for science teaching purposes. The mentality is that the computer laboratories serve for training the students in computer science as users and for internet access where can be provided(the most of them do not have internet access)
- they are used only for teaching and learning informatics in the cities and this is to prepare users only.
- preparing teachers to benefit from ICT use is about more than just technical skills. Teacher technical mastery of ICT skills is not a sufficient precondition for successful integration of ICTs in teaching.
- There are some teachers of all school levels and all subject matters, which make references in his teaching only to textbooks.
- Few teachers have broad “expertise” in using ICTs in their teaching and few teachers are confident in using a wide range of ICTs resources
- Students are more sophisticated in their use of technology than teachers. There appears to be a great disconnect between student knowledge and usage of ICTs the knowledge and abilities of teacher to use ICTs.
- The teachers of the main sciences(mathematics, physics, chemistry) are not able to use the computers for teaching purposes, never have thought about it and their mentality is the same: computers serve to prepare users and for internet access, or just do some work in WORD and EXCEL.
- Computer laboratories are used for teaching purposes in universities for some experiments, especially in physics.

The challenges are:

I. Change the mindset of the teachers by training them how to use computer programs in the teaching and learning process. The first step is taken: in Elbasan is started the first GeoGebra training with the teachers of the secondary schools and it is going on. The results are very positive and full of encouragement

II. Organize other trainings involving and teachers of other sciences and extend it in other cities

III Create the Albanian version of GeoGebra (a lot of work was done so far, but it takes much more time to translate materials from other sources)

IV. Put and build links in national scale between the teachers who use GeoGebra in the teaching process in order to share their achievements and develop their skills

V. integrate the Albanian GeoGebra users in the international community of GeoGebra

2.3 GeoGebra Software and the use in teaching and learning math

As mentioned above, there are many mathematical programs, but one which is being embraced and used by a daily increasing number of users throughout the world is GeoGebra. The recently published software GeoGebra by Markus Hohenwater (2004) explicitly links geometry and algebra. GeoGebra affords a bidirectional combination of geometry and algebra that differs from earlier software forms. The bidirectional combination means that, for instance, by typing in an equation in the algebra window, the graph of the equation will be shown in the dynamic and graphic window. This program is so much preferred because of its three main features: the double representation of the mathematical object (geometric and algebraic), there are not strong requirements as to the age and the knowledge in using it (the students of the elementary school can use it as well) and, it is offered free of charge(simply by downloading it). In this paper we are concentrating in the double representation of the mathematical object and its advantages in explaining and forming mathematical concepts and performing operations.

> GeoGebra is dynamic mathematics software for schools that joins geometry, algebra, and calculs, it is an interactive geometry system, a new technology for teaching and learning mathematics. With Geogebra is possible to do constructions with points, vectors, segments, lines, and conic sections as well as functions while changing them dynamically afterwards. The two characteristic views of GeoGebra are: an expression in the algebra window corresponds to an object in the geometry window and vice versa. GeoGebra's user interface consists of a graphics window and an algebra window. On the one hand […] we can create geometric constructions on the drawing pad of the graphics

window and, on the other hand, we can directly enter algebraic input, commands, and functions into the input field by using the keyboard.

What is the double representation?

While the graphical representation of all objects is displayed in the graphics window, their algebraic numeric representation is shown in the algebra window.

The geometric constructions are done by the mean of the main virtual tools. The virtual tools are found in the set of the toolboxes which have to be opened, selected a tool, activated it and used it during the construction process. In the toolboxes are found the virtual tools with their names linked with their functions like: New point, Move, Line through two points, Segment between two points etc., alongside which is their picture also. There are also buttons like: Delete object, Move drawing pad, Zoom in / Zoom out, Undo / Redo buttons etc… GeoGebra offers more commands than geometry tools.

2.4 The advantages and the power of using GeoGebra with double representation in math teaching and learning process

2.4.1 Easy teaching and easy learning

In geometry window is possible to display a grid and the coordinate axes. The coordinate system facilitates the work with integer coordinates. GeoGebra performs a double representation: the geometric one and the algebraic one (=GeoGebra). One can enter the objects either as geometric objects (via drop down menus) or as algebraic objects – pairs of coordinates, functions – via the entry line. Moving the objects in the Geometry window changes the expressions in the Algebra window accordingly. Editing the expressions in the Algebra Window results in the respective change in the Geometry Window. This is a main feature of GeoGebra meeting the demands of many didactics and educators to provide as many representations forms as possible for the students. Taking advantage of this double representation feature of GeoGebra it is easier for the teachers to explain the mathematical concepts, the properties of algebraic objects and to methodically reason the result of a mathematical operation based on the manipulations with their geometrical representations; on the other hand the students have the possibility to grasp faster, a common model and correctly what is taught and to add more to their knowledge through their experience while they use GeoGebra. Let explain this by the example of teaching and studying the monotony of the function.

2.4.2 Quick and correct grasping of the concept

Because of the double representation feature it is possible to perform dynamic calculus like functions in x, derivatives and integrals and draw conclusions about the properties of the algebraic objects within a short interval of time because there is e dependency between the algebraic object and its respective geometric object in the way that, a change done in the algebraic object is accompanied with the respective change in the geometric object. So, we can enter any function and show a visualization of generating the first derivative. Change f(x) in the algebra window and have other functions. Consequently, within a short time we can present many examples and observe the mutual change of the two objects and draw conclusions of how the two objects relate to one another. Otherwise, it would take a long time for the teacher to cooperate with the students to draw conclusions together with them or to convey his thoughts to the students; also, it would take a much longer time for the students to work and get conclusions on this subject. The fine thing is that the construction is so easy to do that it can be done together with the students. The double representation feature allows the students to quickly grasp mathematical concept. This is a real power of GeoGebra with double representation compared with other mathematical software. Here are several demonstrations with GeoGebra tools performed during the teaching in the chapter of Derivatives.

Note: The following examples are special cases of applets prepared by the teacher to use in the classroom in explaining concepts and demonstrating different properties. They have been part of the frontal work with the classroom which did help them very much. Similar examples were brought by the students (the skilled ones) in the classroom or tried out at their home PCs (by those who had PCs). Their manipulations, in groups, have been very useful for the peers who needed support. The examples brought by the students served as a confirmation of the topics raised up by the teacher.

Demonstration 1 <u>The first example is the concept of Derivative.</u>
Considering the function, f(x) = 1/8*x*(x-2)*(x+4) shown below, along with the secant line passing through points A and B (look at Fig.2.1) it was easily explained and captured the meaning of the derivative of the function at point A. Very helpful was here

the use of GeoGebra applet accompanied by the discussion with the students. While the dot was slid along the graphic the students observed and estimated the rate of change of f(x) from abscissa of movable point B to abscissa of fixed point A. They observed what did happen with the ratio when was tried to put both points exactly on x = -2 and there was discussion of why did that happen! The students understood that problem is: we can't calculate a slope from a single point. But, if we set it up as a limit, the students could see if they could cancel out the "divide by zero" problem. Easier further was demonstrated this concept by linking the slope of that secant line with the "average rate of change of f(x) from abscissa of point B to abscissa of point A." Thinking of the function as representing an object's location (in meters) north or south from a fixed point, at a certain number of seconds before or after a fixed time, t_0, then the slope of the secant line represents the average velocity of the object in that time interval. The students estimated the "instantaneous velocity" exactly at t = -2 seconds (interpreting it as velocity 2 seconds before the moment the study was started).

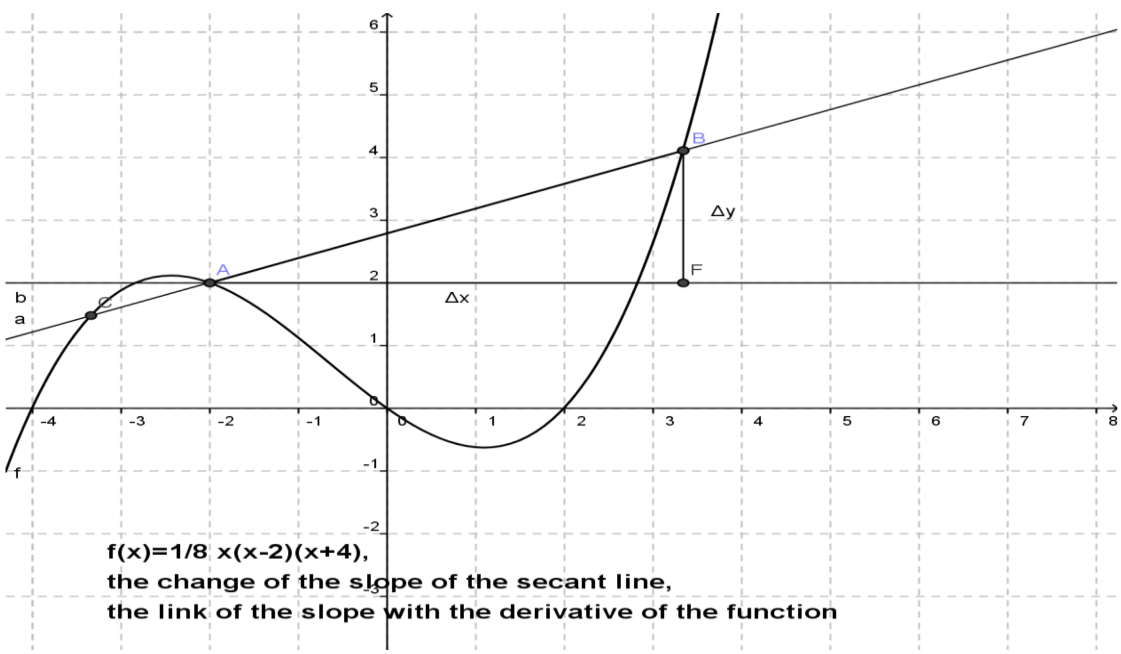


Fig 2.1 Illustration of the concept of the first derivative
(exported by GeoGebra applet)

Demonstration 2 The concept of monotony

We used an applet of GeoGebra to speed up the students' mental process to grasp of the concepts: Strictly Increasing function, also

called "order-preserving" and Strictly Decreasing function, also called "order-reversing."
This was demonstrated by the mean of the slider “a” (look at Fig 2.2) and the movable point A = (a, f(a)). Point B is a fixed point on the graph of the function. By observing what was happening with the ordinate of A while its abscissa was increased (or using the common language: while point A was moved from left to right, simultaneously it was "going up") the students grasped very quickly the definition of the increasing function. In the same pad is demonstrated the concept of the decreasing function by taking the function g(x) = (1/1.3)^x, but it is hidden in the picture below. So, the observation done led to a quick grasping of the concept for the increasing(decreasing) function by the students.

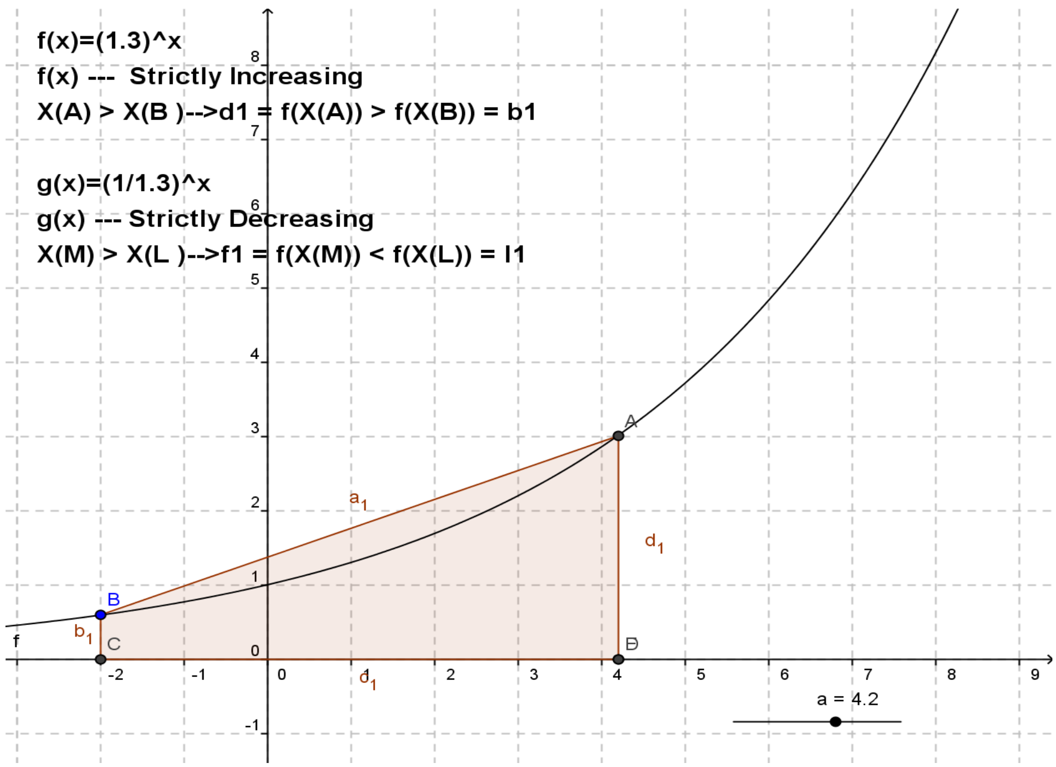


Fig 2.2 Illustration of the concept of monotonous function
(exported by GeoGebra applet)

Demonstration 3 GeoGebra serves as a testing tool for many mathematical concepts or properties

The feature of double representation allows us to perform The First Derivative Test. Consider the theorem: Given that f(x) is differentiable on (a,b): If f′(x) >0 for all x on (a,b) then f(x) is

strictly increasing on (a,b); conversely, if f(x) is strictly increasing on (a,b) then f'(x) =>0 for all x on (a,b) . It is not the purpose to present here the proof of this theorem. We want to show how can be demonstrated the relation between the first derivative of a function and its monotony by displaying their graphs. By using GeoGebra applet (look at Fig.2.3) was so easily demonstrated that on the interval (x(D), x(E)), where the function is strictly decreasing, its first derivative represented by the ordinates of the points of the part of the graph of the first derivative of f(x) corresponding to the mentioned interval, also the ordinates represented by the sign length of the segment A'H, is negative. The students could observe not only geometrically the relation between the first derivative of a function and its monotony but algebraically as well by observing in algebra window how the values of the first derivative change from positive to negative when point A is moving from the increasing interval to the decreasing one.
The demonstration was extended to the intervals where the function is strictly increasing and accompanied with teacher–students discussion. This assertion was also accompanied by the discussion about the measure of the angle between the tangent on the respective point and x-axis related to the monotony of the function. The observation was based on the move of the slider a. The demonstration was as much helpful as even the students of low level in mathematics received a full understanding about the relation under discussion and were active part of it. To the students were given different functions to test the above discussed relation and assignments of selecting functions themselves and testing the relation between the monotony of the function and its first derivative.

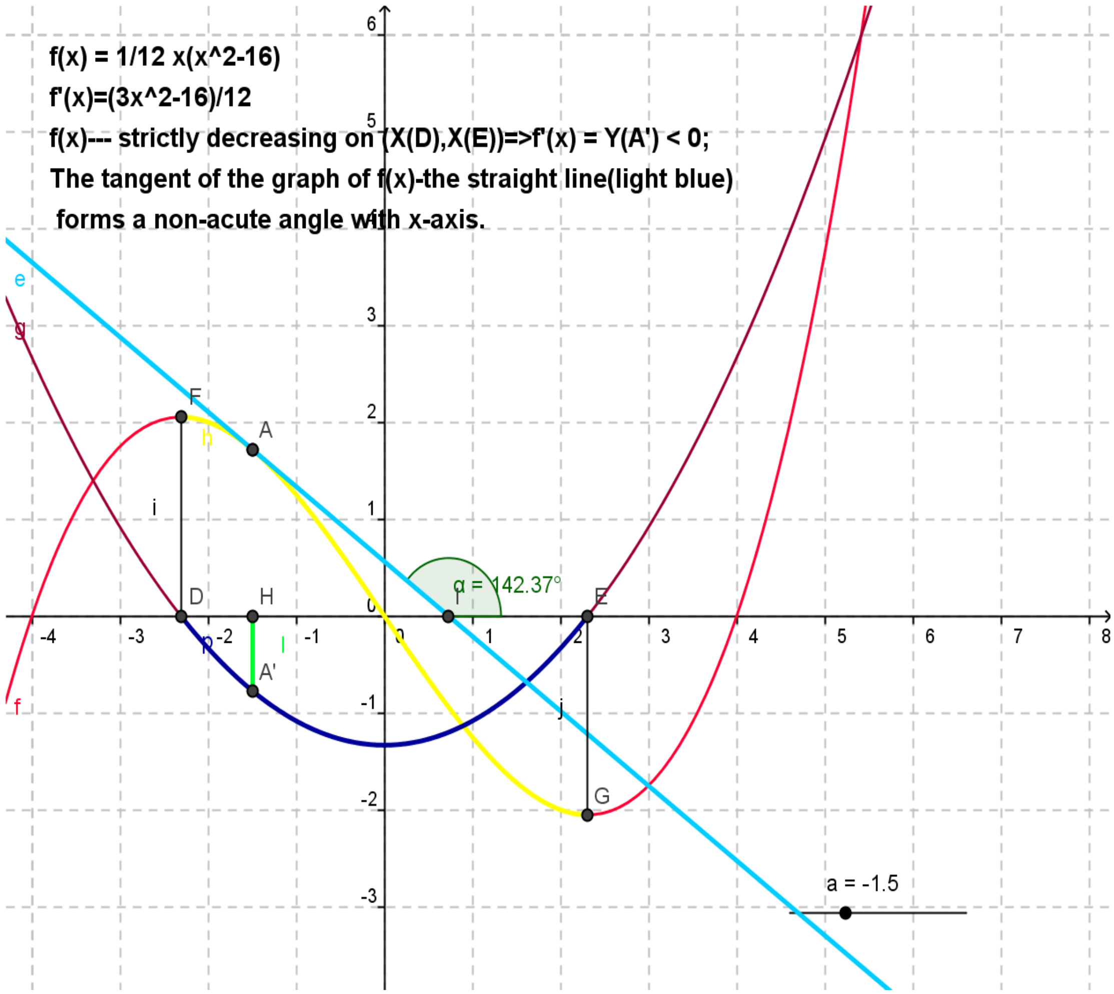


Fig 2.3 Relation between the first derivative of a function and its monotony (exported by GeoGebra applet)

Conversely, were chosen simple examples of functions that on different intervals took values of different signs (for two neighbor intervals the values of the function were positive on one and negative on the other). By applying the command of anti-derivative (which was to be learned in the next chapter) the students could observe that on the interval where the function (seen as derivative) was positive its anti-derivative was strictly increasing and so on. This observation done by using GeoGebra is very helpful in teaching and learning mathematics. It can be used before proving the above theorem: the students observe the relation under discussion for specific functions and later jump to generalization of this relation by proving the theorem for any function. It can be used after the proof of theorem for demonstration purposes. I used it before proving the theorem and the result in regard with grasping the theorem and using it in applications was very successful. Using the example of the Fig.2.3 was demonstrated also the meaning of extremums at the points F and G, also at the points A and D (Fig.2.4).

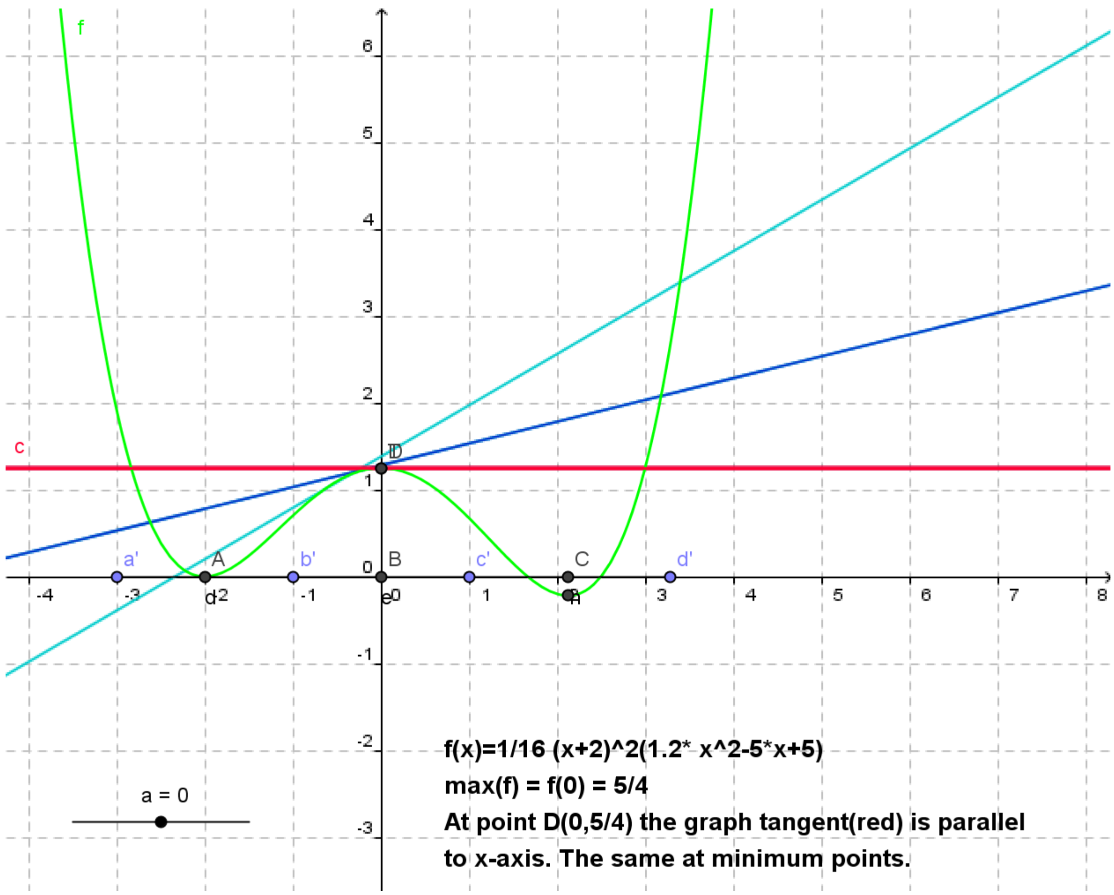


Fig 2.4 Illustration of the concept of local extremums
(exported by GeoGebra applet)

Demonstration 4 Extreme Value Theorem: Let f(x) be a continuous, real-valued function defined in a closed interval, [a′ , d′]. Then, **1)** There exists a point, x_D, in the interval [b′, c′], such that M = f(x_D) >= f(x) for all x in [b′ , c′](see Fig…). **2)** There exists also a point, x_C, in the interval [c′, d′], such that m = f(x_C) <= f(x) for all x in [c′, d′], etc. The strict is a′ < b′ < c′ < d′. M is called the maximum value of f(x) on [a′, d′] and m is called the minimum value of f(x) on [a′, d′]. If the continuous image of [a′, d′] is a single point, then we have m = M.

The Extreme Value Theorem says that there is a maximum (respectively, minimum), and that there is at least one way of achieving that maximum (respectively minimum) within the interval [a', d']. The demonstration of the result of the important lemma that if x_E from (a,b) is abscissa of an extremum point E then $f'(x_E) = 0$, was performed by using the applet of Fig.2.3.
The discussion was linked with the geometrical meaning of the first derivative. It was quite obvious that at the extremum points the tangent of the graph (red line) is parallel to x-axis.
The Max-Min Theorem for Derivatives asserts that:
If f(x) is continuous in the closed interval [a , b], with maximum value of M occurring at x_M, and minimum value of m occurring at x_m then the set S of the values where is achieved extremum is consists of : 1) The left-hand endpoint, a. 2) The right-hand endpoint, b. 3) All values x in (a , b) such that f ' (x) does not exist. 4) All values x in (a, b) such that f ' (x) = 0.
Points of type 3) or 4) are called "Critical Points" of the function f(x) in the interval (a, b). Using applet in Fig.2.5 and leading the discussion with the students, they were convinced that there is maximum at point M but the above lemma is not satisfied (they could observe that the tangent at this point is undefined). The observation was accompanied by the discussion about the left and right limits of the derivatives. The left limit is 6, while the right one is -3.5 (both of them are very far from 0, so cannot equal). By moving the slider they could see in algebra window that on the left side and close to M the values of derivative are positive and much greater than 0 and, on the right side and close to M the values of the first derivative are negative and much less than 0. The students could observe this fact by displaying the graphs of the derivatives. At point N there is minimum(observation showed that tangent at N is parallel to x-axis) and was given for the students the task of testing the sided limits at that point. Additional exercises were given to bring other examples of this type.

After the proof of Max-Min theorem it is important that the teacher emphasize its practical use in many problems, that the second result, known as the Extreme Value Theorem (or the Max-Min Theorem), is linked with important application. This theorem is used in one of the main applications of mathematics today which is the area of problems referred to "optimization problems". Mathematics is used to find an optimal solution to a problem in a particular situation. Word "optimal" means "highest" or "lowest" value of a function such as, minimization of the cost of a production, or maximization of profit in sales etc. In any of these cases, it is not just enough to know what the extreme value exists, but also, how to achieve that value, or even, whether it is possible to achieve an extreme value. For this reason the Extreme Value Theorem is known as an "existence theorem" as well.

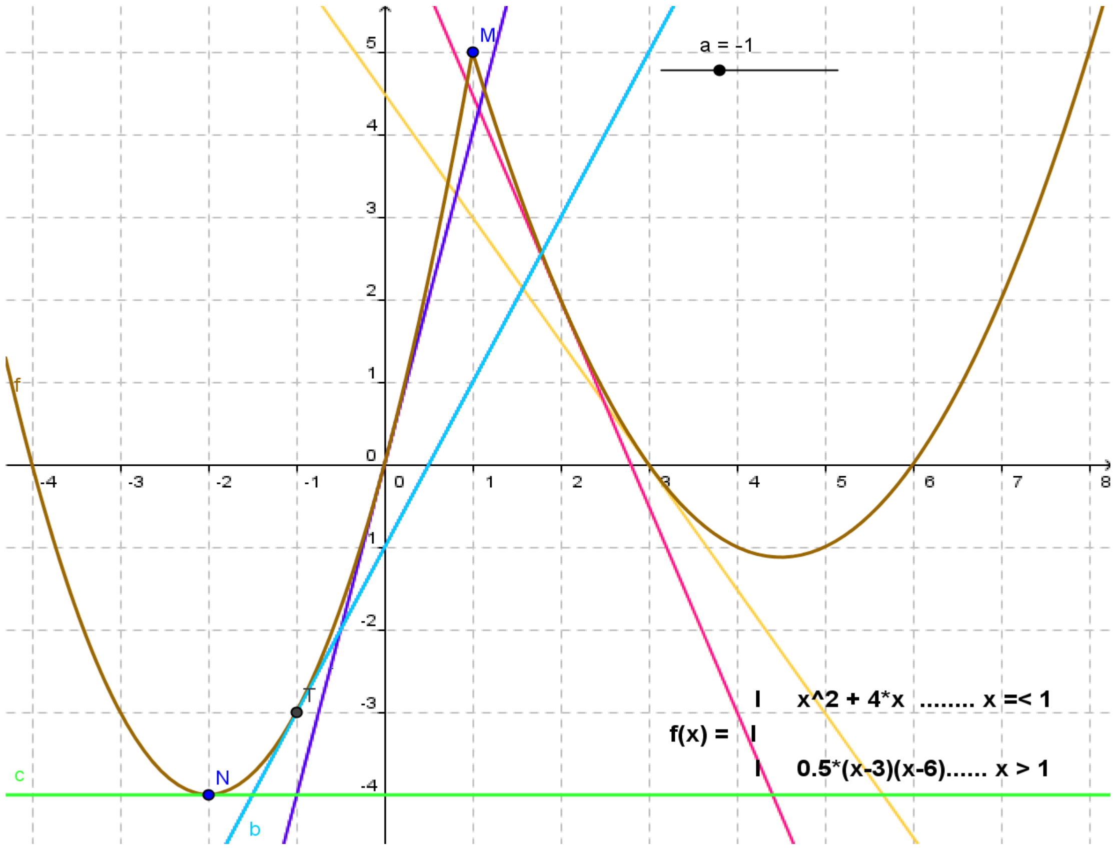


Fig 2.5 Tangent undefined at the local extremum/maximum (exported by GeoGebra applet)

A summary of the procedure for finding the extreme values (if they are achieved) was given to the students and it was very helpful in solving optimization problems or other exercises.

The procedure is:

- First, make sure that the assumptions hold (f(x) is continuous on [a , b].)
- Second, set up a table that you fill with candidates from the above mentioned set, S, starting with the end points.
- Third, finish filling the table with the critical points (make sure that the critical points are in the interval!)
- Finally, evaluate f (x) for all the points in the table (be sure to use f (x), and not f ' (x)).

Demonstration 5 Mean Value Theorem for Derivatives.
Rolle's Theorem: Let f(x) be continuous on [a, b], and differentiable throughout the interior (a, b). Then, for some point c in the interior (a, b), f '(c) = 0. Now that Rolle's Theorem was demonstrated in the above examples and also proved, was proceed to the result we actually wanted, called the Mean Value Theorem for Derivatives: The Mean Value Theorem for Derivatives (MVT):
Let f(x) be continuous on [a , b] and differentiable on (a , b). Then, for some c in the interior: (a , b), the slope of the tangent line at x = c is equal to the slope of the secant line through x = a and x = b:
f ' (c) = (f(b) - f(a))/(b - a) or f(b) - f(a) = f ' (c)*(b - a)
Here in Fig.2.6 is a GeoGebra Applet allowing the students to explore the conclusion of the MVT for Derivatives. It is not my purpose to prove the MVT, but to demonstrate the employing of the common trick in proving this Calculus result which is achieved by taking the general function, and subtracting off the secant line, that is, we consider the assistant function: g(x) = f(x) - secant line. By this employment we are subtracting the general case function down to the case where the values of f(b) and f(a) are equal. The GeoGebra applet demonstrates the "lifting" of the graph of f(x) up or down or twisting it.... With this applet was demonstrated that by geometrical transformation of the graph of f(x) the students could be convinced that there is a point N on the graph of f(x) and between the intersection points of it with its secant AB, at which the tangent of the graph is parallel to the secant AB.

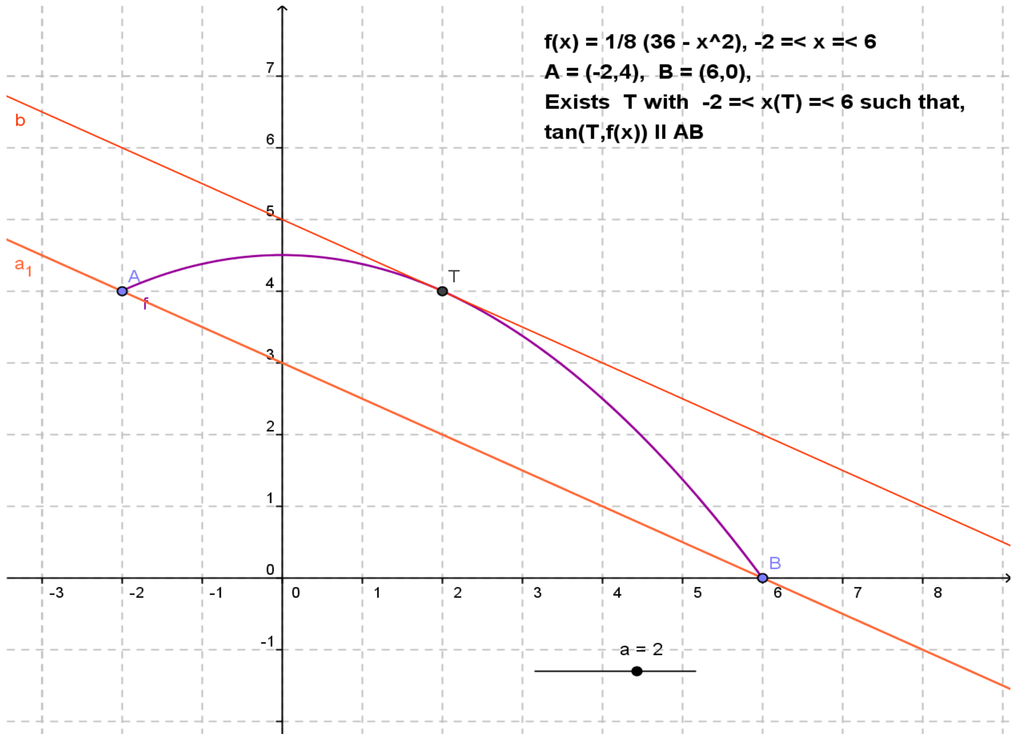


Fig 2.6 Picture of GeoGebra applet illustrating MVT
(exported by GeoGebra applet)

The transformation was performed in such a way that the tangent of g(x) could touch the graph of f(x) where was done possible to plot the intersection point (N). It is clear that during the transformation the tangent remained always parallel to the respective secant. This is provided by one of the most important features of GeoGebra, by that of preserving the relative relationship between two objects where one is dependent on the other. Was needed the construction of a parallel to secant AB, a parallel passing through the point F on the graph of f(x) and close to point A. Then, the movement of the graph was done carefully until the secant CD rested on that parallel. After this the existence of the spoken tangent was obvious (look at Figures 2.7 and 2.8).

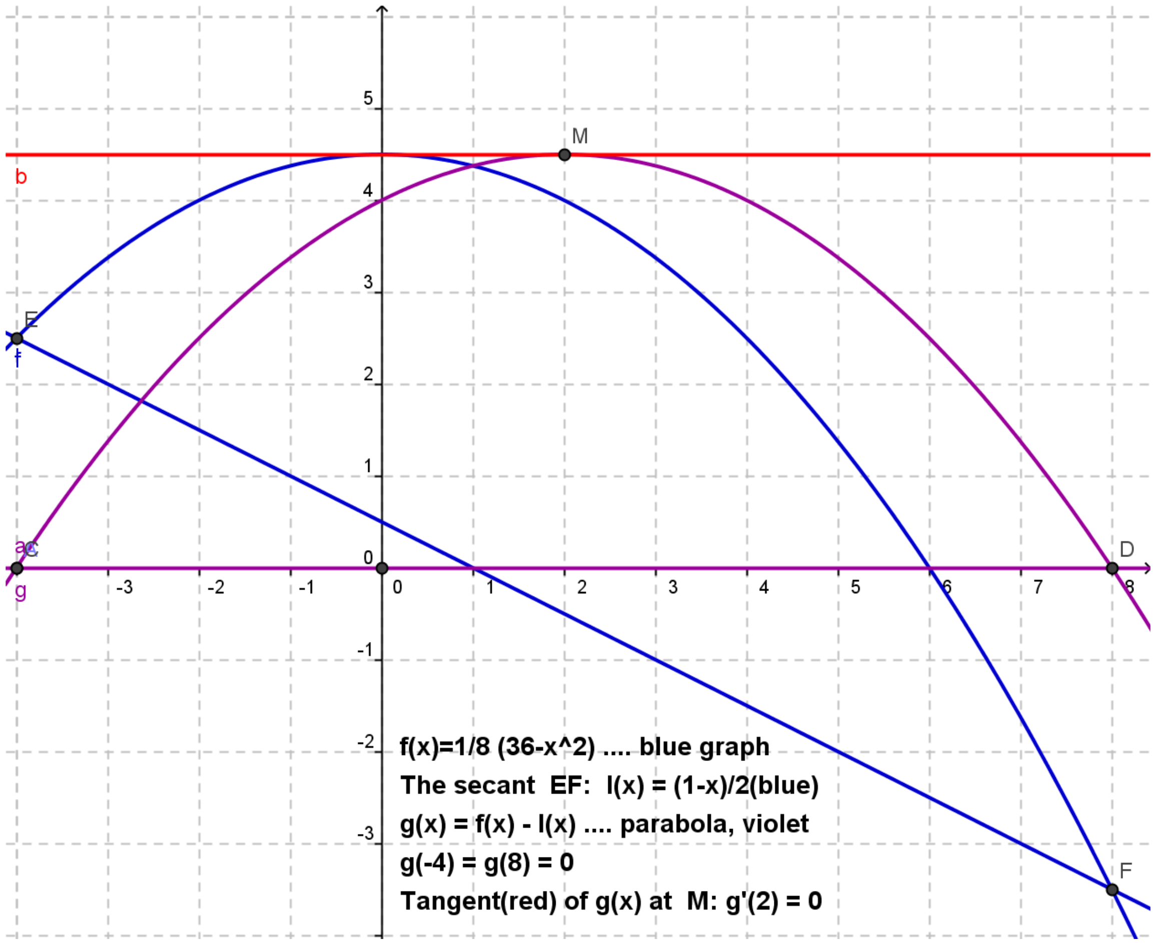


Fig 2.7 Picture of GeoGebra applet illustrating the transformation in MVT (exported by GeoGebra applet)

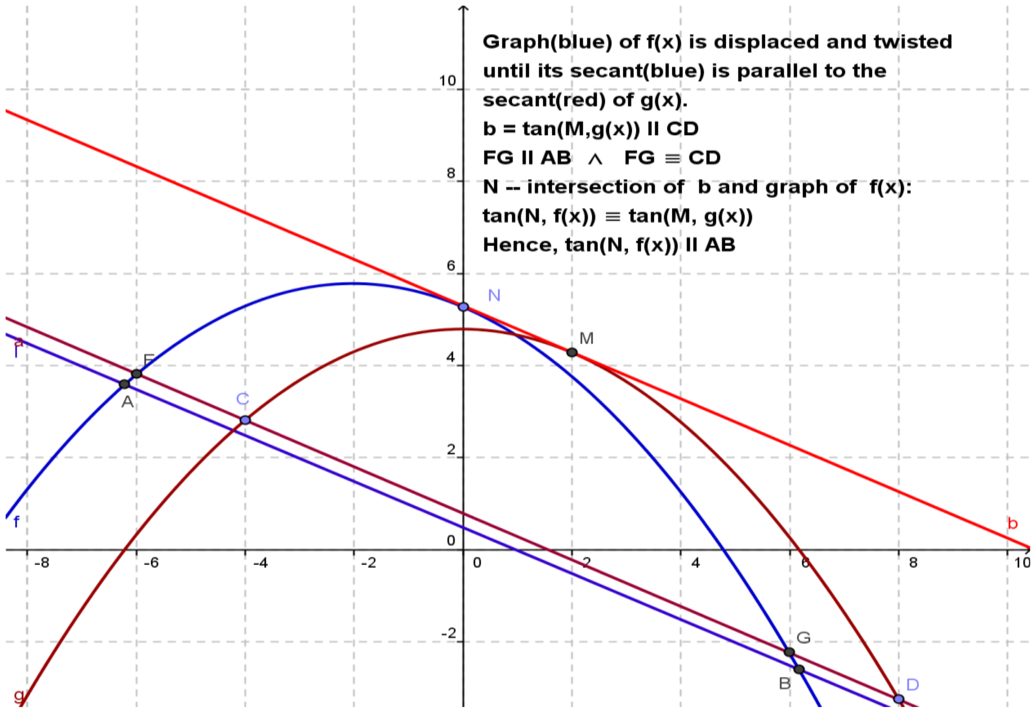


Fig 2.8 Picture of GeoGebra applet illustrating the transformation in MVT (exported by GeoGebra applet)

2.4.3 **Geogebra provides the tools and the conditions for research activity**.

GeoGebra is mainly used as a tool for teaching and researching.
It is used as a checking tool to test and verify thinking, or sometimes, when it is inconvenient to draw graphs on the blackboard, it is used as a demonstration tool to emphasize their impression. GeoGebra offers a very good place for practice and research work.
There is a practice block consisted of a pool of geometry activities of two different difficulty levels: *Basic Tasks* and *Advanced Tasks*.
The student can pick tasks of his/her interest and work on them either on his/her own or together with a colleague. The teacher must draw the attention of the students to investigate different cases by using GeoGebra software if the tangent is defined at any point of the graph. During the teaching we brought examples when in different parts of the domain the functional dependence is different and after plotting the graph it looks quite clear that at the junction point there is tangent and so it is (look at Fig 2.9).

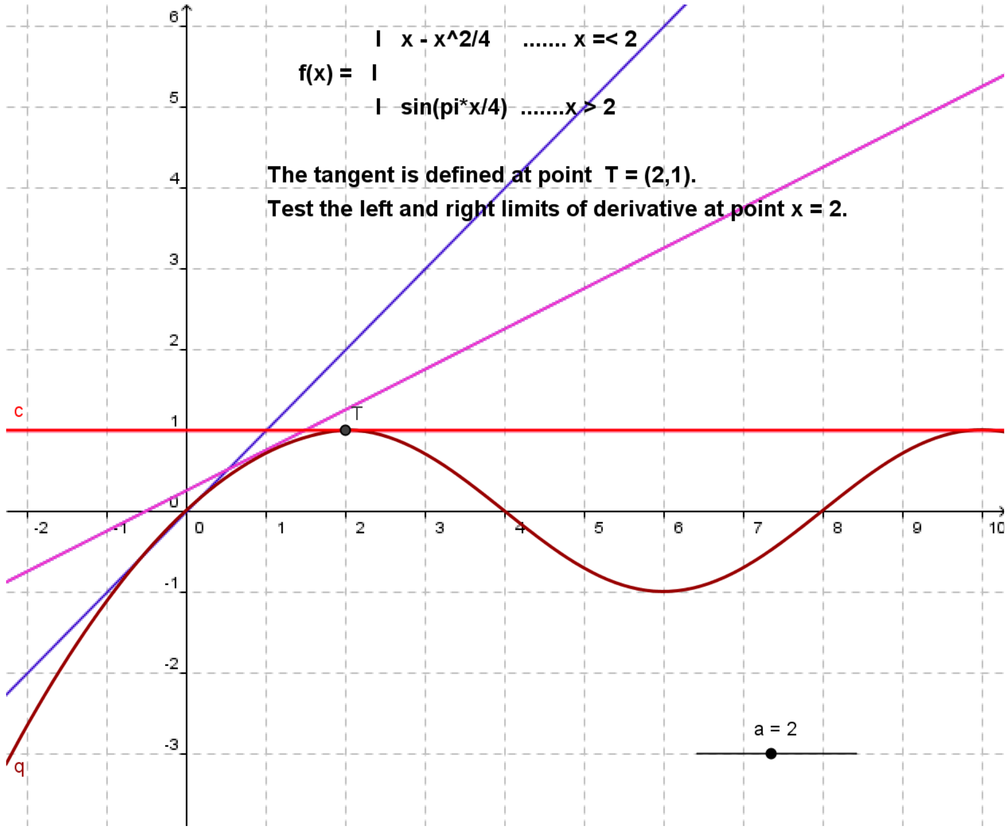


Fig 2.9 Picture of GeoGebra applet illustrating the existence of the tangent (exported by GeoGebra applet)

Also, there are examples when in different parts of the domain the functional dependence is different and after plotting the graph it looks quite clear that at the junction point there is tangent but in reality it is not so (look at Fig 2.10). There is maximum at point T, moving the points A and B of the secants TA and TB towards the point T by the GeoGebra program is produced one single position at point T (the red line b), however the tangent doesn’t exist at this point. The proof is by analytical method: testing the left and right side limit of derivatives.

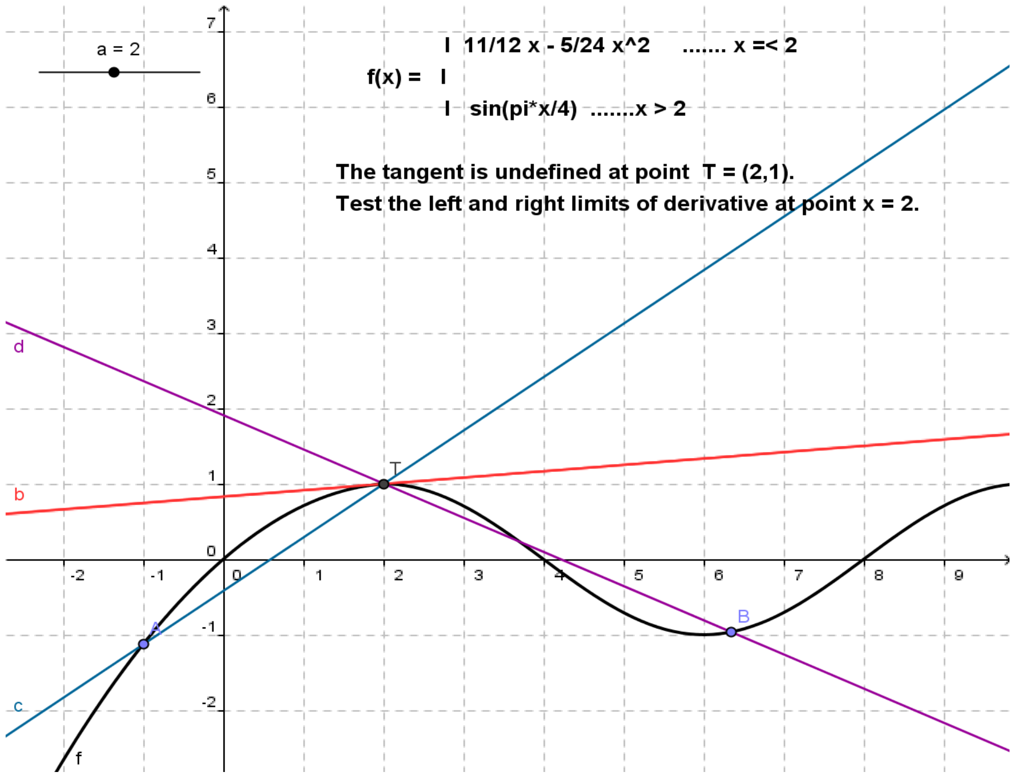


Fig 2.10 Picture of GeoGebra applet illustrating the non-existence of the tangent (exported by GeoGebra applet)

<u>The conclusion is that the existence of the tangent is proved by testing the left and right side limits of the derivative.</u> These two examples stimulated the students to look for other examples in the class. It was not easy for them to find and present examples during the teaching hour, so it was left as assignment for the next class. Some of them who had computer facilities and access brought and presented their examples.

The point here is that, GeoGebra allows to do **research work, to do explorations and, it is a real entertainment** manipulating and playing with the tools provided by GeoGebra and the result is: the students continually add more to their mathematical fund and they get a deeper understanding for the concepts and the methods of mathematics.

*** Another example of research work(<u>this is not part of the experiment</u> done with the experimental class, it helps for the chapter on Integral) is the calculation of areas of plane figures by the integration operator and using GeoGebra software.

Consider the plane surface bordered by the lines: x=a, x=b, y=0 and the curve y=1/x (Fig 2.11). It is known that the area of such plane figure is ln(b)-ln(a)=ln(b/a), where 0<a<b. The double representation allows the students to see a particular case for the lower and upper bounds: selecting them be consecutive powers of 10, and write down the values of the areas calculated or generated by GeoGebra. Making few trials is found out that:

$$\int_{10^3}^{10^4} \frac{1}{x} dx = \ln(10^4) - \ln(10^3) = \ln(10)$$

$$\int_{10}^{100} \frac{1}{x} dx = \ln(100) - \ln(10) = \ln(10)$$

$$\int_{10^{-2}}^{10^{-1}} \frac{1}{x} dx = \ln(10^{-1}) - \ln(10^{-2}) = \ln(10)$$

$$\int_{10^k}^{10^{k+1}} \frac{1}{x} dx = \ln(10^{k+1}) - \ln(10^k) = \ln(10)$$

$$\int_{10^k}^{10^{k+1}} \frac{m}{x} dx = m\ln(10^{k+1}) - m\ln(10^k) = m \cdot \ln(10)$$

The area calculated by GeoGebra program is constant: 2.3

In the figure above are given two cases for the lower bound of the integral, which is 10^n : n=-0.5 for which area is denoted by a and, n=0 for which area is denoted by b. They are equal: a = b = 2.3, approximately ln(10).

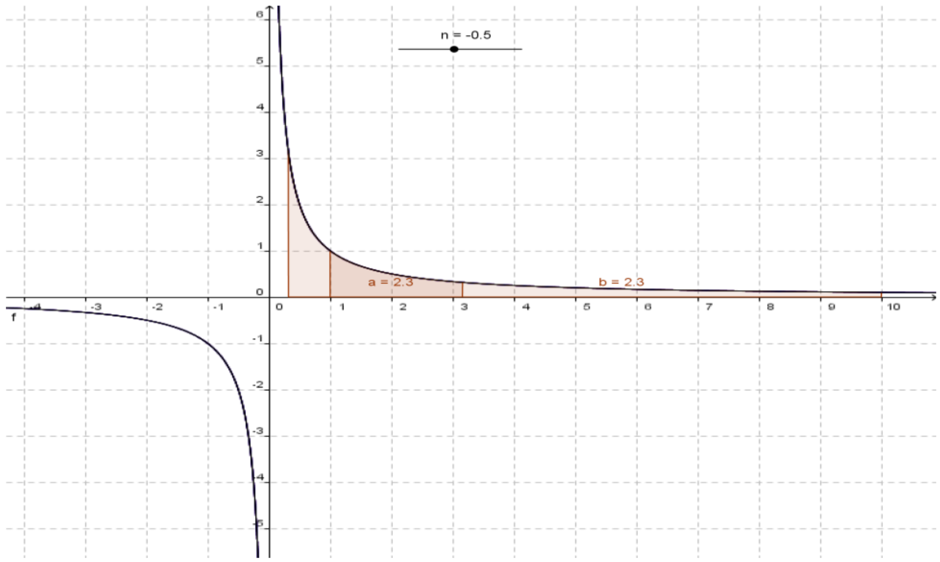


Fig 2.11 Area calculated by GeoGebra programme is constant: 2.3
(exported by GeoGebra applet)

In general, if the bounds of the area are respectively: $a^b..and..a^{b+1}$, where a>0, b any real value, the area is ln(a). Again, it can be demonstrated with GeoGebra that the area of the figure with such lower and upper bound is constant. So, by manipulating with tools of computer programs we are led in analytically proving and formulation of the statement: <u>The area of the plane surface with boundaries f(x)=1/x, y=0, and the perpendiculars with OX in the points of two consecutive powers of number a is constant.</u>

"GeoGebra is mainly used as a tool for ***teaching*** *and* ***researching.*** *It is used as a* ***checking*** *tool…to* ***test*** *and* ***verify*** *thinking, or sometimes, when it is inconvenient to draw graphs on the blackboard, it can be used as a demonstration tool to emphasize their impression".*

2.4.4 GeoGebra and Visible Thinking - Psychological Issues

To master the students in capturing, assimilating and using the above concepts, also executing different techniques of tests is the same as in the case of making sense of formal concept definitions which have to be linked with the concept images. The concept images of the students are based on their prior knowledge got through their different experiences. "The tendency of many students to evoke their concept image[…] in many situations it is desirable to have and evoke rich concept images and,….research shows that

visualization facilitates mathematical understanding” (Handbook of Research on the Psychology of Mathematics Education, Advanced Mathematical Thinking, pg.149). In order the students correctly own the very important analytical concepts treated in the chapter of Derivatives   is acquired prior experience and a very good and effective tool providing the environment for getting such experience is GeoGebra with its double representation feature. The visualization of the concepts leads to visual reasoning, as Gutierrez (1996) summarized much of the discussion on visualization noting that “the visual processes are involved in interpreting: (a) External representations to form mental images and (b) The mental image in order to generate information.” (Handbook of Research on the Psychology of Mathematics Education, The Complexity of Learning Geometry and Measurement, pg. 90). So, by visualization abilities the students form visual reasoning and get the right information in understanding, capturing and owning a mathematical concept, consequently make the right application. Many psychologists and researchers of the mathematics field are strongly stressing the visual reasoning in the work of today’s mathematicians and teachers. Suffice to quote here that “In his 1991 plenary address to the International Group for the Psychology of Mathematics Education, Dreyfus urged mathematicians and mathematics educators to give increased importance to visual reasoning—not to elevate it above analytic reasoning but on an equal level with it. Visual reasoning plays a far more important role in the work of today's mathematicians than is generally acknowledged (Hadamard, 1949; Sfard, 1994). …. Other research, for example, Battista, Wheatley, and Talsma (1989), Brown (1993), Brown and Wheatley (1989, 1990, 1991), Clements and Sarama (this vol.), Reynolds and Wheatley (1992), Wheatley, Brown, and Solano (1994), has shown the power of image-based reasoning in mathematics problem solving. Students who used images in their reasoning were more successful in solving nonroutine mathematics problems than those who approached the tasks procedurally.” (Mathematical Reasoning: Analogies, Metaphors, and Images, pg. 154)
Thinking is pretty much invisible. To be sure, sometimes people explain the thoughts behind a particular conclusion, but often they do not. Mostly, thinking happens within the engine of our mind-brain. The basic strategy of visible thinking is *to make thinking visible* in the context of learning. One reason why thinking develops slowly is that thinking happens inside the head. As a result, children do not 'see' their own cognitive moves. Much harder is for teachers to see them. The most of classroom practices do not engage students in substantive thinking around content very much, and certainly not in ways that make it visible across the classroom. Visible Thinking makes thinking an explicit part of classroom discourse in a natural manageable way by putting the foundations of powerful practices of thinking and learning. *Visible Thinking* is a systematic research-based approach to integrating the development of students' thinking with content learning

across the subject matters. Visible Thinking has a double goal: on the one hand, to deepen subject-matter learning and on the other to cultivate students' disposition toward thinking. Visible Thinking includes a large number of classroom routines, easily and flexibly integrated with content learning, and representing areas of thinking such as understanding, truth and evidence, fairness and moral reasoning, creativity, self-management, and decision making. It also provides tools for integrating the arts with subject-matter content. Finally, it includes a practical framework for how to create "cultures of thinking" in individual classrooms and within an entire school. Every committed educator wants better learning and more thoughtful students. Visible Thinking is a way of helping to achieve that without a separate 'thinking skills' course or fixed lessons. Visible Thinking is a broad and flexible framework for enriching classroom learning in the content areas and fostering students' intellectual development at the same time. It aims in establishing good and solid thinking in the minds of the students.
David Perkins and Shari Tishman write:

> "Traditionally, good thinking has been defined as a matter of cognitive ability or skill. Hence, the term "thinking skills." Certainly, good thinkers have skills. But they also have more. Passions, attitudes, values, and habits of mind all play key roles in thinking, and, in large part, it is these elements that determine whether learners use their thinking skills when it counts. In short, good thinkers have the right "thinking dispositions.""

An individual who understands a disciplinary topic can apply that understanding to new situations, ones that are never encountered before. "In the absence of such performances of understanding, acquired knowledge remains inert—incapable of being mobilized for useful purposes" concludes Howard Gardner. The visible thinking takes an integrated stance toward the teaching of thinking, weaving thinking into the culture of the school and classroom, rather than as a program designed to be implemented. A key condition of the approach is to seek ways to uncover and document students thinking so it can be discussed, reflected upon, and pushed further. Consequently, teachers employ various strategies for documenting the thinking students do.

Some of key achievements at the end of the chapter on Derivatives were:

- Deeper understanding of content
- Greater motivation for learning
- Development of learners' thinking and learning abilities.
- Development of learners' attitudes toward thinking and learning and their willingness to opportunities for thinking and learning.
- A shift in classroom culture toward a community of enthusiastically engaged thinkers and learners.

Visible Thinking involved several practices using GeoGebra tools and different resources like, applets created by the teacher and GeoGebra wikis. The students were challenged and invited to use with their peers a number of "thinking routines" like: simple protocols for exploring ideas and topics linked with the chapter, say operations with functions, the concepts and operations with limits, the concepts and properties about the straight line and so on. Visible Thinking included the attention to understanding, truth, fairness, and creativity. The main emphasizes of the visible thinking was that of making students' thinking visible to themselves and to one another, so that they could improve it and they did. In every step of the teaching we were concerned on the issues:

Were the students explaining things to one another? Were the students offering creative ideas? Were they using the language of thinking? Were they able to make the known interpretations or bring alternative interpretations? Were the students debating about the subject under discussion?"

Based on the observation done with the experimental class and on their end chapter results and using the optimistic language I say that, <u>the students are more likely to show interest and commitment to mathematics when the teaching is incorporated with computer technology which provides visible objects of static or dynamic state</u>. The students begin to display the sorts of attitudes toward thinking and learning we would most like to see in young learners, they are open-minded, curious, not of negative attitude, not skeptical, not satisfied with "just the facts" but wanting to understand.
Another important finding was that skills and abilities are not enough. They are important of course in learning mathematics, but alertness to situations that call for thinking and positive attitudes toward thinking and learning are tremendously important as well. Several times, I have found that the students think in shallow ways not for lack of ability to think more deeply but because they simply did not notice the opportunity or did not care. The good thinking involves *abilities, attitudes, and alertness*. Technically this is called a <u>dispositional view of thinking</u>. Visible Thinking is designed to foster all three components.

The central idea of Visible Thinking is: *making thinking visible and in a broader sense,*

“The visualization helps to: develop spatial and perception skills, predict the theorems and the properties of geometrical figures, increase the intuitive talents, increase divergent thinking and the checking of new ideas, recognize the ‘visible’ proofs, motivate the students activity,

increase the students' enthusiasm" (Elvira Ripco, Teaching Geometry using computer visualizations, TMCS 7(2009)2,Pg. 262).
The students learn best what they can see and hear "visible thinking" means generally available to the senses, not just what can be seen with the eyes). We watch, we listen, we imitate, we adapt what we find to our own styles and interests, we build from there. Learning without seeing would be like learning to play guitar just by describing the strings, the different sounds they produce and so on, without seeing the guitar and the movements done by the teacher. What would be the result of learning a sport without seeing the game???!!!
When thinking is visible, it becomes clear that school is not about memorizing content but exploring ideas. "*The dynamic figure provides plenty of freedom to explore the relations of its mathematical objects and discover mathematical concepts*" (Markus Hohenwarter, Judith Prenier, TMCS 6(2008)2, Pg. 320).
Teachers benefit when they can see students' thinking because misconceptions, prior knowledge, reasoning ability, and degrees of understanding are more likely to be uncovered. Teachers can then address these challenges and extend students' thinking by starting from where they are.

It is important to note here the remark done by Marcus Giaquinto in his book "Visual Thinking in Mathematics": "In fact visual representations are so useful that most books on calculus are peppered with diagrams…. Visualizing may have various roles. For example, visual illustrations may facilitate comprehension of formulas or definitions; they can be reminders of counter-examples to plausible seeming claims; they can serve as stimuli, to speak an idea for a proof. These uses of visual thinking do not involve trusting it to deliver or preserve truth. Visualizing or, seeing a diagram in a particular way, maybe useful when its role is merely to illustrate or to stimulate, but untrustworthy when used a means of discovery" (Pg. 163).

The common view is that, visualizing in analysis, though heuristically useful, is not a means of discovery. But in the cases of illustration and stimulation (we cannot number such cases) it is really important.
The geometric representation is very important for the students of every cycle of education system, further more for primary level of education. To master the students in performing an algorithm is the same as in the case of making sense of formal concept definitions which have to be linked with the concept images. The concept images of the students are based on their prior knowledge got through their different experiences. "The tendency of many students to evoke their concept image[…] in many situations it is desirable to have and evoke rich

concept images and,….research shows that visualization facilitates mathematical understanding"(Handbook of Research on the Psychology of Mathematics Education, Advanced Mathematical Thinking, pg.149). In order the students correctly perform the algorithm of multiplying fractions is acquired prior experience and a very good and effective tool providing the environment for getting such experience is GeoGebra with its double representation feature. The visualization of the algorithm leads to visual reasoning, as Gutierrez (1996) summarized much of the discussion on visualization noting that "the visual processes are involved in interpreting: (a) External representations to form mental images and (b) The mental image in order to generate information." (Handbook of Research on the Psychology of Mathematics Education, The Complexity of Learning Geometry and Measurement, pg. 90).
So, by visualization abilities the students form visual reasoning and get the right information in performing an algorithm and understanding and owning a mathematical concept. Many psychologists and researchers of the mathematics field are strongly stressing the visual reasoning in the work of today's mathematicians and teachers. Suffice to quote here that "In his 1991 plenary address to the International Group for the Psychology of Mathematics Education, Dreyfus urged mathematicians and mathematics educators to give increased importance to visual reasoning—not to elevate it above analytic reasoning but on an equal level with it. Visual reasoning plays a far more important role in the work of today's mathematicians than is generally acknowledged (Hadamard, 1949; Sfard, 1994). …. Other research, for example, Battista, Wheatley, and Talsma (1989), Brown (1993), Brown and Wheatley (1989, 1990, 1991), Clements and Sarama (this vol.), Reynolds and Wheatley (1992), Wheatley, Brown, and Solano (1994), has shown the power of image-based reasoning in mathematics problem solving. Students who used images in their reasoning were more successful in solving non routine mathematics problems than those who approached the tasks procedurally." (Mathematical Reasoning: Analogies, Metaphors, and Images, pg. 154)

Some Conclusions about Understanding Based on Visualizing

The experiment carried out with the third grade of the secondary school in Elbasan, Albania, helpt in giving answer to some methological issues. The computer laboratory and the use of GeoGebra software in the teaching and learning process provided the necessary conditions for investigating, comparing, and refining promising visualizations and determining when and how they improve mathematics learning. We are listing here several benefits and outcomes in the improvement of skills and understanding mathematics, not excluding the benefits of the teachers and students in computer science. At the end of the experiment with GeoGebra is concluded that visualizations can transform mathematical instruction in the following issues:

1. Visualizations considerably improved the level of the experimental group in mathematics. At the end of the chapter the average outcome was higher by one mark.

2. The benefit from visualizations is for all learners (all the students had improvements in mathematical skills and knowledge). The great number of mathematical problems and games and the wide range regarding the difficulty scale that can be solved by the use of GeoGebra software show that it is possible that GeoGebra be used even in the elementary school, hence there is a challenge for all the educators of mathematics to participate in trainings and qualify themselves in the area of cyberlearning.

3. The use of GeoGebra applet (a very good visualizing material) in teaching and learning process is indisputable. All the students got a full understanding about the new concept or property. They enabled the students to learn complex topics and the teachers to easily and effectively transfer the mathematical concepts and ideas from their storage to the storage of students.

4. The use of visualizations increases the effectiveness in the teaching hour:

- more visual representations and better quality(the teacher can prepare the visual material in advance, of high quality and attractive to the students and as many as possible)

- good source of visual and instructional materials is GeoGebra wiki

- within a teaching unit can be covered a larger part of the text(the use of the computers by each student or the projection of the figures and text on the wall-screen provide higher speed than in traditional teaching).

5. Using GeoGebra there was a double benefit: the students and the teachers, both benefited by improving their computer skills. At the end of the chapter, some of them told that they were given the opportunity to use the computer for the first time and extended their gratitude to the teacher and to the skilled students in computers.

6. This experiment and the GeoGebra training organized with teachers served as a basis for generating new ideas. The teachers proposed to design a curriculum of cyber-teaching for training and qualifying the teachers of mathematics (why not of Physics as well?) in order they use computer programs in the teaching process. This is a topic and a project requiring more study, energy, support and investment and which belongs to a near future.

7. The best way to have effectiveness in teaching and explaining a new concept, idea or property is: firstly project on the wall the visual representation and after make the students try themselves by manipulating or playing on geometry window. The teacher must be concentrated with the students having less computer skills or missing them.

8. Visualizations helped a lot in connecting formal and informal teaching and learning. In the formal teaching there is no room in the chapter of limits for visual interpretation. But, benefiting of the use of GeoGebra tools we can visualize topics linked with the existence of the limit or the behavior of a function. After the students were capable of using GeoGebra their attention was drawn also to consider topics of the limit. Taking advantage of the speed and the short time in covering the teaching subject provided by GeoGebra we had time that in a teaching hour to present:

2.4.5 The problem of the behavior of the function at the limit point

The chapter of Limits is before the chapter of Derivatives. Using the skills for the calculation of limits (Maple program helps for calculation of limits) the advanced students were given the assignment of finding the limits of the following functions that are selected in a special way, not only the functions but the point of the limit as well:

$$\lim_{x\to 1}\frac{\sqrt[4]{x}-1}{\sqrt[3]{x}-1}=\frac{3}{4} \qquad \lim_{x\to 16}\frac{\sqrt[4]{x}-2}{\sqrt[3]{\frac{x}{2}}-2}=....=\frac{3}{4}..(16=2^2)$$

For further investigation, was taken the limit:

$$\lim_{x\to 81}\frac{\sqrt[4]{x}-3}{\sqrt[3]{\frac{x}{3}}-3}=.....=\frac{3}{4}..(81=3^4)$$

The function is properly selected for the reasons of having no difficulties in calculations. To see that this is not accidental was taken the general limit of such type:

$$\lim_{x\to a^4}\frac{\sqrt[4]{x}-a}{\sqrt[3]{\frac{x}{a}}-a}=.......=\frac{3}{4}$$

As is seen, the result is the same, and so we have a new statement for the limit of a particular family of functions of the above-mentioned type. Using a computer mathematical program cannot be got this answer for the limit.
For example, if generated by Maple, the answer is:

$$\frac{-(a^4)^{\frac{1}{4}}+a}{-(a^3)^{\frac{1}{3}}+a} \quad \text{!!!???}$$

The reason for such answer is because the program works if the variable is replaced by a numerical value. <u>One is concerned to know how is the behavior of the function close to the point a^4.</u> To have a clearer view about this fact we used GeoGebra to plot the graphics of these functions for integer values of a.
It is known that the domain of these functions is the non-negative real numbers set. Can be seen that while the values of variable x tend to a^4, the respective values of the function approach the number 3/4. This fact on the above limit can be easily observed and detected by using GEOGEBRA program or Maple program, where is possible to plot the graphic of the parametric function, with parameter a. The graphics of the functions of this family are plotted on the same pad. Through GeoGebra program is possible to use a slider for parameter a to see how the graphs are related to one another and, how they approach (as a family) the value 3/4 when x tends to a^4(Fig. 2.12).

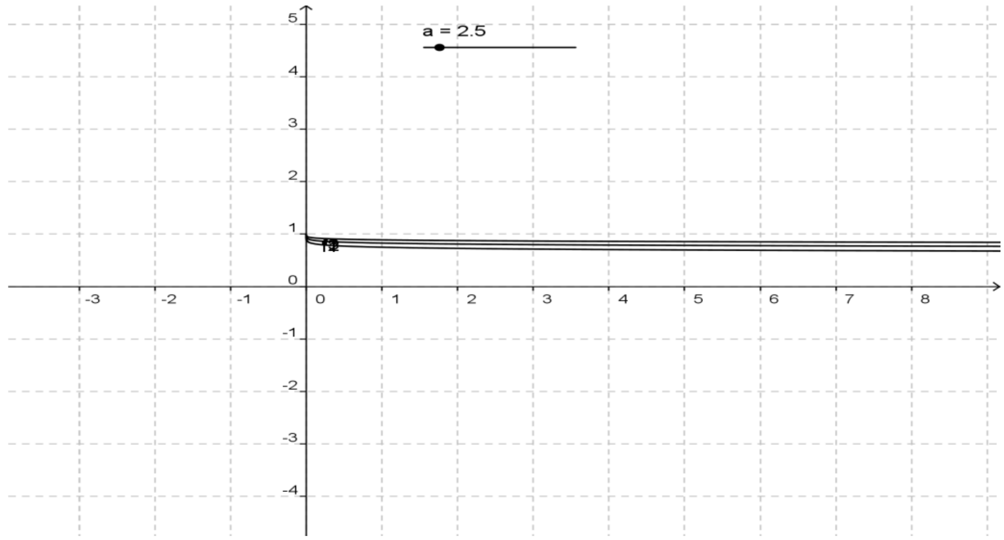


Fig. 2.12 The graphics of the parametric family
(exported by GeoGebra applet)

I am presenting here a separate graph for the function $f(x)=\dfrac{\sqrt[4]{x}-5}{\sqrt[3]{\dfrac{x}{5}}-5}$ and particularly for its position for large values of variable x(x>100) to see how is the behavior of the function, because is known that its limit is 3/4 when x tends to 625. Applying *Zoom Out* command we observe that for 0 < x <625 the values of the function are smaller than 1 and continually becoming smaller. Continuing by applying *Zoom Out* command we observe on the right side of the point 625 the same phenomena: the values of the function continue to become smaller. It is known that the $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x}-5}{\sqrt[3]{\dfrac{x}{5}}-5}=0$.

Staying close to the point (625, 3/4) and manipulating with *Zoom In* command until the program allows we see a cut of the graphic, in other words, we see a large gap between the two parts of the graphic. The point 625 is a disjunction point, but the program shows a gap that is a reflection of a missing segment!
Note: This assignment was part of a research work as well (for the good students).

It is important to note here that, we cannot get true information by GeoGebra, even by other computer program. In the above question the reality or the fact is that in the graph is missing a point alone. The analytical method in calculus gives the right answer and, a mathematically and logically shaped mind only has the power to make the division between points and discern the elements of the micro-world.

2.4.6 GeoGebra is a program for teaching "the feel" to the students (teach them having an eye or sense how to get from "here" - "there")

Teaching is "simply" about getting people from one point in their understanding/ability to a "further" point. In traditional teaching and generally it requires knowing how to figure out what students already can do, figuring out what sorts of things can build from what s/he knows. Usually it requires knowing some ways to get from here – there, knowing how to address someone toward assertion B starting from assertion A.

This task can be compared with the teaching of bicycle riding. It is known that there is not some thing that the kids do that makes them be able to ride; they don't need teaching procedures in order to ride the bicycle. They need to teach "the feel", they do not need some knowledge, not all the components of bicycle teaching or training. The information and the instruction they need is simply this: "balance with your butt, not your shoulders --when you start to fall left, move your butt to the right, not your shoulders; your balance is in your butt, not your shoulders; when you move your shoulders to the right, your butt goes to the left, so you will fall over faster by trying to balance with your shoulders". But the trainer cannot say how far, or when, the kid needs to move his/her butt. The kid has to learn to feel him/herself that; and though the trainer can help him/her learn to feel that, he cannot do it by explaining the physiological or physics steps to the kid, even if he knew what they were.

We as teachers spend too much time by explaining artificially constructed steps of what must be done in proving a theorem, in following a strategy to solve a problem or apply certain ordered steps in performing an algorithm etc. This practice makes teaching more complex than it is.
It makes learning more difficult as well. "Feeling" how to get from here – there or "having an eye or sense" has not to do with knowing "what and how to perceive", it is like an expert estimates and shares the opinion in a certain painting. S/he does not apply rules in judging about the values of a painting: neither takes measurements nor makes comparisons. Just has an eye or sense that the painting is really pleasing. Having an "eye" means recognizing something pleasant or interesting or whatever, not recognizing the components of what might be normally associated with its looking interesting or pleasant or whatever. In such cases we don't have to know what it is about the picture that makes us like it or dislike it; we just have to

"feel" some sort of difference compared with others.
The experiment carried out in the secondary school in regard with the influence of GeoGebra for higher results in mathematics confirmed this kind of teaching and learning “to feel”. The teaching of derivatives was accompanied with the teaching of how to use GeoGebra tools.

GeoGebra is dynamic mathematics software for schools that joins geometry, algebra, and calcules, it is an interactive geometry system. With Geogebra is possible to do constructions with points, vectors, segments, lines, and conic sections as well as functions while changing them dynamically afterwards. The two characteristic views of GeoGebra are: an expression in the algebra window corresponds to an object in the geometry window and vice versa. GeoGebra’s user interface consists of a graphics window and an algebra window. On the one hand […] we can create geometric constructions on the drawing pad of the graphics window and, on the other hand, we can directly enter algebraic input, commands, and functions into the input field by using the keyboard.

While the graphical representation of all objects is displayed in the graphics window, their algebraic numeric representation is shown in the algebra window.

The geometric constructions are done by the mean of the main virtual tools.
The virtual tools are found in the set of the toolboxes which have to be opened, selected a tool, activated it and used it during the construction process. In the toolboxes are found the virtual tools with their names linked with their functions like: New point, Move, Line through two points, Segment between two points etc., alongside which is their picture also. There are also buttons like: Delete object, Move drawing pad, Zoom in / Zoom out, Undo / Redo buttons etc… GeoGebra offers more commands than geometry tools. I can go on by describing other virtual tools which are many. Also it will take a longer time to describe in details their functions and describe step by step the procedure followed for the construction of a figure. Each figure has its own constructing procedure (there are some common steps as well). If I would dare to do such a teaching I had never started the teaching on derivatives within that period planed for that chapter. I had not finished with GeoGebra also. What do I mean by this? I want to say that teaching on GeoGebra was done alongside the teaching on derivatives. This way I was able to achieve two goals: the students learn about derivatives (not just know but able to apply) and, learn about GeoGebra program in the sense of

being able to use it in learning mathematics and other sciences. The achievement of these two goals was granted by the nature and features of GeoGebra: a program for fun, practice, knowledge and research. It is impossible to go through all the steps of a procedure and achieve the main goal linked with derivatives. I just used several tools for the construction of certain figures (they have been simple), the rest has been done by the students. By "the rest" do not mean all the things that can be done using GeoGebra, but those linked with the chapter of derivatives. So, how could students succeed in performing different constructing tasks? <u>They succeeded by "feeling" what tool use and how to use it</u>. They had no prior experience or knowledge on this task. Some of them only had knowledge on using computer and internet, but no computer program or further more about mathematics. This group was a minority compared to the whole experimental group.

The students had an eye or a sense in using GeoGebra for carrying out different task of constructing figures and they made it. Not all of them but the majority had it. They just needed to be taught how the tools were used in the construction of the square, equilateral triangle, of a circumscribed triangle and that was enough for them to start experience Geogebra themselves. Those who were slow or very slow were drawn by the progressive students. They developed by work and experiencing with GeoGebra an eye in that matter. I do think that one can learn how and what to perceive and develop "an eye" in the process. I am not against those who accept and defend the concept that the differences between the perceptions of people have **to do with talent, something inborn**.
Sometimes in mathematics is enough to point out that there is a B, that they must know where B is (or find out if they don't know) and that getting from A means taking steps, and trying to decide if they are getting closer to B. Sometimes might take many steps before realize that B is further away, hence is required a backtrack. This is like teaching people how to engage in independent learning. There are many occasions when a person knows the answer of a problem without making an effort to solve it. While some experienced in rules and procedures is engaged in *writing the givens, formulating questions what s/he needs to know, indicating a formula that containes both knowns and unknowns, substituting and solving and so on, the person having an eye or a sense gives the right answer just by thinking and much faster.*

2.4.7 GeoGebra - an interactive learning environment

GeoGebra provides an interactive learning environment where, "*the pre-requisites are built into the system and, where learners can become* *active, constructing architects of their own learning"* (Papert, 1980, p.117), (Mathematics Education Library, Volume 13, Computer Environments for the Learning of Mathematics, pg.191). The students, manipulating with the tools provided by GeoGebra software and making observations in the two windows, continually have a horizontal growth of knowledge, in which they build links between different representations, but even more they have a powerful vertical growth of knowledge that enables them to explore other aspects. The teacher alone determines the effectiveness of curriculum by his or her decisions, behavior, attitudes, and cognitive processes, no matter how carefully the curriculum has been developed. The high expectations educators once had about the benefits of scientifically developed curricula have been supplanted by a more modest assessment. *"Recent research has placed more emphasis on everyday curriculum in the classroom, on teachers' ideas and subjective theories concerning their quotidian preparation of classes, their subjective learning theories, implicit and explicit objectives, philosophy of mathematics, and the influence of these cognitions on their teaching"* (Mathematics Education Library, Volume 13, Didactics of Mathematics as a Scientific Discipline, pg.52). GeoGebra is a special field to make research on teaching and learning and a strong tool to realize the "didactical triangle" which is: the teacher, the student, and the knowledge taught/learned"(Mathematics Education Library, Volume 13, Chapter 3, Interaction in the Classroom, pg.117) . Epistemology helps researchers make sense of research information transforming it into data detailing how that analysis might be patterned, reasoned, and compiled and shows the belief they have about the nature of the reality they describe (Willis, 2007; Creswell, 2007; Scott and Morrison, 2005). GeoGebra is an answer to the epistemological questions about *"how technology can help to construct an understanding of mathematics and how GeoGebra can be used interactively to scaffold the construction of mathematics knowledge"*. (Yu-Wen Allison Lu,Linking Geometry and Algebra, 2008, pg.22.).

GeoGebra creates an atmosphere where the teacher encourages the students to think creatively and promotes a problem-oriented approach to the teaching of mathematics.

Learning involves the transmission of known stuff from one person to another. A teacher is a person who fosters learning in others. This can sometimes be done with relatively limited subject matter expertise. It involves far more than giving assignments. It includes listening very carefully and restating what has been heard, raising questions about what seems not to be clearly understood (either by the learner or by the teacher), finding sources (including other teachers and subject matter experts) that will help the learner to advance, responding to assignments as an intelligent reader (and perhaps finding experts to respond to content), encouraging effort in times of depression, demanding critical analyses, and a good many other activities. We will take advantage of cyber-learning by creating new tools and a powerful, open-source learning environment. The learning environment will readily integrate new visualizations, incorporate best practices from research, and support researchers, designers, and teachers.

In the interactive learning environment created in the class where the teacher and students communicated all the time by addressing questions and ideas regarding the issues linked with derivatives or GeoGebra was observed that: the student either 1) solved the problem her/himself after the explaination done,
2) found a solution as a consequence of the efforts done by the teacher to understand the question (and intuitively asking good questions in the process) or 3) got the solution as a result of suggestions were made about general problem solving strategies. The students have been so thankful for teaching them as they expected and desired. The very fact that they approached me is evidence of a good and effective teaching, something they count on to help them learn. The questions and general suggestions constitute teaching the learning process, something with generic utility. There have been times in the math class when the students could be allowed to work independently (they were allowed indeed), because that is the environment created in the class of teaching math using GeoGebra, but the guidance by the teacher is very important, it is a matter of effective teaching. Making everyone (specially students) have to discover everything for themselves would take each of us(specially students) more time to learn than each of us has. Many scholars don't think that telling is necessarily teaching or that knowledge is necessarily "transmitted" or fostered by telling, It is vital that in many courses the guidance by the teacher IS to facilitate the learning of a particular body of material (or at least introduce it in a way that students can begin to assimilate it, reflect on it, demonstrate it, apply it, etc.).
There are two main aspects to a good teaching provided in the math lessons: (1) inspiring students to want to learn more, and (2) helping them acquire the skills/knowledge to do so; especially on their own. That was result of making stuff interesting, and that might make it somewhat

memorable; the students were helped understand the stuff as they thought about it; and the thinking was made interesting, stimulating, and fruitful because of the use of GeoGebra tools. This is really challenging to me. I think it is important in most subjects to do both of the above aspects, however.
Finally, I can say that in this chapter was provided an adequate teaching. I mean the course was taught in a way that did help the students learn it in some relatively efficient time frame. For example, when I taught about monotony and extremes values, I took two lessons, of about two hours. The traditional teaching program takes four hours, and doesn't cover much of the stuff I did and the stuff is not efficiently assimilated. That happens because they are not taught adequately and the teaching methods used do not provoke their interests in it and the teachers make them do a lot of unnecessary, and unhelpful work.
The students of the experimental group were taught in such a way that they realized that what they did receive they were able to apply. I say they were taught adequately - not because they did learn a lot of material, but because I did help them UNDERSTAND (and how to do in new situations) mathematics. The big mission in teaching anything is making/keeping it interesting, and making the subject UNDERSTANDABLE. The material or the "instruction" (including the tasks and questions I gave to students) was organized in such a way that the students understood the subject, or I had a framework of the subject, in such a way that they could fill in their own knowledge gaps (for example, gaps in limits).

2.4.8 GeoGebra is an approach to establish a communicative bridge between the textbooks and economy and technology.

In the traditional education, the programs of mathematics are mainly consisted of theory and applications, rarely is found space for research work. It is known that mathematics is the foundation of all the inventions, of all the new discoveries, techniques and technologies. Mathematics is created to meet the needs of our physical and social world.
"*Our mathematical concepts, structures, ideas have been invented as tools to organize the phenomena of the physical, social and mental world. Phenomenology of a mathematical concept, structure or idea means describing it in relation to the phenomena for which it was created, ….and, a way to show the teacher the places where the learner must step into the learning process of mankind"* (Hans Frendenthal, Didactical Phenomenology of mathematical Structures, D. Reidel, Dordrecht, 1983, p. IX).
It is also known that, while these new technologies and techniques are progressing very fast, the programs and the textbooks of mathematics are left behind regarding the reflection of them and

the incorporation with them in the teaching process. John F. Sanford, Les M. Sztandera in their paper "Thoughts on the future of education in information Technology" treat the topic of difficulties faced by educational institutions two of which are:
□□ textbooks are often slow to incorporate new technologies (Reisman, 2005a)
□□ few four-year programs emphasize problem-solving through research of new product capabilities…… Educational programmes in the physical sciences present natural laws coupled with experimental observation. Mathematics is usually taught in a similar way with problems and/ or proofs substituted for experimentation (Pg.28). GeoGebra software provides a perfect link between mathematical textbooks and IT and a very comfortable and desired environment for problem-solving situations through research work.

2.4.9 Teaching in the knowledge society: an art of passion

GeoGebra software is a tool and a platform that can be used by the students of any level. It can be used by the young people, even by the students of the primary school. This is because of the great number of varieties of the exercises and of different types like puzzle and entertaining, construction, testing, research, problem-solving etc. that can be accomplished by using this tool and platform. Young people are game-driven and curiosity problem-driven. GeoGebra software is the right tool and the platform meeting the trends and the needs of this generation not only in the school but in their homes as well or elsewhere, suffice to have internet access. GeoGebra is an open source for teaching and learning, free of charge and for all. Their mathematical formation by using GeoGebra is indisputable in this IT age and knowledge society where, as Miltiadis D. Lytras writes in his paper "*Teaching in the knowledge society: an art of passion", "Young people are computer literate up to an extremely satisfying level, they use advanced software tools and hardware systems, ….. Furthermore, the new amazing communication capacities characterizing our era (blogs, wikis, personal desktops, satellite connections) provide them with a global context. These characteristics of change require a translation in terms of reflective actions. Academic Institutions must develop new flexible approaches for open teaching and learning. We started the International Journal of Teaching and Case Studies with the ultimate objective of providing fresh ideas on how Teaching can be transformed into an Art of Passion*."(Pg.2). The young people potentialities and the communication capacities are present. Is required a respond by the Academic Institutions. GeoGebra software is a very good program and tool to be used by them and Geogebra Wiki is a response added to the other efforts and approaches done for open teaching and learning and for incorporating new techniques and technologies in the teaching and learning process. GeoGebra provides networking, open access, sharing of knowledge and teaching and results, accumulation of experiencies and further development of them. GeoGebra and other sources are very useful tools and opportunities that must be used by the Academic Institutions as the best means to reach communities and use them for global progress and giving answer this way to the questions raised by Miltiadis in the same paper like:

☐ "How can we exploit communities of teachers and learners aiming to match together common interests and to exploit the synergies of differences?

☐ How can we manage the various resources required for teaching, including Content, Technologies, Human Resources, Processes, Skills Competencies, Institutional policies etc.?"(Pg.5)

2.4.10 **Learning in a mediated online environment**

*** GeoGebra is a global platform where the students share together their knowledge and their creative works in the field of mathematics…The web page, GeoGebraWiki, allows the communication between the students and teachers in a global scale. In GeoGebra page the students can work and perform tasks individually or in groups, they are motivated and enabled to take necessary actions for deeper learning, they are involved in discussions that considerably facilitate their individual learning and provide the possibilities for learning from one another and exchanging their experiences not only locally but globally also and, for measuring their own level of knowledge and capabilities. This way, the students become accountable for constructing knowledge and adding to it. As Alison Ruth writes in her paper "Learning in a mediated online environment", "Students may then become engaged within their Zone of Learning Capability, which is analogous to Vygotsky's (1978) 'Zone of Proximal Development'. However, in the Zone of Learning Capability the student is enabled to take the actions necessary to facilitate their own learning rather than being led to a pre-determined point of knowledge. It emphasizes students' epistemic motivation and agency…. or the desire to know (Hatano and Inagaki, 1991), which is central to understanding the Zone of Learning Capability,"(Pg.138)

Chapter 3
GeoGebra Experiment for Teaching and Learning Mathematics

3.1 Platform: Comparative Experimental Study

Theme: Teaching with GeoGebra versus the traditional teaching

The treatment under study: Effect of using GeoGebra software in teaching and learning process.

Purpose: investigate and determine the "cause and effect" of an action-determine if the treatment caused a change in the individuals' responses.. Decide whether the mathematical course taught by using GeoGebra software is as effective as more traditional methods of instruction.

Methodology: particular treatment of a class of students. Are selected two classes of the same secondary school and of the same level. One class receives traditional teaching and instruction, while the other takes the course by using GeoGebra software. At the end of the course, each group takes the same comprehensive exam.

Population: The entire group of the students of the secondary school "Dhaskal Todri", in Elbasan, ALBANIA we want information about by examining a portion of the population, two classes – which are representative of the population (the relevant characteristics of the sample members are generally the same as the characteristics of the population).

Samples: Two Classes of the 3d year of the secondary school "Dhaskal Todri".

Sample sizes: Class A (28 students), class B (29 students) - look at students tables in appendices.

Experimental text-book and chapter: Mathematics 3(text-book for the secondary school); Chapter of Derivatives.

The variable to be measured: one characteristic shared by two samples of one population (the math level represented by the marks in a chapter in two classes treated in different ways).

The data for comparative experimental studies consist of two sets of measurements.

Although there may be more than one variable in a study, we will restrict our attention to the analysis of data collected on one variable for now. We will use Five-Number Summaries and comparative box plots to analyze and interpret data from several different comparative studies.

Sampling practice: random selection in order to remove the bias caused by human involvement in the selection process (there are 6 classes in the third grade, randomly chosen two of them by number marking.

Venue: The computer laboratory of the school (class A) and the classroom (class D).

Year of experimental study: 2010

Statistics is a problem-solving process that seeks answers to questions through data. The experimental process has four components:

1. Ask Questions
2. Collect Appropriate Data
3. Analyze Data
4. Interpret the Results

FORMULATION OF QUESTIONS

1. **For the students:** Are the students aware about the computer programs used in the mathematics class? Are there facilities in the school that the students use internet (computer laboratories)? What purpose do they use the internet? Do the students have internet access in their homes? Are there students able to use computer programs, especially for learning mathematics or playing mathematical games? What kind of computer programs they have used? Is GeoGebra soft-ware more effective than the traditional methods of instruction? What is the cause of the difference between the results got by using GeoGebra in the teaching

process in the experimental class and the results got by using the traditional methods in the comparative class? Why are there differences (i.e., variation) in our measurements? What is the source of this variation? What do the students think about using GeoGebra to learn math? Is GeoGebra a more preferable and helpful tool than the traditional means and tools used in learning math?

2. **For the teachers**: Are the teachers familiar with computer programs for mathematics and which one? Do they use math programs in teaching process and how? What programs they use? Do they use the computer laboratory if there is such in the school? What problems do they face in using the computer laboratory and the internet? (proffesional or lack of equipments and funds?) What do the teachers think about using GeoGebra? Does the use of GeoGebra in the teaching and learning process cause increase in the level of mathematics? Does the use of GeoGebra in the teaching and learning process grow the interest and the activation of the students in the classroom? Is GeoGebra a more effective tool than the traditional? Are got similar answers using the different tools of teaching? Why or why not? Are got identical answers to the same questions of the chapter of the experiment? Why or why not?

COLLECTION OF DATA

- Questionnaries (for students of the two classes and teachers)
- Appropriate forms for collecting data of the students' marks in the experimental chapter
- Register of the class
- Notes kept during the experiment
- Forms with questions about the chapter for the two classes(to compare the answers)
- Short tests
- List of final chapter marks of the comparative class
- List of chapter distributed marks of the experimental class
- List of chapter distributed marks of the comparative class (which class can be controlled and evaluated and marked more?)

STATISTICAL METHODS TO ANALYZE DATA

Appropriate graphical representations of data: histograms, stem and leaf plot, line plots (for studying the distribution of marks), box plots (a representation or another way to compare and discuss the variance between the two data sets when analyzed the results of the two classes), scatter plots, Five-Number Summaries (to find, use, and interpret measures of center and spread, including mean and interquartile range).

INTERPRETATION OF DATA (RESULTS)

Interpretation is necessary in order to provide answers -- to the original question. Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem and leaf plots, box plots, and scatter plots.

Use the observations about differences between the two samples to make conjectures about the population from which the samples are taken. Make conjectures about possible relationships between the two characteristics under study of the two samples on the basis of scatter plots of the data and approximate lines of fit. Use conjectures to formulate new questions and plan new studies to answer them.

Use of Five-Number Summaries and Box Plots helps in comparing two sets of measurements which is not quite as simple as comparing two numbers. The conjectures are based on the estimation of Min, Q1, median, Q3, Max. Because we are comparing a set of many measurements in two samples, any comparison must be based on percentages and not absolute frequencies. A comparison of the Five-Number Summaries is useful, since these quantities divide the ordered data into four groups, with approximately 25% of the data in each group. In order to identify any patterns present in the variation, we **must analyze** our data by organizing and summarizing it. Once this analysis is complete, we can interpret the results to answer our questions about using GeoGebra in teaching and learning process.

CONCLUSIONS: conjectures on mathematical course taught by using GeoGebra software, whether it is as effective as more traditional methods of teaching.

3.2 Data organizing of classes XI-A (control group) and XI-D (experimental group). INTERPRETATION OF DATA RESULTS

When two groups are to be compared, an alternative to superimposition is to draw their two histograms back-to-back (in a similar way to back-to-back stem and leaf plots). These back-to-back histograms are called **population pyramids,** as well.

For this purpose, I have grouped the data as in Tables 3.6 and 3.7 in accordance with the points the students have got in the test, which are taken out of the Table 3.4 of the experimental group and of the Table 3.3 of the control group (look at appendices). The points of the control group have not been available for me, but having at hand the marks of this group is known what class of points any mark belongs to.

Interpretation is necessary in order to provide answers -- to the original question. Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem and leaf plots, box plots, and scatter plots.

Use the observations about differences between the two samples to make conjectures about the population from which the samples are taken. Make conjectures about possible relationships between the two characteristics under study of the two samples on the basis of scatter plots of the data and approximate lines of fit. Use conjectures to formulate new questions and plan new studies to answer them.

Use of Five-Number Summaries and Box Plots helps in comparing two sets of measurements which is not quite as simple as comparing two numbers. The conjectures are based on the estimation of Min, Q1, median, Q3, Max.

Univariate Analysis

Univariate analysis involves the examination across cases of one variable at a time, specially looking at three major characteristics of a single variable that are:

- the distribution
- the central tendency
- the dispersion

The Distribution is a summary of the frequency of individual values or ranges of values for a variable. In the simplest distribution are listed all values of a variable and the number of persons having each value. A typical way to describe the distribution of the marks of the students of the experimental class is as in the respective table above. But one cannot list well the students regarding their level in mathematics because the mark is a round up or down of the points the student has got in the test. Therefore we use the stem and leaf plot where can be seen more details. Distributions may also be displayed using percentages, so we describe the level in mathematics by listing the number or percentage of the students in accordance with the group or range of the points they belong to(the values are grouped into ranges and the frequencies determined). This is called a frequency distribution for grouped data. To compare the two classes I have taken into consideration the marks of the previous chapter, because that was what I had available from the control class in carrying out the experiment. Also, I use the marks of the test done at the end of the chapter on Derivatives. So, from the previous chapter data I got the following distribution of marks and percentages for the experimental and control class (Table 3.5):

Range of points	Mark	Experimental Group Frequencies	Percentage	Control Group Frequencies	Percentage
<35					
35 - < 45	4	1	3.40 %		
45 - < 55	5	3	10.34	2	7.14 %
55 - < 65	6	4	13.79	1	3.57
65 - < 75	7	2	6.90	6	21.43
75 - < 85	8	9	31.03	7	25
85 - < 95	9	9	31.03	6	21.43
95+	10	1	3.40	6	21.43
		Sum = 29		Sum = 28	

Table 3.5 Frequency and percentage distribution table.

As can be seen from the percentages in the table the control group has a better distribution of percentages and a higher level in mathematics (for higher marks, starting with the grade 7, it has higher accumulative percentage). This is the state of the groups in the beginning of the experiment. There is a considerable difference between the two groups. However this is not a problem for making conjectures and drawing conclusions because we do comparisons of the results between the two groups and between the two results of the experimental group got at the beginning of the chapter and at the end. This comparison is more important to judge regarding the new method used in teaching and learning process.

Notice that, in the above table we know about the ranges of points for the experimental group only. For the control group I had available only the marks, not the points. In my case, that is enough to do comparisons between the two groups. Any one knows for a certain mark what range is of. The points help in better understanding the order of the students regarding their knowledge and skills in mathematics within their class. My issue is to compare the classes.

The same frequency distribution I have depicted in a graph that is often referred to as a histogram or bar chart as shown in Figure 3.1.
I have constructed back to back histograms(called bihistogram) for the two groups, using Geogebra tools, and I have put them back to back to make easier the reading and interpretation of data. The histogram above the horizontal axis, is of the experimental group, the histogram below the horizontal axis is of the control group.

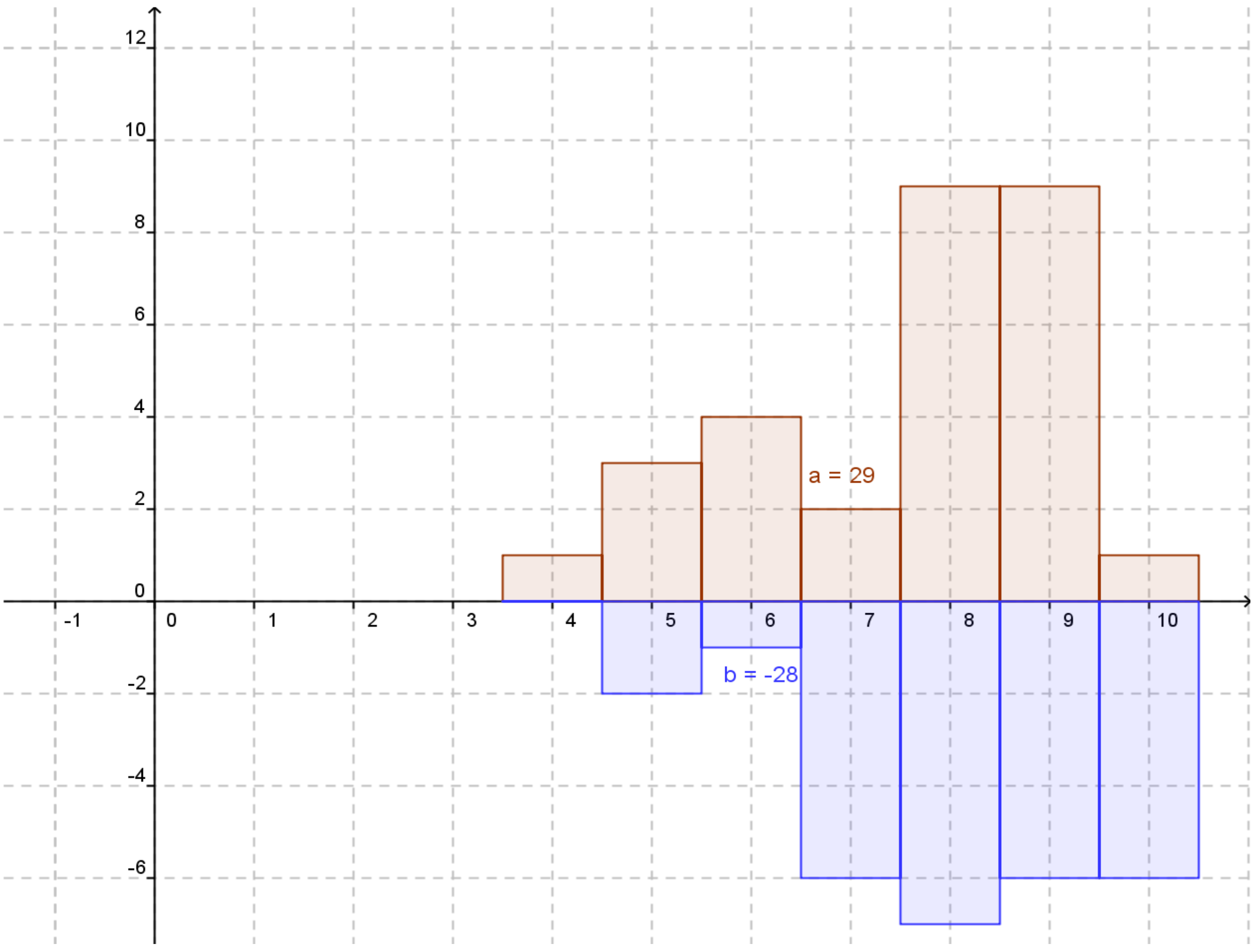


Fig.3.1 Back to back histograms of the two groups(bihistogram).

(exported by GeoGebra applet)

The above-axis histogram does not appear to be that much different from the below-axis histogram. With respect with distributional shape, note that the two histograms are skewed right. Thus the bihistogram reveals that there is not a clear difference between the two histograms with respect to location and distribution, whereas, in regard to variation there is a little change of 1 unit. To get clearer information about data we look at the other estimators.

I have arranged the marks of the two groups in the previous chapter in increasing order (from the lowest to the highest value):

Experimental group: 4 5 5 5 6 6 6 6 7 7 8 8 8 8 8 8 8 8 8 9 9 9 9 9 9 9 9 9 10

Control group: 5 5 6 7 7 7 7 7 7 8 8 8 8 8 8 8 8 9 9 9 9 9 9 10 10 10 10 10 10

The computed means are: for the experimental group….$\overline{X}$ = 7.6, for the

control group…$\overline{X}$ = 8.14

For the experimental group the *__median is 8__* which is the mark dividing into two equal parts the given sequence.

For the control group the median is 8, as well. The two groups have the same median.

Considering the ordered scores as shown above, and then counting each one, is found out that:

The experimental group has two modes (is bimodal) : 8 and 9

The control group has one mode: 8

Summarizing: for the experimental group the mean, median and mode are 7.6, 8, 8 and 9, respectively; for the control group they are 8.14, 8,8, respectively.

If the distribution is truly normal (i.e., bell-shaped), the mean, median and mode are all equal to each other.

The bihistogram can provide answers to the following questions:

1. Is the difference between the experimental and control group significant?
2. Does the difference (if any) have an effect in the level of mathematics at the end of the chapter?
3. Does the location change between the 2 groups?

4. Does the variation change between the 2 groups?
5. Does the distributional shape change between groups?
6. Are there any outliers?

From the above bihistogram, is seen that the initial state of the experiment is: the two sets of data are centered at the value of approximately 8. That indicates that these sets are not displaced from one another, thus it is difficult to conclude that there is a real difference between the two groups. Later, we will see graphically and convincingly what a t-test or analysis of variance would indicate quantitatively.

Five-number summary

The distribution of values in many data sets can be effectively summarized by a few numerical values called **summary statistics** by using a graphical display that is based on five summary statistics called the **5-number summary**.

- The two **extremes of data set** (i.e. the minimum and maximum values).
- Three other values that split the data set into groups that contain (as closely as possible) equal numbers of values are: the **lower quartile**, the **median** and the **upper quartile**.

Box plot

The **box plot** of each one of the sets of values above displays these five values graphically. A box plot splits the data set into four quarters with (approximately) equal numbers of values.

Firstly, must be calculated the quartiles, the other statistics are ready. Quartiles (Q_i) are calculated using the formula for their position:

$$Position(Q_i) = \frac{n \cdot i}{4} + 0.5$$

For the control group:

$$Position(Q_1) = \frac{28 \cdot 1}{4} + 0.5 = 7.5$$

$$Position(Q_3) = \frac{28 \cdot 3}{4} + 0.5 = 21.5$$

Looking at the respective ordered set of data follows that: $Q_1 = 7$, Q_2 = median = 8, $Q_3 = 9$. Inter-quartile range = 9 – 7 = 2.

<u>For the experimental group</u>:

$$Position(Q_1) = \frac{29 \cdot 1}{4} + 0.5 = 7.75$$

$$Position(Q_3) = \frac{29 \cdot 3}{4} + 0.5 = 22.25$$

Looking at the respective ordered set of data follows that: $Q_1 = 6$, Q_2 = median = 8, $Q_3 = 9$.

Inter-quartile range = 9 – 6 = 3.

Here, in Fig 3.2 are the box plots showing the relation of the five values with the respective histogram.

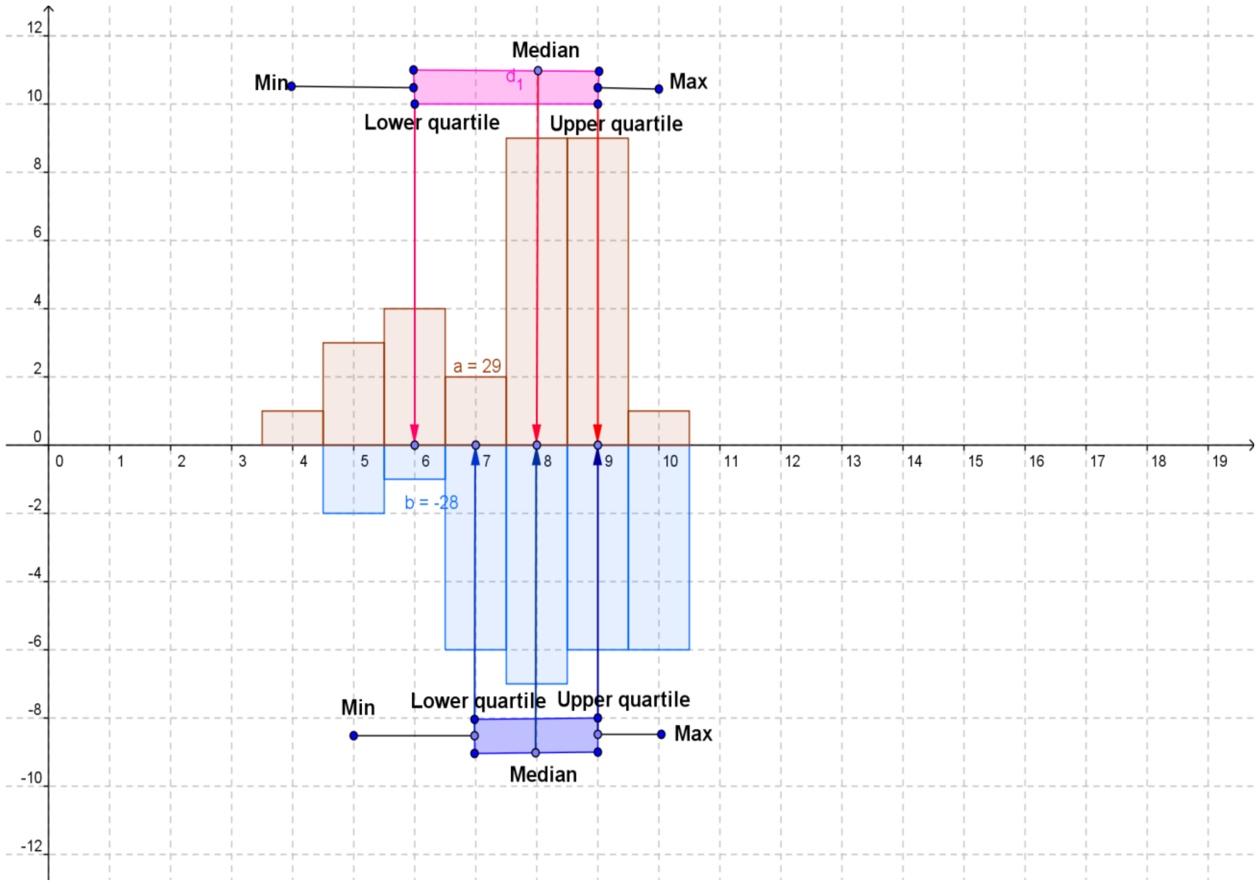


Figure 3.2 Histograms and their respective box plots
(exported by GeoGebra applet)

In summary:

Experimental group five values
Min = 4
Q1 = 6
Med = 8
Q3 = 9
Max = 10

Control group five values
Min = 5
Q1 = 7
Med = 8
Q3 = 9
Max = 10

Looking at the histogram above the horizontal axis in figure, we see that:
Centre: the vertical line inside the box (the median) gives an indication of the centre of the distribution: the centre is 8.
Spread: the width of the box (the interquartile range) gives an indication of the spread of values in the distribution. It is 3.
Shape: the high density of the values is found in the part of histogram having the median and the upper quartile. The right extreme and quartile are closer to the median than the left extreme and quartile, this shows that the distribution is right skew.

Comparing the box plots clearly shows that the difference between the two groups stands for the lower quartiles, minimums and inter-quartiles. The difference between these three values is the same: 1 unit. Considering the part of the experimental group spanning from minimum to the first quartile as the poorest part of this group, we can determine their percentage using GeoGebra tools to calculate the area of that part of the histogram above the horizontal axis. The area of that part is 5.92. Doing the same thing, with the same span, for the part of the histogram below the horizontal axis, corresponding to the control group, the area is 2.50. So, the number of the students of the experimental group and belonging to the poor category is more than two times greater than the number of the students of the control group and belonging to the same category. Also, the number of the students of the experimental group and belonging to the superior category is smaller than the number of the students of the control group and belonging to the this category. Here, it is not the purpose to make other calculations. Conclude that, at the initial stage or state, the control group appears to be better than the experimental group.

Although the box plot of a single data set shows various useful aspects of the distribution of values, it is no more informative than a dot plot, stem and leaf plot or histogram. However box plots are often used when two or more sets of data are compared. The most important differences between the sets are usually precisely the aspects we are interested in, as can be understood by the analysis done above. In reality, the differences between the individual values are more prominent but the box plots hide them, on the other side a box plot cannot give any indication of clusters in a data set. If clustering is present, a box plot should not be used to summarize the data. The existence of clusters is examined by using a dot plot, stem and leaf plot or histogram.

Range is simply the difference between the highest value and the lowest value. In our case, for the experimental group the range is: 10 – 4 = 6,

for the control group is: 10 – 5 = 5

Outliers: there are no.

The **Standard Deviation** is a more accurate and detailed estimate of dispersion because there are cases of ordered data with outliers that can greatly exaggerate the range (the outlier values stand apart from the rest of the values). The Standard Deviation shows the relation that set of scores has to the mean of the sample. For the tests of our two groups the standard deviations are calculated in the tables below.

The calculation of the Means and Standard Deviations of data from the tests done with the experimental and control group.

Symbols:

X_i ...... each score (points or mark)

$\overline{X}$ ...... the mean or average

N = $\sum f(X_i)$ .... the number of values

$f(X_i)$ ...... the frequency of value(mark) X_i

$S^2 = \frac{\sum f(X_i)(X_i - \overline{X})^2}{\sum f(X_i) - 1}$ .... Variance

$S = \sqrt{\frac{\sum f(X_i)(X_i - \overline{X})^2}{\sum f(X_i) - 1}}$ .... Standard Deviation

$\sum$ .... means the sum across the values

Here below are the calculation of Standard Deviation and one, two, three-St. Deviation intervals of data from each test for each group.

Class XI-control (Data of the previous chapter, Table 3.6)

Variance = $\frac{\sum f(X_i)(X_i-\overline{X})^2}{\sum f(X_i)-1} = \frac{57.48}{27} = 2.13$, Std. Deviation = $\sqrt{2.13} = 1.46$

$\overline{X} \pm 1\, Std.Deviation = 8.14 \pm 1.46 = (6.68, 9.60)$

$\overline{X} \pm 2\, Std.Deviation = 8.14 \pm 2.92 = (5.22, 11.06)$

$\overline{X} \pm 3\, Std.Deviation = 8.14 \pm 4.38 = (3.86, 12.52)$

Having not at hand the points got by the students of the control group in the previous chapter, also at the end of the chapter on Derivatives, I cannot calculate the percentages of the values in each of the intervals above.

Class XI-experimental (Data of the previous chapter, Table 3.8)

Variance = $\frac{\sum f(X_i)(X_i-\overline{X})^2}{\sum f(X_i)-1} = \frac{70.84}{28} = 2.53$, Std. Deviation = $\sqrt{2.53} = 1.59$

$\overline{X} \pm 1\, Std.Deviation = 7.6 \pm 1.59 = (6.01, 9.19)$

$\overline{X} \pm 2\, Std.Deviation = 7.6 \pm 3.18 = (4.42, 10.78)$

$\overline{X} \pm 3\, Std.Deviation = 7.6 \pm 4.77 = (2.83, 12.37)$

We observe that in the first interval are approximately 72.4 % of the values; in the second and third interval are 100 % of the values.

Class XI-experimental (Data of the test at the beginning of the chapter, Table 3.9)

Variance = $\frac{\sum f(X_i)(X_i-\overline{X})^2}{\sum f(X_i)-1} = \frac{80}{28} = 2.857$, Std. Deviation = $\sqrt{2.857} = 1.69$

$\overline{X} \pm 1\, Std.Deviation = 7 \pm 1.69 = (5.31, 8.69)$

$\overline{X} \pm 2\, Std.Deviation = 7 \pm 3.38 = (3.62, 10.38)$

$\overline{X} \pm 3\, Std.Deviation = 7 \pm 5.07 = (1.93, 12.07)$

We observe that in the first interval are approximately 65.5 % of the values; in the second and third interval are 100 % of the values.

Class XI-experimental (Data at the end of chapter on Derivatives, Table 3.10)

Variance = $\frac{\sum f(X_i)(X_i-\overline{X})^2}{\sum f(X_i)-1} = \frac{73.3104}{28} = 2.6$, Std. Deviation = $\sqrt{2.6} = 1.62$

$\overline{X} \pm 1\, Std.Deviation = 8.24\ \pm 1.62 = (6.62, 9.86)$

$\overline{X} \pm 2\, Std.Deviation = 8.24\ \pm 3.24 = (5, 11.48)$

$\overline{X} \pm 3\, Std.Deviation = 8.24\ \pm 4.86 = (3.38, 13.10)$

We observe that in the first interval are approximately 82.7 % of the values; in the second and third interval are 100 % of the values.

Class XI-experimental (Data of the test on GeoGebra skills, Table 3.11)

Variance = $\frac{\sum f(X_i)(X_i-\overline{X})^2}{\sum f(X_i)-1} = \frac{71.04}{28} = 2.537$, Std. Deviation = $\sqrt{2.537} = 1.593$

$\overline{X} \pm 1\, Std.Deviation = 8.40\ \pm 1.593 = (6.807, 9.993)$

$\overline{X} \pm 2\, Std.Deviation = 8.40\ \pm 3.186 = (5.214, 11.586)$

$\overline{X} \pm 3\, Std.Deviation = 8.40\ \pm 4.779 = (3.621, 13.179)$

We observe that in the first interval are approximately 75.8 % of the values; in the second interval are 96.5 % of the values and in third interval are 100 % of the values.

All these results are a confirmation that data under study have not a normal distribution.

Taking the standard deviations of data from the previous chapter ready from the respective tables above, we have:

for the experimental group the Standard Deviation is:
Std. Deviation = $\sqrt{2.53} = 1.59$,

for the control group the Standard Deviation is:
Std. Deviation = $\sqrt{2.13} = 1.46$

There is a little difference between them. This fact confirms again that to do good and right interpretation and a good comparison of the two groups we have to take into consideration all statistics describing the data under study.

Based on the five-number summary and in the analysis of the bihistogram with the box plots done earlier the conclusion is: at the initial stage or state, the control group appears to be better than the experimental group.

For the normal distribution,

- approximately 68% of the scores in the sample fall within one standard deviation of the mean
- approximately 95% of the scores in the sample fall within two standard deviations of the mean
- approximately 99% of the scores in the sample fall within three standard deviations of the mean

3.3 Comparision of two data sets of experimental group

Using the stem and leaf plot

My main concern is making conjectures and drawing conclusions about the two results of the experimental group got at the beginning of the chapter and at the end. This comparison is more important to judge regarding the new method of using GeoGebra software in teaching and learning process of mathematics. I do this starting with the information given by stem and leaf plot (Fig 3.3). The stem and leaf plot contains more details about the values than the corresponding stacked dot plot, but this extra information rarely helps to understand the data. In many situations, stem and leaf plots have few advantages over stacked dot plots as graphical displays of data. Here are displayed the stem and leaf plots of points got by the students of the experimental group in three tests.

Stem and Leaf Plot of Points
(Beginning of Chapter)

4	224
5	2238
6	24448
7	1237788
8	012234
9	246
10	0

Stem and Leaf Plot of Points
(End of Chapter)

5	2478
6	48
7	1148
8	0223468
9	224566778888

Stem and Leaf Plot of Points
(GeoGebra Skills)

5	157
6	2448
7	77
8	02246788
9	22566667778
10	0

Fig 3.3 Stem and leaf plot of data of experimental group
(exported by GeoGebra applet)

Looking at one of the Stem and Leaf plots, for example, at the first one: the notation 5 | 2 represents 52 points that a student has got out of 100 points, 5|8 represents 58 points that a student has got out of 100 points and so on. The stem and leaf plot shows the distribution of points well. It allows the teacher and any student to determine exactly his/her place in the class. For example, a student who got 83 out of 100 in the test at the beginning of the chapter can easily count that 5 students got a higher mark (or the same) in the class. Comparing the stem and leaf plots (the first and

<u>the third) it is clear that there is a shift toward higher marks within the experimental group.</u>

Stacked dot plot A **stacked dot (circles) plot** uses the perpendicular axis more directly to show density. A stacked dot plot is obtained by:

1. **grouping values** into classes, then
2. **stacking** the circles in each class on top of each other.

Stacking the circles shows density better. This kind of plot can provide an effective display of ranges of high and low densities of values. However, there are cases when the randomness of data can be disconcerting and can be easily seen by the geometrical display (stacked circles). The diagram below (Fig 3.4) illustrates a normal stacking. This diagram of stacked circles corresponds to the stem and leaf plot of points got by the students in the test at the beginning of the chapter.

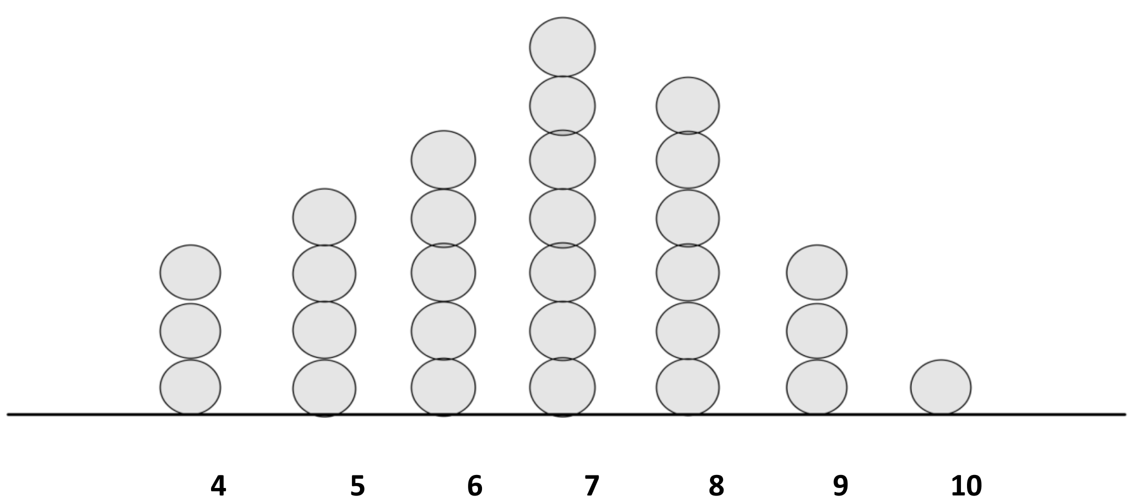


Fig 3.4 Stacked circles plot of data of experimental group
(exported by GeoGebra applet)

I could display here the stacked circles plots of the other two tests and comment about the type of stacking but I have left it out. Can be easily seen from the stem and leaf plots that the other two stacked circles plots (if I had displayed), corresponding to the test at the end of the experimental chapter and to the test on GeoGebra, respectively, would show that they both are right skewed.

Comparing histograms

Let look at the bihistogram below (Fig 3.5). The above-axis histogram corresponding to the data of the experimental group from the test purposely done at the beginning of the chapter appears to be different from the below-axis histogram corresponding to the data of the experimental group from the test done at the end of the chapter on Derivatives. With respect to distributional shape, note that the two histograms look different in their skew. Thus the bihistogram reveals that there is a clear difference between the two histograms with respect to location and distribution, whereas, in regard to variation will be spoken in the next section. It is sure that there is a change in their variations. To get clearer information about these two groups of data we look also at bihistogram of percentages distribution of the two sets of values (Fig 3.6). It is very obvious by the two bihistograms that moving from the above-axis histogram corresponding to the data of the test done at the beginning of the chapter to the below-axis histogram corresponding to the data of the test done at the end of the chapter on Derivatives, there is a displacement of the values from left to right, also the concentration of the percentages of the values in the percentages below-axis histogram is in the right half of the histogram. This shows that there is a considerable increase in the level of the experimental group in mathematics. I am not concentrating too much in the information given by these two bihistograms and in the analysis based in this geometrical display. In the next section I will perform a paired t-test to observe the difference between the two groups of data (beginning and end of chapter). The benefits of performing a t-test is that it gives more accurate information, it is easy to understand and generally easy to perform.

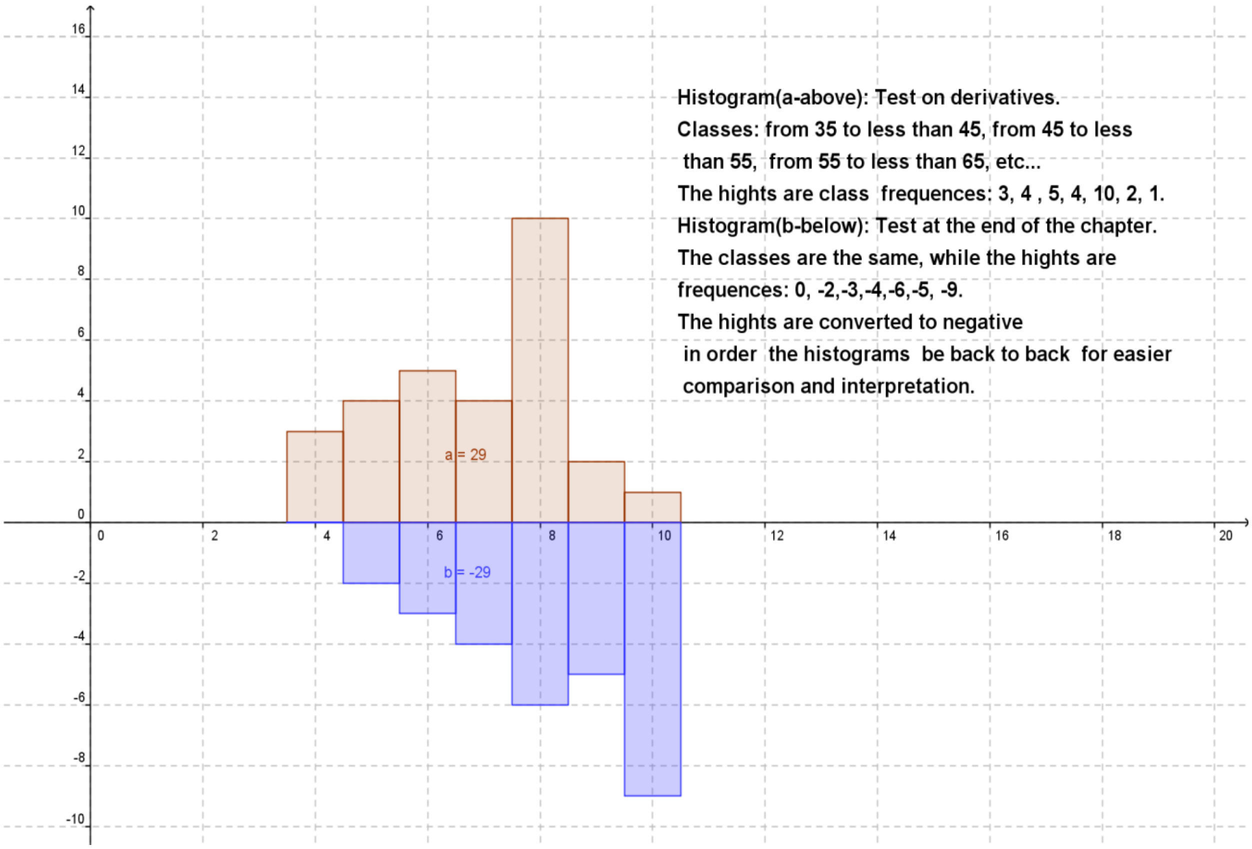


Fig 3.5. Bihistogram
(exported by GeoGebra applet)

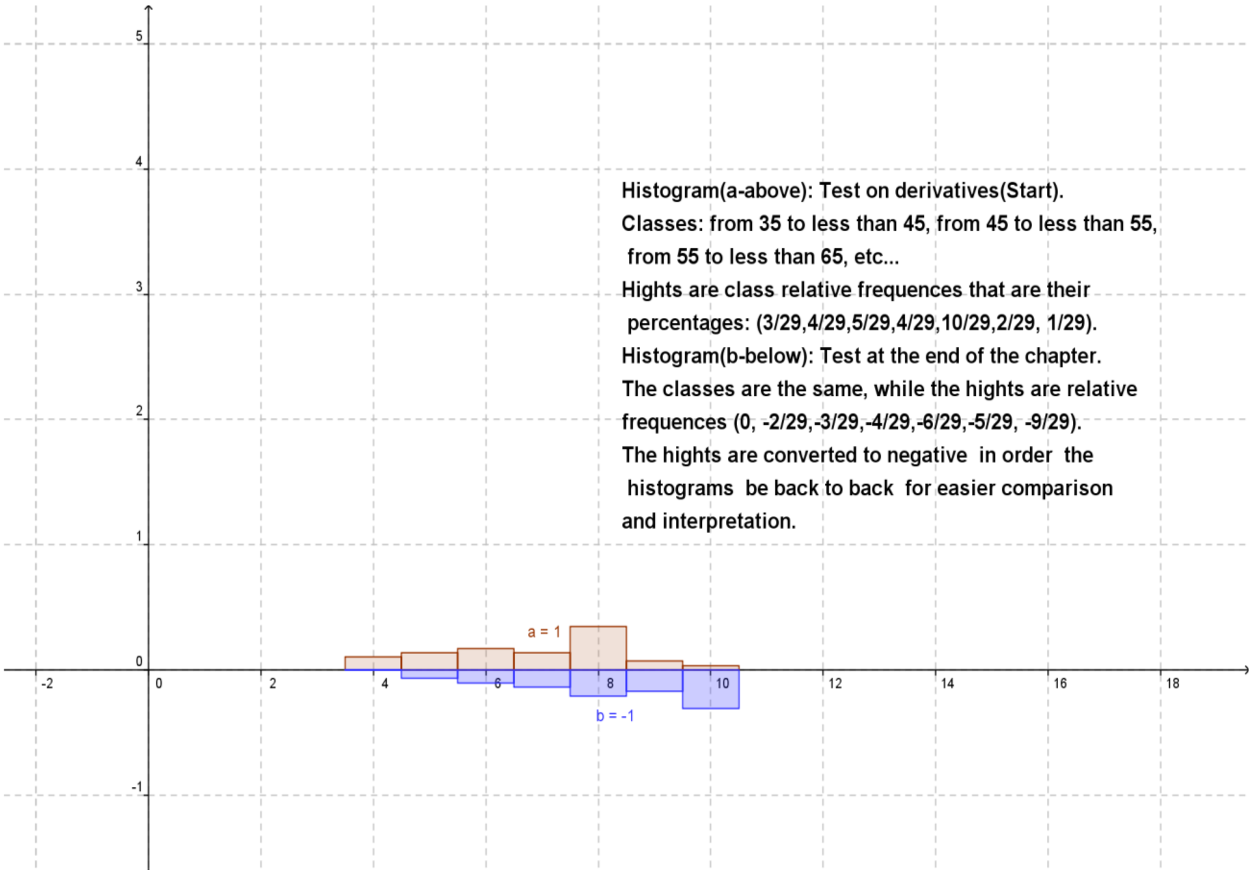


Fig 3.6. Percentages bihistogram
(exported by GeoGebra applet)

To compare the distributions of the two groups of values (e.g. measurements for the experimental group and control group) at the end of the chapter, histograms for the two groups were superimposed on the same axes. We got this way the so-called bihistogram. Color or shading are used to help distinguish the two histograms -- in ordinary black-and-white histograms it can be difficult to tell which lines belong to which histograms. In the Figure 3.7 below, the light rose histogram, above the horizontal axis, corresponds to the experimental group and the blue histogram, below the horizontal axis, corresponds to the control group. In the same picture we have displayed the box plot and the five summary numbers (two extremes, median and two quartiles).

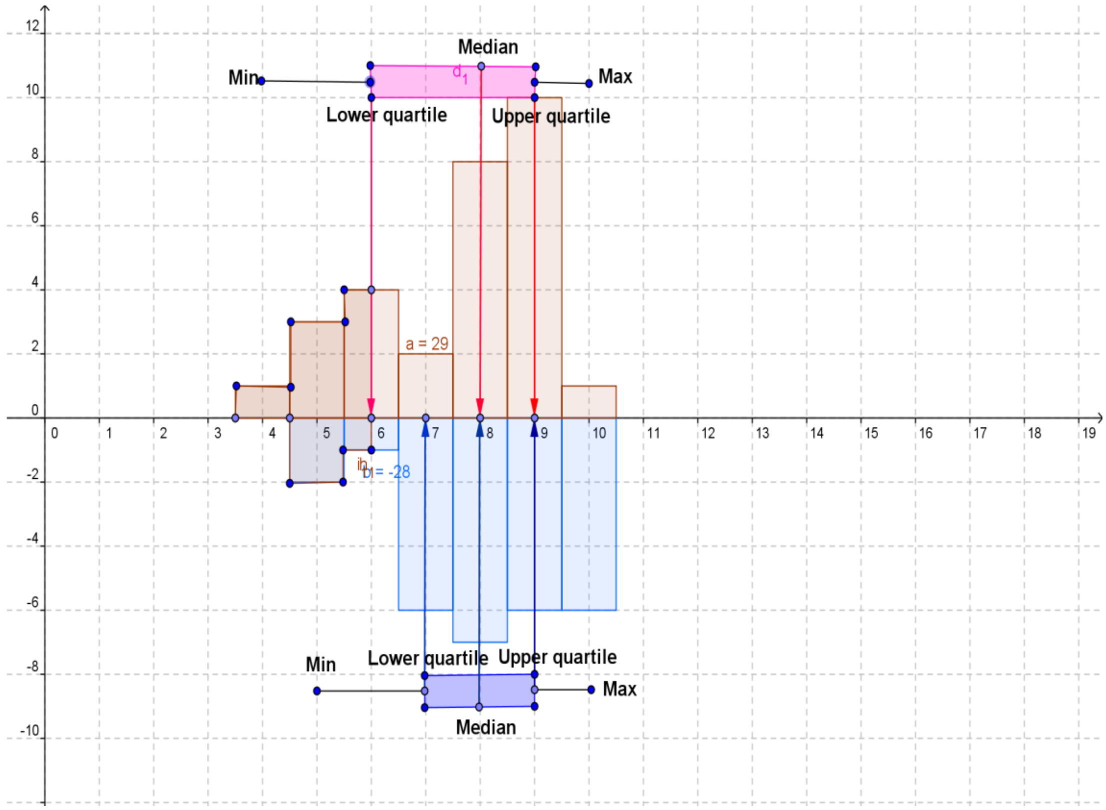


Figure 3.7 Histograms(bihistogram) and their respective box plots
(exported by GeoGebra applet)

Looking at the histogram above the horizontal axis in Figure 3.7, we see that:
Centre: the vertical line inside the box (the median) gives an indication of the centre of the distribution: the centre is 8.
Spread: the width of the box (the interquartile range) gives an indication of the spread of values in the distribution. It is 3.
Shape: the high density of the values is found in the part of histogram having the median and the upper quartile.

The histograms show that at the end of the chapter the difference between the two groups regarding the level in mathematics is reduced a lot, a confirmation of the positive influence of GeoGebra in the teaching and learning mathematics process. <u>The right extreme and quartile are closer to the median than the left extreme and quartile, this shows that the distribution is right skew and this fact is interpreted as a shift toward higher results in math.</u> Although the box plot of a single data set shows various useful aspects of the distribution of values, it is no more informative than a dot plot, stem and leaf plot or histogram. However box plots are often used when two or more sets of data are compared. The most important differences between the sets are usually precisely the aspects we are interested in.

Having into consideration the usual practice for defining the statistics by arranging data in increasing order(from the lowest value to the highest), ordering after which are defined the positions of median and quartiles, we arranged the rectangles of the histogram(green) in a vertical way. This is achieved by interchanging the positions of the rectangles of the histogram (green) with the respective rectangles above them in the accumulated histogram (light pink). The new positions of the rectangles of the histogram are those with blue color (look at Fig. 3.8). It is clear that these rectangles have only one vertex in common and the sum of their area is 29, equal to the area of the rightmost rectangle of the accumulated histogram. Arranging the histogram rectangles this way makes easy for us the defining of the statistics. The same thing was done for the control group but I didn'fing reasons to show here.
I employed another statistical tool, called "ruler of statistics", to measure the above statistics after the construction of the cumulative histogram.
The procedure for the construction of the "ruler of statistics" is as follows:

1. *Construct a segment of length 29 perpendicular to X – axis, (BC). This segment is used to measure the statistics, and I suggest to call it* ***"Ruler of statistics".***
2. *Divide it in four equal parts. The division points D, E, F correspond to upper quartile, median and lower quartile respectively. The division of the segment in equal parts is done by using GeoGebra tools and mathematical assertions.*
3. *In points C, D, E, F, B construct the verticals p_1, k_1, l_1, m_1,n_1, respectively(they are perpendicular to segment BC).*
4. *Define the intersections of these verticals with the middle height of the respective rectangle (blue). The respective blue rectangle of a vertical is that one which is intersected by the vertical. Is taken the middle height of the rectangle because it is known that the value of a class is the middle value(point) of that class.*
5. *From the intersection points construct the perpendiculars to X-axis. In Fig 3.8 they are represented by the arrows and for quartiles and the median only.*

*** The end points of the arrows on X-axis show the values of the quartiles and median which are: 9(upper), 8 and 6(lower). Also, it is easy to see that the extreme values are: 10(the highest) and 4(the lowest). From the picture can be understood that the three important statistics divide the set of ordered data into four equal parts. This property can be easily demonstrated by comparing the four parts in which the three verticals divide the right side rectangle of the accumulated histogram (it has a height of 29 units). We can construct polygons with sides the sides of each part and read the numbers corresponding to their areas (this is another feature of GeoGebra, showing in algebra window the algebraic representative of the geometrical object). These numbers are approximately equal.

In summary:

Experimental group five values
Min = 4
Q1 = 6
Med = 8
Q3 = 9
Max = 10

Control group five values
Min = 5
Q1 = 7
Med = 8
Q3 = 9
Max = 10

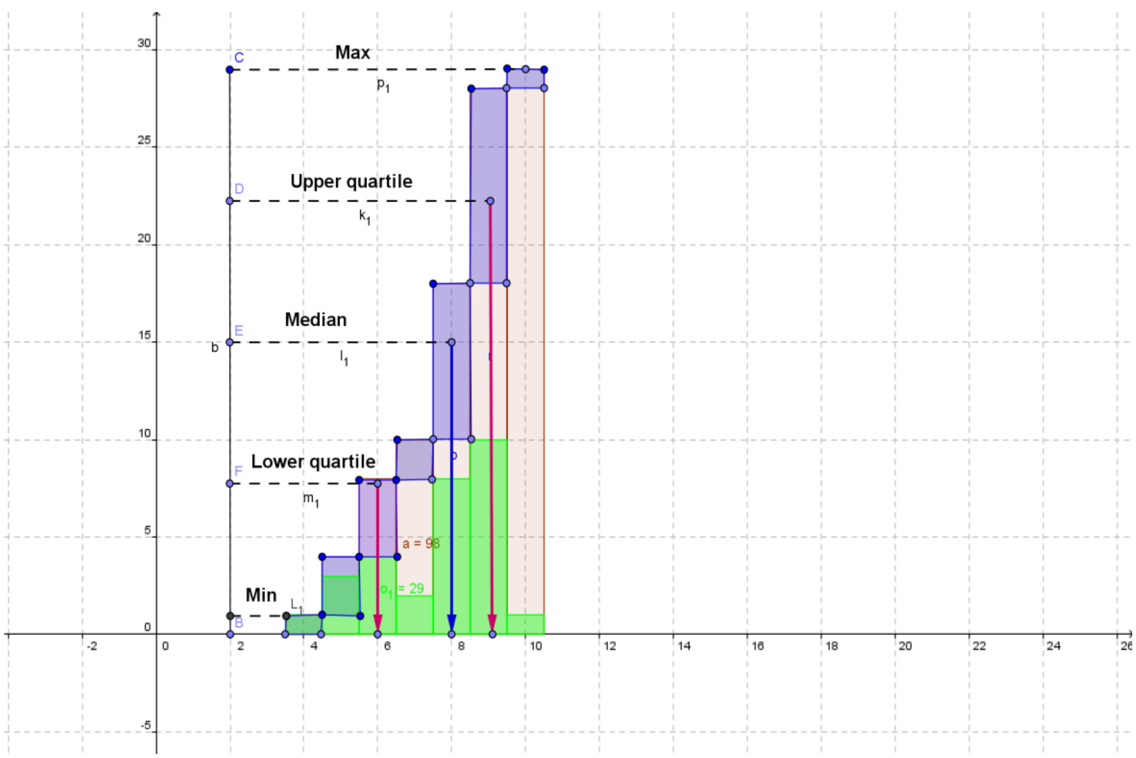


Figure 3.8 Histogram, accumulated histogram and the ruler of statistics
(exported by GeoGebra applet)

I want to emphasize here the advantages of using GeoGebra for geometrical representation and analysis of data and the interpretation:

1. GeoGebra software provides a quick construction of the histogram and a nice picture(different styles and colors)
2. GeoGebra provides a geometrical interpretation of the statistics that cannot easy be performed by other program.
3. The use of the ruler of statistics makes easier and faster the procedure for finding the central statistics, also is measured a sufficient accurate value of them.
4. The use of the ruler of statistics helps the students not make mistakes in reading the values of statistics in each case of the frequency distribution versus the method of using formulas for the grouped data. When there are gaps between the classes they can make mistakes in defining the values of statistics by using the usual formulas.
5. Teachers and students have the possibility to dynamically demonstrate and understand the relation between the frequencies and statistics.

Chapter 4

Statistical Comparison of the Two Data Sets of the Experimental Group

4.1 Introduction

A common form of scientific experimentation is the comparison of two groups. This comparison could be of two different treatments, the comparison of a treatment to a control, or a before and after comparison. My case is the comparison of the last type. The preliminary results of experiments that are designed to compare two groups are usually summarized into a means or scores for each group, in my case they are the points(marks) collected from the previous chapter and the points(marks) got by the students in the experimental chapter. My main interest is that after summarizing this data **compare the two sets of data of the experimental group:** compare the marks at the beginning of the chapter with the marks at the end of the chapter, chapter in which is used a new teaching and learning method in mathematics based on GeoGebra software. The comparison will show or prove if the observed differences between the two sets of data are real or just a chance difference caused by the natural variation within the measurements? A common way to approach that question is by performing a statistical analysis.

The two most widely used statistical techniques for comparing two groups, **where the measurements of the groups are *normally distributed*,** are the **Independent Group t-test and the Paired t-test**. What is the difference between these two tests and when should each be used?

For the normal distribution,

- approximately 68% of the scores in the sample fall within one standard deviation of the mean
- approximately 95% of the scores in the sample fall within two standard deviations of the mean
- approximately 99% of the scores in the sample fall within three standard deviations of the mean

Besides the normality assumption, another requirement of the Independent Group t-test is that the variances of the two groups be equal. That is, if we are to plot the observed data from each of the two groups, the resulting bell-shaped histograms would have approximately the same shape.
The above numerical and graphical characteristics show that the two samples (groups) under study are not of bell-shaped, they have not a real normal distribution.

As can be seen by the back to back histograms our data are not normally distributed, but they bare just a sample. On the other side, it is reasonable by the experience to consider that the set of data consisted of the marks from the population of students has a normal distribution. Comparing the means of Data of the test in the beginning of the chapter on Derivatives and of Data of the test at the end of the chapter on Derivatives (they are 7 and 8.24, respectively), it is seen that there is a difference. Our purpose is to study the effect of using GeoGebra software in teaching and learning process, investigate and determine if the treatment with GeoGebra software in teaching and learning process caused a change in the individuals' math knowledge and skills. We have to investigate and decide whether the mathematical course taught by using GeoGebra software is as effective as more traditional methods of instruction.

Our testing hypothesis is related to the means of two methods of instruction: Do two methods have the same mean?

In my case the subjects for the two groups are the same or matched. That is, the same subjects are observed twice: at the beginning of the chapter and at the end of it. The intervention taking place between the two measures is the use of GeoGebra software in teaching and learning mathematics.

In these conditions, <u>the commonly used type of t-test is the Paired t-test</u>. One advantage of using the same subjects is that experimental variability is less than the independent group case. For this test the mean difference between the two repeated observations is observed and compared. <u>If the difference is sufficiently great then there is evidence that the treatment (the new teaching and learning method) caused some change in the observed variable</u>. A paired t-test is performed and the observed difference between the groups is summarized in a p-value. The benefits of performing a t-test is that it is easy to understand and generally easy to perform. However, the fact that these tests are so widely used does not make them the correct analysis for all comparisons.

4.2 The basis of the paired t-test

In general given two random samples of measurements, $\boldsymbol{Y_1}$, ..., $\boldsymbol{Y_N}$ and $\boldsymbol{Z_1}$, ..., $\boldsymbol{Z_N}$ from two independent tests (the Y's are sampled from test 1 and the Z's are sampled from test 2), there are three types of questions regarding the true means linked with the two methods that are often asked.

1. Are the means from the two methods the same?

2 + (3). Is the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional method?

The question being addressed is whether the mean, μ_2, of the new method is greater than the mean, μ_1, for the traditional method.

We choose to test hypothesis 3 (H_0: μ_1> or equal to μ_2), in the hope that we will reject this null hypothesis and thereby feel we have a strong degree of confidence that the new method of using GeoGebra software is an improvement worth implementing. Based on data above we have:

The basic statistics for the test are the sample means. *Basic statistics from the two tests regarding the two teaching methods are:*

$$\bar{Y} = \frac{1}{N_1}\sum_{i=1}^{N_1} Y_i \qquad \bar{Z} = \frac{1}{N_2}\sum_{i=1}^{N_2} Z_i$$

In our case: $\bar{Y} = \frac{203}{29} = 7 \ \ and \ \bar{Z} = \frac{239}{29} = 8.24$, respectively.

Where as, the sample standard deviations

$$s_1 = \sqrt{\frac{\sum_{i=1}^{N_1}(Y_i - \bar{Y})^2}{N_1 - 1}}$$

$$s_2 = \sqrt{\frac{\sum_{i=1}^{N_2}(Z_i - \bar{Z})^2}{N_2 - 1}}$$

with degrees of freedom $\nu_1 = N_1 - 1$ and $\nu_2 = N_2 - 1$ respectively hence,

$Variance = \frac{\sum f(X_i)(X_i - \overline{X})^2}{\sum f(X_i) - 1} = \frac{80}{28} = 2.857 \Rightarrow S_1 = \sqrt{2.857} = 1.69$ and,

Variance $= \frac{\sum f(X_i)(X_i - \overline{X})^2}{\sum f(X_i) - 1} = \frac{73.3104}{28} = 2.6 \Rightarrow S_2 = \sqrt{2.6} = 1.62$, respectively.

We cannot prove that the standard deviations from the two methods are equivalent therefore, the test statistic is

$$t = \frac{\bar{Y} - \bar{Z}}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

$$\Rightarrow t = \frac{7 - 8.24}{\sqrt{\frac{2.857}{29} + \frac{2.6}{29}}} = \frac{-1.24}{\sqrt{0.188}} = -2.8585$$

The degrees of freedom are not known exactly but can be estimated using the Welch-Satterthwaite approximation:

$$\nu = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{s_1^4}{N_1^2(N_1 - 1)} + \frac{s_2^4}{N_2^2(N_2 - 1)}}$$

Hence, $\nu = \frac{0.188^2}{\frac{2.857^2}{28 \cdot 29^2} + \frac{2.6^2}{28 \cdot 29^2}} = \frac{0.0353}{0.0006337} = 56$

The strategy for testing the hypothesis 3 above is to calculate the appropriate t statistic from the formulas above, and then perform a test at significance level α, where α is chosen to be small; we choose it 0.05. The hypothesis associated with this case is rejected if $t \le -t_{\alpha;\nu}$.

Our test is an one-sided test at the 5% significance level, so we read at the t - table for 5 % significance level, looking up the critical value for degrees of freedom ν= 56. This critical value is 1.673. Consequently, hypothesis (3) is rejected because the test statistic (t = -2.8585) is smaller than 1.673 and,

therefore, we conclude the new method of using GeoGebra software in teaching and learning math has increased the level of math knowledge and skills over the traditional method used in teaching and learning process.

4.3 Test of differences

We can try another way to compare the new method with the traditional one by analyzing the paired observations (one done at the beginning of the chapter and the other at the end of the chapter). We have two random samples, $\boldsymbol{Y_1},\ldots,\boldsymbol{Y_N}$ and $\boldsymbol{Z_1}, \ldots, \boldsymbol{Z_N}$ consisted of the marks got at the beginning of the chapter and at its end. In our case, are made "before" and "after" measurements with the scale on *N* objects, so it is possible to decide if there is a difference between "before" and "after" measurement. Each "before" measurement is paired with the corresponding "after" measurement, and the differences

$d_i = Y_i - X_i \quad (i = 1, \cdots, N)$ are calculated. Basic statistics for the test are: the mean and the standard deviations for the differences calculated from the formulas

$\bar{d} = \frac{1}{N}\sum_{i=1}^{N} d_i$ and $s_d = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2}$, respectively and, with N – 1 degrees of freedom.

In the case of paired sample (t – *distribution)* a t-test is used to test for the difference of two means before and after the treatment and the test statistic is:

$$t = \frac{\bar{d}}{s_d / \sqrt{N}}$$

Our example is one-tailed test. Our purpose is to try out a new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process and carrying out two tests in a randomly chosen classroom of 29 students: one at the beginning of the chapter and the other at the end of the chapter. We *have* measured the level of the group in mathematics b*efore applying the new method*. At the end of the chapter, in which is used GeoGebra software, we have measured the level in mathematics again. We have to do a comparison with an average increase in the level of mathematics of this group in this chapter by using the traditional method. But, this is impossible because we have a state program for the schools that must be fulfilled and rigorously observed, so there is no room for repeating the chapter. For this reason, we use the average increase in the level of mathematics of the control group in this chapter where is used the traditional method. The question is: did the new method cause an increase in the level of mathematics? We use a significance level of 0.05, again.

Taking into consideration the tables at Appendices with systemized data for the experimental group we have the calculations for the differences of data as below.

Test on derivatives		**End of Chapter on Derivatives**		**$d_i = Z_i - Y_i$**	**$d_i - \bar{d}$**	**$(d_i - \bar{d})^2$**
Points	Mark(Y_i)	Points	Mark(Z_i)			
	$\bar{X}$ =**7**	**2383 (82)**	$\bar{X}$ = **8.24**	$\bar{d}$ = **1.24**		**Sum: 13.3104**

$$S^2 = \frac{\sum(d_i - \bar{d})^2}{N-1} = \frac{13.3104}{28} = 0.475$$

The standard deviation: **S = 0.69**

Whereas, for the control group the calculations for the differences of data are as in the following:

No.	**Previous chapter Mark**	**End of Chapter on Derivatives**	
	Y_i	**Z_i**	**$d_i = Z_i - Y_i$**
	$\overline{X} = 8.14$	$\overline{X}$ =**8.25**	$\overline{d_o}$ =**0.107**

This way**,** supposing that the average increase in the level of mathematics of the control group represents the average increase of the population of the students of the secondary schools in this chapter by using the traditional method, then it is known for us this quantity, which is 0.107. Therefore, we have:

null hypothesis: H_0: Δ ≤ 0.107

alternative hypothesis: H_a: Δ > 0.107

Δ represents the increase of the level in mathematics.
For the paired sample *t*-test the test statistic used to test for the difference of two means before and after a treatment is:

$$t = \frac{\overline{d} - \overline{d_o}}{S/\sqrt{N}} = \frac{1.24 - 0.107}{0.69/\sqrt{29}} = 8.84$$

In our case the problem has n – 1 = 28 degrees of freedom. The test is one-tailed test and we are concerned if there is increase only. From the t-table we read that: $t_{0.05,28}$ = 1.701. Means that, the computed *t*-value of 8.84 is larger than 1.701 therefore, the null hypothesis can be rejected.
Consequently, the test has provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes much more increase in

the level of knowledge and skills in mathematics than the traditional method used in this process.

4.4 Correlation: Correlation between the observed variables

Correlation between the variable linked with the marks at the end of the chapter and the variable linked with the marks in GeoGebra test.

The correlation is one of the most common and most useful statistics. A correlation is a single number that describes the degree of relationship between two variables. Let's work through with the results (marks) of the students got in the special test done for knowing their level regarding GeoGebra software and skills and later, after the pre-preparation work in the table, show how this statistic is computed.

In the Table 3.12 at appendices is made the data up to illustrate the meaning of correlation and compute it. X is variable of the students' results in GeoGebra test, Y is the variable of the students result at the end of the chapter.

The formula for calculating the correlation is:

$$r = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[N\Sigma x^2 - (\Sigma x)^2][N\Sigma y^2 - (\Sigma y)^2]}}$$

Where:

N	=	number of pairs of scores
Σxy	=	sum of the products of paired scores
Σx	=	sum of x scores
Σy	=	sum of y scores
Σx^2	=	sum of squared x scores
Σy^2	=	sum of squared y scores

The symbol **r** stands for the correlation. The values of r will always be between -1 and +1. If the correlation is negative, there is a negative relationship; if it is positive, the relationship is positive.

Plugging the values calculated in the respective table into the formula given above, we get the following:

$$r = \frac{29 \cdot 2079 - 244 \cdot 239}{\sqrt{(29 \cdot 2124 - 244^2) \cdot (29 \cdot 2143 - 239^2)}} = \frac{1975}{3218} = 0.61$$

For our problem the correlation is positive, meaning that the increased level of knowledge and skills in GeoGebra is accompanied with an increase of the level of knowledge and skills in mathematics. The value calculated shows that there is correlation. It is not perfect, however it is considered a strong positive correlation. Closer to the value 1 be it, much stronger is the correlation between the two variables. Myself, I believe that there is a relationship between the computer programs for math (Geogebra software also) and mathematics in regard with mastering them. Using the computer programs the students will not be delved in long time performing calculations, solving equations or systems of equations and plotting graphs, or other algorithms.

"*The computer does all the tedious work which leaves the teacher and the pupils with enough time to discuss the problem, try out multiple ideas and approaches to solving and, finally compare and analyze them. The students are freed from uninspiring and time-consuming solving by hand, so they have more time to learn the important points*"(Dragoslav Herceg, Dorote Herceg, TMCS 6(2008)2, Pg. 377).

The students will benefit of this programs too much, releasing the valuable time for analytical and logical proofs and interpretations. The students have much more available time to explore new or old topics and subjects of mathematics.

4.5 Conjectures and Conclusions on the results' tests

Our testing hypothesis is related to the means of two methods of instruction: Do two methods have the same mean?

In my case the subjects for the two groups are the same or matched. That is, the same subjects are observed twice: at the beginning of the chapter and at the end of it. The intervention taking place between the two measures is the use of GeoGebra software in teaching and learning mathematics.

1. Comparing the histograms of the two groups at the end of the chapter is seen that the right extreme and quartile are closer to the median than the left extreme and quartile for both of them. For the experimental group the comparison of the histogram at the end of the chapter with the one at the beginning shows that the distribution is more right skew at the end of the chapter, and this fact is interpreted as a shift toward higher results in math.
2. The comparison of the main statistics at the end of the chapter for the two groups shows that there is a little difference between them:

Experimental group five values (Min = 4, Q1 = 6, Med = 8, Q3 = 9, Max = 10)

Control group five values (Min = 5, Q1 = 7, Med = 8, Q3 = 9, Max = 10)

3. In our first test hypothesis (3): Is the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional method? is rejected because the test statistic ($t = -2.8585$) is smaller than 1.673(the critic value), therefore, we conclude

   that the new method of <u>use of GeoGebra software in teaching and learning math has increased the level of math knowledge and skills over the traditional method</u> used in teaching and learning process.
4. In the test of differences where each "before" measurement is paired with the corresponding "after" measurement, we achieved the same conclusion. <u>*The test has provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes increase in the level of knowledge and skills in mathematics.*</u>
5. Regarding the correlation, for our problem the correlation is positive, meaning that <u>the increased level of knowledge and skills in GeoGebra is associatied with an increase of the level of knowledge and skills in mathematics</u>. The value calculated shows that there is correlation. It is not perfect, however it is considered a strong positive correlation. Closer to the value 1 be it, much stronger is the correlation between the two variables. The three test performed are a confirmation to one another. It counts that GeoGebra be used in the teaching and learning process.
6. Comparing the stem and leaf plots (the first and the third) it is clear that there is a shift toward higher marks within the experimental group.
7. Another indication of progress for the experimental class is the means. In the beginning of the experiment they were: The computed means for the experimental group, $\overline{X}$ = 7.6, for the control group…$\overline{X}$ = 8.14.

   At the end of the chapter they were: The computed means are for the experimental group….$\overline{X}$ = 8.24, for the control group…$\overline{X}$ = 8.25.

   Almost equal!!!
8. Criterion that a group of data has a the normal distribution is,

   - approximately 68% of the scores in the sample fall within one standard deviation of the mean

- approximately 95% of the scores in the sample fall within two standard deviations of the mean
- approximately 99% of the scores in the sample fall within three standard deviations of the mean.

Based on the calculations done for the standard deviations of all the tests done for the two groups resulted that none of them met the above conditions.

All the results got are a confirmation that data under study have not a normal distribution.

Taking the standard deviations of data from the previous chapter ready from the respective tables above, we have:

for the experimental group the Standard Deviation is:

$$\text{Std. Deviation} = \sqrt{2.53} = 1.59,$$

for the control group the Standard Deviation is:

$$\text{Std. Deviation} = \sqrt{2.13} = 1.46$$

This is an indication that there is a little difference between them. This fact confirms again that to do good and right interpretation and a good comparison of the two groups we have to take into consideration all statistics describing the data under study.

Based on the five-number summary and in the analysis of the bihistogram with the box plots done earlier the conclusion is: at the initial stage or state, the control group appears to be better than the experimental group.
The difference between the average marks of control group and experimental group was approximately 1 unit (mark). The influence of GeoGebra is so obvious.

Chapter 5
Repetition of Experiment

(REPETITION OF GEOGEBRA EXPERIMENT FOR TEACHING AND LEARNING MATHEMATICS)

School year 2010 – 2011, ALBANIA

Three classes are involved in the experimental study: Class X – A (Secondary school "Dhaskal Todri", Elbasan – teacher Shpetim Sulanjaku); Class X-A (Secondary school "Jani Kilica" , Fier – teacher Lefter Leka); Class X-B (The General Secondary School, Librazhd, teacher: Luljeta Blloshmi). One class is the control class: Class X – B (Secondary school "Dhaskal Todri", Elbasan - teacher Shpetim Sulanjaku). The four classes are considered groups and labeled respectively A, B, C and D (D – the control class)

5.1 Criterions for the selection of the classes and preliminary work with data

1. The selection of the classes is based on the known relationships between the experimenter (Pellumb Kllogjeri) and the teachers and the availability and willingness of the teachers to be involved with the experiment.
2. The experiment is performed in the same chapter "Systems of equations and inequations". This chapter is taught in the second year of the middle school.
3. The classes have not much difference in their means regarding the quality in math.

The classes have totally different backgrounds, coming from different towns (Elbasan, Fier and Librazhd). The control class is from Elbasan, as well. They have different teachers, experimenting for the first time – teaching with GeoGebra. The groups are independent from one another.

Because our task is to compare the means of several groups (four) and get conclusion about which teaching and learning method is better we perform ANOVA test. One-way **AN**alysis **O**f **VA**riance (**ANOVA**) is used when we want to compare more than two means. It is a technique that generalizes the two-sample *t* procedure which compares two means to a situation with more than two sample means. The statistic corresponding to ANOVA test is F-statistic (Fisher statistic) defined by the ratio $F = \frac{MS_B}{MS_W}$, where the nominator is the between-groups mean square, whereas the denominator is the within-groups mean square. More precise they are defined below the following table.

The preliminary work having all is needed to perform the test is in the following table, containing all the by hand computations. Next are the other figures necessary for the estimation of F-statistic.

Summary table: of the sizes of the sums, of the sum of squares for each group and the totals.

A	B	C [	D	All groups combined
$N_A=36$	$N_B=29$	$N_C=29$	$N_D=38$	$N_T=132$
$\Sigma X_{Ai}=291$	$\Sigma X_{Bi}=223$	$\Sigma X_{Ci}=218$	$\Sigma X_{Di}=276$	$\Sigma X_{Ti}=1007$
$\Sigma X^2_{Ai} = 2473$	$\Sigma X^2_{Bi} = 1793$	$\Sigma X^2_{Ci}= 1639$	$\Sigma X^2_{Di} =2080$	$\Sigma X^2_{Ti}= 7985$
$\overline{X_A}=8.08$	$\overline{X_B}=7.69$	$\overline{X_C}=7.52$	$\overline{X_D}=7.26$	

SS_{BG} - Sum of Squares Between Groups

$$SS_{BG} = \frac{(\Sigma X_{Ai})^2}{N_A} + \frac{(\Sigma X_{Bi})^2}{N_B} + \frac{(\Sigma X_{Ci})^2}{N_C} + \frac{(\Sigma X_{Di})^2}{N_D} - \frac{(\Sigma X_{Ti})^2}{N_T}$$

$$= \frac{(291)^2}{36} + \frac{(223)^2}{29} + \frac{(218)^2}{29} + \frac{(276)^2}{38} - \frac{(1007)^2}{132}$$

$$= 28.27$$

SS_{WG} - Sum of Squares Within Groups

$$SS_{WG} = \sum X_{Ai}{}^2 - \frac{(\sum X_{Ai})^2}{N_A} + \sum X_{Bi}{}^2 - \frac{(\sum X_{Bi})^2}{N_B} + \sum X_{Ci}{}^2 - \frac{(\sum X_{Ci})^2}{N_C} + \sum X_{Di}{}^2 - \frac{(\sum X_{Di})^2}{N_D} =$$

$$= 2473 - \frac{291^2}{36} + 1793 - \frac{223^2}{29} + 1639 - \frac{218^2}{29} + 2080 - \frac{276^2}{38} = 274.54$$

$$MS_B = \frac{SS_B}{df_B} = \frac{28.27}{3} = 9.423333$$

$$MS_W = \frac{SS_W}{df_W} = \frac{274.54}{128} = 2.1478$$

$$F = \frac{MS_B}{MS_W} = \frac{9.423333}{2.1478} = 4.39$$

The summary table is:

	SS	**df**	**MS**	**F**
Between groups(B)	**28.27**	**3**	**9.423333**	**4.39**
Within groups(W)	**274.54**	**128**	**2.1478**	
Total	**302.80**	**131**		

Performing the F test: One-way test (ANOVA) is used to test whether the means of all groups are equal. In the ANOVA test, samples are drawn from each population and the data is used to test the null hypothesis that the populations are all equal against the alternative that not all are equal. If we reject the null, we need to perform some further analysis to draw conclusions about which population means do differ.

5.2 Assumptions of the ANOVA and the test of null hypothesis

1. The data is normally distributed.
2. The population standard deviations are equal.

In our experiment, the random variable representing the marks has approximately a normal distribution. It is confirmed by many statistical studies and this fact is considered in many text books. In many problems, the population standard deviations are considered equal, but what we do with our experiment?

Recall: Our second assumption in the ANOVA model was that our population standard deviations are all equal. The official test is quite complicated and not practical, also statistical official data do not help, so we use the following rule of thumb:

If the largest standard deviation is less than twice the smallest standard deviation, we can use methods based on the assumption of equal standard deviations and our results will still be approximately correct. So we compare 2 x smallest std. dev to the largest std. dev.---we want 2 x smallest std. dev > largest std. dev.

The two hypotheses tested by ANOVA procedure are:
$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4,$ all the means are the same.

H_a: Not all the sample means are equal (at least one is different).

I have tested these hypotheses at three significance levels: ɑ = 0.050, ɑ = 0.025 and ɑ = 0.010. The tabled values of $F = \frac{MS_B}{MS_W}$, in accordance

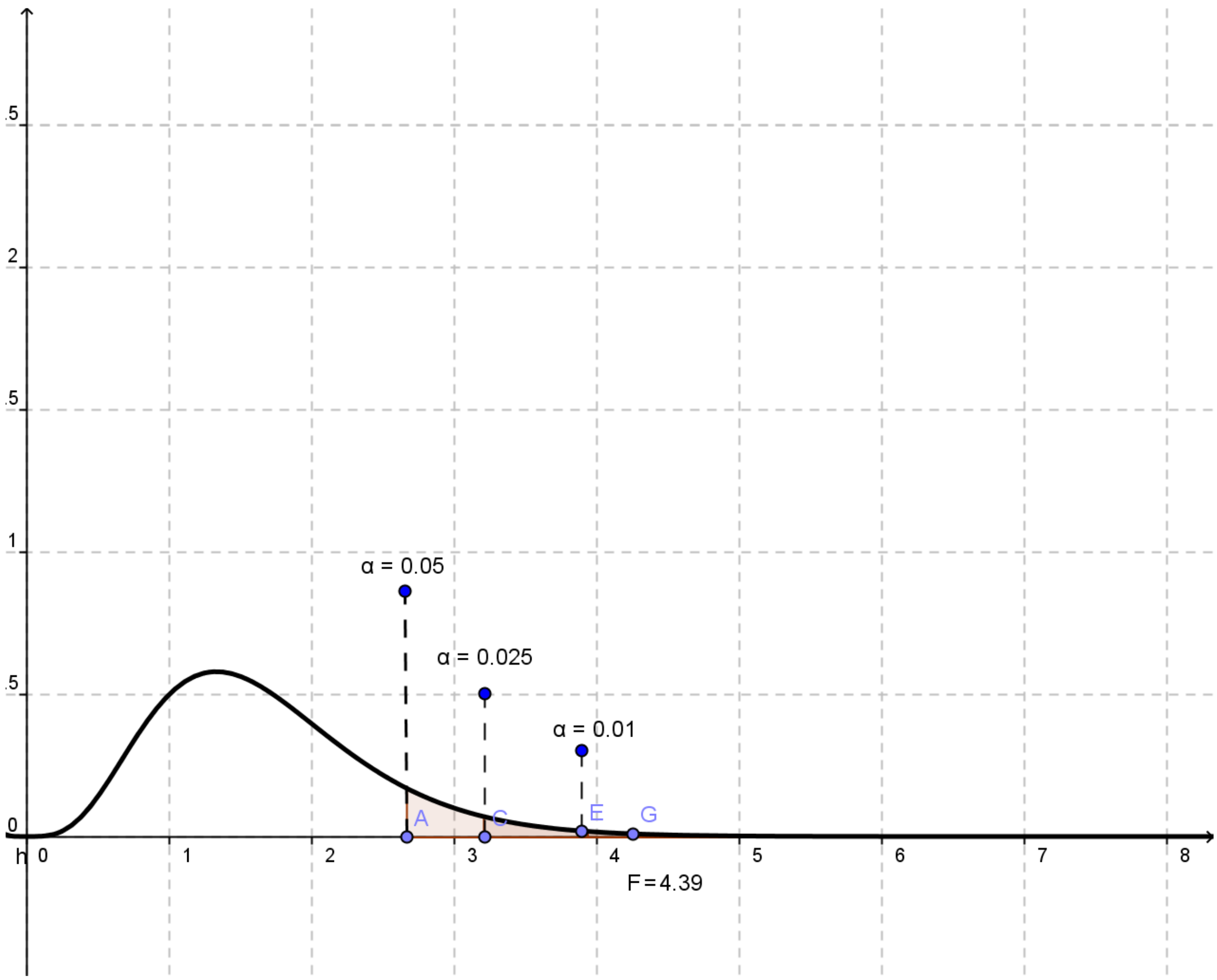
with the degrees of freedom of nominator and denominator of the ratio, taken out from the respective tables are:

$$F(3,128) = \begin{cases} 2.67 & for \;\; \alpha = 0.050 \\ 3.21 & for \;\; \alpha = 0.025 \\ 3.94 & for \;\; \alpha = 0.010 \end{cases}$$

In the experiment carried out, the estimated value of F, called an F statistic, is 4.39 (look at the summary table, above).

It tells us how much more variability there is between treatment groups than within treatment groups. The larger that ratio, the more confident we feel in rejecting the null hypothesis, which is that all means are equal and meaning that there is no effect. As can be seen by the figure of the probability density function for F, the estimated value of F falls in the three areas of rejection of H_0 for the three levels of ɑ.

Therefore we **reject H_0 and accept H_a**, concluding that **the means of the four groups are not equal**.



(Exported by GeoGebra applet)

5.3 Comparing the Means and Interpretation

Although the ANOVA F test may be significant, (i.e. we reject H_0) it does not tell us specifically which means differ from each other. We can look at the difference graphically or by formal inference. We use the method of:

Simultaneous Confidence Intervals for Differences between Means

This method is used **ONLY AFTER the rejection of H_0** with the F test. All combinations of means are compared. Knowing that there are differences between the means, we naturally want to know which means are significantly different. This is **post-hoc analysis**. One of the post-hoc analyses, which is the most common choice, is the HSD (Honestly Significant Difference) test of Tukey. Shortly, the HSD test is performed in the following way: is computed something analogous to a t-score for each pair of means, but they are not compared to the Student's t distribution. Instead, is used a new distribution called the **studentized range** or **q distribution**.

Caution: The post-hoc analysis is performed only if the ANOVA test shows a p-value less than chosen α. The p-value corresponding to the estimated F is less than each chosen α. If p>α, we don't know whether the means are all equal or not, so we cannot be sure for unequal means. In our case, using the p-value calculator for the Fisher F-test is found out that the p-value = 0.005628 which is much, much less than each chosen α. (Look at http://www.danielsoper.com/statcalc/calc07.aspx).

We want to know not just which means differ, but by how much they differ in order to see the effect size**.** The easiest thing is to compute the confidence interval first, and then interpret it for a significant difference in means. It is known that the relationship between a test of significance at α level and a 1−α confidence interval is interpreted as follows:

- **If the endpoints of the CI have the same sign** (they are both, positive or both negative), then 0 is not in the interval and we can conclude that **the means are different.**
- **If the endpoints of the CI have opposite signs, then** 0 is in the interval but we **can't determine whether the means are equal or different**.

The confidence interval can be computed similarly to the confidence interval for the difference of two means, but using the q distribution which avoids the problem of inflating α: because **testing multiple hypotheses increases α dramatically.** Even with just three treatments, the effective α is almost three times the nominal α and this is unacceptable. On the other side we cannot lower α for its decrease is associated with increase of β, which is chance of a Type II error. β represents the probability of a **false negative**, failing to find a difference in our means when there actually is a difference. This, too, is unacceptable.

To test all the pairs of means at the same time, in one test we extend the t test to multiple samples, and that is called ANOVA. The confidence interval for each difference of paired means is:

$$\bar{x}_i - \bar{x}_j \pm q(\alpha, r, df_w)\sqrt{\frac{MS_w}{2}\cdot\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$$

where $\bar{x}_i$ and $\bar{x}_j$ are the two sample means, n_i and n_j are the two sample sizes, MS_W is the within-groups mean square from ANOVA table, and q is the **critical value** of the studentized range for α, the number of treatments or samples *r*, and the within-groups degrees of freedom df_W. The square-root term is called the **standardized error**.

The studentized range, developed by Tukey, overcomes the problem of inflating significance level (Look at ENGINEERING STATISTICS HANDBOOK, NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook/, date. CHAPTER 7: Product and Process Comparisons, 7.4.3.5. Confidence intervals for the difference of treatment means).*

The assumptions of Tukey's test:

1. The observations being tested MUST BE INDEPENDENT
2. The means come from normally distributed populations
3. Observations have almost equal variations

These assumptions in our case are met. The random variable representing the marks has approximately a normal distribution. It is confirmed by many statistical studies and this fact is considered in many text books. The variances differ slightly from one another. The value of q is function of the number of treatments, of the total number of data points and α level.

The estimation for the differences of the means and their respective confidence intervals is as in the following table:

	$\bar{x}_i - \bar{x}_j$	Critical q $q(\alpha,r,df_W)$	Standardized error	95% Conf. Interval for $\mu_i - \mu_j$		Signif. at 0.05?
A – B	0.39	3.6805	0.258	−0.56	1.34	
A – C	0.56	3.6805	0.258	−0.39	1.51	
A – D	0.82	3.6805	0.2406	−0.055	1.705	**YES**
B – C	0.17	3.6805	0.272	- 0.83	1.170	
B – D	0.43	3.6805	0.255	- 0.51	1.370	
C – D	0.26	3.6805	0.255	- 0.673	1.20	

Explanation about the table:

1. The first column shows **which group means are being compared.**
2. The next column gives the **point estimate of difference**, which is the difference of the two sample means. The sample means of A and B are 8.08 and 7.69, so their difference is 0.39 and so on.
3. Third column belongs to **critical q**. Looking at formula is understood that q (α,r,df_W) depends on the number of treatments and total number of data points, not on the individual treatments, so it's the same for all rows in any given experiment. In the experiment carried out in Albania regarding the effect of GeoGebra in teaching and learning math, there are four groups. Choosing $\alpha = 0.05$, we find on the table of critical values for the studentized range that $q(0.05,4,129) = 3.6805$.

4.Fourth column contains the **standardized error** given from Tukey's formula for confidence interval $\sqrt{\frac{MS_w}{2}\cdot\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}$.

In the experiment we are talking, the sample sizes are unequal so, the standardized error varies for comparing different pairs of groups. For the first difference, A – B, we have:

$\sqrt{[(MS_W/2)\cdot(1/N_A+1/N_B)]} = \sqrt{[(2.1478/2)\cdot(1/36 + 1/29)]} = 0.258$ and, so on…

5. Fifth column contains the two endpoints of the confidence interval computed for each difference by the formula,

$$\bar{x}_i - \bar{x}_j \pm q(\alpha, r, df_w)\sqrt{\frac{MS_w}{2}\cdot\left(\frac{1}{n_i}+\frac{1}{n_j}\right)}.$$

6. The last column applies to the relation between confidence interval and significance test in order to see whether there's a significant difference between the two groups. If the confidence interval includes the value 0, then that pair of means will not be declared significantly different, and vice versa.
   Looking at the difference of the means of groups A and D, that is A – D , the left endpoint of the interval is almost 0. Consequently, we don't make a big mistake saying that the endpoints of the confidence interval are both positive. This means that 0 is not in this interval and ***we reject the null hypothesis*** of equality of the respective means (by noting YES in the respective row of that difference). In this table, only groups A and D have a significant difference.

Interpretation: The means of the groups A and D (the respective classes) are not equal. Moreover, the mean of the experimental class is greater than that of the control class and <u>we are 95% confident that teaching with GeoGebra gives higher results than the teaching of the traditional way</u>.
The confidence intervals of the other differences go from a negative to a positive, so they do include zero. That means that the two respective means might be equal or different, so we can't say whether there is a difference between them. However, the interval center of each one is a positive number (0.9 approx.), leading us to say that there are differences. For each pair of the groups the tendency is the same. The effectiveness of the method ˝teaching

with GeoGebra ˝ is easily obvious when compare group A with group D. But, it is not so when we compare the groups B, C and D. **I believe that one of the causes is the lack of experience of the teachers with GeoGebra software.** The first experimental class (group A) is from Elbasan, a city in which there is a 3-year experience using GeoGebra: training with teachers and diploma themes for the students of Elbasan University. The other two experimental classes are from schools of other towns where Geogebra was introduced and used for the first time. Teachers themselves of these schools have faced difficulties in teaching with GeoGebra. Even of such difficulties the respective experimental classes have higher results (higher means) than that of the control class.

5.4 Summary of the two experiments, Problems and suggestions

Year 2010, February

The experiment was carried out in Elbasan, in the secondary school "Dhaskal Todri". <u>The scientific experimentation was the comparison of two groups. This comparison could be of two different treatments, the comparison of a new treatment to a control, or a before and after comparison</u>.

Were selected two classes for the experiment. The experimental class was taught by me (the first chapter on Derivatives using GeoGebra and geometrically and visually demonstrating the concepts and properties of monotonous functions, extreme values, the mean value theorem etc). <u>The reason, why this class was taught by me, was because there was no math teacher able to teach using GeoGebra</u>. The other class (again of third year) was part of the experiment for comparing the results between the two classes where this chapter was taught in the traditional way by another teacher. The last class served as control group. The control group appeared better than the experimental group at the beginning of the chapter. Its mean in the previous chapter was a little higher. In **the first way** for comparing the two groups at the end of the chapter we used the main statistics like, the mean and median of the two groups, also displaying their results(scores) with bihistogram and using box-plot. At the end of the chapter the experimental group showed to be better than the control one.

This was the first evidence that teaching with GeoGebra is more effective than the traditional method of teaching.

Another way to compare the new method with the traditional one was by analyzing the paired observations (one done at the beginning of the chapter and the other at the end of the chapter).

After summarizing the data into a means or scores for each group of results, which were the points(marks) collected from the previous chapter and the points(marks) got by the students in the experimental chapter, were **compared the two sets of data of the experimental group:** compared the marks at the beginning of the chapter with the marks at the end of the chapter (chapter in which was used a new teaching and learning method in mathematics based on GeoGebra software).

The assumption was that the set of data, consisted of the marks from the population of students, had a normal distribution (this was reasonable and acceptable by the experience regarding the distribution of the marks in the population of students). A paired t-test was performed and the observed difference between the groups was summarized in a p-value.

Three types of questions regarding the true means linked with the two methods of teaching were:

1. Were the means from the two methods the same?

2 (3). Was the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional method?

These were "before" and "after" measurements with the scale on *N* objects, and the experiment was performed to decide if there was a difference between "before" and "after" measurement. The technique: each "before" measurement was paired with the corresponding "after" measurement, and the differences

$d_i = Y_i - X_i \quad (i = 1, \cdots, N)$ are calculated.

The idea is to do a comparison with an average increase in the level of mathematics of this group and in the same chapter by using the traditional method one time and by using the GeoGebra teaching the second time. But, this is impossible because we have a state program for the schools that must be fulfilled and rigorously observed, so there was no room for repeating the chapter. For this reason, we used the average increase in the level of mathematics of the control group in this chapter where was used the traditional method. The paired sample *t*-test, the test statistic used to test for the difference of two means before and after a treatment, *provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes much more increase in the level of knowledge and skills in mathematics than the traditional method used in this process.*

In the first test hypothesis "Was the mean of marks got by method of using GeoGebra software less (greater) than the mean of the marks got by traditional method?" was concluded that the new method of using GeoGebra software in teaching and learning math increases the level of math knowledge and skills over the traditional method used in teaching and learning process.

In the test of differences where each "before" measurement was paired with the corresponding "after" measurement, was achieved the same conclusion. *The test provided evidence that the new teaching and learning method in mathematics based on GeoGebra software by using this software in teaching and learning process causes much more increase in the level of knowledge and skills in mathematics than the traditional method used in this process.*

Year: 2011, March(repeated experiment)

The main reasons of repeating the experiment were:

1. In the first experiment the teacher of the experimental class was the experimenter as well.
2. The experiment was based in two classes only, so there was not sufficient evidence of drawing right and trusted conclusions.

Criterions for the selection of the classes:

1. The selection of the classes is based on the known relationships between the experimenter (Pellumb Kllogjeri) and the teachers and the availability and willingness of the teachers to be involved with the experiment.

2. The experimenter is not the teacher of the classes
3. In the experiment are involved four classes.
4. The experiment is performed in the same chapter “Systems of equations and inequations”. This chapter is taught in the second year of the middle school.
5. The classes have not much difference in their means regarding the quality in math.
6. The classes have totally different backgrounds, coming from different towns (Elbasan, Fier and Librazhd). The control class is from Elbasan, as well. They have different teachers, experimenting for the first time – teaching with GeoGebra. The groups are independent from one another. The experiment range is wider.

Because our task was to compare the means of several groups (four) and get conclusion about which teaching and learning method is better we performed ANOVA test. One-way **AN**alysis **O**f **VA**riance (**ANOVA**) is used when we want to compare more than two means.

The statistic corresponding to ANOVA test is **F-statistic** (Fisher statistic) defined by the ratio $F = \frac{MS_B}{MS_W}$, where the nominator is the between-groups mean square, whereas the denominator is the within-groups mean square. In the ANOVA test, samples are drawn from each population and the data is used to test the null hypothesis that the populations are all equal against the alternative that not all are equal.

Assumptions of the ANOVA:

1. The data is normally distributed.
2. The population standard deviations are equal.

In the experiment carried out during this year (March), the estimated value of F, called an F statistic which tells us how much more variability there is between treatment groups than within treatment groups, was in favor of Geogebra teaching. The larger that ratio, the more confident we feel in rejecting the null hypothesis, which is that all means are equal and meaning that there is no effect. <u>The test showed that the estimated value of F fell in the three areas of rejection of H_0 for the three levels of α that were purposely chosen(look at figure above).</u>

Although the ANOVA F test may be significant, (i.e. we reject H_0) it does not tell us specifically which means differ from each other. So, we looked at their differences using the method of of **Simultaneous Confidence Intervals for Differences between Means** which is used **ONLY AFTER the rejection of H_0** with the F test. All combinations of means are compared. We want to know which means are significantly different. This is **post-hoc analysis** and, the most common choice, is the HSD (Honestly Significant Difference) test of Tukey.

The assumptions of Tukey's test:

1. The observations being tested MUST BE INDEPENDENT
2. The means come from normally distributed populations
3. Observations have almost equal variations

These assumptions in our case were met. The random variable representing the marks has approximately a normal distribution. The variances differ slightly from one another. The value of q is function of the number of treatments, of the total number of data points and α level. Looking at the difference of the means of groups A and D, that is A – D (D is the control class), the left endpoint of the interval was almost 0. Consequently, we don't make a big mistake saying that the endpoints of the confidence interval are both positive. This means that 0 is not in this interval and ***we reject the null hypothesis*** of equality of the respective means. In this table, only groups A and D had a significant difference.

Interpretation: The means of the groups A and D (the respective classes) are not equal. Moreover, the mean of the experimental class is greater than that of the control class and <u>we are 95% confident that teaching with GeoGebra gives higher results than the teaching of the traditional way</u>.

The confidence intervals of the other differences went from a negative to a positive, so they did include zero. That means that the two respective means might be equal or different, so we couldn't say whether there was a difference between them. However, the interval center of each one is a positive number (0.9 approx.), leading us to say that <u>there were differences</u>. For each pair of the groups the tendency was the same. The effectiveness of the ˝teaching with GeoGebra ˝ method was easily obvious when compared group A with group D. But, it was not so when we compared the groups B, C and D. **I believe that one of the causes is the lack of experience of the teachers with GeoGebra software.** The first experimental class (group A) is from Elbasan, a city in which there is a 3-year experience using GeoGebra: training with teachers and diploma themes for the students of Elbasan University. The other two experimental classes are from schools of other towns where GeoGebra was introduced and used for the first time. Teachers themselves of these schools have faced difficulties in teaching with GeoGebra. Even of such difficulties the respective experimental classes have higher results (higher means) than that of the control class.

Problems, lessons and suggestions

**** The first problem regarding the results and inferences of the first experiment was the teacher of the experimental class (who was me). I was the experimenter and the teacher, so there are strong reasons of no believing in the results and the inferences got at the end of the experiment. It is right to think that, in getting conclusions is not missing subjectivism.*

**** Another questionable topic is: if there is a good positive difference between the results at the end of a chapter and the results at the end of the previous chapter, is this an evidence that the improving scores are result of the new teaching and learning method?? My opinion is that the conclusions about the new method not be depended on this kind of comparison (by comparing the scores in different chapters).*

**** The case of making comparison with an average increase in the level of mathematics of a class in the same chapter by using the traditional method one time and by using the GeoGebra teaching the second time cannot happen. T<u>his is impossible because we have a state program for the schools that must be fulfilled and rigorously observed, so there is no room for repeating the chapter</u>. The other problem is that by repeating a second time the chapter it is expected and believed that the results must be higher(the results are correlated with the repetition process) . For this reason, we used the average increase in the level of mathematics of the experimental group where was used "teaching with GeoGebra" and of the control group (in the same chapter) where was used the traditional method.*

**** The second (repeated) experiment showed again that the <u>training of the math teachers with Geogebra</u> is very important for the implementation of "teaching and learning with GeoGebra" method in the teaching and learning* *process and, to draw right inferences about the experiment. The new method of teaching was based in one class only, in the first try, because the only person in Albania who was able to do such teaching was me. The need of training the teachers showed up again, this year, in the other two towns where GeoGebra was introduced the first time where no experience with GeoGebra was there.*

**** The F-test: One-way test (ANOVA), used to test whether the means of several groups are equal, is more trustful than the t-test of any kind. The F-test is based on the measurements done in at least three groups (more many groups better the conclusions).*

**** As mentioned above, the selection of the classes was based on the known relationships between the experimenter and the teachers and the availability and willingness of the teachers to be involved with the experiment. This is a violation of the important requirement and principal on randomness in carrying out the experiment. Therefore, when carried out an experiment special attention must be paid to the randomness (each member of the population must have the same chance of being member of the sample). In the case of experimenting with the teaching process must be thought well about what kind of test perform and how to independently select classes involved in the experiment. My suggestion is that, a good solution is the cooperation with the ministry of Education and with the Regional Directorates of Education.*

Chapter 6
GeoGebra Surveys –Qualitative Variables

Course: GeoGebra for Teaching and Learning Derivatives

6.1 Survey No. 1
Teacher Professional Evaluation

(On teacher knowledge, preparation, and presentation).

This is questionary for the evaluation survey on GeoGebra course, chapter of Derivatives that focuses on teacher knowledge, preparation, and presentation. This course evaluation survey asks students to how much they agree with the following statements:

- The teacher is well prepared for class sessions.
- The teacher answers questions carefully and completely.
- The teacher uses examples to make the materials easy to understand.
- The teacher stimulated interest in the course.
- The teacher made the course material interesting.
- The teacher is knowledgeable about the topics presented in the course.
- The teacher treats students respectfully.
- The teacher is fair in dealing with students.
- The teacher makes students feel comfortable about asking questions.
- Course assignments are interesting and stimulating.
- The instructor's use of technology enhanced learning in the classroom.
- Directions for course assignments were clear.
- The difficulty level of this course evaluation survey was appropriate for me.
- This course evaluation survey is one of the most difficult I have taken.
- I would recommend this course evaluation to others.

Please tell us what you liked or disliked about this course. ………………………………………
……………………………………………………………………………………………………

……………………………………………………………………………………………………
……………………………………………………………………………………………………

These questionaries are linked with the students' attitude toward teacher and this not the issue of my study. I am concerned about the impact of GeoGebra software in teaching and learning mathematics process.

6.2 Survey No. 2

GeoGebra software impact in teaching and learning

This is questionary for the evaluation survey on GeoGebra course, chapter of Derivatives that focuses on GeoGebra software impact in teaching and learning. This course evaluation survey asks students to how much they agree with the following statements:

This survey example contains the following questionnaire items:

1. Course name or number
2. Was this a required course?
3. What did you except after taking this course?
4. What is your current computer knowledge?
5. The syllabus clearly described the course content.
6. The syllabus clearly defined assignments.
7. The syllabus clearly described class activities.
8. Course assignments were at an appropriate level of difficulty.
9. The teacher was good at facilitating class discussion.
10. All students became familiar with GeoGebra.
11. The size of the class was appropriate.
12. The notes and other readings appropriately covered the course content.
13. The teacher effectively used GeoGebra tools and GeoGebra-wiki to teach the course.
14. My knowledge and skills in mathematics are increased more by using GeoGebra tools than the usual tools in teaching and learning.
15. My interest on mathematics is increased much more now that I can use Geogebra in learning math.
16. The current textbooks should continue to be used.
17. I would recommend this course to other students.
18. I would recommend this teacher to other teachers and students.
19. I have a stronger interest in mathematics because of this course.
20. Please describe what you liked most about this course.
21. Please describe what you liked least about this course.

*** In the following tables are registered the data representing the answers of the students to the questionaries.

Questionary on GeoGebra software impact in teaching and learning

Please tell us how much you agree or disagree with the following statements.

	Statements	Student's Answers				
No		Strongly Disagree(1)	Disagree(2)	Undecided(3)	Agree(4)	Strongly Agree(5)
1	This was a required course.			5	8	16
2	I met my expectations by taking this course.		5	2	18	4
3	I improved a lot my computer skills by taking this course.		4	1	16	8
4	The syllabus clearly described the course content.		11	6	10	2
5	The course clearly defined assignments.		3	8	12	6
6	The course clearly described class activities.	2	2	5	12	8
7	Course		6	1	14	8

	assignments were at an appropriate level of difficulty.					
8	The teacher was good at facilitating class discussion.			3	12	14
9	All students became familiar with GeoGebra.			2	14	13
10	All students had fun with GeoGebra.			3	10	16
11	The size of the class was appropriate.	4	9	8	8	
12	The notes and other readings appropriately covered the course content.	2	6	5	12	4
13	The teacher effectively used GeoGebra tools and GeoGebra-wiki to teach the course.			3	16	10
14	My knowledge and skills in mathematics are increased more by using GeoGebra tools than the usual tools in teaching and		6	4	11	8

	learning.					
15	I gain more math knowledge and skills by experimenting with GeoGebra.		7	5	12	5
16	I believe the other classes will increase their math level by using GeoGebra.		8	6	12	3
17	My interest on mathematics is increased much more now that I can use Geogebra in learning math.		5	4	14	6
18	The current textbooks should continue to be used.		4	3	12	10
19	The difficulty level of this course was appropriate for me.		3	2	10	14
20	This course is one of the most difficult I have taken.	10	12	7		
21	I feel more secure in math by exploring with GeoGebra		4	2	16	7
22	I would recommend			7	15	7

	this course to other students and teachers.					

Please describe what you liked most about this course ……………………………………..
…………………………………………………………………………………………………….

Please describe what you liked least about this course ……………………………………….

……………………………………………………………………………………………………

Computing Scale Score Values for Each Item
The next step is to analyze the rating data. For each statement, we do compute the Median and the Interquartile Range.

The median is the value above and below which 50% of the ratings fall. The first quartile (Q1) is the value below which 25% of the cases fall and above which 75% of the cases fall -- in other words, the 25th percentile. The median is the 50th percentile. The third quartile, Q3, is the 75th percentile. The Interquartile Range is the difference between third and first quartile, or Q3 - Q1. The figure above shows a histogram for a single item and indicates the median and Interquartile Range. You can compute these values easily with any introductory statistics program or with most spreadsheet programs. To facilitate the final selection of items for our scale, we sort the table of medians and Interquartile Range in ascending order by Median and, within that, in descending order by Interquartile Range. For the items in this example, we got a table like the following:

Statement Number	Median	Q_1	Q_3	Interquartile Range
20	2	1	2.25	1.25
12	3	2	4	2
11	3	2	4	2
4	3	2	4	2
6	4	3	5	2
16	4	2	4	2
14	4	3	5	2
18	4	3.75	5	1.25
7	4	3.75	5	1.25
15	4	2.75	4	1.25
5	4	3	4	1
8	4	4	5	1

9	4	4	5	1
19	4	4	5	1
17	4	3	4	1
13	4	4	5	1
22	4	3.25	4	0.75
2	4	3.75	4	0.25
21	4	4	4.25	0.25
3	4	4	4	0
10	5	4	5	1
1	5	4	5	1

Selecting the Final Scale Items. Now, we have to select the final statements for our scale. The rule is to select statements that are at equal intervals across the range of medians. In our example, we have to select one statement for each of the four median values. Within each value, we select the statement that has the smallest Interquartile Range. This is the statement with the least amount of variability across judges (the students in our case). You don't want the statistical analysis to be the only deciding factor here. Look over the candidate statements at each level and select the statement that makes the most sense. If you find that the best statistical choice is a confusing statement, select the next best choice. We skip the first statement, it is just one, and it makes no sense for the case in study. We have taken two statements from the set of statements with median 4(they form the biggest set), those with the smallest interquartile range, which are making sense as well.

Going through our statements, we come up with the following set of items for our scale:

No.4 The syllabus clearly described the course content(3).

No. 3 I improved a lot my computer skills by. (4).

No.21 I feel more secure in math by exploring with GeoGebra(4).

No. 1 This was a required course (5).

It is seen that are missing the values 1 and 2, meaning: we do not have an item with scale value of 1 and 2 and, that we have two with values of 4.

When we take the average scale values for these four items, we get a final value of 4. This is where the class estimation on GeoGebra softw-are impact in learning mathematics would fall on our "yardstick" that measures attitudes towards GeoGebra soft-ware.

6.3 Survey No. 3
Type of learning and commitment

What type of learning do the students use, how much time do they commit for learning mathematics and GeoGebra out of school, and the impact of GeoGebra in learning math and computer science (Done at the end of the course).

	Statements	Student's Estimations				
No		1	2	3	4	5
1	Please rank the following types of learning you use out of school by importance with 1 being least important to 5 being most important: a - In- Work Groups(collaborative learning groups). ………. b - Alone and independent ………………………………. c - With tutor …………………………………………….. d - Online Chat/Discussion Groups………………………. e - Other: math. Courses………………………………….	7 13 4	6 3 8 7 2	4 7 5 6 4	4 9 5 3 3	8 10 11 16
2	Please rank the following benefits of collaborative learning by importance with 1 being least important to 5 being most important: a - Learning to Work in Groups………………………………….. b - Help to Learn Course Content……………………………….. c - Increase Between-Student Communication …………………….. d - Increase Student Involvement	2 4 2 3	3 5 2 4 3 4	8 6 5 4 6 9	8 11 10 8 12 9	8 7 8 11 8 4

	……………………………….. e - Increase Social Behavior …………………………………… f - Foster Integration (gender, friendly relations, etc) ……………...					
3	Please rank the following benefits of indipendent learning by importance with 1 being least important to 5 being most important. a - Learning to Work by Myself …………………………….. b - Help to Learn Course Content ………………………………. c - Increase Knowledge in many fields ……………………….. d - Increase Student Contribution ………………………………. e - Increase Social Value ………………………………………… f - Foster Persistence(will and attitude) …………………………..	3 4 2 6 7 5	2 4 4 4 6 5	7 5 4 4 6 4	7 6 7 6 4 8	10 10 12 9 6 7
4	Please rank the following methods usually used to form study collaborative groups: a - Student Choice…..………………………………………. b - Parental Advise …………………………………………. c - By Ability ………………………………………………. d - By Seating in the Class-room ………………………………. e - Other:………………………………………………………….	5 4 6 8	7 4 2 2	5 8 3 4	6 6 8 8	6 7 10 7
5	Please rank the span of time you worked independently to learn and solve GeoGebra problems during the course by commitment with 1 being none (not involved), 2(little), 3(some), 4(much) and 5(too much) - a -……………………………………………………….. b- Rank the time you use to learn math and solve problems as well	 4	6 4	7 6	8 6	8 9
6	Please rank the span of time you worked with groups to learn and solve GeoGebra problems during the course by commitment with 1 being					

	none (not involved), 2(little), 3(some), 4(much) and 5(too much) a - ………….. ……………………………………….. b - Rank the time you use to learn math and solve problems as well..	2 2	4 6	4 4	8 8	11 9
7	Please rank the value of attendance of any training sessions or workshops on computer programs by importance with 1 being least important to 5 being most important……………………………	6	4	4	7	8
8	Please rank the school involvement to provide assistance for computer programmes for the students by interest with 1 being least interested to 5 being most interested……………………………..	10	6	8	5	
9	Please rank the impact of GeoGebra software in learning mathematics by influence with 1 being of least influence to 5 being of most influence………………………………………………..		4	4	12	9
10	Please rank the impact of GeoGebra software in stimulating you to learn computer science by influence with 1 being of least influence to 5 being of most influence…………………………………			6	10	13

To facilitate the final selection of items for our scale, we sort the table of medians and Interquartile Range in ascending order by Median and, within that, in descending order by Interquartile Range. For the items in this example, we have the table like the following:

Statement Number	Median	Q_1	Q_3	Interquartile Range
8	2	1	3	2
1-d	2	1	3	2
1-a	3	1.75	5	3.25
3-e	3	1.75	4	2.25
3-f	3	2	4.25	2.25
4-b	3	2	4.25	2.25
4-a	3	2	4	2
2-f	3	2.75	4	1.25
4-d	4	1	4.25	3.25
4-c	4	2	5	3
5-b	4	2	5	3
6-b	4	2	5	3
7	4	2	5	3
3-d	4	2	5	3
1-c	4	2	5	3
3-b	4	2	5	3
3-a	4	3	5	2
2-e	4	3	5	2
2-d	4	3	5	2
2-c	4	3	5	2
2-a	4	3	5	2
1-b	4	3	5	2
5-a	4	3	5	2
6-a	4	3	5	2
9	4	3	5	2
3-c	4	3	5	2
2-b	4	3	4.25	1.25
10	4	4	5	1
1-e	5	3	5	2

Now, we have to select the final statements for our scale. The rule is the same as in the above case: select statements that are at equal intervals across the range of medians. In our example, we have to select one statement for each of the four median values. Within each value, we select the statement that has the smallest Interquartile Range. This is the statement with the least amount of variability across judges (the students in our case). We don't want the statistical analysis to be the only deciding factor here. We look over the candidate statements at each level and select the statement that makes the most sense. If we find that the best statistical choice is a confusing statement, select the next best choice.

Going through our statements, we come up with the following set of items for our scale:

1-d. The type of learning the student use more out of school is Online Chat/Discussion Groups(2)

2-f. The most important benefits of collaborative learning is: Foster Integration - gender, friendly relations, etc (3)

10. GeoGebra software has really stimulated the students to learn computer science (4)

1-e. ANOTHER type of learning the student use out of school and consider it very important is math. Course organized by the teachers (5)

If we take the average of these four items, we get a final value of 3.5. <u>This is where the class estimation on the learning types out of school and GeoGebra software influence in learning mathematics would fall on our "yardstick" that measures attitudes towards these two things</u>.

It counts to note here that the surveys are linked with the qualitative methods and they are of special importance because in drawing conclusion about the teaching methods and the results and improving them for the benefit of society is engaged the society itself. In this case we have not a one-sided evaluation and estimation of the situation in education (the teachers only), instead the teachers and the society act as components of a mutual concern. Now-days, "there is a general preference for qualitative methods, and there is a wide agreement that research should be socially minded."(10th International Congress on Mathematical Education, Pg.81)

5.4 Concluding Thoughts, Recommendations for Future Research

I believe that there is a relationship between computer programs for math (Geogebra software, also) and mathematics in regard with mastering them for the reasons I have presented above. The four tests performed are a confirmation to one another. It counts very much that GeoGebra be used in the teaching and learning process. This fact is confirmed by the surveys result as well: When we took the average scale values for the above four items (survey No.2), **we got a final value of 4**. The highest is 5. This is where the class estimation on GeoGebra software impact in learning mathematics falls on our "yardstick" that measures attitudes towards GeoGebra software. In the survey No. 3 we took the average of the four items and we got a final value of 3.5. This is where the class estimation on the learning types out of school and GeoGebra software influence in learning mathematics would fall on our "yardstick" that measures attitudes towards these two things. The students' attitudes and estimations regarding GeoGebra are positive.

The challenges are:

I. Change the mindset of the teachers by training them how to use computer programs in the teaching and learning process. The first step is taken: in Elbasan is started the first GeoGebra training with the teachers of the secondary schools and it is going on. The results are very positive and full of encouragement

II. Organize other trainings involving and teachers of other sciences and extend it in other cities

III Create the Albanian version of GeoGebra(a lot of work os done so far, but it takes much more time to translate materials from other sources)

IV. Put and build links in national scale between the teachers who use GeoGebra in the teaching process in order to share their achievements and develop their skills

V. Integrate the Albanian GeoGebra users in the international community of GeoGebra

Imperative task for teachers using GeoGebra is:

Teachers need a support system and professional development to improve their skills in teaching mathematics using GeoGebra (Hohenwarter and Preiner, 2007). With this guidance and support from the International GeoGebra Institute (IGI), GeoGebra enhances teachers' willingness to integrate this new technology into their teaching practices and helps collaboration between teachers and researchers and provides professional development for teachers (Hohenwarter and Lavicza, 2007).

As to Yu-Wen Allison Lu, there are four different stages that teachers possibly go through from learning to use GeoGebra to teaching mathematics with GeoGebra: **Stage 1**: teachers have to get comfortable with the software alone at home, using the software to create nice pictures for tests; **Stage 2:** teachers use GeoGebra as a presentation tool; **Stage 3:** teachers do use GeoGebra to visualize what has been discussed and, to get students interact with one another. So students do some kind of exercise and GeoGebra can be used as a checking tool. They type in what they think is the answer and show it to the class and compare different answers. **Stage 4:** teachers can ask much more open questions. Students can play with GeoGebra to come up with conjectures. So not just checking the conjecture but also developing the conjectures.

(Yu-Wen Allison Lu, Linking Geometry and Algebra: A multiple-case study of Upper-Secondary mathematics teachers' conceptions and practices of GeoGebra in England and Taiwan, 2008, pg. 52).

REFERENCES

1. ALBERT GOODMAN, School of Engineering & Information Technology: **Introduction to Data Collection and Analysis**, Deakin University Chapter, Copyright © Deakin University , 1995, 2003. Chapter: Data Graphical Presentation.
2. Bibliographie nationale de la r: articles de la ..., volume 25, issues 4-7 (biblioteka kombëtare -albania) - reference – 1985, pg. 164
3. Biblioteka kombëtare (albania), national bibliography of p.r.a., volumes 35, 37, 40,41,42.
4. Biblioteka shkencore e universitetit shteteror te tiranes(scientific library of the state university of tirana), tirana, vol. 18, pg. 54.

5. Böhm Josef (2008), Linking Geometry, Algebra and Calculus with GeoGebra, ACDCA, DUG and Technical University of Vienna.

6. Edwin e. jackues, the albanians-an ethnic history from prehistoric times to the present, manufactured in usa, mcfarland&company, inc. publishers, 1995, pg. 326
7. Bulletin ”studime shqiptare”(studia albanica), vol. 25 - pg. 213
8. Dragoslav Herceg, Dorote Herceg, TMCS 6(2008)2, Pg. 377.
9. Elvira Ripco, Teaching Geometry using computer visualizations, TMCS 7(2009)2,Pg. 262.
10. Enciklopedia shqiptare(the albanian encyclopedia), 1988

http://www.akad.edu.al/index.php?option=com_content&view=article&id=87&itemid=98

11. Gardner, H. (1987). Beyond the IQ: Education and human development,Developing the spectrum of human intelligences*. Harvard Educational Review, 57*, 187-193.
12. Giaquinto M., Visual Thinking in Mathematics(An Epistemological Study), Published in the United States by Oxford University, Press Inc., New York, 2007)Pg. 163 – 185)

13. Grotzer, T.A. (2002, March-April). Expanding our vision for educational technology: Procedural, conceptual, and structural knowledge. *Educational Technology*, (Pg.52-59).
14. Gutierrez Angel and Boero Paolo(2006), Handbook of Research on the Psychology of Mathematics Education, Published by: Sense Publishers, The Netherlands(Pg.31-64)
15. Hohenwarter Judith, Hohenwarter Markus(2008), Introduction to GeoGebra **(Pg. 37-42)**
16. Hans Frendenthal, Didactical Phenomenology of mathematical Structures, D. Reidel, Dordrecht, 1983, p. IX

17. http://www.sciencedirect.com/science?_ob=publication

18. *Int. J. Teaching and Case Studies, Vol. 1, Nos. 1/2, 2007* 23 Copyright © 2007 Inderscience Enterprises

19. John F. Sanford, Les M. Sztandera: Thoughts on the future of education in information technology(pp. 14-24)

20. Kllogjeri P. (2008), THE POWER OF DOUBLE REPRESENTATION OF GEOGEBRA, published in the scientific bulletin DOKTORANDUSZOK FURUMA, 2008, Miskolci Egyetem, November 5 (pp. 117-122)

21. Kllogjeri P. (2010), GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND USING THE SYNERGY OF MATHEMATICIANS, published in Technology Enhanced Learning: Quality of Teaching and Educational Reform, TECH-EDUCATION 2010, ATHENS, GREECE, MY 2010, PROCEEDINGS. P. 681-687. TECHNOLOGY ENHANCED LEARNING. QUALITY OF TEACHING AND EDUCATIONAL REFORM, Communications in Computer and Information Science, Volume 73, pp. 681-687, DOI: 10.1007/978-3-642-13166-0_95.

22. Kllogjeri P., Shyti B. (2010, GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND USING THE SYNERGY OF MATHEMATICIANS – extended version, published in Inderscience Journals, Int. Journal of Teaching and Case Studies (IJTCS), *Vol. 2, Nos. 3/4, 2010*(Pg. 225-236). **DOI:** 10.1504/IJTCS.2010.033318.

23. Kllogjeri P., Kllogjeri Q.(2011), GEOGEBRA – A VERY EFFECTIVE TOOL FOR TEACHING MATHEMATICAL CONCEPTS AND PROPERTIES, published in the Proceedings Volume of International GeoGebra Conference for Southeast Europe (Međunarodna GeoGebra Konferencija), Novi Sad, 15-16 Jan 2011, pp. 90-98 [Izdaje: Departman za matematiku i informatiku, Prirodnomatemati; ki fakultet u Novom Sadu, 21000 Novi Sad, Trg Dositeja Obradivica 4, tel (021)458136, Štampa: Stojkov, Novi Sad. Tiraž: 250 primeraka]

24. Kllogjeri P, Kllogjeri A. (2012), Use of GeoGebra in Teaching about Central Tendency and Spread Variability, published in the Carpathian Journal for Mathematics in Romania – CREATIVE MATH. & INFORMATICS, Volume **21**, No. 1, PP. 57 – 64. Online version available at http://creative-mathematics.ubm.ro/ Print Edition: ISSN 1584 - 286X Online Edition: ISSN 1843 - 441X/e , published by Springer-Verlag: Zentralblatt MATH Database 1931 – 2014, c 2014 European Mathematical Society, FIZ Karlsruhe & Springer-Verlag/ Zbl 1274.97047

25. Kllogjeri P, Kllogjeri A. (2015), Statistical Inferences Supporting the Hypothesis of Teaching with GeoGebra; *Open Access Library Journal*, **2**: e1255. http://dx.doi.org/10.4236/oalib.1101255

26. Körtesi P., Kllogjeri P. (2008), Extended Help Files in GeoGebra, published in the International Journal for Technology in Mathematics Education, Volume 16, No 1. pp. 37-41

27. Körtesi P., Kllogjeri P. (2010), Remarks on Help Files in GeoGebra, published in **RoJEd** (**Romanian Journal of Education)**, ISSN: 2067–8347**,** Volume 1 Number 2, pp. 15-22., responsible editor: Iuliana Marchis

28. Lu Yu-Wen Allison(2008),Linking Geometry and Algebra: A multiple-case study of Upper-Secondary mathematics teachers' conceptions and practices of GeoGebra in England and Taiwan Faculty of Education, University of Cambridge

29. Lyn D. English - editor. Publisher: Lawrence Erlbaum Associates (1997), Mathematical Reasoning: Analogies, Metaphors, and Images. Place of Publication: Mahwah, NJ. (Pg. 76-105)

30. Markus Hohenwarter, Judith Prenier, TMCS 6(2008)2, Pg. 320

31. Matematika dhe Fizika ne shkolle(mathematics and physics in school), 1978, 1980, 1982, 1983, 1986, 1989, 1991, 1996

32. Mathematics Education Library, Volume 13(2002),KLUWER ACADEMIC PUBLISHERS: NEW YORK, BOSTON, DORDRECHT, LONDON, MOSCO(Pg. 24-48)

33. Murray R. Spiegel, Statistics 2/ed, Copyright 1988, McGRAW-HILL BOOK COMPANY, Pg: 58-86

34. Perkins, D.N. (1998). What is understanding? In Wiske, M.S. (Ed.), *Teaching for understanding: Linking research with practice* (pg. 39-57). San Francisco, CA: Jossey-Bass.

35. School Of Intensive Courses in Novi Sad, Development of Computer-aided Methods in Teaching Mathematics and Science, April 4-9 2008- Novi Sad: Prirodno-matematicki fakultet, 2008, ISBN 978-86-7031-166-4, www.im.ns.ac.yu(Dynamical geometry of Dr. Dragoslav herceg, Pg.143-177)

36. Studime Shqiptare(studia albanica), magazine of academy of sciences of albania, vol. 26, 1988 - pg. 212-215;

37. Studime Shqiptare(studia albanica), magazine of academy of sciences of albania vol. 19, 1982 – pg. 245-249

38. 10th International Congress on Mathematical Education(ICME 10, 2004), Copenhagen, Denmark; Editor: Mogens Niss, department of Science, Systems and Models, Roskilde University, Denmark; ISBN: 978-87-7349-733-3

39. Tishman, S. (2001). A dispositional perspective on thinking. In A. Costa (Ed.), *Developing minds: A resource book for teaching thinking*. (Rev. Ed.). Association for Supervision and Curriculum Development (ASCD). (Vol. 3, pp. 72-75).

40. Zsolt Lavicza, Markus Hohenwarter,.., Establishing a Professional Development Network…., National Center for Excellence in the Teaching of Mathematics, Pg. 16-18.

41. website of the faculty of natural sciences, university of tirana. http://www.fshn.edu.al//

42. Winner, E., & Gardner, H.(1993). Metaphor and irony: Two levels of understanding. Invited chapter prepared for A. Ortony (Ed.), *Metaphor and thought* (2nd ed.).Cambridge: Cambridge University Press.(Pg. 80-146)

APPENDICES

1. Curriculum Vitae
2. REPORT ON DOCTORAL STUDIES AND WORK WITH GEOGEBRA
3. Tables of calculations
4. Tables of points and marks of the classes
5. Samples of the Survey Questionaries
6. Scanned copies of Journals of publications
7. Video-record of the experiment done and pictures

1. CURRICULUM VITAE

NAME: PELLUMB KLLOGJERI

TITLE: LECTURER IN UNIVERSITY "ALEKSANDER XHUVANI"

PLACE: ELBASAN, ALBANIA

DEPARTAMENT OF MATHEMATICS AND INFORMATICS

ADDRESS: L. "KONF. LABINOTIT", P.61, EL BASAN

e-mail: pkallogjeri@gmail.com

CeL. 00355-692086191

Education: University of Tirana,

Faculty of the Natural Sciences

Branch of mathematics

Specialty: Statistics and Probability, Graph Theory, Linear Programming

Degree and Research:

I have a Master degree on Statistics in the field of **Forecast** (Time Series and Forecast Methods).

Research Work (Years: 2004 – 2006):

Finding a model of good fit regarding the relation between the added weights of crossed lambs and the number of the days after their birth.

As result of the research done for three years was found a valuable dependance of the lamb weight on the number of the days of their normal nutrition in order that the farmers forecast when is the right time to make a decision to slayghter or sell the lamb.

Work Experience

1. Math Teacher in the Secondary School, Peqin(1977 – 1987)
2. ,, in the Secondary School „Kostandin Kristoforidhi", Elbasan(1987 – 1995)
3. Math Teacher (Part-Time) in the Dep. of Mathematics, University „Aleksander Xhuvani", Elbasan(1989 – 2004).
4. Math Teacher (Full-Time) in the Dep. of Mathematics and Informatics, University „Aleksander Xhuvani", Elbasan(2005 - ....).

Other Experiences and Qualifications

1. Competency in the English Language
2. Competency in Russian Language for the specialty of math
3. Knowledge of Italian Language
4. Computer knowledge and capabilities, Competency and licensed in Computer Math Programs: European Computer Algebra Driving License(ECADL)

Academic Work:

1) University publishing: Mathematics(Second Part), useful for the students of Low Cycle in Elbasan and for the teachers of 9-year school (authors: Pellumb Kllogjeri, Islam Braja)

2) Preparation for the first time of lectures on Descriptive Statistics(useful for the students of Social Sciences, University „Aleksander Xhuvani", Elbasan, ALBANIA).—Year 2006

3) Preparation for the first time of lectures from Graph Theory (useful for the students of Math. Branch, University „Aleksander Xhuvani", Elbasan, ALBANIA).—Year 2007

4) Preparation for the first time of lectures from Linear Programming (useful for the students of Math. Branch, University „Aleksander Xhuvani", Elbasan, ALBANIA).—Year 2007

5) Preparation for the first time of the lectures for the subject „Methodology of Teaching Mathematics(useful for the students of Math. Branch, University „Aleksander Xhuvani", Elbasan, ALBANIA).—Year 2009)

6) Tutorship of students for diploma work in the subject:

- of Statistics with theme: Covariance and Regression Theory.(Spring 2008),
- on Algebra: Use of Mathematical Induction Method in Mathematical Statements(July 2009) ;
- Computer programs: Presentation on GeoGebra software, July 2009, July 2011
- on Geometry: Dynamic methods in teaching Geometry(GeoGebra), July 2010
- GeoGebra and Statistics, July 2011

Participation and Contribution in National and International Activities

1. Participation in the Exchange Program between Universities (Mobility) of **CEEPUS** in the University of Miskolc, Hungary(one month - February - year: 2007): teaching in Statistics and Calculus (6 hrs/week) with the students of Construction Enginery(first Year) and seminar presentation in the Department of Analysis and the Institute of Mathematics, University of Miskolc, theme on Regression Analysis for my research work done in Albania.
2. Participation in Joint Conference of SEFI and IGIP organized in the University of Miskolc From 1 - 4 July 2007, two main European organizations for promoting Engineering education with aim: "Joining Forces in Engineering Education Towards Excellence".
3. Participation and Scientific Presentation in the First International Conference for Analysis and Algebra, June 2008, organized by the Dep. of Math., University „ A. Xhuvani" . Place: Elbasan . Presentation: Application of the Generalizing Functions in the Counting Problems.
4. Participation in the Summer University organized in the University of Miskolc, Hungary, from 08.08.2008 to 20.08.2008, with the position: **CEEPUS Teacher**. My teaching was on the partition of a set with n elements into k classes with the same number of elements using the method of exchanging elements between them(including the cases (a) n divisible by k and (b) n not divisible by k).

5. Participation in the Second International Conference for Analysis and Algebra, May 2009, organized by the Dep. of Math., University „ A. Xhuvani” . Place: Elbasan. Presentation: „Partition of a set with n elements into k classes with different number of elements”.
6. Participation in the conference „Computer Algebra and Dynamic Geometry Systems in Mathematics Education” (July 11 – 13, 2009, Hagenberg, University of Linz, Austria). Presentation: European Computer Algebra Driving License”(authors: Peter Kortesi[1]: Pellumb Kllogjeri[2], University of Miskolc, Hungary[1] ; University of Elbasan, Albania[2]).
7. Participation in the conference of GeoGebra(July 14 – 15, 2009, Hagenberg, University of Linz, Austria). Presentation: „Extended Help Files in GeoGebra” (authors: Pellumb Kllogjeri, Albania and Peter Kortesi, Hungary).
8. Third pan-Albanian Science Conference (31 August – 01 September 2009, Tetovo, Macedonia. Presentation: Partition of a Set with n Elements in Classes in Accordance with the Expansion of Number n as Sum of k Natural Summonds (author: Pellumb Kllogjeri)
9. Mathematical International Congress in Ohrid Macedonia (MICOM): 16 – 20 September, 2009, Ohrid, Republic of Macedonia;
   Presentation: Computer Programmes lead to new mathematical statements and to new considerations in the teaching and learning math science (author: Pellumb Kllogjeri);
10. XXIV microCAD International Scientific Conference, University of Miskolc, 18-20 March 2010; a) Section of Physics and Physics Education, Presentation: GeoGebra for Presenting and Interpreting Grouped Data and Solving Problems of Physics(author: Pellumb Kllogjeri). b) Section of Mathematics and Computer Science, Presentation: Mathematical Games(authors: Pellumb Kllogjeri[1], Peter Kortesi[2], University of Elbasan, Albania[1]; University of Miskolc, Hungary[2])
11. 3rd International Conference on Algebra and Analysis (Elbasan, Albania, 2010, May 14-15)
12. 1st International Conference, TECH-EDUCATION 2010(Athens, Greece, May 19-21, 2010).Technology Enhanced Learning: Quality of Teaching and Educational Reform; Presentation: GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND

USING THE SYNERGY OF MATHEMATICIANS (authors: Pellumb Kllogjeri[1], Bederiana Shyti[2], University of Elbasan, Albania[1] ; University of Elbasan, Albania[2])

13. Conference of History of Mathematics and Teaching of Mathematics(Szeged, 2010, May 20-22); Poster: The Albanian Mathematicians by the Flowside of the Mathematicians of the World(*Author: Pellumb Kllogjeri,* University of Elbasan, Albania *)*

14. 7th International Conference on applied mathematics (ICAM); Baia Mare, Romania(1-5 September 2010). Presentation: Use of GeoGebra in Teaching about Central Tendency and Spread Variability *(Author: Pellumb Kllogjeri,* University of Elbasan, Albania *)*

15. 2nd International Conference on TECHNOLOGY ENHANCED LEARNING, QUALITY OF TEACHING and REFORMING EDUCATION (*18th-20th May 2011, Corfu-Greece)* Presentation: Computer programs - new considerations in the teaching and learning mathematics science

16. History of Mathematics and Teaching of Mathematics, Sárospatak, Hungary, 2012. May 23-27. Presentation: Elbasan in the focus of education and the contribution of Hungarian mathematicians-Pellumb Kllogjeri.

17. GeoGebra Conference Budapest 2014 – Annual Dynamics Mathematics Software Conference 23-25 Jan. 2014. http://lanyrd.com/2014/ggbconf/scwfqt. Presentation: "GeoGebra in Studying Similarities and Differences Between Different Geometries", authors: Pellumb Kllogjeri, Qamil Kllogjeri

18. The 10th Annual OpenCourseWare (OCW) Consortium Global Conference – Open Education for a Multicultural World, Ljubjana, Slovenia, 23-25 April 2014. http://conference.ocwconsortium.org/2014/proceedings Presentation: GeoGebra: A Vital Bridge Linking Mathematics with other Sciences. Authors: Pellumb Kllogjeri, Qamil Kllogjeri.

19. PhD Course of Didactical approach to different environments *Novi-Sad, Faculty of Science,* VI International Conference of Teaching and Learning mathematics *Department of mathematics and informatics January 23-25, 2015,* http://www.dmi.uns.ac.rs/ipa/docs/ICTLM/Conference%20program.pdf

Publications

1. Properties of Some Closed Polar Curves (Pellumb Kllogjeri); Mathematical Magazine, OCTOGON 8, Vol.16, No. 1A, April 2008 (P.247-257), ISSN 1222-5657, ISBN 978-973-88255-2-9 http://www.uni-miskolc.hu/~matsefi/Octogon/
2. Partition of a Set S in Classes of Equal Number of Elements and one Residual Class by the Method of Exchanging Elements(Pellumb Kllogjeri); Scientific Bulletin of Elbasan University, 2008/4 (Pg. 34-43)
3. GeoGebra for Presenting and Interpreting Grouped Data and Solving Problems of Physics (author: Pellumb Kllogjeri, University of Elbasan) – published in the Proceedings of XXIV microCAD International Scientific Conference, University of Miskolc, 18-20 March 2010; Volume of Section I: Physics and Physics Education(Pg. 63-70)
4. Mathematical Games (authors: Pellumb Kllogjeri[1], Peter Kortesi[2], University of Elbasan, Albania[1]; University of Miskolc, Hungary[2]) **;** published in the Proceedings of XXIV microCAD International Scientific Conference, University of Miskolc, 18-20 March 2010; ISBN 978-963-661- 916-9; Volume of Section H: Mathematics and Computer Science(Pg. 85-90)
5. Extended Help Files in Software GeoGebra (Peter Kortesi-University of Miskolc, Pellumb Kllogjeri-University of Elbasan)- published in Disputationes Scientificae Universitatis Catholicae in Ruzomberok: katolicka univerzita, 2010, roe 10, e 1(Vol 10, No 1), ISSN 1335-9185, pp 134-142.
6. Extended Help Files in GeoGebra (Peter Kortesi-University of Miskolc, Pellumb Kllogjeri-University of Elbasan)- published in International Journal for Technology in Mathematics Education, Volume 16, No 1. pp. 37-41
7. Remarks on Help Files in GeoGebra. Authors: Péter Körtesi, University of Miskolc, Miskolc (Hungary), Pellumb Kllogjeri, University of Elbasan "Aleksander Xhuvani", Elbasan (Albania), Published in **RoJEd** (**Romanian Journal of Education)***,* ISSN: 2067–8347**,** Volume 1 Number 2, Published: 28 June 2010. pp. 15-22. Responsible editor: Iuliana Marchis
8. THE POWER OF DOUBLE REPRESENTATION OF GEOGEBRA (author: Pellumb Kllogjeri), published in the scientific bulletin DOKTORANDUSZOK FURUMA, Nov. 2008, Miskolci Egyetem, (pp. 117-122)
9. GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND USING THE SYNERGY OF MATHEMATICIANS –extended version, authors: Pellumb Kllogjeri and Bederiana Shyti, University of Elbasan, Albania, Published in Inderscience Journals, Int. Journal of Teaching and Case Studies (IJTCS), *Vol. 2, Nos. 3/4, 2010*(Pg. 225-236). **DOI:** 10.1504/IJTCS.2010.033318

43. GEOGEBRA: A GLOBAL PLATFORM FOR TEACHING AND LEARNING MATH TOGETHER AND USING THE SYNERGY OF MATHEMATICIANS (author: Pellumb Kllogjeri), published in Technology Enhanced Learning: Quality of Teaching and Educational Reform, TECH-EDUCATION 2010, ATHENS, GREECE, MAY 2010, PROCEEDINGS. P. 681-687.

TECHNOLOGY ENHANCED LEARNING. QUALITY OF TEACHING AND EDUCATIONAL REFORM, Communications in Computer and Information Science, 2010, Volume 73, 681-687, DOI: 10.1007/978-3-642-13166-0_95. ISBN: 978-3-642-13165-3 (Print) 978-3-642-13166-0 (Online).

10. The Albanian Mathematicians by the Flowside of the Mathematicians of the World *(Author: Pellumb Kllogjeri). Edited in Proceedings of the Conference History of Mathematics and Teaching of Mathematics, May 2010, Szeged, Hungary; Editor: Department of Analysis of the Univeristy of Miskolc, CD form, ISBN 978-963-661-929-9, 22 pages.*
11. Geogebra for Solving Problems of Physics,(Authors: Pellumb Kllogjeri, Adrian Kllogjeri), published in Springer, ORGANIZATIONAL, BUSINESS, AND TECHNOLOGICAL ASPECTS OF THE KNOWLEDGE SOCIETY, Communications in Computer and Information Science, 2010, Springer ISBN: 978-3-642-16323-4, ISSN: 1865-0929, eBook ISBN: 978-3-642-16324-1, Volume 112, pp. 424-428, DOI: 10.1007/978-3-642-16324-1_50 + The Distributed (TDG) Scholar.
12. Use of GeoGebra in Teaching about Central Tendency and Spread Variability AUTHORS: Pellumb Kllogjeri (University of Elbasan, Albania); Adrian Kllogjeri (University of Kent, UK) - published in the Carpathian Journal for Mathematics in Romania- CREATIVE MATH. & INFORMATICS, Volume **21** (2012), No. 1, 57 – 64. Online version available at http://creative-mathematics.ubm.ro/ Print Edition: ISSN 1584 - 286X Online Edition: ISSN 1843 - 441X/e + Published by Springer-Verlag: Zentralblatt MATH Database 1931 – 2014, c 2014 European Mathematical Society, FIZ Karlsruhe & Springer-Verlag/ Zbl 1274.97047
13. GEOGEBRA – A VERY EFFECTIVE TOOL FOR TEACHING MATHEMATICAL CONCEPTS AND PROPERTIES Author: Pellumb Kllogjeri, published in the Proceedings volume of International GeoGebra Conference for Southeast Europe (Međunarodna GeoGebra Konferencija), Novi Sad, 15-16, January, 2011, pp. 90-98 [Izdaje: Departman za matematiku i informatiku, Prirodnomatemati; ki fakultet u Novom Sadu, 21000 Novi Sad, Trg Dositeja Obradivica 4, tel (021)458136, Štampa: Stojkov, Novi Sad. Tiraž: 250 primeraka]
14. GeoGebra: Calculation of Centroid. Authors: **Qamil Kllogjeri, Pellumb Kllogjeri,** Published in **European Researcher** (Multidisciplinary Scientific Journal), September, 2012, Vol.(30), № 9-3; **ISSN:** 2219-8229, **E-ISSN:** 2224-0136, indexed Impact Factor 2, Pg. 1527-1537)
15. Partition of a Set with N Elements into K Blocks with Number of Elements in Accordance with the Composition of Number N As a Sum of Any K Natural Summands (Another Representation of Stirling Number). Authors: Pellumb Kllogjeri and Adrian Kllogjeri, Published in Recent Science- the International Journal of Advanced Computing (Impact Factor 2.31), August 2013, ISSN 2051-0845, Volume No 46, Issue No 3, Pg. 1278 -1284.
16. The Quasi-Squares and their Limit Curve, Authors: Pellumb Kllogjeri, Adrian Kllogjeri. Published in the International Journal of Mathematical Analysis and Applications (American Association for Science and Technology-AASCIT), ISSN: 2375-3927, Vol 1, No.2, June 2014, pp. 31-37; Published online June 10, 2014 (http://www.aascit.org/journal/ijmaa), Pages: 31-37

17. Dynamic models for multiplication and division offered by GeoGebra; Lindita Kllogjeri, Pellumb Kllogjeri; American Journal of Software Engineering and Applications 2015; 4(2-1):1-6; Published online October 19, 2014 (http://www.sciencepublishinggroup.com/j/ajsea), doi: 10.11648/j.ajsea.s.2015040201.11; ISSN: 2327-2473 (Print); ISSN: 2327-249X (Online)

18. Statistical Inferences Supporting the Hypothesis of Teaching with GeoGebra. Adrian Kllogjeri, Pellumb Kllogjeri; *Open Access Library Journal*, **2**: e1255. Jan. 2015; http://dx.doi.org/10.4236/oalib.1101255

I. List of Published Books

1. An Introduction To Quasi-Quadrilaterals, ISBN-13: 978-3-659-42488-5, ISBN-10:3659424889, EAN:9783659424885, August 2013; Authors: Pellumb Kllogjeri and Adrian Kllogjeri**,** Number of pages:140, **Publishing house:** LAP LAMBERT Academic Publishing, **Website:** https://www.lap-publishing.com/, Heinrich-Böcking-Str. 6-8, 66121, Saarbrücken, Germany.

   Deutsche National Bibliothek, Leipzig:

   http://d-nb.info/1038643848

2. REPORT ON DOCTORAL STUDIES AND WORK WITH GEOGEBRA

Pellumb Kllogjeri

PhD Student in the University of Debrecen, Institute of Mathematics,

Doctoral School

Math Teacher in the University "Aleksander Xhuvani ", Elbasan, Albania

2.1 Translation of GeoGebra:.

- I have translated the five (5) properties files of GeoGebra using Attesoro software for the translation. The translation is reviewed after the remarks and suggestions of Judith Hohenwarter(codesigner of

GeoGebra with **Dr. Markus Hohenwarter)** and, recently is pre-released the Albanian version for GeoGebra.

- Also, are translated some basics of GeoGebra based on the Introductory Book of Markus and Judith Hohenwarter.

2.2 Training on GeoGebra. I have started the teacher training with GeoGebra - two hours teaching and practice every month. February is the fourth month. There are 20 teachers participating in this training.

*** In the beginning there was a talk and exchange of thoughts. The conclusions of the talk were:

- none of the teachers had heard about GeoGebra

- watching some simple applications of GeoGebra in constructing geometrical figures they appreciated very much it and were moved to immediately start the training

- they considered GeoGebra as a mean to be involved and used in a program for <u>further qualification of the math teachers of the secondary schools</u>

- after this proposal there was a talk with the Head of the Department of Mathematics and Informatics, prof. Agron Tato, and with the Directorate of Education of Elbasan District and is agreed to build such a program including GeoGebra and Maple, Analysis and Algebra.

2.3 Pedagogical Experiment. I have finished an experiment with a class(third year) of a secondary school where I taught the first chapter on Derivatives using GeoGebra and geometrically and visually demonstrating the concepts and properties of monotonous functions, extreme values, the mean values etc. For comparing the results there was another testing class where this chapter is taught in the traditional way. To draw right conclusions were kept detailed notes. Now I am working for finalization of my research work and fitting it in my Thesis.

2.4 Diploma with theme on GeoGebra. The other fact regarding GeoGebra is that, last summer there was a dissertation from a student in the last year of mathematics branch whose subject was in GeoGebra and under my tutorship. His presentation was very much appreciated from the commission of the diploma and graded with top mark. The commission asked him how to help in introducing GeoGebra and making available for them.

- Two other Diplomas of a student with theme from GeoGebra was led by me in May 2010 and June 2011

2.5 Beneficiary of CEEPUS scholarships

I am granted several CEEPUS scholarships of one month mobility term as a visiting teacher in the University of Debrecen, Institute of Mathematics, Hungary during the period 2009 – 2010 in which I have met all the requirements of the program:

- teaching activity: 7 working days of at least 6-hour teaching with the groups of the students of 3rd and 4th year and a PhD student dealing with GeoGebra software(My teaching has been within the framework of: Active Methods in Teaching and Learning Mathematics and Informatics – Use of GeoGebra).

- Meetings and consultations with different professors and doctors of Mathematics as part of the exchange program between universities for gaining knowledge and exchanging experience and as part of my PhD studies.

- Use of the Library of the Institute of Mathematics for purposes and in support of my PhD studies

- Visit with the coordinator of the International Relations of the Center of Arts, Humanities and Sciences of the University of Debrecen for the purpose of possible future cooperation between the two universities in the field of University studies and research work(particularly with Prof. Zsolt Pales, on the subject of PhD program; Dr. Peter Kortesi, my scientific advisor, etc.).

- Visit in the University College of Nyiregyhaza (computer laboratories, lecture rooms, outside venues etc) and talk with Dr. Zoltan Kovacs(vice rector) in the subject of cooperation for applying GeoGebra program in Dynamic Geometry.

Dr. Andras Kovacs, the local coordinator of CEEPUS, has been my supervisor in all these one month mobility terms.

The periods of my one month mobilities and the respective scholarships have been:

a) 10.09.2009 – 10.10.2009(CII-HU-0028-03-0910-M-32245)

b) 08.02.2010 – 08.03.2010(CII-HU-0028-03-0910-M-35404)

c) 31.05.2010 – 30.06.2010(CII-HU-0028-03-0910-M-41649)

2.6 Future Plans (revised)

- Revision of my Thesis (August 2011)
- Start of a new Teacher Training on GeoGebra in Fier(November 2011)
- Participations and Presentations in other GeoGebra conferences and TECH-EDUCA conferences: http://www.open-knowledge-society

3. Tables of calculations

No.	Test on derivatives		End of Chapter on Derivatives		$d_i = Z_i - Y_i$	$d_i - \overline{d}$	$(d_i - \overline{d})^2$
	Points	Mark(Y_i)	Points	Mark(Z_i)			
1	78	8	98	10	2	0.76	0.5776
2	42	4	52	5	1	-0.24	0.0576
3	64	6	64	6	0	-1.24	1.5376
4	52	5	58	6	1	-0.24	0.0576
5	62	6	71	7	1	-0.24	0.0576
6	83	8	96	10	2	0.76	0.5776
7	72	7	80	8	1	-0.24	0.0576
8	73	7	78	8	1	-0.24	0.0576
9	80	8	95	10	2	0.76	0.5776
10	53	5	68	7	2	0.76	0.5776
11	92	9	98	10	1	-0.24	0.0576
12	94	9	97	10	1	-0.24	0.0576
13	81	8	82	8	0	-1.24	1.5376
14	44	4	57	6	2	0.76	0.5776
15	78	8	88	9	1	-0.24	0.0576
16	96	10	98	10	0	-1.24	1.5376
17	100	10	98	10	0	-1.24	1.5376
18	52	5	71	7	2	0.76	0.5776
19	42	4	54	5	1	-0.24	0.0576
20	64	6	74	7	1	-0.24	0.0576
21	64	6	83	8	2	0.76	0.5776
22	84	8	96	10	2	0.76	0.5776
23	82	8	97	10	2	0.76	0.5776

24	58	6	84	8	2	0.76	0.5776
25	68	7	86	9	2	0.76	0.5776
26	77	8	92	9	1	-0.24	0.0576
27	77	8	92	9	1	-0.24	0.0576
28	82	8	94	9	1	-0.24	0.0576
29	71	7	82	8	1	-0.24	0.0576
		$\overline{X}$ =**7**	**2383 (82)**	$\overline{X}$ = **8.24**	$\overline{d}$ = **1.24**		**13.3104**

Table 3.1 Calculation of paired differences(experimental group)

No.	Previous chapter Mark	End of Chapter on Derivatives	
	Y_i	Z_i	$d_i = Z_i - Y_i$
1	10	10	0
2	9	8	-1
3	8	8	0
4	9	9	0
5	9	10	1
6	7	7	0
7	7	8	1
8	5	5	0
9	6	6	0
10	10	10	0
11	8	9	1
12	10	10	0
13	10	10	0
14	10	9	-1
15	9	9	0
16	8	7	-1
17	5	5	0
18	9	9	0
19	7	8	1
20	8	8	0
21	10	10	0
22	8	9	1
23	7	7	0
24	9	9	0
25	7	7	0
26	8	9	1
27	8	8	0
28	7	7	0
	$\overline{X} = 8.14$	$\overline{X}$ =**8.25**	$\overline{d_o}$ = **0.107**

Table 3.2 Calculation of paired differences (control group)

Control Group(XI-A)

No.	Previous chapter Mark		End of Chapter on Derivatives
1	10		10
2	9		8
3	8		8
4	9		9
5	9		10
6	7		7
7	7		8
8	5		5
9	6		6
10	10		10
11	8		9
12	10		10
13	10		10
14	10		9
15	9		9
16	8		7
17	5		5
18	9		9
19	7		8
20	8		8
21	10		10
22	8		9
23	7		7
24	9		9
25	7		7
26	8		9
27	8		8
28	7		7
	$\overline{X} = 8.14$		$\overline{X} =$**8.25**

Table3.3 The marks of control group in two tests

Experimental Group (XI-D)

No .	Data of Preivious Chapter		Test on derivatives		Test on the use of GeoGebra Tools		End of Chapter on Derivatives	
	Points	**Mark**	**Points**	**Mark**	**Points**	**Mark**	**Points**	**Mark**
1	77	8	78	8	96	10	98	10
2	47	5	42	4	57	6	52	5
3	63	6	64	6	64	6	64	6
4	51	5	52	5	62	6	58	6
5	68	7	62	6	77	8	71	7
6	79	8	83	8	96	10	96	10
7	76	8	72	7	88	9	80	8
8	64	6	73	7	82	8	78	8
9	88	9	80	8	97	10	95	10
10	56	6	53	5	68	7	68	7
11	94	9	92	9	96	10	98	10
12	92	9	94	9	98	10	97	10
13	78	8	81	8	84	8	82	8
14	46	5	44	4	55	6	57	6
15	86	9	78	8	92	9	88	9
16	93	9	96	10	100	10	98	10
17	96	10	100	10	97	10	98	10
18	60	6	52	5	64	6	71	7
19	42	4	42	4	51	5	54	5
20	72	7	64	6	77	8	74	7
21	76	8	64	6	82	8	83	8
22	87	9	84	8	97	10	96	10
23	91	9	82	8	92	9	97	10
24	78	8	58	6	80	8	84	8
25	88	9	68	7	86	9	86	9
26	87	9	77	8	95	10	92	9
27	81	8	77	8	87	9	92	9
28	89	9	82	8	96	10	94	9
29	78	8	71	7	88	9	82	8
	2179 (75)	$\bar{X}$ **= 7.6**		$\bar{X}$ **=7**	**2404 (83)**	$\bar{X}$ **= 8 .4**	**2383 (82)**	$\bar{X}$ **= 8 .24**

Table3.4 The points and marks of experimental group in four tests

Range of points	Mark	Experimental Group Frequencies	Control Group Frequencies
<35			
35 - < 45	4	1	

45 - < 55	5	3	2
55 - < 65	6	4	1
65 - < 75	7	2	6
75 - < 85	8	8	7
85 - < 95	9	10	6
95+	10	1	6
		Sum = 29	**Sum = 28**

Table 3.5 Frequency distribution of classes for the two groups

X_i *Mark*	$f(X_i)$ *Frequency*	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	0	0	-4.1	16.81	0
5	2	10	-3.1	9.61	19.22
6	1	6	-2.1	4.41	4.41
7	6	42	-1.1	1.21	7.26
8	7	56	-0.1	0.01	0.07
9	6	54	0.9	0.81	4.86
10	6	60	1.9	3.61	21.66
$\bar{X} = 8.14$	$\sum f(X_i) = 28$	$\sum$ =**228**			$\sum$ =**57.48**

Table 3.6 Grouped data of control group (previous chapter)

X_i *Mark*	$f(X_i)$ *Frequency*	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	0	0	-4.25	18.0625	0
5	2	10	-3.25	10.5625	21.1250
6	1	6	-2.25	5.0625	5.0625
7	5	35	-1.25	1.5625	7.8125
8	6	48	-0.25	0.0625	0.3750
9	8	72	0.75	0.5625	4.5000
10	6	60	1.75	3.0625	18.3750
$\bar{X} = 8.25$	$\sum f(X_i) = 28$	$\sum$ =**231**			$\sum$ =**57.25**

Table 3.7 Grouped data of control group (end of chapter)

X_i *Mark*	$f(X_i)$	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	1	4	-3.6	12.96	12.96
5	3	15	-2.6	6.76	20.28

6	4	24	-1.6	2.56	10.24
7	2	14	-0.6	0.36	0.72
8	8	64	0.4	0.16	1.28
9	10	90	1.4	1.96	19.60
10	1	10	2.4	5.76	5.76
$\overline{X} = 7.6$	$\sum f(X_i) = 29$	$\sum$ =**221**			$\sum$ =**70.84**

Table3.8 Grouped data of experimental group(previous chapter)

X_i Mark	$f(X_i)$	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	3	12	-3	9	27
5	3	15	-2	4	12
6	5	30	-1	1	5
7	4	28	0	0	0
8	10	80	1	1	10
9	2	18	2	4	8
10	2	20	3	9	18
$\overline{X} = 7$	$\sum f(X_i) = 29$	$\sum$ =**203**			$\sum$ =**80**

Table 3.9 Grouped data of experimental group (beginning of chapter)

X_i Mark	$f(X_i)$	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	0	0	-4.24	17.9776	0
5	2	10	-3.24	10.4976	20.9952
6	3	18	-2.24	5.0176	15.0528
7	4	28	-1.24	1.5376	6.1504
8	6	48	-0.24	0.0576	0.3456
9	5	45	0.76	0.5776	2.8880
10	9	90	1.76	3.0976	27.8784
$\overline{X} = 8.24$	$\sum f(X_i) = 29$	$\sum$ =**239**			$\sum$ =**73.3104**

Table 3.10 Grouped data of experimental group(end of chapter)

X_i Mark	$f(X_i)$	$f(X_i).X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$f(X_i).(X_i - \bar{X})^2$
4	0	0	-4.4	19.36	0
5	1	5	-3.4	11.56	11.56

6	5	30	-2.4	5.76	28.80
7	1	7	-1.4	1.96	1.96
8	6	48	-0.4	0.16	0.96
9	6	54	0.6	0.36	2.16
10	10	100	1.6	2.56	25.60
$\bar{X} = 8.4$	$\sum f(X_i) = 29$	$\sum$ =244			$\sum$ =71.04

Table 3.11 Grouped data of experimental group(test on GeoGebra)

No.	X	Y	X*Y	X^2	Y^2
1	10	10	100	100	100
2	6	5	30	36	25
3	6	6	36	36	36
4	6	6	36	36	36
5	8	7	56	64	49
6	10	10	100	100	100
7	9	8	72	81	64
8	8	8	64	64	64
9	10	10	100	100	100
10	7	7	49	49	49
11	10	10	100	100	100
12	10	10	100	100	100
13	8	8	64	64	64
14	6	6	36	36	36
15	9	9	81	81	81
16	10	10	100	100	100
17	10	10	100	100	100
18	6	7	42	36	49
19	5	5	25	25	25
20	8	7	56	64	49
21	8	8	64	64	64
22	10	10	100	100	100
23	9	10	90	81	100
24	8	8	64	64	64
25	9	9	81	81	81
26	10	9	90	100	81
27	9	9	81	81	81
28	10	9	90	100	81
29	9	8	72	81	64
Sums	**244**	**239**	**2079**	**2124**	**2143**
	$\bar{X}$ = **8.4**	$\bar{Y}$ = **8.24**			

Table 4.3 Calculation of correlation(experimental group)

4. Tables of points and marks of experimental and control groups

Control group (Class XI-A)- year 2010

No.	Name and Family Name	Previous chapter Mark		End of Chapter on Derivatives
		Mark		Mark
1	Aldo Qoli	10		10
2	Anri Gongo	9		8
3	Arda Tabaku	8		8
4	Aurora Gaxhja	9		9
5	Brunilda Sharra	9		10
6	Bruno Hoxha	7		7
7	Bora Ballkoci	7		8
8	Elvis Gjevori	5		5
9	Elvis Hajrullaj	6		6
10	Enjo Cangonji	10		10
11	Eni Ceka	8		9
12	Erion Suparaku	10		10
13	Ira Maria Paralloj	10		10
14	Ismail Muzhaqi	10		9
15	Jalta Franko	9		9
16	Jerin Musai	8		7
17	Kosat Laro	5		5
18	Kristina Gjica	9		9
19	Kristjana Doka	7		8
20	Ledi Hasa	8		8
21	Marie Tomai	10		10
22	Ornela Deliu	8		9
23	Raisa Koka	7		7
24	Reisi Kotollaku	9		9
25	Rinor Rrasa	7		7
26	Romina Luniku	8		9
27	Semena Alushi	8		8
28	Tedi Turku	7		7
		$\overline{X} = 8.14$		$\overline{X}$ =8.25

Experimental Group(XI-D) – year 2010

No.	Name and Family Name	Data of Preivious Chapter		Test on derivatives		Test on the use of GeoGebra Tools		End of Chapter on Derivatives	
		Points	Mark	Points	Mark	Points	Mark	Points	Mark
1	Alketa Sinanaj	77	8	78	8	96	10	98	10
2	ALeks Kapja	47	5	42	4	57	6	52	5
3	Anisa Goro	63	6	64	6	64	6	64	6
4	Arlind Ruda	51	5	52	5	62	6	58	6
5	Borni Mice	68	7	62	6	77	8	71	7
6	Brunilda Ballkoci	79	8	83	8	96	10	96	10
7	Ejona Samarxhi	76	8	72	7	88	9	80	8
8	Emanuela Kullolli	64	6	73	7	82	8	78	8
9	Eni Duka	88	9	80	8	97	10	95	10
10	Eros Shegani	56	6	53	5	68	7	68	7
11	Emeralda Sinanaj	94	9	92	9	96	10	98	10
12	Fjorela Demi	92	9	94	9	98	10	97	10
13	Ilda Hajdari	78	8	81	8	84	8	82	8
14	Irdi Zjarri	46	5	44	4	55	6	57	6
15	Istriva Balla	86	9	78	8	92	9	88	9
16	Klara Cipa	93	9	96	10	100	10	98	10
17	Klaudia Biba	96	10	100	10	97	10	98	10
18	Mardov Dushku	60	6	52	5	64	6	71	7
19	Mariglen Hyra	42	4	42	4	51	5	54	5
20	Mernisa Allkja	72	7	64	6	77	8	74	7
21	Odeta Elezi	76	8	64	6	82	8	83	8
22	Orienta Dulja	87	9	84	8	97	10	96	10
23	Raselda Rusta	91	9	82	8	92	9	97	10
24	Rigels Guzi	78	8	58	6	80	8	84	8
25	Ruzhdije Asllani	88	9	68	7	86	9	86	9
26	Suada Ballkoci	87	9	77	8	95	10	92	9
27	Xhenxana Rrapa	81	8	77	8	87	9	92	9
28	Xhilda Zyma	89	9	82	8	96	10	94	9
29	Zenaida Shkalla	78	8	71	7	88	9	82	8
		2179 (75)	$\bar{X}$ = **7.6**		$\bar{X}$ =**7**	**2404 (83)**	$\bar{X}=$ **8.4**	**2383 (82)**	$\bar{X}=$ **8.24**

Year 2011

Here are the lists of marks for two experimental classes(year 2011) from Fier and Librazhd and two more lists of Elbasan: one of the experimental class and the other of the control class. They are attached to the appendix in a scanned form.

Day, 21.02.2011

KLASA X-A, SHKOLLA “Jani Kilica”, Fier- 2011(School “Jani Kilica”, Fier, Class X-A)

Nr	Emri, Mbiemri(full name)	Kapitulli i meparshem(previous chapter)	Fund i kapitullit (experimental chapter)	Verejtje (Remarks)
1	Ana Kolina		5	
2	Antonela Prifti		8	
3	Armela Toska		5	
4	Arsilda Bundo		10	
5	Donald Skenderaj		10	
6	Egidiius Hysenbelliu		6	
7	Elda Caushaj		9	
8	Elorja Celaj		7	
9	Emine Kaja		9	
10	Eni Basha		10	
11	Eni Hodaj		7	
12	Enid Bidaj		7	
13	Enxhi Gjeli		5	
14	Esmeralda Buzi		8	
15	Evja Shehaj		9	
16	Henri Hasani		6	
17	Hersa Duraj		9	
18	Ina Shehu		9	
19	Klajdi Mataj		8	
20	Klea Shyti		7	
21	Kristi Noska		9	
22	Ledia Ndreu		10	
23	Niko Goga		7	
24	Renalda Kastrati		8	
25	Saimir Peli		10	
26	Silvi Lera		5	
27	Stela Shyti		6	
28	Xhilda Coko		7	
29	Xhuliana Arapi		7	

Emri i mesuesit: Lefter Leka(the teacher)

KLASA X-B, Shkolla e Mesme e Pergjithshme, Librazhd-2011(General Secondary School-Librazhd, class X-B)

Nr	Emri, Mbiemri(full name)	Kapitulli i meparshem(previous chapter)	Fund i kapitullit (experimental chapter)	Verejtje (Remarks)
1	Albana Rrumbullaku		7	
2	Amarilda Kotorri		8	
3	Aurora Almadhi		8	
4	Aurora Dhamato		8	
5	Bitila Tusha		8	
6	Denisa Arapi		9	
7	Denisa Trifka		7	
8	Elvira Qosja		7	
9	Egla Cullahi		10	
10	Emanuela Cela		7	
11	Eni Kurti		9	
12	Esmeralda Brahja		8	
13	Gazela Vladi		8	
14	Griselda Qosja		5	
15	Ina Bucka		8	
16	Izabela Gllava		7	
17	Jonida Hoxha		8	
18	Julina Mira		7	
19	Olejda Collaku		7	
20	Olta Sado		7	
21	Ornela Teta		7	
22	Paola Pali		8	
23	Silvana Hoxha		7	
24	Suela Abazi		7	
25	Suzana Nikaj		7	
26	Tatiana Merko		7	
27	Xhensila Sado		8	
28	Xhuljeta Kora		7	
29	Griselda Latollari		7	

Emri i mesuesit: Luljeta Blloshmi(the teacher)

Following table(scanned copy): The Secondary School "Dhaskal Todri", Elbasan(The marks of previous chapter and end of experimental chapter-systems of equations; class 10-A)

Shkolla "[illegible] Todri"

2010 - 2011

Klasa 10 A

Sistemet e Ekuacioneve

Nr	Emri mbiemri	Detyra pa programin	Detyra pas programit	Verejtje
1	Ana Boja	9	10	
2	Anxhela Sadushi	7	8	
3	Anxhela Sulanjaku	10	10	
4	Aurela Hulloli	9	10	
5	Besnik Pallugi	7	7	
6	Dejvi Qogi	4	4	
7	Demiren Vila	6	9	
8	Endi Hreka	8	9	
9	Evi Hadaifeiu	4	7	
10	Erxhelina Çapja	6	8	
11	Hasan Biçja	6	9	
12	Ina Xibraku	7	6	
13	Iris Pilo	7	9	
14	Jona Hruja	8	10	
15	Klajdi Gryksht	8	8	
16	Klaudio Gica	7	9	
17	Kristi Janku	8	9	
18	Maringlen Dyrma	7	7	
19	Meliana Shingjergji	7	10	
20	Myrteza Shavra	5	5	
21	Nadire Cankja	8	9	
22	Nikola Hristo	6	7	
23	Petrika Tarusha	5	5	
24	Petrit Samarxhiu	10	10	
25	Rexhina Musaku	7	7	
26	Roxhers Muçaj	8	9	
27	Sabina Xhaferri	6	10	
28	Sindi Brakollari	8	8	
29	Sindi Samarxhiu	8	10	
30	Stela Danga	6	8	
31	Vasil Gjini	10	10	
32	Xhesjana Muça	8	10	
33	Xhejsila Tashi	7	6	
34	Xhuljeta Blloku	8	9	
35	Xhuljana Hysa	6	5	
36	Klejdi Kurti	4	4	

Nota mes: 7,08 n. mes: 8,05

Kalueshmeria: 91,7% ; 94,4%

Mësuesi lëndës

Shpëtime Sulanjaku

Shkolla "Ahmed Tochi"
2010 – 2011

Sistemet e Ekuacioneve

Klasa 10 B

Nr	Emri mbiemri	Detyra pa programin	Detyra pas programit	Verejtje
1	Aldo Luta	5		
2	Ana Vrioni	9		
3	Anjeza Doka	9		
4	Anjeza Piligriu	5		
5	Anita Rucl	8		
6	Anxhela Cili	4		
7	Anxhela Imeri	9		
8	Arlisa Salliu	5		
9	Demokrat Meme	6		
10	Denisida Drezhi	9		
11	Endi Balza	6		
12	Flaminda Simoni	7		
13	Franceska Pepa	6		
14	Iris Dedja	7		
15	Jonalda Ruçi	8		
16	Kejvin Kolo	10		
17	Klementjan Mirako	8		
18	Margo Masa	6		
19	Marsit Arapi	7		
20	Mehmet Ipeku	6		
21	Metin Pepa	8		
22	Nensi Skënderi	9		
23	Olsi Bicaku	10		
24	Qamil Jolla	8		
25	Renata Asllani	8		
26	Renato Bolo	8		
27	Sabina Hasa	7		
28	Vilma Bici	6		
29	Xhafize Kalemaj	7		
30	Xhuljana Pasha	8		
31	Irida Thomoli	8		
32	Kristjan Guxha	7		
33	Dejon Dedi	8		
34	Stiljano Belegu	7		
35	Sazana Thaçi	8		
36	Aldi Stringa	6		
37	Arbër Dumani	6		
38	Klevis Kodzodej	7		

Nota mes.: 7,26
Kalueshmëria: 97%

Mësuesi i lëndës
Sefetim Sulaqola

Above Table(scanned copy): The Secondary School “Dhaskal Todri”, Elbasan(The marks of end of chapter on Systems of Equations without GeoGebra method; class 10-B (control group)).

5. Samples of the Survey Questionaries

No	Questions	Qualitative Estimation				
		Excellent	Good	Fair	Poor	Undecided
1	How would you rate the overall quality of this course?					
2	Overall, how would you rate the teacher?					

Questionary on Teacher Professional Evaluation

Please tell us how much you agree or disagree with the following statements.

	Statements	Student’s Answers				
No		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	The teacher is well prepared for class sessions.					
2	The teacher answers questions carefully and completely.					
3	The teacher uses examples to make the materials easy to understand.					
4	The teacher stimulated interest in the course.					
5	The teacher made the course material interesting.					
6	The teacher is knowledgeable about the topics presented in the course.					
7	Was current with developments in field.					
8	Integrated theory with practice.					
9	Communicated clearly					

	mathematical topics.					
10	Course assignments are interesting and stimulating.					
11	The teacher's use of technology enhanced learning in the classroom.					
12	Directions for course assignments were clear.					
13	Used helpful examples and references.					
14	The difficulty level of this course was appropriate for me.					
15	This course is one of the most difficult I have taken.					
16	The notes and readings required for the course were appropriate.					
17	Showed expertise in the subject of mathematical analysis.					
18	I would recommend this course to others.					
19	I would recommend this teacher to other teachers and students.					

Teacher Attitude Evaluation

	Statements	Student's Answers				
No	The Teacher:	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1	Showed respect for all students.					
2	Was open and receptive to students' ideas.					
3	Maintained the work in small groups.					
4	Showed sensitivity to the educational differences.					
5	The teacher treats students respectfully..					
6	The teacher is fair in dealing with students.					
7	The teacher makes students feel comfortable about asking questions.					
8	Maintained communication confidentiality.					
9	Communicated candidly and constructively.					
10	Has advanced my knowledge					

	of the subject.					
11	Showed enthusiasm toward the subject.					
12	Maintained lasting relations: student-teacher.					
13	Encouraged student interaction.					
14	Was prepared for each class.					
15	Maintained positive classroom environment:					
16	Made class enjoyable as well as educational.					
17	Would you take another class from this professor? .					

Questionary on GeoGebra software impact in teaching and learning.

	Statements	Student's Answers				
No		Strongly Disagree(1)	Disagree(2)	Undecided(3)	Agree(4)	Strongly Agree(5)
1	This was a required course.			5	8	16
2	I met my expectations by taking this course.		5	2	18	4
3	I improved a lot my computer skills by taking this course.		4	1	16	8
4	The syllabus clearly described the course content.		11	6	10	2
5	The course clearly defined assignments.		3	8	12	6
6	The course clearly described class activities.	2	2	5	12	8
7	Course assignments were at an appropriate level of difficulty.		6	1	14	8
8	The teacher was good at facilitating class discussion.			3	12	14
9	All students became familiar with GeoGebra.			2	14	13
10	All students had fun with GeoGebra.			3	10	16

11	The size of the class was appropriate.	4	9	8	8	
12	The notes and other readings appropriately covered the course content.	2	6	5	12	4
13	The teacher effectively used GeoGebra tools and GeoGebra-wiki to teach the course.			3	16	10
14	My knowledge and skills in mathematics are increased more by using GeoGebra tools than the usual tools in teaching and learning.		6	4	11	8
15	I gain more math knowledge and skills by experimenting with GeoGebra.		7	5	12	5
16	I believe the other classes will increase their math level by using GeoGebra.		8	6	12	3
17	My interest on mathematics is increased much more now that I can use Geogebra in learning math.		5	4	14	6
18	The current textbooks should continue to be used.		4	3	12	10
19	The difficulty level of this course was appropriate for me.		3	2	10	14
20	This course is one of the most difficult I have taken.	10	12	7		
21	I feel more secure in math by exploring with GeoGebra		4	2	16	7
22	I would recommend this course to other students and teachers.			7	15	7

6. Scanned copies of Journals of publications

The scientific bulletin DOKTORANDUSZOK FORUMA, MISKOLC 2009, November 5 , where is published the paper: THE POWER OF DOUBLE REPRESENTATION OF GEOGEBRA(author: Pellumb Kllogjeri).

TARTALOMJEGYZÉK

Scanned copy from the Journal: DOKTORANDUSZOK FORUMA

IRODALOM

[1] Karlheinz Roth: **Tervezés katalógussal**. *Budapest, Műszaki Könyvkiadó,* 1989.

[2] G. Pahl-W. Beitz: **Engineering Design**. *London, The Design Council,* 1984.

[3] VDI Richtlinie 2222 Bl. 1: **Methodisches Entwickeln von Lösungsprinzipien**. *Düsseldorf*, 1997.

[4] H.-J. Franke, - M. Deimel: **Selecting and combining methods for complex problem solving within the design process.** *International design conference – design 2004* Dubrovnik, May 18 - 21, 2004.

[5] Nagy, L – Jakab, E.-Takács, Gy.: **Mechatronikai szerkezetek funkcióanalízise,** *MicroCAD 2007. Konferencia kiadvány*, Miskolc, 2007. március 22-23.

[6] Tajnafői J: **Szerszámgéptervezés II.** *Budapest, Tankönyvkiadó,* 1990.

[7] **Alternators and Starter Motors.** *Stuttgart, Robert Bosch GmbH,* 2003.

[8] Birkhofer, H.: **Analyse und Synthese der Funktionen technischer Produkte. Fortschr. Ber. VDI-Z.** *Reihe 1, Nr. 70. Düsseldorf: VDI-Verlag* 1980.

THE POWER OF DOUBLE REPRESENTATION OF GEOGEBRA

Pellumb Kllogjeri
MS, Lecturer of Probability and Statistics
University "Aleksander Xhuvani ", Elbasan, Albania.
Faculty of Natural Sciences
Department of Mathematics and Informatics

Abstract

In present we witness the high-speed progress of computer-based education, which allows educators and students to use educational programming language and e-tutors to teach and learn, to interact with one another and share together the results of their work. The mathematical programmes, teaching and learning software are probably one of the most important tools the computer-based education is based on. Beside many mathematical programmes, one which is used by a daily increasing number of users throughout the world is GeoGebra (published by Markus Hohenwater,(2004) explicitly linking geometry and algebra. The software affords a bidirectional combination of geometry and algebra that differs from earlier ones. The bidirectional combination means that, for instance, by typing in an equation in the algebra window, the graph of the equation will be shown in the dynamic and graphic window. This programme is preferred so much, probably because of its three main features: the double representation of the mathematical object (geometric and algebraic), no strong requirements as to the age and the pre-knowledge in using it (the students of the elementary school can use it as well) and it is offered free of charge (simply by downloading it). In this paper we concentrate on the double representation of the mathematical object and its advantages in explaining and forming mathematical concepts and performing operations.

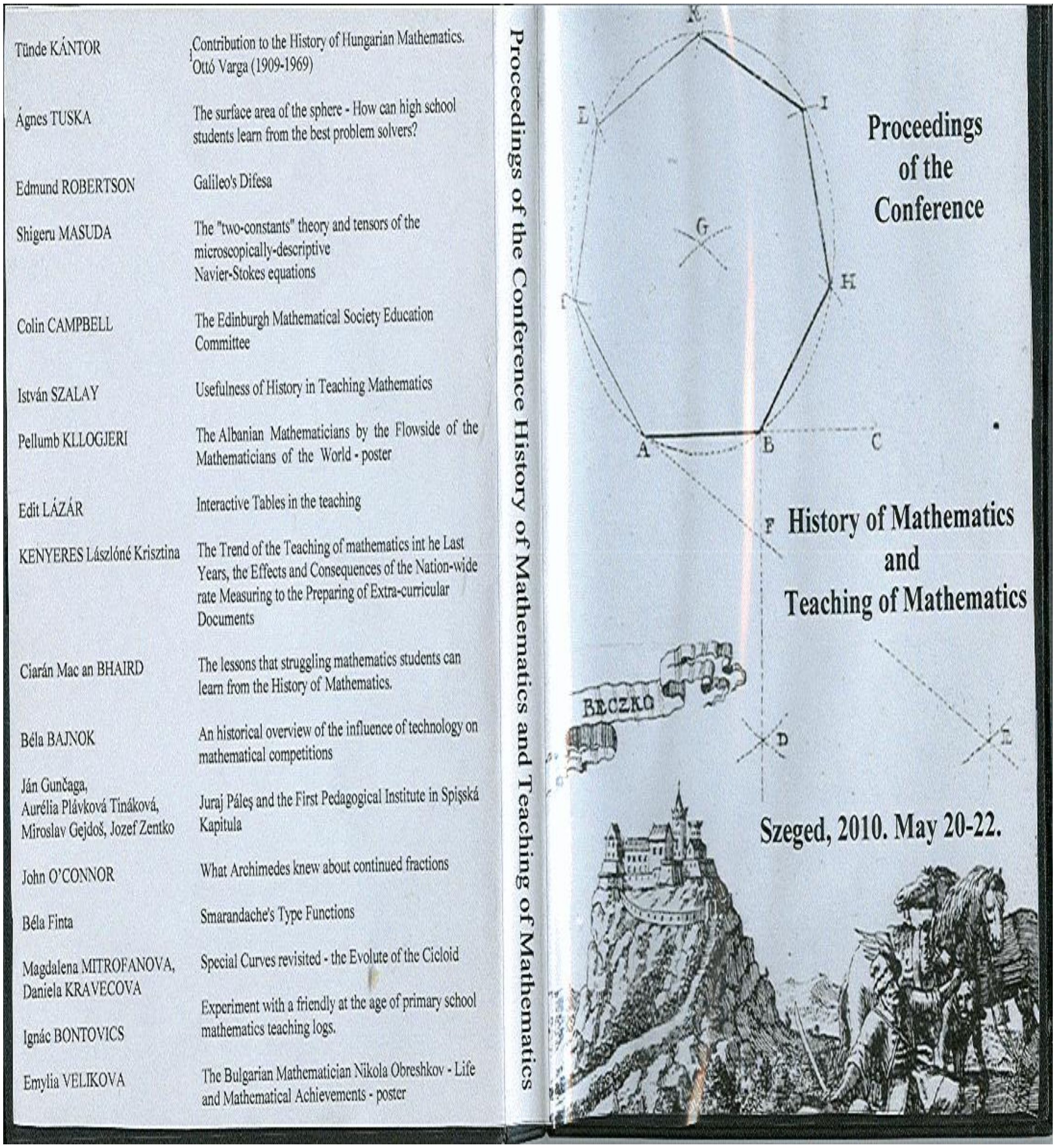
ACKNOWLEDGEMENTS

I am grateful to Dr. Péter Körtesi (University of Miskolc, Hungary, Co-chair of Department of Analysis), my scientific adviser for my PhD studies in the field of Mathematical and Statistical Didactics, who has encouraged me to work with didactical tools and methods and write didactical papers. Also, I want to thank Dr. András Kovács (University of Debrecen, Hungary, Institute of Natural Sciences and Informatics), who, during the time I was in the University of Debrecen as CEEPUS visiting teacher (Sept. 2009), helped me by drawing my attention on many didactical and methodical books.

What is GeoGebra, what are its main characteristics and tools?

GeoGebra is a dynamic mathematics software for schools that joins geometry, algebra, and calculus, it is an interactive geometry system. With Geogebra is possible to do constructions with points, vectors, segments, lines, and conic sections as well as functions while changing them dynamically afterwards. The two characteristic views of GeoGebra are: an expression in the algebra window

Scanned copy from the Conference : History of Mathematics, Szeged, Hungary. The Albanian Mathematicians by the Flowside of the Mathematicians of the World



Tünde KÁNTOR	Contribution to the History of Hungarian Mathematics. Ottó Varga (1909-1969)
Ágnes TUSKA	The surface area of the sphere - How can high school students learn from the best problem solvers?
Edmund ROBERTSON	Galileo's Difesa
Shigeru MASUDA	The "two-constants" theory and tensors of the microscopically-descriptive Navier-Stokes equations
Colin CAMPBELL	The Edinburgh Mathematical Society Education Committee
István SZALAY	Usefulness of History in Teaching Mathematics
Pellumb KLLOGJERI	The Albanian Mathematicians by the Flowside of the Mathematicians of the World - poster
Edit LÁZÁR	Interactive Tables in the teaching
KENYERES Lászlóné Krisztina	The Trend of the Teaching of mathematics int he Last Years, the Effects and Consequences of the Nation-wide rate Measuring to the Preparing of Extra-curricular Documents
Ciarán Mac an BHAIRD	The lessons that struggling mathematics students can learn from the History of Mathematics.
Béla BAJNOK	An historical overview of the influence of technology on mathematical competitions
Ján Gunčaga, Aurélia Plávková Tináková, Miroslav Gejdoš, Jozef Zentko	Juraj Páleş and the First Pedagogical Institute in Spişská Kapitula
John O'CONNOR	What Archimedes knew about continued fractions
Béla Finta	Smarandache's Type Functions
Magdalena MITROFANOVA, Daniela KRAVECOVA	Special Curves revisited - the Evolute of the Cicloid
Ignác BONTOVICS	Experiment with a friendly at the age of primary school mathematics teaching logs.
Emylia VELIKOVA	The Bulgarian Mathematician Nikola Obreshkov - Life and Mathematical Achievements - poster

Proceedings of the Conference History of Mathematics and Teaching of Mathematics

Proceedings of the Conference

History of Mathematics and Teaching of Mathematics

Szeged, 2010. May 20-22.

Scanned copy from Disputationes Scientificae Universitatis Catholicae in Ruzomberok: katolicka univerzita, where is published the paper "Extended Help Files in Software GeoGebra"(authors: P. Kortesi, P. Kllogjeri)

Rozšírenie súborov pomocníka v softvéri GeoGebra

Extending Help Files in Software GeoGebra

Péter Kőrtesi, Pellumb Kllogjeri

Resume

Mathematics software users must know the features of different programs like Maple, GeoGebra, Matlab,..., and they must be easily adopted to each one of them. We are trying to make use of some commands and technique of Maple for GeoGebra in completing the already existing summary help files in order to have a set of instructions for the teachers and the students who use GeoGebra ,such meeting some of the most necessary needs of them in using GeoGebra tools and commands for doing work.

Key words: Plotting functions, Maple, GeoGebra, help files, inverse of a function.

MathEduc subject classification: E 10, I 10, I 20

1. INTRODUCTION

While using Maple in CEEPUS Summer University in Miskolc we found extremely useful the help files of the software. Comparing Maple and GeoGebra software (also considering other types of math software) one can see the differences regarding the commands, the tools, the help files, the syntax used and the possibilities of carrying out a task. We can also see that there are tasks that can be carried out using Maple but not with GeoGebra and reversely. Our suggestion is that the math software users must know these two useful kinds of software, others also, and they must be easily adopted to each one of them. We try to put together something similar for GeoGebra in completing the already existing summary help files, especially for constructing graphs, in order to have a set of instructions for the teachers and the students who use GeoGebra, meeting in this way some of the most necessary needs of them in using GeoGebra tools and commands for doing work.

2. CONSIDERATIONS REGARDING THE PLOTTING

2.1 With Maple,

graphs of the functions can be plotted by putting conditions on their domain and range, on the graphs colors etc.

There are different options of plotting:

1. No boundaries on function domain
   The syntax and command are: < Plot(sin(t),t); < Plot(3/x,x), ...
2. Boundaries on function domain and range
   < Plot(sin(t),t=-Pi..Pi); < Plot(sin(x)+cos(x),x=0..Pi);
   < Plot(tan(x),x=-2*Pi..2*Pi,y=-4..4); < Plot(sin(x),x=0..infinity); .....
3. Several graphs on a pad and different graphs colors(multiple plots)
   < Plot((sin(x),x-1/6*x^3),x=0..2,color=(red,blue),style=(point,line));
4. Point plots

< 1=((n,sin(n)),n=1..10)

*** There are other plots also.

2.2 With GeoGebra

Using GeoGebra there are many advantages vs Maple:

1 it is possible to see the interaction between algebra objects and the respective geometric objects
2 there are many tools at one's disposal for constructing geometric objects and applying commands
3 a theorem or a math statement or a property can be tested
4 research work can be done and conclusions on different math topics can be drawn
5 GeoGebra can also be used by the students of early age, and they can use it as a playing tool
6 There are many other things to mention but it is not our purpose here.

3. PROBLEMS WITH PLOTTINGS, SOLUTIONS, EXTENDED FILES FOR PLOTTING A PARAMETRIC FUNCTIONS FAMILY

As far as we have seen, using GeoGebra the graphs of any functions can be plotted using the following syntax and the commands, and with the options:

1. No boundaries on the domain: In the input bar, type (for ex.) f(x)= =x+sin(x)

2. Boundaries on function domain (having two alternatives): In the input bar, type f(x)=Function(x+sin(x),-pi,4*pi)

Or, using the If command: f(x)=If(x>2,1/3*x-1/3), where is constructed the graph of the given linear function for x>2. (one command If)

Or, f(x)=If(x<2,If(x>0,2*x,cos(x))). (Here we have two commands If).

Teachers and students need explanation and instruction on this case.

In this case, the first If, conditioned by x<2, defines the domain of the function having two representations: 2*x and cos(x). The second If, conditioned by x>0, defines a sub-domain of the first domain and which serves as domain for the first function in the bracelets. By this we mean that the first function in the bracelets(2*x) is related to the second If with x>0, whereas the second

Copies from the Romanian Journal where is published the paper "Remarks on Help Files in GeoGebra". Authors: PÉTER KÖRTESI, UNIVERSITY OF MISKOLC, MISKOLC (HUNGARY)

PELLUMB KLLOGJERI, *UNIVERSITY "ALEKSANDER XHUVANI", ELBASAN (ALBANIA)*

RoJEd Romanian Journal of Education

despre revistă

echipa de redacţie

informaţii pentru autori

articole

Volumul 1 numărul 1

Volumul 1 numărul 2

autorii noştri

contact

ISSN: 2067–8347

Learning Mathematics, Doing Mathematics: Deductive Thinking and Construction Tasks with *The Geometer's Sketchpad*

A. Homero Flores Samaniego, Colegio de Ciencias y Humanidades-UNAM, Conjunto Habitacional Los Culhuacanes (Mexico)

Abstract. A deductive thinking can be considered as the concatenation of ideas, each one determined by the previous one. In mathematics, deduction is the way in which we validate a conjecture, using general facts to justify less general or particular facts. All individuals possess this way of constructing thoughts and jumping into conclusions, but we need to practice it in order to develop it deeper and make more educated decisions.

In construction tasks with The Geometer's Sketchpad (GSP) it is possible to explore the construction itself, make conjectures and try to validate them, putting ourselves in a theoretical context and making use of deduction as the means of getting the right answers.

Learning Mathematics, Doing Mathematics, is a teaching model in which I propose, among other things, the fostering of a deductive thinking mainly through construction tasks in Euclidian Geometry. In this paper I present the main features of the model and some GSP activities that are helpful in the fostering of a deductive thinking.

Keywords: learning Mathematics; Geometer's Sketchpad; deductive thinking

P. 7-14 PDF file

---

3. Remarks on Help Files in GeoGebra

PÉTER KÖRTESI, *UNIVERSITY OF MISKOLC, MISKOLC (HUNGARY)*

PELLUMB KLLOGJERI, *UNIVERSITY "ALEKSANDER XHUVANI", ELBASAN (ALBANIA)*

Abstract. Math software users must know the features of different math software like Maple, GeoGebra, Matlab. Comparing Maple and GeoGebra software (also considering other math software) one can see the differences regarding the commands, tools, the help files, the syntax used and the possibilities of caring out a task. Also, we see that there are tasks that can be carried out using Maple but not with GeoGebra and Matlab, and they must be easily adopted to each one of them. We try to make use of some commands and techniques of Maple for GeoGebra in completing the already existing summary help files in order to have a set of instructions for the teachers and the students who use GeoGebra, meeting this way some of the most necessary needs of them in using GeoGebra tools and commands for doing work.

Keywords: Math software; dynamical Geometry program; GeoGebra; computer Algebra system

P. 15-22 PDF file

---

4. New possibilities of education – the adaptive eLearning environment

Kateřina Kostolányová, University of Ostrava, Ostrava (Czech Republic)

Jana Šarmanová, University of Ostrava, Ostrava (Czech Republic)

Abstract: The basic idea of the adaptive eLearning development is to respect and support different learning styles of students who can be thus offered a more effective, more-user friendly teaching environment of better quality. This contribution outlines existing situation in the area of the Learning Management System – LMS) and its potential future implementation with regards to the system adaptability in terms of students and the author of the course.

Key words: eLearning, LMS, learning styles, study materials, multimedia

P. 23-26 PDF file

Link :
http://inderscience.metapress.com/app/home/contribution.asp?referrer=parent&backto=issue,3,9;journal,3,8;linkingpublicationresults,1:120778,1

GeoGebra: a global platform for teaching and learning math together and using the synergy of mathematicians

Pellumb Kllogjeri [A1] and Bederiana Shyti [A2]

[A1] Faculty of Natural Sciences, Department of Mathematics and Informatics, University 'Aleksander Xhuvani', L. Konf. Labinotit, P.61, Elbasan, Albania.
[A2] Faculty of Natural Sciences, Department of Mathematics and Informatics, University 'Aleksander Xhuvani', Rr. Rinia, Elbasan, Albania

Abstract:

The computer-based education allows educators and students to use educational programming language and e-tutors to teach and learn, to interact with one another and share together the results of their work. It is done possible by special electronic tools such as the mathematical programmes. One of them that is embraced and used by a daily increasing number of users throughout the world is GeoGebra, because of three main features: the double representation of the mathematical object, there are not strong requirements as to the age and the knowledge in using it and, it is offered free of charge. In this paper we are concentrating in the double representation of the mathematical object and its advantages in explaining and forming mathematical concepts and performing operations, in the global opportunities for using GeoGebra and the benefits of using it by cooperating and sharing experiences.

Keywords:

GeoGebra, double representation, virtual tools, dynamic demonstration, computer programming, interactive environment, knowledge sharing, communicative bridge, e-learning, electronic learning, online learning, mathematics education, maths education, math education, mathematical concepts

Link: http://adsabs.harvard.edu/abs/2010telq.conf..681K

GeoGebra: A Global Platform for Teaching and Learning Math Together and Using the Synergy of Mathematicians

Kllogjeri, Pellumb

Technology Enhanced Learning. Quality of Teaching and Educational Reform, Communications in Computer and Information Science, Volume 73. ISBN 978-3-642-13165-3. Springer-Verlag Berlin Heidelberg, 2010, p. 681

In present age we are witnesses and practioners of computer-based education which is highly speed progressing. The computer-based education allows educators and students to use educational programming language and e-tutors to teach and learn, to interact with one another and share together the results of their work. The computer-based education is done possible by special electronic tools among which the most important are the mathematical programmes. There are many mathematical programmes, but one which is being embraced and used by a daily increasing number of users throughout the world is GeoGebra. The recently published software GeoGebra by Markus Hohenwater (2004) explicitly links geometry and algebra. GeoGebra affords a bidirectional combination of geometry and algebra that differs from earlier software forms. The bidirectional combination means that, for instance, by typing in an equation in the algebra window, the graph of the equation will be shown in the dynamic and graphic window. This programme is so much preferred because of its three main features: the double representation of the mathematical object (geometric and algebraic), there are not strong requirements as to the age and the knowledge in using it(the students of the elementary school can use it as well) and, it is offered free of charge(simply by downloading it). In this paper we are concentrating in the double representation of the mathematical object and its advantages in explaining and forming mathematical concepts and performing operations, in the global opportunities for using GeoGebra and the benefits of using it by cooperating and sharing experiences.

Keywords: Geogebra, double representation, virtual tools, dynamic demonstration, research work using computer programmes, interactive environment, platform of sharing knowledge and results, communicative bridge

DOI: 10.1007/978-3-642-13166-0_95

Scan of Author Mapper

AuthorMapper

http://www.authormapper.com/search.aspx?val=subject%3AComputer+Science&sort=asc&val=country%3AAlbania

SEARCH

Subjects

Countries

- Albania [x]8(%)
- Austria1(%)
- France1(%)
- Italy1(%)
- Japan1(%)

INSTITUTION

Geogebra for Solving Problems of Physics
Organizational, Business, and Technological Aspects of the Knowledge Society (2010) 112: 424-428, January 01, 2010
By Kllogjeri, Pellumb; Kllogjeri, Adrian

Download PDF | Post to Citeulike

Today is highly speed progressing the computer-based education, which allowes educators and students to use educational programming language and e-tutors to teach and learn, to interact with one another and share together the results of their work. In this paper we will be concentrated on the use of GeoGebra programme for solving problems of physics. We have brought an example from physics of how can be used GeoGebra for finding the center of mass(centroid) of a picture(or system of polygons). After the problem is solved graphically, there is an application of finding the center of a real object(a plate)by firstly, scanning the object and secondly, by inserting its scanned picture into the drawing pad of GeoGebra window and lastly, by finding its centroid. GeoGebra serve as effective tool in problem-solving. There are many other applications of GeoGebra in the problems of physics, and many more in different fields of mathematics.

7. Pictures from teaching with GeoGebra process

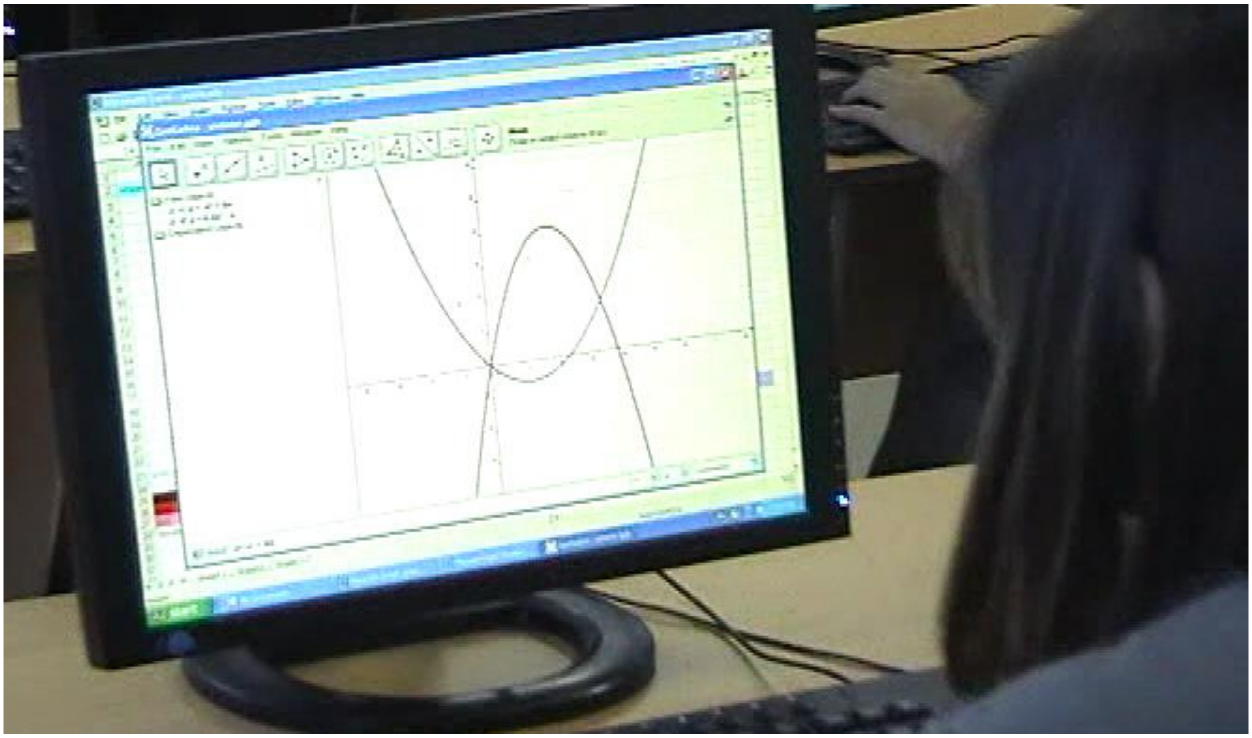
School “Dhaskal Todri”, Elbasan

School “Dhaskal Todri”, Elbasan

School “Dhaskal Todri”, Elbasan

School “Dhaskal Todri”, Elbasan

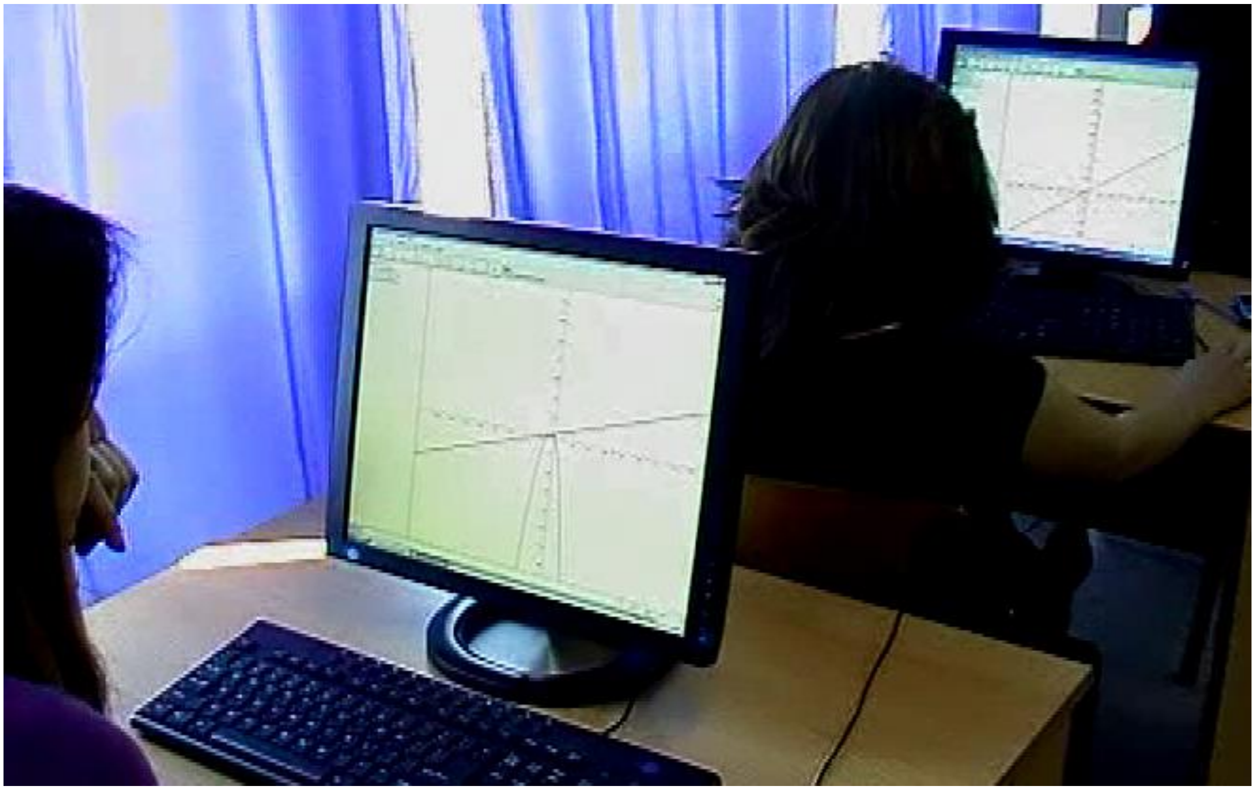
School “Dhaskal Todri”, Elbasan



School "Jani Kilica”, Fier

School “Jani Kilica”, Fier

School “Jani Kilica”, Fier



School “Jani Kilica”, Fier

University of Tirana

Faculty of Natural Sciences