Introduction to Corporate Financial Decisions for Engineers and Engineering Managers

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1. An Introduction to Financial Decisions

"A dollar today is worth more than a dollar tomorrow, because the dollar today can be invested to start earning interest immediately. This is the first basic principle of finance (Brealey – Myers, 2005)."

In the following chapters, we try to describe how we can cope with routine financial decisions and how we can make good investment decisions. In this chapter, we examine the difference between investment and financing decisions and the importance of investment.

1.1. What is Finance? Corporate Finance - Financial Decisions

Why is finance important to you? Every day we have to make financial decisions; for example, we go to the supermarket and purchase different goods, we can save our money, and firms have to make investment decisions.
We can distinguish two main types of corporate financial decisions:

- **Financing decisions and**
- **Investment decisions**

Investment decisions: Investment decisions include managing assets, such as deciding what real assets to hold and what projects to invest in. As we will see later on, one of the objectives of corporate finance is to choose assets and projects that yield a return greater than the minimum acceptable rate.

Financing decisions: Financing decisions include raising cash, purchasing real assets and deciding how the chosen investment should be financed.

1.2. The importance of investments

Purchases of new plant, buildings, vehicles and machinery along with additions to stocks, are called **investment** (I).

1.2.1. Investment at the macroeconomic level

The amount of investment is a component of the aggregate demand in an economy. In our macroeconomics studies we learnt how important investment is in an economy. The amount of investment has a multiplier effect on the Gross Domestic Product (GDP).

Picture 1.

Planned investment expenditure is the amount firms intend and are able to spend on new capital goods in order to realize profit. Firms purchase new capital goods, such as tools, machinery, vehicles and buildings.

Investment decisions depend on profit expectations (β) and the real interest rate (i):

Real interest rate (i) $$I(i,\ \beta)\ .$$

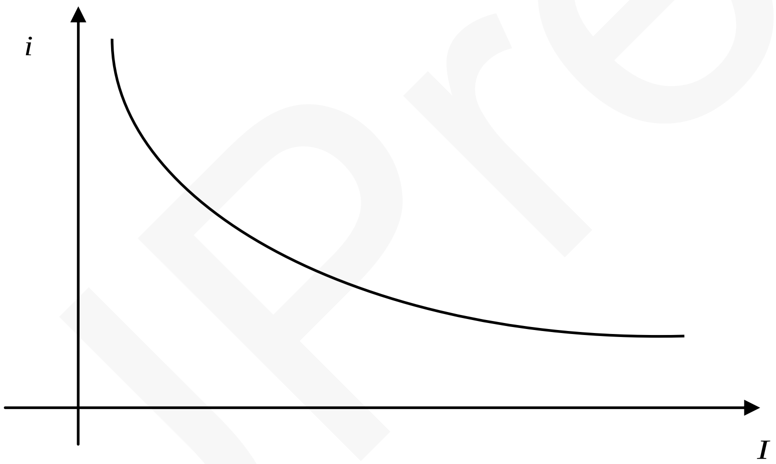
The real interest rate is the interest rate paid by a borrower and received by a lender after taking into account changes in the value of money resulting from inflation.

The higher the real interest rate, the lower the amount of investment expenditure, other things remaining unchanged. A lower interest rate makes a greater number of alternative possibilities more profitable.

$$\frac{\partial I}{\partial i} < 0$$

The investment demand function shows the relationship between the real interest rate and the level of planned investment, other things remaining unchanged.

Figure 1.1 The planned investment (I) demand function at different interest rates



The rate of interest represents a key factor in the decision about whether the investment should be carried out. If the interest rate is lower, further alternative investment - such as banking investment – may be accepted, which means a greater chance of reaching the profit threshold.

Picture 2.

Furthermore, investments are, in many cases, financed from credit, which means that if the real interest rate is higher, loans are more expensive, and the demand for loans decreases so the amount of investments falls.
There is a difference between nominal and real interest rates. According to the Fisher equation, the real interest rate is approximately equal to the difference between the nominal interest rate (r) and the rate of inflation (π):

$$i = r - \pi.$$

Suppose that the nominal interest rate is 5.8 percent and assume Peter lends out 100 HUF for one year. Peter will receive 105.8 forints in a year's time when the interest is due and the loan is repaid:

$$loan = 100\ HUF,$$
$$interest = 100 \cdot 0.058 = 5.8\ HUF,$$

$$100 + 100 \cdot 0.058 = 105.8\ HUF.$$

If the rate of inflation is zero - which means that prices are unchanged – Peter benefits by 5.8 forints. Suppose the rate of inflation is 2.5 percent; Peter will receive 105.8 HUF, which has a lower purchasing power than if the rate of inflation were zero. In the case of an inflation rate of 2.5 percent, Peter could buy products in exchange for 100 HUF, but one year later the price of these products would be 102.5 forints. This means the real interest of the loan is 3.3 percent.

Appendix 1

Irving Fisher recommended an accurate estimate of the calculation of the real interest rate:

$$i = \frac{M_{t+1} - M_t}{M_t} = \frac{\frac{M_t \cdot (1+r)}{1+\pi} - M_t}{M_t},$$

where M_{t+1} (M_t) shows the value of money in period $t+1$ (t). π is the percentage change in the price level from period t to period ($t+1$).

$$i = \frac{(1+r)}{1+\pi} - 1,$$

$$(i+1) \cdot (1+\pi) = 1 + r,$$

$$i + 1 + i \cdot \pi + \pi = 1 + r,$$

$$i + i \cdot \pi + \pi = r,$$

$$i = r - i \cdot \pi - \pi,$$

if the real interest rate and the inflation rate are low, the multiplication of the two rates ($i \cdot \pi$) produces a low result, too, so we can ignore it:

$$i \cdot \pi \approx 0,$$

we get

$$i = r - \pi.$$

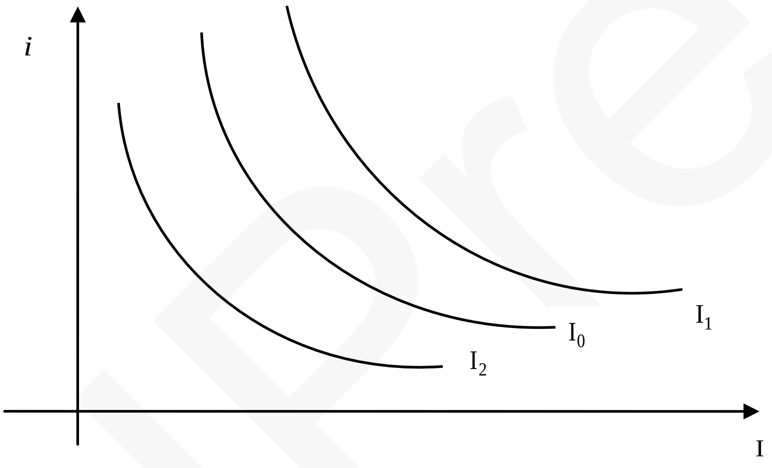
Profit expectations (β)

Many factors influence economic actors' expectations about the future development of the economic environment. Among the main affecting factors are prices, taxes on labour income or company profits, the instability of the regulatory system and the degree to which existing capital is being utilized, the global environment and political stability. For example, the collapse of the Soviet Union and the emergence of the new republics of Central and Eastern Europe are likely to have had a large impact on profit expectations in the 1990s.

Sometimes firms are pessimistic about future profits, sometimes they are optimistic, and sometimes their expectations are average. When firms become optimistic or their optimism increases, the investment demand curve shifts to the right; it shifts to the left as pessimism grows, or firms' expectations become pessimistic (Figure 1.2). There is a positive relationship between investment and expectations about the future:

$$\frac{\partial I}{\partial \beta} > 0$$

Figure 1.2 Planned investment (I) demand function by different profit expectations



Suppose the investment demand curve is I_0 with average profit expectations (Figure 1.2). If the firms' profit expectations rise, firms become more optimistic, and planned investment increases at each expected real interest rate level. The investment demand curve shifts to the right, to I_1. Otherwise, if the firms' profit expectations decrease, planned investment decreases, too, and the investment demand curve shifts to the left, to I_2 (Figure 1.2).

1.2.2. Investment at the microeconomic level at the firm level

At the firm level, investment is determined by many factors such as the funds available, expected benefits, profit expectation, economic environment, and the views/attitudes of stakeholders. We can distinguish different corporate stakeholders such as:

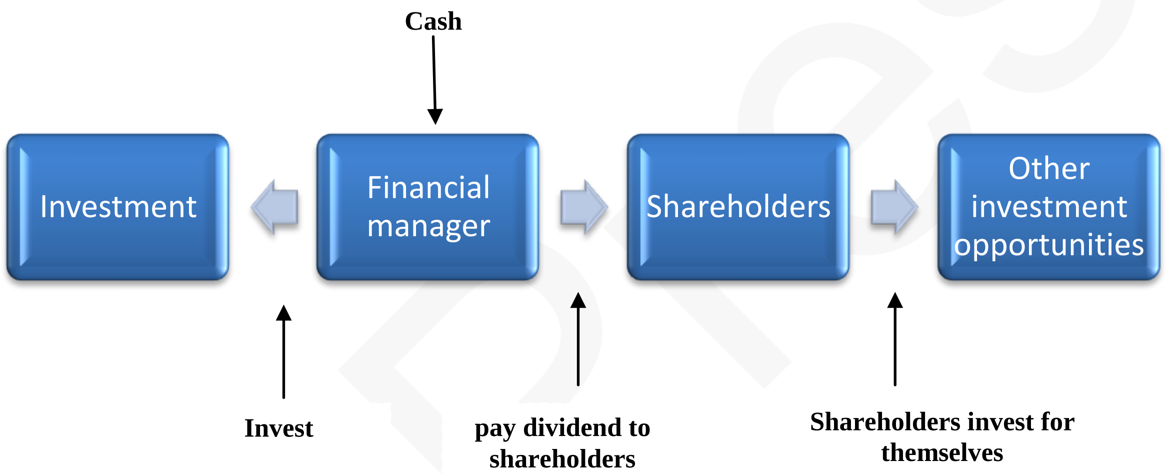
- Shareholders,
- Employees,
- Government,

Market actors, such as

- Customers,
- Production factor suppliers,
- Investors,
- Competitors.

Corporate stakeholders can influence corporate operation and production. However, the corporate stakeholders can, in their turn, be affected by corporate operation and production. The financial manager tries to decide to what extent short-term and long-term investments can be accepted. If the financial manager decides not to invest, more dividends may be paid out to investors, or added to the capital (Figure 1.3).

Figure 1.3 Investment decisions



Source: Brealey – Myers - Allen, 2014
Arrows represent possible cash flows or transfers.

However, if the financial manager decides to approve the investment, different economic actors can be affected by the investment. If the financial manager wants to carry out the investment, the decision can be a good or a bad decision for the shareholders. The result of the decision depends on the rate of return on the investment and on the rates of return from other projects. If the rate of return on the investment selected is higher than the rate of return from other opportunities, the shareholders would accept the investment offered by the financial manager (Figure 1.3). If the investment to be carried out by the financial manager yields higher returns than the shareholders can realize in the market, the shareholders will be satisfied and the stock price will increase. The minimum acceptable rate of return is called the hurdle rate (cost) of capital. The minimum acceptable rate of return is the opportunity cost of capital. If the financial manager decides to carry out the investment, the shareholders lose the opportunity cost of capital, i.e. the possibility to reinvest money on their own. The firm creates value if the investment selected yields higher returns than the opportunity cost of capital.

1.3. Terms and Questions

cost of capital,
financing decision,
Fisher theory,
hurdle rate of capital,
investment decision,
investment,
nominal interest rate,
opportunity cost of capital,
profit expectations,
rate of return,
real interest rate,
shareholders,
stakeholders.

Problems

Theoretical questions

1. What is the difference between the real and the nominal interest rate?

2. Give the definition of the opportunity cost of capital.

3. How can we calculate the real interest rate?

4. Explain what the relationship is between the nominal interest rate and the real interest rate.

5. What is the difference between a financing decision and an investment decision? Give an example of each.

6. Give the main factors influencing investment.

7. Explain why investment is important in an economy.

8. Give an example of a corporation's stakeholders.

9. In large firms, the management and shareholders are generally different. What is the reason for the separation of management and ownership?

Calculation exercise

1.

Suppose that your firm has 400 000 EUR to finance your employees' support now and next year. You would like to provide the same amount of support in the two periods. The annual interest rate is 10%.

a) Compute the amount of the support in each period.
b) How much should you invest now?

2. The Future Value of Money

We will study the different valuation methods in order to evaluate investment possibilities. The first part of this chapter deals with the calculation of the future value of money, and in the subsequent sections we study how to evaluate and choose from among the different investment possibilities. If we try to evaluate an investment, a bond or a stock, we generally determine the time value of expected future cash flows.
In financing decisions and investment decisions it is very important to take into account the time value of money. To make good decisions, we should know how to determine the time value of future cash flows.

2.1. Cash flows

Picture 3.

We can distinguish different types of cash flow patterns:
- lump-sum amount,
- annuity,
- uneven cash flows.

Suppose you deposit 100 000 EUR in your bank account. In this case, we talk about a lump-sum amount. The lump-sum amount is a single payment that is made now or in the future.

Lump-sum amount

A single payment of money that is made (or received) either today or at some date in the future. (Besley - Birgham, 2015)

Suppose that you pay in 10 000 EUR at the beginning of each year for the next 15 years. This series of payments represents an annuity. For example, a credit repayment instalment can be considered an annuity.

Annuity

"A series of payments of equal amounts at fixed equal intervals for a specified number of periods". (Besley - Birgham, 2015)

We can distinct two types of annuity:
- ordinary annuity and
- annuity due.

If the payments of equal amounts are made at the beginning of each year, the annuity is called an annuity due. However if the payments occur at the end of each year, the annuity is called an ordinary annuity. If you repay your housing loan, generally you have to make repayments of equal amounts at the beginning of each month or at the end of each month. These payments correspond to an annuity.

Uneven cash flows

"Multiple payments of different amounts over a period of time". (Besley - Birgham, 2015)

2.2. Future value of a lump-sum amount

Suppose you deposit 100 000 EUR in your bank account. The interest rate is 6 percent per year. How much will you have in one year? How much will you have in your account after 6 years? In this case, we talk about a lump-sum amount. The lump-sum amount is a single payment that is made now or in the future.

The present value of your money is 100 000 EUR; however, its future value is different.

Future value (FV)

Future value is the amount to which a cash flow or a series of cash flows will grow over a given period of time (t) when **compounded** at a given interest rate (r).

If you invest 100 000 EUR for one year at a 6% annual interest rate, your investment will grow to $1.06 \cdot 100\,000\ EUR$.

At the end of the first year, you will have 106 000 EUR:

$$FV_1 = 100\,000 + 100\,000 \cdot 0.06 = 100\,000 \cdot 1.06 = 106\,000\ EUR.$$

Suppose you reinvest the entire amount of your money for one year, when the annual interest rate remains the same. You will have 112 360 EUR at the end of the second year:

$$FV_2 = 106\,000 + 106\,000 \cdot 0.06 = 106\,000 \cdot 1.06 = 112\,360\ EUR.$$

At the end of the second year you will get both your money 106 000 EUR and the interest on your money (106 000 EUR). The future value of 100 000 EUR at the end of the second year is:

$$FV_2 = 100\,000 \cdot 1.06^2 = 112\,360\ EUR.$$

How much will you have if you deposit 100 000 EUR in the bank and leave it for 15 years at a 6 percent annual interest rate? The future value of your money in 15 years is the following:

$$FV_{15} = 100\,000 \cdot 1.06^{15} = 239\,655.8193\ EUR.$$

In the case of a compound interest calculation, the future value of C_0 invested for t periods at an interest rate r per period is:

$$FV_t = C_0 \cdot (1 + r)^t. \qquad (2.1)$$

In applying the equation (2.1) to compound interest calculation, suitable payment periods and interest rates should be taken into account. If the annual interest rate is given, t represents the number of years. However, if a monthly interest rate is given, t represents the number of months.

*An investment yields **compound interest** if the interest earned in the previous periods is left in the bank account. Compound interest is the interest earned on both the initial principal and the interest reinvested.*

2.3. Compound interest and simple interest

So far we have supposed that the interest calculation is a compound interest calculation. However, we can distinguish two types of interest calculation:

- compound interest and
- simple interest.

Picture 4.

In the case of simple interest, the interest is calculated only on the original principal. This means that the interest earned is not reinvested in the next time period. In the case of a simple interest calculation, the future value of C_0 invested for t periods at an interest rate r per period is:

$$FV_t = C_0 \cdot (1 + r \cdot t). \qquad (2.2)$$

Example 2.1:

How much will you have if you deposit 200 000 EUR in the bank, and leave it in your bank account for 10 years at a 12 percent annual interest rate? In the case of compound interest, the future value of 200 000 EUR invested after 10 years is:

$$FV_{10} = 200\,000 \cdot 1.12^{10} = 621\,169.6417\ EUR.$$

Assuming a simple interest calculation, the future value of 200 000 EUR invested after 10 years is:

$$FV_{10} = 200\,000 \cdot (1 + 0.12 \cdot 10) = 440\,000\ EUR.$$

Interest earned remains the same over the years when simple interest is applied. However, if the interest earned is calculated as compound interest, it increases as the number of periods increase (Table 2.1).

Table 2.1 Future value (simple interest, compound interest)

	Simple interest			Compound interest		
Years	**Principal**	**Interest earned**	**Future value**	**Principal**	**Interest earned**	**Future value**
1	200 000	200000·0.12= =24 000	224 000	200 000	$200\,000 \cdot 0.12 = 24\,000$	224 000
2	224 000	200000·0.12= =24 000	248 000	224 000	$224\,000 \cdot 0.12 = 26\,880$	250 880
3	248 000	24 000	272 000	250 880	30 105.6	280 985,6
4	272 000	24 000	296 000	280 985.6	33 718.272	314 703.872
5	296 000	24 000	320 000	314 703.872	37 764.4646	352 468.3366
6	320 000	24 000	344 000	352 468.3366	42 296.2004	394 764.537
7	344 000	24 000	368 000	394 764.537	47 371.7444	442 136.2815
8	368 000	24 000	392 000	442 136.2815	53 056.3538	495 192.6353
9	392 000	24 000	416 000	495 192.6353	59 423.1162	554 615.7515
10	416 000	24 000	440 000	554 615.7515	66 553.8902	621 169.6417

Example 2.2:
How much will you have if you deposit 400 000 EUR in the bank and leave it for 10 years at a 4 percent monthly interest rate? Give your result

a) in the case of compound interest.

b) in the case of simple interest.

In the case of compound interest, the future value of 400 000 EUR invested over 10 years is:

$$FV_{10} = 400\,000 \cdot 1.04^{10 \cdot 12} = 44\,265\,024.32\ EUR.$$

Assuming a simple interest calculation, the future value of 400 000 EUR invested over 10 years is:

$$FV_{10} = 400\,000 \cdot (1 + 0.04 \cdot 120) = 2\,320\,000\ EUR.$$

2.4. Frequency of payments, continuous compounding

Interest can be compounded yearly, quarterly, monthly, weekly or daily. If you invest C_0 at a rate of *r* per year compounded *m* times a year, the future value of your money after *t* years is:

$$FV_t = C_0 \cdot (1 + (\frac{r}{m})^{m \cdot t}.$$

Generally there is no upper limit to the frequency of interest payment.

Example 2.3:
Suppose you invest 20 000 EUR in the bank and leave it for 4 years at a rate of 12 percent per year compounded
a) quarterly
b) monthly
c) weekly.
How much will you have at the end of year 4?

a) The future value of 20 000 EUR, if the interest is compounded quarterly:

$$FV_4 = 20\ 000 \cdot [1 + \left(\frac{0.12}{4}\right)]^{4 \cdot 4} = 32\ 094.12878\ EUR.$$

b) The future value of 20 000 EUR, if the interest is compounded monthly:

$$FV_4 = 20\ 000 \cdot [1 + \left(\frac{0.12}{12}\right)]^{12 \cdot 4} = 32\ 244.52155\ EUR.$$

c) The future value of 20 000 EUR, if the interest is compounded weekly:

$$FV_4 = 20\ 000 \cdot (1 + \left(\frac{0.12}{52}\right)^{52 \cdot 4} = 32\ 303.61935\ EUR.$$

Continuous compounding
If the interest is compounded continuously, we assume that the number of periods of the interest compounded approaches infinity. If you invest C_0 at a rate of r per year compounded continuously, the future value of your money after *t* years is:

$$FV_t = C_0 \cdot \lim_{m \to \infty} (1 + (\frac{r}{m})^{m \cdot t} = C_0 \cdot e^{r \cdot t}$$

Example 2.4:
How much will you have if you invest 20 000 EUR in the bank and leave it for 5 years at a 15 percent interest rate compounded continuously?

$$FV_5 = 20\ 000 \cdot e^{0.15 \cdot 5} = 42\ 340\ EUR.$$

If the interest is paid more frequently, the effective interest rate is not equal to the quoted rate.

2.5. Future value of an annuity

Suppose you invest your money in an asset, and it pays a fixed sum each year for a specified period. The future value of these payments depends on whether the payments occur at the end of each year or at the beginning of each year. If the payments of equal amounts occur at the end of each period, the annuity is called an ordinary annuity. However, if the payments of equal amounts occur at the beginning of each year the annuity is called an annuity due.

Picture 5.

Ordinary Annuity

Let us try to define and determine the future value of an ordinary annuity.

Annuity

"An annuity is an asset that pays a fixed sum each year for a specified number of years. The equal-payment house mortgage or instalment credit agreements are common examples of annuities" (Brealey – Myers, 2005).

The cash-flow streams of the fixed sum (C) at the end of each year beginning in year 1.

Year	Cash flow
1	C
2	C
3	C
.	C
.	C
.	C
n	C

The future value of the cash-flow streams for n years is the following:

$$FV = C + C \cdot (1 + r)^1 + C \cdot (1 + r)^2 + \cdots + C \cdot (1 + r)^{n-1},$$

where *r* is the interest rate.

The sum can be written in a simpler form in several ways. One of the possibilities is to apply the sum formula of a geometrical series, because if we look carefully at the addend terms of the future value, they represent the elements of a geometrical series. The sum of a finite geometric series (S_n) is the following:

$$S_n = a_1 \cdot \frac{q^n - 1}{q - 1},$$

where a_1 is the first element of the geometric series and q is the quotient of the series. Substitute the suitable elements into the formula:

$$FV = C \cdot \frac{(1+r)^n - 1}{(1+r) - 1} = C \cdot \frac{(1+r)^n - 1}{r}.$$

Annuity Due

The future value of an annuity due can be written as follows:

$$FV = C \cdot (1+r) + C \cdot (1+r)^2 + C \cdot (1+r)^3 + \cdots + C \cdot (1+r)^n.$$

Applying the formula of the sum of a geometric series we get:

$$FV = C \cdot (1+r) \cdot \frac{(1+r)^n - 1}{(1+r) - 1} = C \cdot (1+r) \cdot \frac{(1+r)^n - 1}{r}.$$

Example 2.5:
Suppose you deposit 1 000 EUR

a) at the end of each year
b) at the beginning of each year
for the next five years in a bank account paying 12 percent annual interest. How much will you have in five years?

The future value of an annuity due can be calculated as the following:

$$FV = C \cdot (1+r) \cdot \frac{(1+r)^n - 1}{r} = 1\,000 \cdot 1.12 \cdot \frac{1.12^5 - 1}{0.12} = 7\,115.19 \text{ EUR}.$$

The future value of an ordinary annuity is the following:

$$FV = C \cdot \frac{(1+r)^n - 1}{r} = 1\,000 \cdot \frac{1.12^5 - 1}{0.12} = 6\,352.85 \text{ EUR}.$$

2.6. Effective annual rate

The interest earned is higher when the interest is compounded more than once per year. This means that the total interest earned is greater if the interest is compounded semi-annually than annually.

Example 2.6:
Suppose you invest 1 000 EUR in a bank account paying 12 percent annual interest. How much will you have in six years
a) if the interest is compounded annually?
b) if the interest is compounded semi-annually?

If the interest is compounded annually, the future value of your money is:

$$FV = 1\,000 \cdot 1.12^6 = 1\,973.82 \text{ EUR}.$$

However, if the interest is compounded semi-annually, you will have:

$$FV = 1\,000 \cdot \left(1 + \frac{0.12}{2}\right)^{2 \cdot 6} = 2\,012.2 \text{ EUR}.$$

Interest compounded semi-annually is more favourable since it yields a greater benefit than interest compounded annually.

In this example the 12 percent is called a quoted (or stated) interest rate. Although the quoted interest rate is the same in the two calculations, we have found there is a greater benefit when the interest is compounded semi-annually than annually. This means that the effective annual rates are different.

> ***Effective Annual Rate***
> "The interest rate expressed as if it were compounded once per year" (Ross et al., 1993).
>
> "The annual rate of interest actually being earned as opposed to the quoted rate, considering the compounding of interest" (Besley – Brigham, 2015).

If the quoted rate (*r*) and the number of compounding periods per year (*m*) are given, we can determine the effective annual rate (r_{EAR}):

$$r_{EAR} = \left(1 + \frac{r}{m}\right)^{m} - 1.$$

In example 2.6, the effective annual rate (r_{EAR}) is:

$$r_{EAR} = \left(1 + \frac{0.12}{2}\right)^{2} - 1 = 0.1236.$$

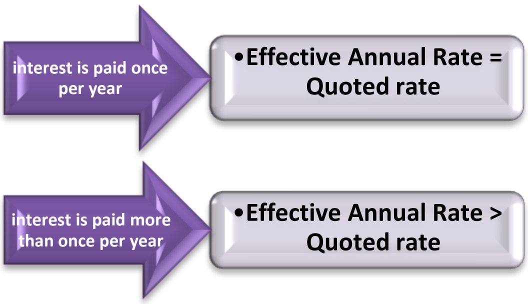
If you invest 1 000 EUR in a bank account paying a 12 percent interest rate semi-annually, your benefit is equal to the investment paying 12.36 percent annually:

$$FV = 1\,000 \cdot \left(1 + \frac{0.12}{2}\right)^{2 \cdot 6} = 2\,012.2 \text{ EUR},$$

$$FV = 1\,000 \cdot 1.1236^{6} = 2\,012.2 \text{ EUR}.$$

If the interest is compounded annually the effective annual rate is equal to the quoted rate. In this case the interest is paid once per year. However, if the interest is compounded more than once per year, the effective annual rate is greater than the quoted rate.

Figure 2.1 The relationship between the effective annual rate and quoted rate



2.7. Terms and Questions

annuity,
annuity due,
compound interest,
continuous compounding,
effective annual rate,
frequency of payments,
future value,
future value of an annuity,
future value of a lump-sum amount,
lump-sum amount,
ordinary annuity,
quoted interest rate,
single interest,
uneven cash flows.

Problems

Theoretical questions

1. What is the difference between the quoted interest rate and the effective annual rate?

2. Give the definition of an annuity.

3. How can we calculate the future value of an annuity?

4. Explain what the relationship is between the effective annual rate and the quoted interest rate.

5. What is the difference between a single interest calculation and a compound interest calculation? Give an example of each.

6. Give the main factors influencing the future value of an annuity.

7. What is the difference between an annuity due and an ordinary annuity? Give an example of each.

8. Give an example of an annuity due.

9. Give the definition of the effective annual rate.

10. How can we calculate the future value of a single sum by using continuous compounding?

11. Give the definition of uneven cash flows.

Calculation exercise

1.
Suppose that you have 1 500 EUR invested in a bank account and banks are currently paying an interest rate of 6 percent per year on deposits. How much will you accumulate in your account after 5 years?

2.
You deposit 200 000 EUR in your bank account. The interest rate is 6 percent per year.

a) How much will you have in your account after 10 years if the bank pays simple interest?
b) How much will you have in your account after 10 years if the bank pays compound interest?

3.
What monthly interest rate will result in 600 EUR growing to 1 223.9324 in three years?

4.
How long will it take for 1000 EUR to double at a 10 percent interest rate compounded annually?

5.

How long will it take for 50 000 EUR to grow to 81 000 EUR if the bank pays compound interest and the annual interest rate is

a) 2 percent?
b) 5 percent?
c) 10 percent?
d) 12 percent?

6.

Determine the future value of a 440 000 EUR bank deposit if $t = 12$ years and

a) the bank pays interest at 15% compounded annually.
b) the bank pays interest at 12.5% compounded semi-annually.
c) the bank pays interest at 12% compounded continuously.

7.

Suppose that you pay 12 000 EUR at the beginning of each year for the next 12 years. The annual interest rate is 2 percent.

a) Determine the future value of this stream of payments.

8.

Suppose you would like to invest 18 500 EUR in a bank account.
You have two offers: Bank A pays a 6 percent interest rate, compounded quarterly, on time deposits. Bank B pays 5.2 percent, compounded monthly.

a) Determine the effective interest rates.
b) In which bank would you prefer to deposit your money?

9.

Suppose that you pay 10 000 EUR at the end of each year for the next 10 years. The monthly interest rate is 0.5 percent.

a) Determine the future value of this stream of payments.

10.

Determine the amount to which 1 000 EUR will grow in three years under each of the following conditions:

a) 8 percent compounded annually.
b) 8 percent compounded semi-annually.
c) 8 percent compounded quarterly.
d) 8 percent compounded monthly.
e) 8 percent compounded daily.
f) 8 percent compounded continuously.

11.

Suppose you want to invest 10 500 EUR in a bank account.

You have two offers: Bank A pays a 10 percent interest rate, compounded annually, on time deposits. Bank B pays 8 percent, compounded semi-annually.

a) Determine the effective interest rates.
b) In which bank would you prefer to deposit your money?

12.

What annual interest rate will result in 1 200 EUR growing to 1 685 in five years?

13.

How long will it take for your savings to double at an 8 percent interest rate compounded annually?

14.

Your parents invested 10 000 EUR 12 years ago at 10 percent, compounded semi-annually. How much have they accumulated?

15.

You deposit 100 000 EUR in your bank account.

a) How much will you have in your account after 3 years if the bank pays simple interest? The interest rate is 5 percent per year.
b) How much will you have in your account after 3 years if the bank pays simple interest and the interest rates are the following:

Year	interest rate (%)
1	4
2	5
3	6

16.

Determine the amount to which 3 500 U.S. dollar will grow in 30 months under each of the following conditions:

a) 10 percent compounded annually.
b) 10 percent compounded semi-annually.
c) 10 percent compounded monthly.
d) 10 percent compounded daily.

17.

Determine the amount to which 1 200 Swiss francs will grow in 5 years under each of the following conditions:

a) 5 percent compounded annually.
b) 3 percent compounded quarterly.
c) 1.5 percent compounded monthly.

3. Present value of Money

How do we decide if an investment is worth implementing? Revenues and expenditures occur at different times and as a result we have to take account of the time value of money. In the previous chapter, we saw that it is very important take into consideration the time value of money in financial decisions. The time value of money depends on several influencing factors such as:

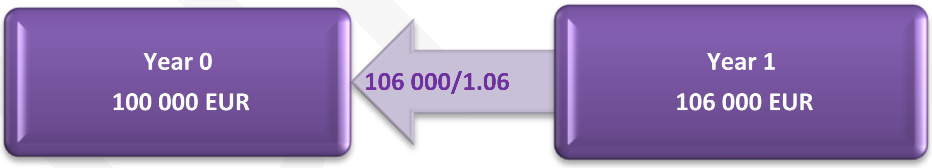
Picture 6.

- the interest rate,
- the time interval,
- the amount of the principal.

In our decision, we take the time value of money into account; this means that we have to determine the value of money for the same time period (year, month, week etc.). Considering that we take decisions in respect of the investment at the present moment, we calculate the current time value of money.

3.1. Present value of a lump sum amount

In the previous chapter (Chapter 2), we calculated the future value of 100 000 EUR for one year at a 6% annual interest rate, and saw that the investment will grow to $1.06 \cdot 100\,000\ EUR = 106\,000\ EUR$ (Chapter 2.2). We will receive 106 000 EUR after one year; however, we know that the present value of 106 000 EUR is 100 000 EUR. We can calculate the present value of 106 000 EUR (at a 6 percent rate) as the following:



$$\mathrm{PV} = \frac{106\,000}{1.06} = 100\,000\ \mathrm{EUR}.$$

The present value of C_1 one year from now equals:

$$\mathrm{PV} = \frac{C_1}{(1+r)^1}.$$

The present value of a future payment can be determined by multiplying the payment by the discount factor $(\frac{1}{1+r})$:

$$\mathrm{PV} = C_1 \cdot \frac{1}{(1+r)^1}.$$

The present value of C_2 two years from now equals:

$$\mathrm{PV} = \frac{C_2}{(1+r)^2}.$$

The present value of C_n n years from now equals:

$$PV = \frac{C_n}{(1+r)^n}.$$

We discount the expected payment in the *n*-th year by the discount rate (r) to determine the present value of C_n.

Present value of a lump sum amount
To determine the present value of a future amount, we discount the future payment by the interest rate. The present value of a future lump-sum amount which becomes due at the end of n year can be calculated as the following:

$$PV = \frac{C_n}{(1+r)^n}.$$

The discount rate is called the opportunity cost of capital because it is the return foregone by investing in the project rather than investing in securities (Brealey – Myers, 2005).
Suppose that you lend your friend (John) 1 000 EUR, and the yearly interest rate is 5 percent. According to your agreement, John will repay 1 200 EUR in three years. Calculate the present value of 1 200 EUR.
The present value of 1 200 EUR 3 years from now equals:

$$PV = \frac{1\,200}{1.05^3} = 1\,036.6 \text{ EUR}.$$

3.2. Present value of a stream of future cash flows

3.2.1. Present value and net present value of a stream of future cash flows at a constant discount rate

To determine the present value of a stream of future cash flows, we add the present values of all future payments. The present value (PV) of the future cash flows ($C_1 \dots C_n$) of an investment is the following, assuming that the discount rate (r) is constant over time:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \dots + \frac{C_n}{(1+r)^n} = \sum_i^n \frac{C_i}{(1+r)^i}.$$

If we suppose the initial investment expenditure occurs now, the net present value is equal to the difference between the present value of future cash flows and the required initial investment C_0.

$$NPV = C_0 + PV = C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \dots + \frac{C_n}{(1+r)^n} = C_0 + \sum_i^n \frac{C_i}{(1+r)^i}.$$

Discount cash flow method (DCF formula)
The discounted cash flow method is used to evaluate the value of an investment possibility:

$$PV = \sum_i^n \frac{C_i}{(1+r)^i}.$$

Example 3.1:
Consider the cash flows shown in the following table:

Year	Revenues minus Expenditures (million HUF)
0	-50
1	12
2	18
3	23
4	20
5	20

Suppose you would like to invest 50 000 000 HUF in a project now, and the annual difference between the future revenues and the expenditures of your project can be found in the table above. The discount rate is 12 percent. Determine the value of your project.
The present value of the stream of future cash flows is the following (million HUF):

$$PV = \frac{12}{1.12} + \frac{18}{1.12^2} + \frac{23}{1.12^3} + \frac{20}{1.12^4} + \frac{20}{1.12^5} = 65.5.$$

We get the net present value of the cash flows if we add the initial cost of the investment to the present value of future cash flows (million HUF):

$$NPV = -50 + \frac{12}{1.12} + \frac{18}{1.12^2} + \frac{23}{1.12^3} + \frac{20}{1.12^4} + \frac{20}{1.12^5} = 15.5.$$

The result shows us that the investment is profitable, because it yields a 15.5 million HUF profit.

3.2.2. Present value and net present value of a stream of future cash flows at a changing discount rate

We have assumed that the opportunity cost of capital is constant. Now, we ignore the condition of the constant discount rate and we assume that the rate changes over time.
Suppose that the discount rates are r_1, r_2,...,, r_n and future cash flows are C_1 ... C_n in period 1, 2,...,n.

Year	Discount rate (%)	Cash flow
1	r_1	C_1
2	r_2	C_2
3	r_3	C_3
.	.	.
.	.	.
.	.	.
n	r_n	C_n

Determine the present value of the future cash flows step by step. The present value of cash flow C_1 is the following:

$$PV = \frac{C_1}{(1+r_1)^1}.$$

If we want to determine the first year value of C_2 occurring in year 2, we have to determine the discounted value of C_2 using the second year discount rate r_2:

$$C_2 \text{ value in year } 1 = \frac{C_2}{(1+r_2)}.$$

To give the present value of the cash flow in year 2, we have to discount the value in year 1 by the discount rate r_1. The present value of the cash flow C_2 is the following:

$$PV = \frac{C_2}{(1+r_1)\cdot(1+r_2)}.$$

The present value (PV) of the future cash flows ($C_1 \ldots C_n$) of an investment is the following, assuming that the discount rate (r) changes over time.

$$PV = \frac{C_1}{(1+r_1)^1} + \frac{C_2}{(1+r_1)\cdot(1+r_2)} + \frac{C_3}{(1+r_1)\cdot(1+r_2)\cdot(1+r_3)} + \cdots + \frac{C_n}{(1+r_1)\cdot(1+r_2)\cdot\ldots\cdot(1+r_n)}.$$

Example 3.2:

Calculate the present value and net present value of the investment examined in Example 3.1 under the changing interest rate condition. The interest rates in different years are shown in the following table.

Year	Revenue-Expenditure (million HUF)	Interest rate (%)
0	-50	
1	12	4
2	18	5
3	23	4
4	20	6
5	20	3

Determine the value of your project.

The present value of the stream of future cash flows is the following (million HUF):

$$PV = \frac{12}{1.04} + \frac{18}{1.04\cdot1.05} + \frac{23}{1.04\cdot1.05\cdot1.04} + \frac{20}{1.04^2\cdot1.05\cdot1.06} + \frac{20}{1.04^2\cdot1.05\cdot1.06\cdot1.03} = 81.02.$$

We get the net present value of the cash flows if we add the initial cost of the investment to the present value of future cash flows (million HUF):

$$NPV = -50 + \frac{12}{1.04} + \frac{18}{1.04\cdot1.05} + \frac{23}{1.04\cdot1.05\cdot1.04} + \frac{20}{1.04^2\cdot1.05\cdot1.06} + \frac{20}{1.04^2\cdot1.05\cdot1.06\cdot1.03} = 31.02.$$

The result shows us that the investment is profitable, because it yields a 31.02 million HUF profit.

3.2.3. Present value of annuity cash flows

In this subsection, we examine the present value of annuity cash flows.

"An annuity is an asset that pays a fixed sum each year for a specified number of years. The equal-payment house mortgage or instalment credit agreements are common examples of annuities" (Brealey – Myers, 2005).

An annuity is a series of constant cash flows that occurs at the beginning or at the end of each period. Let us assume you get 1 000 EUR at the end of each of the next five years. What is the present value of these future cash flows?

Firstly, we determine the general form of the present value of an annuity. Suppose that we have to pay, or we receive, the fixed sum (C) at the end of each of the next *n* years.

Year	Cash flow
1	**C**
2	**C**
3	**C**
.	**C**
.	**C**
.	**C**
n	**C**

The present value of the constant cash-flow streams for n years is the following:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^n}.$$

Let us try to determine the sum of the cash-flow streams. If we look carefully at the addend terms of the present value, the addend terms represent the elements of a geometrical series. The sum of a finite geometric series (S_n) is the following:

$$S_n = a_1 \cdot \frac{q^n - 1}{q - 1},$$

where a_1 is the first element of the geometric series and q is the quotient of the series. Substitute the suitable elements into the formula:

$$PV = \frac{C}{(1+r)^1} \cdot \frac{(\frac{1}{1+r})^n - 1}{\frac{1}{1+r} - 1} = \frac{C}{(1+r)^1} \cdot \frac{(\frac{1}{1+r})^n - 1}{\frac{-r}{1+r}} = \frac{C}{r} \cdot (1 - \frac{1}{(1+r)^n}).$$

The present value of an annuity can be written as follows:

$$PV = \frac{C}{r} \cdot (1 - \frac{1}{(1+r)^n}),$$

or

$$= C \cdot (\frac{1}{r} - \frac{1}{r \cdot (1+r)^n}),$$

where the expression in brackets is the annuity factor $(\frac{1}{r} - \frac{1}{r \cdot (1+r)^n})$ which is the present value at a discount rate r of an annuity of 1 HUF paid at the end of each of n periods.
We can determine the present value of the constant cash flows that occur at the end of each year in other ways. Earlier, we analysed the future value of an ordinary annuity. If the payments of equal amounts occur at the end of each period, the annuity is called an ordinary annuity.
The future value of an ordinary annuity in year n is the following:

$$FV_n = C \cdot \frac{(1+r)^n - 1}{r}.$$

The present value of an amount that is due in year n (C_n) can be calculated as the following:

$$PV = \frac{C_n}{(1+r)^n}.$$

Apply the present formula for the future value of an ordinary annuity:

$$PV = \frac{C \cdot \frac{(1+r)^n - 1}{r}}{(1+r)^n} = \frac{C}{r} \cdot \frac{(1+r)^n - 1}{(1+r)^n} = \frac{C}{r} \cdot (1 - \frac{1}{(1+r)^n}).$$

If the constant series of cash flows occurs at the beginning of each year instead of at the end of each year, we have to multiply the present value equation by (1+r). The present value of constant cash flows occurring at the beginning of each year is:

$$PV = \frac{C}{r} \cdot (1 + r) \cdot (1 - \frac{1}{(1+r)^n}).$$

Example 3.3:
Assume that you would like to purchase a new car, which costs 5 000 000 HUF. You get a favourable offer. According to the offer, you have to repay the price in 60 equal annual instalments of 150 000 HUF. The offer is equivalent to a loan that you have to pay over 5 years, and the monthly instalment is 150 000 HUF. Determine the present value of your future payments if the monthly interest rate is 2%.
You have to pay the instalments at the end of each month. The present value of your constant series of payments is the following:

$$PV = \frac{C}{r} \cdot \left(1 - \frac{1}{(1+r)^n}\right) = \frac{150\ 000}{0.02} \cdot \left(1 - \frac{1}{1.02^{60}}\right) = 5\ 214\ 133 \text{ HUF}.$$

If you pay the price of the car in cash instead of as a loan, you have to pay 214 133 HUF less.

3.3. Terms and Questions

annuity,
changing discount rate,
constant discount rate,
discount factor,
discount rate,
discounted cash flow method
net present value,
opportunity cost of capital,
present value,
present value of an annuity cash flows,
present value of a lump-sum amount,
present value of a stream of future cash flows,
series of payments.

Problems

Theoretical questions

1. What is the difference between present value calculation and future value calculation?

2. Explain the discounted cash flow method.

3. How can we calculate the present value of an annuity?

4. How can we calculate the net present value of an investment possibility?

5. What is the difference between the opportunity cost of capital and the interest rate?

6. Explain what the relationship is between the present value of cash flows and the interest rate.

7. What is the difference between annuity cash flows and a constant series of cash flows? Give an example of each.

8. Give the main influencing factors of the present value of a stream of future cash flows.

9. Give an example of an annuity cash flow.

10. Give the definition of an annuity.

11. How can we calculate the present value of a lump sum amount?

12. How can we calculate the present value of a stream of future cash flows?

13. What is the opportunity cost of capital?

14. What is the discount factor?

Calculation exercise

1.
Suppose that the annual interest rate is 4 percent and an investment produces the following cash flows (million HUF):

Year	Cash flow
0	-30
1	12
2	14
3	15.5
4	16

a) Calculate the present value of the investment opportunity.
b) Calculate the net present value of the investment opportunity.

2.
Determine the present value of 1 000 EUR due four years from today, under the following conditions:

a) 10 percent interest rate compounded annually.
b) 8 percent interest rate compounded semi-annually.

c) 2 percent interest rate compounded quarterly.

3.

Suppose that you pay 12 000 EUR at the beginning of each year for the next 10 years. The annual interest rate is 6 percent.

a) Determine the future value of this stream of payments.
b) Determine the present value of this stream of payments.

4.

Calculate the present value of the following cash flow streams (million HUF):

Year	Cash flow
0	25
1	30
2	40
3	60

a) The annual interest rate is 10 percent.
b) The annual interest rate is 1 percent.

5.

Suppose that you pay 5 000 EUR at the end of each month for the next 5 years. The monthly interest rate is 0.8 percent.

a) Determine the future value of this stream of payments.
b) Determine the present value of this stream of payments.

6.

Consider the following information.

Present value	Future value	Time period (years)
1 200	2 326.533	4
5 600	11 263.6	5
6 000	6 756.975	6
12 000	13 230	2

a) Determine the annual interest rate.

7.

Suppose that you have borrowed 5 000 EUR in student loans at a monthly interest rate of 1 percent. The loan requires monthly payments. If you repay 200 EUR per month, how long will it take you to repay the loan?

8.

Suppose that the interest rate varies yearly. Consider the following information about an investment opportunity:

Year	Cash flow (million HUF)	Annual interest rate (percent)
0	-150	
1	50	12
2	50	13
3	60	14
4	60	15

a) Calculate the present value of the given investment opportunity.
b) Calculate the net present value of the given investment opportunity.

9.

You pay a five-year ordinary annuity of 150 EUR. The annual interest rate is 12 percent.
a) What is the present value of the annuity?
b) What is the future value of the annuity?
c) How can we determine the future value of the annuity from its present value?
d) What would the future value be if the annuity were an annuity due?
e) What would the present value be if the annuity were an annuity due?

10.

You have agreed to pay 5 500 HUF per month for 2.5 years for a mobile phone.

a) What is the equivalent price of this mobile phone, if the interest rate is an annual 1.2 percent?
b) What is the equivalent price of this mobile phone, if the interest rate is 0.5 percent monthly?

11.

You are saving to purchase a car at the end of six years. If the car costs 16 000 EUR and you can earn 8 percent a year on your savings, how much do you need to put aside over the six years?

12.

Suppose you are on holiday in a very elegant hotel and you win a competition organised by the hotel. As a winner, you can choose one of the following prizes:

a) 15 000 EUR now.
b) 10 000 EUR now and 6 000 EUR at the end of three years.
c) 20 000 EUR at the end of four years.
d) 1 500 EUR annually, forever.
e) 1 600 EUR for each of the next eight years.

Which one would you choose? The annual interest rate is 10 percent.

13.

Assume an investor offers you an investment possibility. According to the investor's information, you will realise 15 000 EUR annually for the next six years. The opportunity cost of capital is 8 percent.

a) What is the maximum amount you would pay for the investment?

14.

Assume that you get 1 000 EUR at the end of each subsequent year for five years. The first cash flow will be paid out four years from today. The annual interest rate is 10 percent.

a) Determine the present value of this stream of cash flow.

15.

Your uncle John is going to transfer 100 000 HUF at the end of the next three months to your account. The monthly interest rate is 2 percent.

a) What is the value at the end of three months of this cash flow?
b) What is the present value of this cash flow?

16.

Try to determine the present value of a perpetuity of 1 000 EUR per year. The discount rate is 5 percent.

4. Making investment decisions

A firm makes countless financial decision during its operation, including, among others: issuing bonds, borrowing, paying dividends, and increasing capital or wages, as well as decisions relating to expenditures on assets, expansion, and replacement of assets. In the following chapters, we examine different financial indicators, and we study how we can make investment decisions. In this chapter we analyse both the independent projects and the mutually exclusive projects that are part of capital budgeting decisions. A capital budgeting decision is the process of analysing investments and deciding which are profitable and acceptable, and which of these acceptable investments should be realised.

Capital budgeting decisions can be both replacement decisions and expansion decisions. Generally, a firm has to decide whether to buy a new machine instead of the old one, or purchase new assets to expand its production. Efficiency can be improved in both cases; however, capacity can be enhanced when the firm expands.

Picture 7.

Replacement decisions involve determining whether capital investments should be purchased to take the place of existing assets that might be worn out, damaged, or obsolete (Besley – Brigham, 2015).

Expansion decisions involve determining whether to purchase capital investments and add them to existing assets so as to increase existing production (Besley – Brigham, 2015).

We can distinguish between independent investments and mutually exclusive investments. ***Independent investments*** are investments whose cash flows are not affected by any other investments, so the acceptance of one investment does not affect any decisions to accept another investment (Besley – Brigham, 2015).

Mutually exclusive investments are investments where the acceptance of one investment means that other investment cannot be accepted (Besley – Brigham, 2015).

Suppose that we have to decide on an investment possibility. Is it good or not? Is it worth realizing? Should we accept the investment offer or is it better if we refuse it, because it is not profitable?

Let us start our examination with three conditions; later two limitations will be removed.

Condition 1

We have to examine only one investment possibility. This means that we will not have to compare our possibility with another.

Condition 2

We know the future cash flows of our project. This means that we have information about the revenues and costs of our project, or we know the benefits per year.

Condition 3

Assume that the discount rate is constant.

4.1. Present value (PV) and net present value (NPV)

4.1.1. Net present value of an investment at a constant discount rate

In this sub-section, we will try to apply the present value and net present value calculation to evaluate an investment possibility. Let us start our examination of the following example.

Example 4.1:

The cash flows for our investment possibility are as follows:

Year	Initial expenditure	Costs (HUF)	Revenues (HUF)
0	80 000 000		
1		12 500 000	22 750 000
2		11 800 000	28 000 000
3		8 800 000	32 500 000
4		8 200 000	33 000 000
5		8 200 000	32 000 000
6		8 200 000	32 000 000

Our project requires an initial investment of 80 000 000 HUF, followed by cash outflows and inflows over the next six years. Suppose the opportunity cost of capital is 10%. In the first step, give the value of the difference between revenues and expenditures per year. The benefits per year are the following:

Year	Initial expenditure (HUF)	Revenue-Expenditure (HUF)
0	80 000 000	
1		10 250 000
2		16 200 000
3		23 700 000
4		24 800 000
5		23 800 000
6		23 800 000

In our calculation, it is necessary to take into account the time value of money. The values of money at different times are different.

As we have seen in subsection 3.2.1, the present value of future cash flows can be calculated as the following formula:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_n}{(1+r)^n} = \sum_i^n \frac{C_i}{(1+r)^i},$$

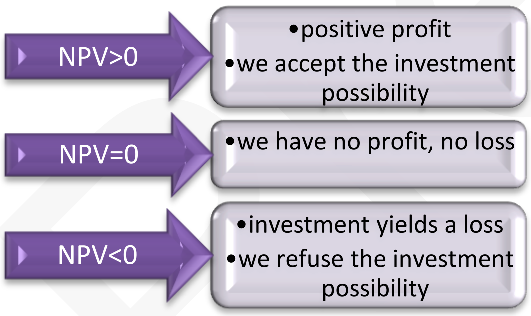
where $C_1 \dots C_n$ are the future cash flows in period 1...n, and r is the discount rate.
If we suppose the initial investment expenditure occurs now, net present value is equal to the difference between the present value of future cash flows and the required initial cost (C_0).

$$NPV = C_0 + PV = C_0 + \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots + \frac{C_n}{(1+r)^n} = C_0 + \sum_i^n \frac{C_i}{(1+r)^i}.$$

An investment yields a profit if the net present value is a positive number.

Net present value rule: It is worth accepting investments that have positive net present values.

Figure 4.1 Evaluation of the net present value of an investment possibility



Let us go back to the original problem (3.1) and solve it.

Year	Initial expenditure (HUF)	Net Cash Flow (HUF)
0	80 000 000	
1		10 250 000
2		16 200 000
3		23 700 000
4		24 800 000
5		23 800 000
6		23 800 000

Let us calculate the present value of the future cash flows at 10 percent. We determine the value of the investment by discounting the net cash flows back to the present; the total value in million HUF is the following:

$$PV = \frac{10.25}{1.1^1} + \frac{16.2}{1.1^2} + \frac{23.7}{1.1^3} + \frac{24.8}{1.1^4} + \frac{23.8}{1.1^5} + \frac{23.8}{1.1^6} = 85.664.$$

The net present value is (million HUF):

$$NPV = C_0 + PV = -80 + 85.66 = 5.664.$$

The net present value of the investment is positive; this means that 5.66 million HUF value is created by undertaking the investment. In this case, it is recommended to accept the investment opportunity. It is worth accepting an investment if it creates value for its owners.

4.1.2. Net present value of an investment at a changing discount rate

We supposed under Condition 3 that the opportunity cost of capital is constant. Now we ignore the condition of the constant discount rate and we suppose that the rate changes over time.

Example 4.2:

Year	Discount rate (%)	Initial expenditure (HUF)	Net Cash Flow (HUF)
0		80 000 000	
1	12		10 250 000
2	11		16 200 000
3	10		23 700 000
4	12		24 800 000
5	13		23 800 000
6	14		23 800 000

Calculate the present value and the net present value of the net cash flows (million HUF):

$$PV = \frac{10.25}{1.12^1} + \frac{16.2}{1.12 \cdot 1.11} + \frac{23.7}{1.12 \cdot 1.11 \cdot 1.1} + \frac{24.8}{1.12 \cdot 1.11 \cdot 1.1 \cdot 1.12} + \\ + \frac{23.8}{1.12 \cdot 1.11 \cdot 1.1 \cdot 1.12 \cdot 1.13} + \frac{23.8}{1.12 \cdot 1.11 \cdot 1.1 \cdot 1.12 \cdot 1.13 \cdot 1.14} = 81.52.$$

The net present value is equal to the sum of the initial expenditure and the present value of the net cash flows (million HUF):

$$NPV = C_0 + PV = -80 + 81.52 = 1.52.$$

If the net present value of the project is positive, it yields a benefit. However, if the net present value of an investment possibility is negative, the investment is loss-making.
We can see that the net present value strongly depends on the value of the opportunity cost of capital. The lower the opportunity cost of capital, the larger the net present value.

4.1.3. The relationships between present value, time periods and discount rates

The present value of a given amount or future cash flows depends on the discount rate and time periods.

Example 4.3:
Let us calculate the present value and net present value of the project examined in Examples 4.1 and 4.2 with two discount rates: a 12% and a 2 % annual discount rate.

The present value of the project (million HUF) at a 12% discount rate is the following:

$$PV = \frac{10.25}{1.12^1} + \frac{16.2}{1.12^2} + \frac{23.7}{1.12^3} + \frac{24.8}{1.12^4} + \frac{23.8}{1.12^5} + \frac{23.8}{1.12^6} = 80.259.$$

The net present value of the project (million HUF):

$$NPV = C_0 + PV = -80 + 80.259 = 0.259.$$

If the discount rate is only 2%, the net present value of the project is greater than the net present value at a 12% discount rate. At a 2% discount rate the project yields a positive profit.

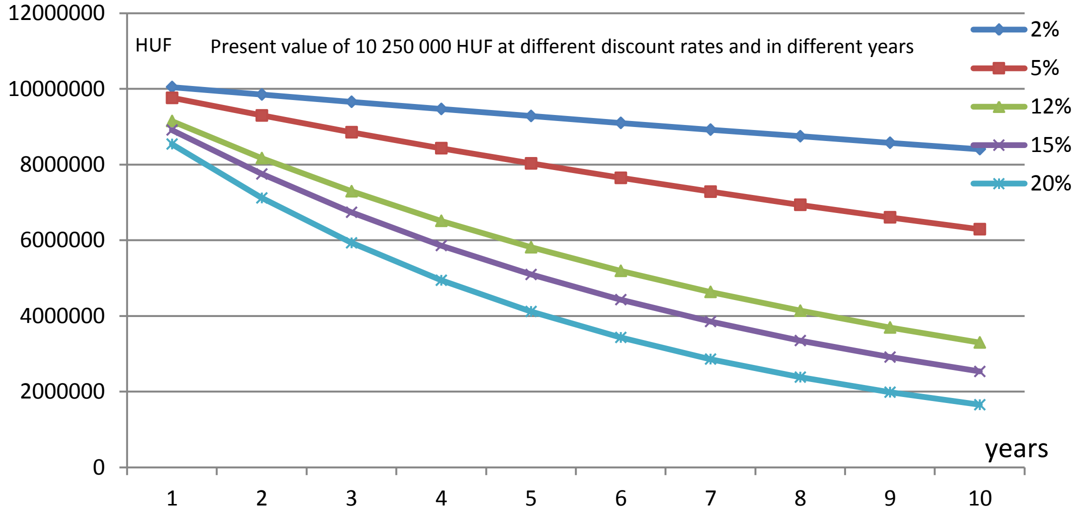
$$PV = \frac{10.25}{1.02^1} + \frac{16.2}{1.02^2} + \frac{27.7}{1.02^3} + \frac{24.8}{1.02^4} + \frac{23.08}{1.02^5} + \frac{23.08}{1.02^6} = 113.55.$$

The net present value of the project (million HUF):

$$NPV = C_0 + PV = -80 + 113.55 = 33.55.$$

We can see from the examples in 4.1 and 4.3 that the lower the discount rate is, the more profitable the project becomes.
If we determine the present value of a given amount at different discount rates, we can see that the present value is greater at a lower discount rate (Figure 4.2). The present value of a given amount is lower in subsequent periods (Figure 4.2).

Figure 4.2 Relationship among present value, discount rate and time



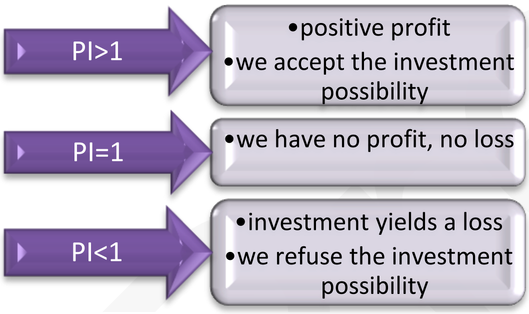
4.2. Profitability index (PI)

The profitability index (PI) shows the amount of the present value of future cash flows (PV) for only one unit of the required initial investment (C_0). The profitability index (PI) is equal to the present value of an investment's cash flows (PV) divided by the absolute value of the initial expenditure of the required investment (C_0):

$$PI = \frac{PV}{|C_0|}.$$

> ***The profitability index rule:*** *It is worth accepting investments that have a larger profitability index than one.*

Figure 4.3 Evaluation of the profitability index (PI) of an investment possibility



Any decision taken under the profitability index rule leads to the same decision taken under the net present value rule. Consider the following table:

Profitability index (PI)	multiply by C_0	Net present value (NPV)	Decision
$\frac{PV}{|C_0|} > 1$	$PV > |C_0|$	$NPV > 0$	**acceptance**
$PI = \frac{PV}{|C_0|} = 1$	$PV = |C_0|$	$NPV = 0$	**deliberation**
$PI = \frac{PV}{|C_0|} < 1$	$PV < |C_0|$	$NPV < 0$	**rejection**

Example 4.4:
Let us calculate the present value of the cash-flow streams of the investment examined in example 4.1.
The initial expenditure of the investment is 80 000 000 HUF.

Year	Net Cash Flow (HUF)
0	
1	10 250 000
2	16 200 000
3	23 700 000
4	24 800 000
5	23 800 000
6	23 800 000

Suppose the opportunity cost of capital is 10%. Compute the profitability index of the project:

$$\mathrm{PI} = \frac{\frac{10.25}{1.1^1}+\frac{16.2}{1.1^2}+\frac{23.7}{1.1^3}+\frac{24.8}{1.1^4}+\frac{23.8}{1.1^5}+\frac{23.8}{1.1^6}}{80} = \frac{85.664}{80} = 1.0708.$$

The solution means that the present value of the future cash-flow streams is greater by 7.08 percent than the initial requirement investment. Considering that the profitability index is greater than 1, the project makes a profit. The value of the profit is equal to 7.08 percent of the initial expenditure (80 000 000 HUF):

$$\text{Profit} = 80\,000\,000 \cdot 0.0708 = 5\,664\,000\ \text{HUF}.$$

Picture 8.

The net present value of the project shows the amount of profit of an investment. According to the result of the net present value calculation in chapter 3.2.1, the net present value of the project is equal to our result (5.664 million HUF). We can establish that the results of the net present value calculation and profitability index give the same conclusion in relation to the investment. On the basis of the conclusion, the investment is profitable.

Can the profitability calculation apply to all investment possibilities without any restrictions? How can we make a decision, if we have to choose only one of various investment possibilities?

Do we have to choose the investment that indicates the highest profitability index?

Example 4.5:

Consider the following two projects and determine the net present value and the profitability index, if the opportunity cost of capital is 12 percent.

Year	Project 1 (HUF)	Project 2 (HUF)
0	-200 000	-20 000 000
1	800 000	40 000 000

The net present value and the profitability index for the two projects is the following:

Project 1

$$PV = \frac{800\,000}{1.12^1} = 714\,285.7143 \text{ HUF},$$

$$NPV = -200\,000 + \frac{800\,000}{1.12^1} = 514\,285.7143 \text{ HUF},$$

$$PI = \frac{714\,285.7143}{200\,000} = 3.57.$$

Project 2

$$PV = \frac{40\,000\,000}{1.12^1} = 35\,714\,285.71 \text{ HUF},$$

$$NPV = -20\,000\,000 + \frac{40\,000\,000}{1.12^1} = 15\,714\,285.71 \text{ HUF},$$

$$PI = \frac{35\,714\,285.71}{20\,000\,000} = 1.786.$$

Year	Project 1 (HUF)	Project 2 (HUF)
0	**-200 000**	**-20 000 000**
1	**800 000**	**40 000 000**
PV	714 285.7143	35 714 285.71
NPV	514 285.7143	15 714 285.71
PI	3.57	1.786

Based on the foregoing analysis, it can be seen that the profitability index of the first project is higher than the second project; however the net present value (benefit) of the second project is greater. If we have no capital limitation, the investment with the largest profit is a more advisable choice. The application of the profitability index may lead to an incorrect decision in comparison with mutually exclusive investments. Later (in Chapter 4.6), we will examine the problem of mutually exclusive investments.

We used a given discount rate in the net present value and the profitability index calculations; however, in general, we would like to know the rate of return of our project. In the following chapter we aim to understand how to calculate the internal rate of return of a given investment opportunity.

Addendum:

Modified profitability index (PI*)

In the literature, another definition of the profitability index can be found. The new profitability index (PI^*) is different only in the sense that it shows the amount of the net present value for only one unit of the initial investment:

$$PI^* = \frac{NPV}{C_0}.$$

The modified profitability index (PI^*) gives the net benefit per one unit of initial investment, if the net present value is positive. The difference between the profitability index and the modified profitability index (PI^*) is equal to one:

$$PI^* - PI = \frac{NPV}{C_0} - \frac{PV}{C_0} = \frac{C_0 + PV}{C_0} - \frac{PV}{C_0} = 1.$$

This means that our decision is not affected by the choice between the profitability index (PI) and the modified profitability index (PI^*). The decision produced by the profitability index rule is equivalent to the decision based on the modified profitability index.

4.3. Internal rate of return (IRR)

If you invest your money in a bank account, you are always informed about the rates of return on your investment. However, if you invest your money in a capital investment, the bank does not provide information on the rate of return of the investment. We will try to determine the internal rate of return of an investment. Let us assume you deposit 100 000 EUR in your bank account and you get 110 000 EUR in a year. In this case, the present value and the net present value of your money can be written as the following:

$$PV = 100\,000 = \frac{110\,000}{1 + \text{rate of return}},$$

$$NPV = -100\,000 + \frac{110\,000}{1 + \text{rate of return}} = 0.$$

We have no information about the discount rate; however, we can compute the rate of return of your investment:

$$\text{rate of return} = \frac{110\,000}{100\,000} - 1 = 0.1.$$

The rate of return of your investment is 10%. The rate of return of a project which has only one year of cash flow, is the following:

$$\text{Rate of return} = \frac{\text{cash flow in year 1}}{\text{required investment}} - 1,$$

$$\text{Rate of return} = \frac{C_1}{-C_0} - 1,$$

$$NPV = C_0 + \frac{C_1}{1 + \text{internal rate of return}} = 0.$$

The internal rate of return of an investment is equal to the discount rate, which gives a zero net present value. The internal rate of return represents the discount rate at which the present value of the future cash flows of an investment is equal to the initial costs of the same investment.

$$NPV = C_0 + \frac{C_1}{1 + \text{internal rate of return}} = 0.$$

The previous calculation is very simple, because only one year cash flow is given. How can we calculate the rate of return of an investment possibility which has two years of cash flows?

Example 4.6:
Suppose an investment's initial cost is 18 million HUF. The investment yields a net revenue of 15 million HUF at the end of the first year, and 18 million HUF at the end of the second year. Let us determine the internal rate of return of the project, if the discount rate is 10.5%.
The internal rate of return can be calculated as the following:

$$\text{NPV} = \text{C}_0 + \text{PV} = 0,$$

$$\text{NPV} = -18 + \frac{15}{(1+\text{internal rate of return})^1} + \frac{18}{(1+\text{internal rate of return})^2} = 0,$$

$$\text{internal rate of return} = 0.5.$$

Supplement to the calculation:

$$\text{NPV} = -18 + \frac{15}{(1+\text{internal rate of return})^1} + \frac{18}{(1+\text{internal rate of return})^2} = 0,$$

let x be equal to (1 + internal rate of return):

$$1 + \text{internal rate of return} = \text{x},$$

$$-18 + \frac{15}{\text{x}^1} + \frac{18}{\text{x}^2} = 0,$$

$$-18 \cdot \text{x}^2 + 15 \cdot \text{x} + 18 = 0,$$

$$-6 \cdot \text{x}^2 + 5 \cdot \text{x} + 6 = 0,$$

$$\text{x}_1 = 1.5$$
$$\text{x}_1 = -0.667.$$

The internal rate of return of the investment is 50%. It is greater than the given opportunity cost of capital, so this means the investment yields a larger profit than an alternative investment opportunity.
The calculation of the internal rate of return of a project similar to the investment examined with two year cash flows in example 4.6 is very simple. However, in the case of a project with a longer life, the calculation of the rates of return can be more complicated.
"The internal rate of return is defined as the rate of discount which makes NPV=0. This means that to find the IRR for an investment project lasting n years, we must solve for *IRR* in the following expression (Brealey – Myers, 2005)":

$$NPV = C_0 + PV = C_0 + \frac{C_1}{(1+IRR)^1} + \frac{C_2}{(1+IRR)^2} + \frac{C_3}{(1+IRR)^3} + \cdots + \frac{C_n}{(1+IRR)^n} = -C_0 + \\ + \sum_i^n \frac{C_i}{(1+IRR)^i} = 0.$$

The solution of the IRR equation can be given by the iteration method, and it can be computed using Excel. Examine the following example.

Example 4.7:

Consider the investment possibility in Example 4.1. The initial cost of the project is 80 000 000 HUF, and the future benefits are shown in the following table:

Table 4.1 Investment projected cash flows

Year	Initial expenditure (HUF)	Revenue-Expenditure (HUF)
0	80 000 000	
1		10 250 000
2		16 200 000
3		23 700 000
4		24 800 000
5		23 800 000
6		23 800 000

Suppose that the expected return is 10%. Compute the internal rate of the cash flows represented in Table 4.1. The net present value is equal to zero (cash flows are given in million HUF):

$$NPV = -80 + \frac{10.25}{(1+IRR)^1} + \frac{16.2}{(1+IRR)1^2} + \frac{23.7}{(1+IRR)^3} + \frac{24.8}{(1+IRR)^4} + \frac{23.8}{(1+IRR)^5} + \frac{23.8}{(1+IRR)^6} = 0.$$

The value of IRR can be calculated using Excel. Write the initial cost (- 80 000 000) into the A1 cell of a new Excel worksheet. Next, type the subsequent values of the given cash flow into the cells A2:A7. To calculate the internal rate of return, go to the B1 cell, select the functions on the toolbar and choose the financial functions in the function category. You will find IRR among the financial functions. You have to provide information about the values: A1:A7.

0	-80000000	12.1%
1	10250000	
2	16200000	
3	23700000	
4	24800000	
5	23800000	
6	23800000	

$$IRR = 12.1\%.$$

If the discount rate is equal to the internal rate of return of the project, the net present value is equal to zero. However, if the discount rate is lower than the internal rate of return, the net present value is greater than zero. If the internal rate of return is less than the discount rate, the project is not profitable, because the net present value of the project is negative (Figure 4.4).

Figure 4.4 The net present value of a project

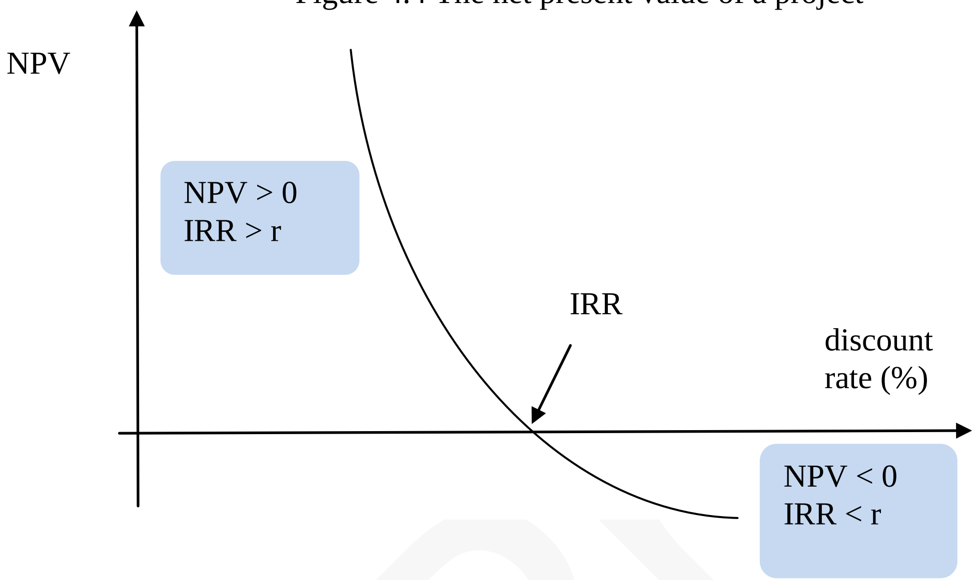
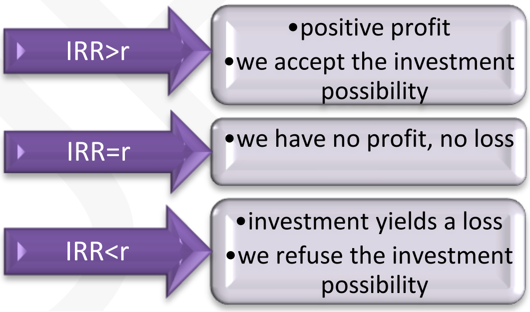


Figure 4.5 Evaluation of the internal rate of return (IRR) of an investment possibility



The IRR decision rule is the following:

"Invest as long as the return on the investment exceeds the rate of return on equivalent investments in the capital market (Brealey - Myers, 2005)".

Internal rate of return (IRR) rule: *It is worth accepting investments that have a greater internal rate of return than the opportunity cost of capital (expected returns).*

The internal rate of return method has several deficiencies; as a result, the use of the internal rate of return for evaluating the profitability of investments can be misleading. We will try to analyse the main shortcomings of the IRR calculation step by step.
The first case is the dichotomy of lending and borrowing

Example 4.8:
Consider the cash flows of the following two projects. Suppose that the interest rate is 20 percent.

Year	Project I (million HUF)	Project II (million HUF)
year 0	-500	500
year 1	800	-800

Take a closer look at the cash flows of the two projects. We can see that the initial cost of the first project is 500 million HUF, and it produces a cash inflow of 800 million HUF in year 1. However, in the case of the second project we can realize 500 million HUF now, and we have to pay for this 800 million HUF benefit in a year. Compute the internal rate of return of the two projects. The internal rate of return is the discount rate that makes the net present value zero:

Project I:

$$\mathrm{NPV} = -500 + \frac{800}{(1+\mathrm{IRR})^1} = 0,$$
$$\mathrm{IRR} = 60\%.$$

Project II:

$$\mathrm{NPV} = 500 - \frac{800}{(1+\mathrm{IRR})^1} = 0,$$
$$\mathrm{IRR} = 60\%.$$

According to the calculations, the internal rates of return of the two projects are equal. This means that it does not matter which project is realised. However, if we choose the second project, we receive 500 million HUF and we have to pay back 800 million HUF in a year. We borrow money at a 60 percent interest rate. If ***Bank A*** provides a loan at a 60% interest rate, we refuse ***Bank A***'s offer, and try to find better one, because the interest rate is very high. It is estimated that the first project will lend 500 million HUF and we will receive 800 million HUF at the end of the first year. Which one is the better choice: to provide a loan at a 60% percent interest rate or to borrow money at 60% percent interest rate?
It is very important that decision-makers have enough knowledge about the financial data of an investment opportunity and in the decision-making process they take into account the cash flow forecast of the project.

However, if we calculate the net present values of the two investment possibilities, we can recognize the loss-making investment possibility (Figure 3.5):

Project I

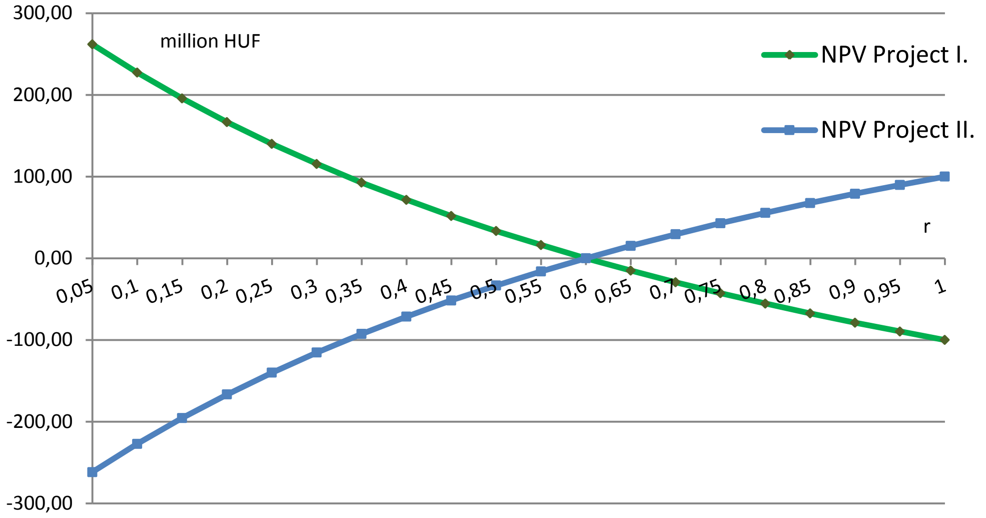
$$NPV = -500 + \frac{800}{1.2} = 166.67,$$

Project II

$$NPV = 500 - \frac{800}{1.2} = -166.67.$$

The net present values of the two projects as a function of the interest rate can be shown in Figure 4.5.

Figure 4.5 The net present value of project I and project II



It can be seen from the net present value calculations that the net present value of the first project is positive if the discount rate is less than the internal rate of return. However, the same could not be said with regard to the second project, because the net present value is negative when the discount rate is less than the internal rate of return of Project II.
The second problem could arise from the multiple solutions of a polynomial.

The second case is the problem of the multiple rates of return

Example 4.9:
Consider the following investments' cash-flow streams.

Year	Investment I (million HUF)	Investment II (million HUF)
0	-200	-1 000
1	520	1 100
2	-336	1 940
3		-2 184

Calculate the internal rates of return of the two investments.

Investment I:

$$\text{NPV} = -200 + \frac{520}{(1+\text{IRR})^1} - \frac{336}{(1+\text{IRR})^2} = 0,$$

$$-200 \cdot (1+\text{IRR})^2 + 520 \cdot (1+\text{IRR})^1 - 336 = 0.$$

We have two solutions for IRR (%):

$$\text{IRR}_1 = 40\%,$$
$$\text{IRR}_2 = 20\%.$$

Investment II:

$$\text{NPV} = -1\,000 + \frac{1\,100}{(1+\text{IRR})^1} + \frac{1\,940}{(1+\text{IRR})^2} - \frac{2\,184}{(1+\text{IRR})^3} = 0,$$

$$-1\,000 \cdot (1+\text{IRR})^3 + 1\,100 \cdot (1+\text{IRR})^2 + 1\,940 \cdot (1+\text{IRR})^1 - 2\,184 = 0.$$

We have three solutions for IRR (%):

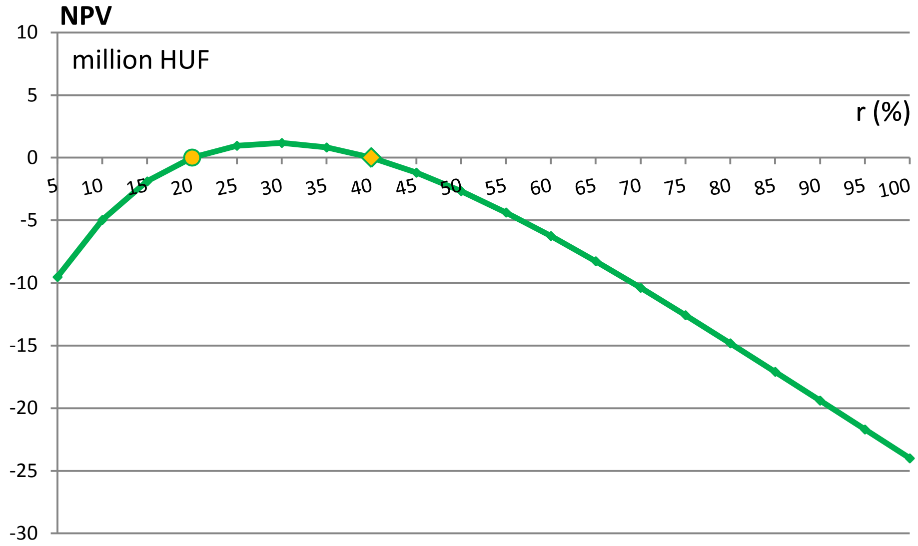
$$\text{IRR}_1 = 20\%$$
$$\text{IRR}_2 = 30\%$$
$$\text{IRR}_3 = -240\%.$$

If we use Excel, we get the smallest positive solution (20% and 20%). However, according to the correct calculation, we have three solutions for Investment II. Which one is the proper internal rate of return? If a negative number cannot be the rate of return of the investment, we have two different solutions for investment II. The two solutions raise the question of whether the first or the second internal rate of return is the better one to use in our decision-making. If we examine a project with a longer duration, we could find more solutions.

"By Descartes's "rule of signs" there can be as many different positive solutions to a polynomial as there are changes of sign. There can be as many different internal rates of return for a project as there are changes in the sign of the cash flows (Brealey – Myers, 2005)."

Examine Investment I more carefully. As can be seen in Figure 4.6, a further problem arises from the two solutions for the internal rate of return. According to the internal rate of return rule, we accept an investment if the opportunity cost of capital is less than the internal rate of return of the investment. As shown in Figure 4.6, if the opportunity cost of capital is less than the lower solution, the net present value is negative. This means that we cannot apply the rule mentioned in decision-making in every case.

Figure 4.6 The net present value of investment I



It is best if, in our decision-making process, we apply the net present value method too.

The third shortcoming is the problem of mutually exclusive projects.

In the following section, we will return to the question of how to choose from mutually exclusive projects. What do we mean by mutually exclusive, or mutually exclusive investments?

Mutually exclusive: A situation where the acceptance of one alternative simultaneously excludes the other alternatives. We talk about mutually exclusive investments when the acceptance of one investment possibility involves the exclusion of other investment possibilities.

For example, if you have a parcel of land in the suburbs of your town, you can build leisure facilities for children or for sportspeople. If you accept the first option, you cannot choose the second.

Example 4.10:

Let assume you are given the cash flows of the following two mutually exclusive investments.

Year	Investment I (million HUF)	Investment II (million HUF)
0	-1 200	-1 050
1	800	200
2	400	400
3	600	600
4	300	700
5	300	600

Determine the internal rates of return of the two mutually exclusive investments.

Investment I:

$$\text{NPV} = -1\,200 + \frac{800}{(1+\text{IRR})^1} + \frac{400}{(1+\text{IRR})^2} + \frac{600}{(1+\text{IRR})^3} + \frac{300}{(1+\text{IRR})^4} + \frac{300}{(1+\text{IRR})^5} = 0,$$

$$-1\,200 \cdot (1+\text{IRR})^5 + 800 \cdot (1+\text{IRR})^4 + 400 \cdot (1+\text{IRR})^3 + 600 \cdot (1+\text{IRR})^2 + 300 \cdot$$
$$\cdot (1+\text{IRR})^1 + 300 = 0.$$

Only one sign changes from negative to positive; we have only one solution for the internal rate of return. The internal rate of return of Investment I is:

$$\text{IRR} = 36\%.$$

Investment II:

$$\text{NPV} = -1\,050 + \frac{200}{(1+\text{IRR})^1} + \frac{400}{(1+\text{IRR})^2} + \frac{600}{(1+\text{IRR})^3} + \frac{700}{(1+\text{IRR})^4} + \frac{600}{(1+\text{IRR})^5} = 0,$$

$$-1\,050 \cdot (1+\text{IRR})^5 + 200 \cdot (1+\text{IRR})^4 + 400 \cdot (1+\text{IRR})^3 + 600 \cdot (1+\text{IRR})^2 + 700 \cdot$$
$$(1+\text{IRR})^1 + 600 = 0.$$

The internal rate of return of Investment II, calculated using Excel, is:

$$\text{IRR} = 31\%.$$

The two investments are mutually exclusive; we can choose only one of them. According to the internal rates of return calculation, we can say that Investment I is the better choice, because the internal rate of return for Investment I is greater than for Investment II. If we determine the net present values of the two investments, we can see that Investment I is not always the better choice. Let the discount rate be between 5% and 100%, and determine the net present values of the two projects (Table 4.2; Figure 4.7).

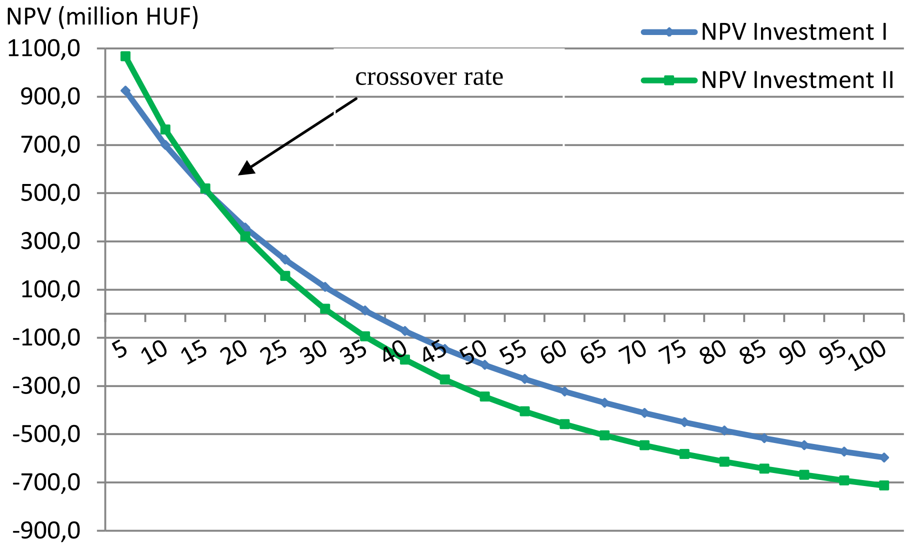
Table 4.2 The net present values of mutually exclusive investments

Discount Rate (%)	NPV Investment I (million HUF)	NPV Investment II (million HUF)
5	924.9	1067.6
10	699.8	763.8
15	513.3	519.4
20	356.9	320.4
25	224.4	156.5
30	111.0	20.3
35	13.2	-94.0
40	-72.0	-190.6
45	-146.5	-273.0
50	-212.3	-343.8
55	-270.7	-405.0
60	-322.9	-458.2
65	-369.7	-504.8
70	-411.8	-545.8
75	-450.0	-582.0
80	-484.8	-614.1

85	-516.5	-642.8
90	-545.5	-668.5
95	-572.2	-691.6
100	-596.9	-712.5

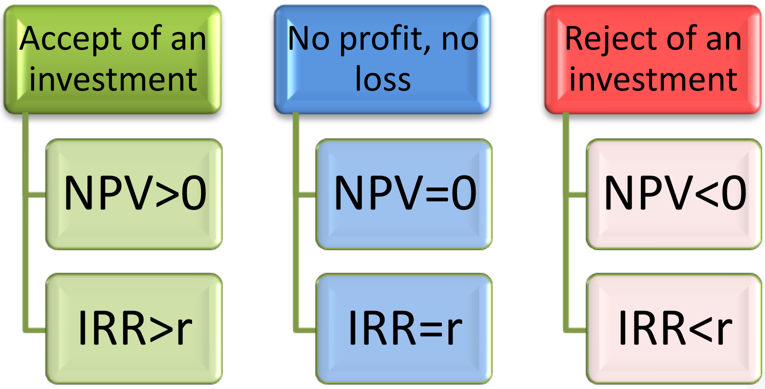
The internal rate of return can be found where the net present value function crosses the horizontal axis (Figure 4.7). We can see that the net present values of Investment II are greater if the discount rate is less than or equal to 15 percent (Table 4.2; Figure 4.7). The higher net present value represents higher benefits, and this means that investment I yields a greater profit at a discount rate under 15 percent. In this case a choice made on the basis of the IRR calculation will be different from one made on the basis of the net present value calculation (Table 4.2; Figure 4.7). A decision made only on the basis of the IRR calculation can be misleading in the case of mutually exclusive investments. The crossover rate can be found at the discount rate where the net present values of the two investments are equal (Figure 4.7).

Figure 4.7 The net present values of mutually exclusive investments



Despite the shortcomings of the internal rate of return calculation, it is very popular in practice. Generally speaking, investors or financial analysts inquire about the rates of return of an investment rather than the net present value that is expressed in Hungarian forints, Euros, or dollars. We must point out that it is worth using the IRR calculation; however, we have to make more calculations before making decisions involving mutually exclusive projects. In the case of independent investments, the net present value and internal rate of return calculations lead to the same decision to accept or refuse. In the case of independent investments, if the net present value of an investment is positive, the internal rate of return is greater than r (Figure 4.8). However, if the net present value is less than zero, the internal rate of return is less than r. Finally, if the net present value is equal to zero, the internal rate of return is equal to r (Figure 4.8).

Figure 4.8 Evaluation of the internal rate of return (IRR) and the net present value of independent invetments



4.4. Modified internal rate of return (MIRR)

Very often investors and analysts give priority to the internal rate of return, because it is very easy to understand. As we have seen, there are various shortcomings with the internal rate of return calculation. One of them is the multiple results which can result from the solution of the equation for the internal rate of returns. If we want to analyse an investment possibility whose life duration is *n* years, according to the Descartes rule, we can obtain n solution for the IRR, depending on the number of changes in the sign. We introduce a new indicator to handle the problem of multiple rates of return.
Let us examine again Investment II, as shown in Example 4.9

Year	Investment II (million HUF)
0	-1 000
1	1 100
2	1 940
3	-2 184

Investment II:

$$NPV = -1\,000 + \frac{1\,100}{(1+IRR)^1} + \frac{1\,940}{(1+IRR)^2} - \frac{2\,184}{(1+IRR)^3} = 0, \qquad (4.4.1)$$

$$-1\,000 \cdot (1+IRR)^3 + 1\,100 \cdot (1+IRR)^2 + 1\,940 \cdot (1+IRR)^1 - 2\,184 = 0.$$

We have three solutions for the IRR (%):

$$IRR_1 = 20\%$$
$$IRR_2 = 30\%$$
$$IRR_3 - 240\%.$$

We have two positive solutions, because there are two changes in the sign.

Additional information:

As we can see, we have three solutions for the equation (4.4.1). According to Descartes' rule we can determine the number and sign of the solution. Let us examine the following equation, which is equivalent to the equation (4.4.1) if we write x instead of (1+IRR):

$$x = 1 + IRR$$

If we write positive x, instead of x, we can see that there are two changes in sign (from negative to positive, then positive to negative).

$$f(x) = -1\,000 \cdot x^3 + 1\,100 \cdot x^2 + 1\,940 \cdot x^1 - 2\,184 = 0 \qquad (4.4.2)$$

This means that equation (4.4.2) has two positive solutions ($x = 1.2$ *and* $x = 1.3$). However, if we write (-x) instead of x, we get the following:

$$f(-x) = -1\,000 \cdot (-x)^3 + 1\,100 \cdot (-x)^2 + 1\,940 \cdot (-x)^1 - 2\,184 = 0$$

In equation (4.4.3), there is only one change in sign (from positive to negative), thus we have only one negative solution ($x = -1.4$).

$$f(-x) = 1\,000 \cdot x^3 + 1\,100 \cdot x^2 - 1\,940 \cdot x^1 - 2\,184 = 0$$

We eliminate the changes in the sign to introduce the ***modified internal rate of return (MIRR).*** More specifically, we leave only one change in the sign. Supposing that the required rate of return is 15%, determine the future value of future cash flows at the end of year three (million HUF):

Year	Investment II (million HUF)	FV_3
0	-1 000	
1	1 100	$1\,100 \cdot 1.15^2 = 1\,454.75$
2	1 940	$1\,940 \cdot 1.15^1 = 2\,231$
3	-2 184	$-2\,184 \cdot 1.15^0 = -2\,184$
Terminal value		1 501.75

$$FV_3 = 1\,100 \cdot 1.15^2 + 1\,940 \cdot 1.15^1 + -2\,184 \cdot 1.15^0 = 1\,501.75$$

The net present value can be written as the following:

$$NPV = -1\,000 + \frac{1\,501.75}{(1+MIRR)^3} = 0,$$

$$\text{MIRR} = 14.52\%.$$

We have determined the modified rate of return when we have assumed that future cash flows can be reinvested at the firm's required rate of return. The general formula for the calculation of the modified internal rate of return (MIRR) is the following (Besley – Brigham, 2015):

$$\text{Present value of cash outflows} = \frac{\text{Future value of cash inflows}}{(1+\text{MIRR})^n} = \frac{\text{TV}}{(1+\text{MIRR})^n}, \qquad (4.4.3)$$

where TV is the terminal value. The terminal value equals the future value of the cash inflows. Let COF represent the cash outflows and CIF represent the cash inflows; the equation (4.4.3) can be written as the following (Besley – Brigham, 2015):

$$\sum_{t=0}^{n} \frac{\text{COF}_t}{(1+r)^t} = \frac{\sum_{t=0}^{n} \text{CIF}_t \cdot (1+r)^{n-t}}{(1+\text{MIRR})^n}, \qquad (4.4.4)$$

where *r* is the required rate of return. The left hand side of the equation (4.4.4) represents the present value of the investment outlays when discounted at the project's or firm's required rate of return. The numerator of the right hand side of the equation (4.4.4) shows the future value of the cash inflows compounded to the end of the life of the investment, assuming that these inflows are reinvested at the required rate of return (Besley – Brigham, 2015).

Modified internal rate of return (MIRR): *The discount rate at which the present value of an investment's costs is equal to the present value of its terminal value, where the terminal value is equal to the sum of the future values of the cash inflows compounded at the firm's required rate of return.*

(Besley – Brigham, 2015)

Modified internal rate of return (MIRR) rule: *It is worth accepting investments that have a greater modified internal rate of return than the required rate of return (opportunity cost of capital).*

IRR versus MIRR

The calculation of the modified internal rate of return eliminates the problem of multiple solutions in the calculation of the internal rate of return. Both the IRR and the MIRR assume that the cash flows are reinvested; however, in the case of the MIRR the cash flows are reinvested at the required rate, while in the case of IRR, the cash flows are reinvested at the investment's internal rate of return. In the case of mutually exclusive investments, the MIRR provides the same investment decisions as the net present value if the investment's duration and size are equal. If investments are of equal size but have different lives, the MIRR always leads to the same decision as the NPV if the modified internal rate of return for investments are calculated using the life of the longer investment as the terminal year (Besley – Brigham, 2015). In this case we can fill zeros for the missing cash flows of the shorter investment. In chapter 4.6, we will examine the problem of mutually exclusive investments, and how we can make investment decisions.

4.5. The role of time in investment decisions (payback and discounted payback period)

In the previous examinations, we have analysed different indicators to measure the profitability of an investment:

- Net present value (NPV) informs us about the benefit or the loss of an investment;
- Internal rate of return (IRR) shows the rate of return for an investment;
- Modified internal rate of return (MIRR) shows the discount rate where the present value of an investment's cost equals the terminal value;
- Profitability index (PI) (modified profitability index - PI*) shows the present value (net present value) per unit of required investment.

Generally, financial analysts and investors inquire about the period during which the chosen investment pays off. In the following sub-sections we will define and analyse the payback period and the discounted payback period.

4.5.1. Payback period – Non-discounted payback period

The (traditional or non-discounted) payback period of an investment shows how long it will take an investment to cover its initial costs from the cash flows it is expected to generate in the future. The payback period indicates the number of years it takes before the cumulative forecast cash flow equals the initial investment. On the basis of the traditional payback period method, we add up the expected cash flows for each period until the cumulative value is equal to the initial investment cost. The payback period can be calculated by using the following formula (Besley – Brigham, 2015):

Picture 9.

$$Payback\ period =$$
$$= Number\ of\ years\ just\ prior\ to\ the\ year\ of\ full\ recovery\ of\ the\ initial\ investment$$
$$+ \frac{Amount\ of\ the\ initial\ investment\ that\ is\ unrecovered\ at\ the\ start\ of\ the\ recovery\ year}{total\ cash\ flow\ generated\ during\ the\ recovery\ year}$$

Let us consider the following example of the calculation of the payback period:

Example 4.11:
Suppose we can choose among three investment opportunities (INV1, INV2, INV3). The three investments have different lifetime periods, but the initial expenditure is identical for each project:

Year	INV1 (million HUF)	INV2 (million HUF)	INV3 (million HUF)
0	-8 000	-8 000	-8 000
1	4 500	8 000	1 000
2	4 500	1 400	4 000
3	4 200		8 000
4			18 000

Let us determine the payback period for the three projects.
Determine the cumulative sum at which the future cash flow will reach the value of the initial outlay.

Year	INV1 (million HUF)		INV2 (million HUF)		INV3 (million HUF)	
	Cash flow	Cumulative net cash flow	Cash flow	Cumulative net cash flow	Cash flow	Cumulative net cash flow
0	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000
1	4 500	-3 500	8 000	0	1 000	-7 000
2	4 500	+1 000	1 400		4 000	-3 000
3	4 200				8 000	+ 5 000
4					18 000	
Payback period (year)	$PB = 1 + \frac{3\,500}{4\,500} = 1.78$		1		$PB = 2 + \frac{3\,000}{8\,000} = 2.375$	

Investment INV2 has the shortest payback period (1 year), and investment INV3 has the longest payback period with 2.375 years. Which investment is the best? It could be argued that we should accept the investment that has the shortest payback period. This means that, according to our calculation, we have to accept the INV 2 project. Generally the payback period is compared with the maximum cost recovery time determined by the firm. If the firm accepts projects with a payback of two years or less, INV 1 and INV 2 would be acceptable, although INV 3 would not.
In order to get a full picture, let us calculate the net present value for each project. Suppose the opportunity cost of capital is 15%.

Year	INV1 (million HUF)		INV2 (million HUF)		INV3 (million HUF)	
	Cash flow	Cumulative net cash flow	Cash flow	Cumulative net cash flow	Cash flow	Cumulative net cash flow
0	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000
1	4 500	-3 500	8 000	0	1 000	-7 000
2	4 500	+1 000	1 400		4 000	-3 000
3	4 200				8 000	+ 5 000
4					18 000	
Payback period (year)	1.78		1		2.375	
NPV (million HUF)	2 077.3		15.1		11 445.8	

We can see that INV 3 has the largest net present value, although it is not a good investment, because it has the longest payback period. What can we do in this case? Why has the decision made on the basis of the payback period and net present value in this calculation led to contradictory results? The main reason for the contradictory results is that the payback period calculation has certain shortcomings.

- "The payback rule ignores all cash flows after the cut-off date" (Brealey – Myers, 2005). If the cut-off date is one year, the payback rule rejects projects INV2 and INV3, regardless of the size of the cash inflows after year 1 (Brealey – Myers, 2005).
- The payback rule gives equal weight to all cash flows before the cut-off date (Brealey – Myers, 2005).
- The method employed by the payback calculation ignores the time value of money and the opportunity cost of capital.

The traditional payback period rule: An investment possibility is acceptable if the payback period of the investment is less than the recovery period. The recovery period is established by the firm.

If we use the traditional payback period to evaluate the profitability of an investment, this method could lead to incorrect decisions. A decision made on the basis of the net present value leads to a better decision than a decision made on the basis of the payback period.
In the next sub-section (3.6.2), we examine a new payback period indicator that corrects one of the shortcomings of the traditional payback period.

4.5.2. The discounted payback period

The discounted payback period method takes account of the time value of money; as a result it leads to better decisions than the traditional payback period.
The discounted payback period shows the length of time required for an investment's discounted cash flows to repay its initial cost.
The discounted payback period rule: An investment is acceptable if the discounted payback period of the investment is less than the expected year determined by the firm or the investment's life.

"The discounted-payback rule asks, How many periods does the project have to last in order to make sense in terms of net present value" (Brealey – Myers, 2005)?

Let us determine the discounted payback period for the investment examined in example 3.11.

Example 4.12:
We can choose from among three investment opportunities (INV1, INV2, INV3). Suppose that the discount rate is 15 percent.

Table 4.3 The discounted payback period

Year	INV1 (million HUF)			INV2 (million HUF)			INV3 (million HUF)		
	Cash flow	Discounted cash flow	Cumulative net cash flow	Cash flow	Discounted cash flow	Cumulative net cash flow	Cash flow	Discounted cash flow	Cumulative net cash flow
0	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000	-8 000
1	4 500	3 913	-4 087	8 000	6 956.5	-1 043.5	1 000	869.6	-7 130.4
2	4 500	3 402.7	-684.3	1 400	1 058.6	15.1	4 000	3 024.6	-4 105.8
3	4 200	2 761.6	2 077.2				8 000	5 260.1	1 154.3
4							18 000	10 291.6	
Discounted Payback period (year)	$DPB = 2 + \frac{684.3}{2\,761.6} = 2.25$			$DPB = 1 + \frac{1043.5}{1058.6} = 1.99$			$DPB = 2 + \frac{4\,105.9}{5\,260.1} = 2.78$		

The discounted payback period of investment INV1 (Table 4.3):

$$8000 - \frac{4\,500}{1.15^1} - \frac{4\,500}{1.15^2} = 684.3.$$

$$\text{discounted payback period (year)}_{\text{INV1}} => 2 + \frac{684.3}{2\,761.6} = 2.25.$$

The discounted payback period of project INV1 is 2.25 years.

The discounted payback period of investment INV2 (Table 3.3):

$$8000 - \frac{8\,500}{1.15^1} = 1043.5.$$

$$\text{discounted payback period (year)}_{\text{INV2}} => 1 + \frac{1\,043.5}{1\,058.6} = 1.99.$$

The discounted payback period of project INV2 is 1.99 years.

The discounted payback period of investment INV3 (Table 3.3):

$$8000 - \frac{1\,000}{1.15^1} - \frac{4\,000}{1.15^2} = 4\,105.9.$$

$$\text{discounted payback period (year)}_{\text{INV2}} => 2 + \frac{4\,105.9}{5\,260.1} = 2.78.$$

The discounted payback period of project INV3 is 2.78 years.

The disadvantage of the discounted payback period calculation is the following:

- The discounted payback rule considers all cash flows, but only during the discounted payback period. The method ignores the present value of money after the discounted payback period.

Advantages:

- The discounted payback rule does not give equal weight to all cash flows before the payback period.
- The payback calculation takes into account the time value of money. It does not ignore the present value of the future cash-flow streams. It takes into account the opportunity cost of capital.

4.6. Independent and mutually exclusive projects

As we have mentioned in the introduction to Chapter 4, mutually exclusive projects are a set of projects where the acceptance of a project means that other projects can be refused. In this subsection we study how we can make mutually exclusive investment decisions. In the case of independent projects, if the firm accepts a project, this decision does not affect the acceptance of other projects.

In the previous subsections we studied the calculation of net present value, the profitability index, the internal rate of return, the modified internal rate of return, the payback, and the discounted payback period. So far, we have mainly examined only one investment opportunity. We have not discussed a case in which we have to choose one of several investment opportunities.

Below, you will find the first examination of a situation in which we can choose from two investment opportunities. We can distinguish the following two cases:

- Choosing between investments with identical life-times;
- Choosing between investments with different life-times.

Suppose the company has to make a choice between two machines, A and B. The two machines are made differently but have identical capacities and do exactly the same job.

4.6.1. First case: Evaluating investments with identical lives

Assume that the firm has to make a choice between two assets, Machine A and Machine B. The two machines are made differently but have identical capacities and do exactly the same job.

If the investment opportunities have the same duration of life, there is no special change in the calculations. For example, if we make a decision on the basis of the net present value of the projects, we have to choose the one with the greatest positive net present value.

Example 4.13:

Two machines are offered to the *BEST PRODUCT COMPANY*. The cash-flow streams of the two projects are the following:

Year	Machine A (thousand HUF)	Machine B (thousand HUF)
0	**-56 000**	**-65 000**
1	**18 500**	**16 000**
2	**18 500**	**18 000**
3	**20 000**	**20 000**
4	**20 500**	**25 000**
5	**25 400**	**28 500**

In the first step, calculate the net present values of the two cash flow streams so as to determine which one generates a greater profit, and specify which one is the better choice. Suppose the opportunity cost of capital is 8%.
First, let us calculate the net present value of each investment (million HUF).

$$NPV_A = -56 + \frac{18.5}{1.08^1} + \frac{18.5}{1.08^2} + \frac{20}{1.08^3} + \frac{20.5}{1.08^4} + \frac{25.4}{1.08^5} = 25.22.$$

$$NPV_B = -65 + \frac{16}{1.08^1} + \frac{18}{1.08^2} + \frac{20}{1.08^3} + \frac{25}{1.08^4} + \frac{28.5}{1.08^5} = 18.9.$$

Both of the two investments have a positive net present value; consequently, these investments opportunities generate profit. However, investment A is better than investment B because the net present value of investment A is greater than that of investment B. The present value of investment A is less than B; however, the present value of future inflows per unit initial cost is greater for investment A than B.
In the second step, let us calculate the traditional internal rate of return for each investment:

$$NPV_A = -56 + \frac{18.5}{(1+IRR)^1} + \frac{18.5}{(1+IRR)^2} + \frac{20}{(1+IRR)^3} + \frac{20.5}{(1+IRR)^4} + \frac{25.4}{(1+IRR)^5} = 0.$$
$$IRR_A = 23\%.$$

$$NPV_B = -65 + \frac{16}{(1+IRR)^1} + \frac{18}{(1+IRR)^2} + \frac{20}{(1+IRR)^3} + \frac{25}{(1+IRR)^4} + \frac{28.5}{(1+IRR)^5} = 0.$$
$$IRR_B = 17.39\%.$$

The internal rate of return for the two investments exceeds the 8 percent opportunity cost of capital. According to the IRR results, we can see that the internal rate of return for the new Machine A is greater than for Machine B. Based on the internal rate of return rule, the purchase of Machine A is a better choice than the purchase of Machine B. This result is consistent with the decision taken on the basis of the net present value calculation.

Finally, let us compute the profitability index for Machine A and Machine B:

$$PI_A = \frac{\frac{18.5}{1.08^1}+\frac{18.5}{1.08^2}+\frac{20}{1.08^3}+\frac{20.5}{1.08^4}+\frac{25.4}{1.08^5}}{56} = \frac{81.22}{56} = 1.45,$$

$$PI_B = \frac{\frac{16}{1.08^1}+\frac{18}{1.08^2}+\frac{20}{1.08^3}+\frac{25}{1.08^4}+\frac{28.5}{1.08^5}}{65} = \frac{83.9}{65} = 1.29.$$

As we can see, the benefit per one unit cost exceeds one; this means that the two investment possibilities are profitable. If we purchase Machine A, the present value of future inflows per one unit initial cost is 1.45, so the present value of benefits exceeds the initial cost by 45 percent. By contrast, for Machine B the present value of benefits is 29 percent greater than the initial cost. If the firm purchases Machine A, it can gain a higher profit than if it purchases Machine B. Based on the net present value, the internal rate of return and the profitability

index calculations, we can come to the same decision to choose the more profitable investment opportunity.
In the next subsection, we will continue our examination with a case involving equipment with different life-times.

4.6.2. The second case: The lifetime of the investment opportunities is different

Suppose we would like to expand our firm's production, and therefore we want to purchase a new machine. We can choose between two machines. The two machines are made differently but have identical capacities and are able to do exactly the same job. The lifetime of the two machines is different. Consider the following example.

Example 4.14:
We want to obtain a new machine in order to increase the production capacity. Two machines (NEW1 and NEW2) are offered. Machine NEW1 and machine NEW2 have the same capacity and these two machines are able to carry out the same job. There are only two differences between the two investments:

- The lifetimes of the two investments are different, (the lifetime of the NEW1 machine is 6 years; and the lifetime of the NEW2 machine is 8 years – the machines have to be replaced after they expire).
- The acquisition costs and the annual operating costs are different.

Since the two machines are able to perform the same function of profit generation, the future benefits are the same.
The annual opportunity cost of capital is 6%.
We have the following information about the acquisition costs and the annual operating costs.

Year	NEW1 (million HUF)	NEW2 (million HUF)
0	110	100
1	14	16
2	14	16
3	14	16
4	14	16
5	14	16
6	14	16
7		16
8		16

Which machine should we purchase? Give reasons for your answer.

As we have mentioned, the two machines are able to perform the same production function and future cash inflows are the same for the two machines, so it is sufficient to concentrate only on the different cash outflows and lifetimes.

In the first step, determine the present value of the cash outflows by using the 6 percent opportunity cost of capital. The present value of the operating costs plus the single acquisition cost is the following (million HUF):

$$PV_{NEW1} = 110 + \frac{14}{1.06^1} + \frac{14}{1.06^2} + \frac{14}{1.06^3} + \frac{14}{1.06^4} + \frac{14}{1.06^5} + \frac{14}{1.06^6} = 178.84,$$

or using by the formula of annuity

$$PV_{NEW1} = 110 + \frac{14}{0.06} \cdot (1 - \frac{1}{1.06^6}) = 178.84,$$

$$PV_{NEW2} = 100 + \frac{16}{1.06^1} + \frac{16}{1.06^2} + \frac{16}{1.06^3} + \frac{16}{1.06^4} + \frac{16}{1.06^5} + \frac{16}{1.06^6} + \frac{16}{1.06^7} + \frac{16}{1.06^8} = 199.36\ ,$$

or

$$PV_{NEW2} = 100 + \frac{16}{0.06} \cdot \left(1 - \frac{1}{1.06^8}\right) = 199.36\ .$$

We can see that the present value of the total cost of the machine NEW1 is 20.52 million HUF lower; we might think, therefore, that it would be worth buying the NEW1 machine. However, the present value of the total cost for the NEW2 machine is spread across eight years, while the NEW1 machine’s lifetime is only 6 years.

This means that it is necessary to divide the present value of the cost by the asset life with regard to the time-value of the money. However, this does not mean we have to divide the computed present value by 6, if the life-time of NEW1 is 6 years. In this case the time-value of money would be ignored. We calculated the equivalent annual cost by using an annuity with the same present value as the lifetime costs of NEW1.

In the next step we calculate the present value of the total cost per year, so we determine the equivalent annual costs. The present value of the equivalent annual cost is equal to the present value of the operating costs plus the single acquisition cost.

$$PV_{NEW1} = 110 + \frac{14}{1.06^1} + \frac{14}{1.06^2} + \frac{14}{1.06^3} + \frac{14}{1.06^4} + \frac{14}{1.06^5} + \frac{14}{1.06^6} = \frac{C_{NEW1}}{1.085^1} + \frac{C_{NEW1}}{1.06^1} + \frac{C_{NEW1}}{1.06^2} + \frac{C_{NEW1}}{1.06^3} + \frac{C_{NEW1}}{1.06^4} + \frac{C_{NEW1}}{1.06^5} + \frac{C_{NEW1}}{1.06^6}.$$

Calculate the equivalent of annual cost (C_{NEW1}) (million HUF):

$$PV_{NEW1} = 110 + \frac{14}{1.06^1} + \frac{14}{1.06^2} + \frac{14}{1.06^3} + \frac{14}{1.06^4} + \frac{14}{1.06^5} + \frac{14}{1.06^6} = \frac{C_{NEW1}}{0.06} \cdot (1 - \frac{1}{1.06^6}).$$

To express the value of C_{NEW1} (million HUF), we can write:

$$C_{NEW1} = \frac{110 + \frac{14}{1.06^1} + \frac{14}{1.06^2} + \frac{14}{1.06^3} + \frac{14}{1.06^4} + \frac{14}{1.06^5} + \frac{14}{1.06^6}}{\frac{1}{0.06} \cdot (1 - \frac{1}{1.06^6})} = \frac{PV_{NEW1}}{\frac{1}{0.06} \cdot (1 - \frac{1}{1.06^6}))}.$$

$$C_{NEW1} = \frac{178.84}{\frac{1}{0.06} \cdot (1 - \frac{1}{1.06^6})} = \frac{178.84}{4.917} = 36.37.$$

The equivalent annual costs for the NEW2 machine are the following:

$$C_{NEW2} = \frac{100+\frac{16}{1.06^1}+\frac{16}{1.06^2}+\frac{16}{1.06^3}+\frac{16}{1.06^4}+\frac{16}{1.06^5}+\frac{16}{1.06^6}+\frac{16}{1.06^7}+\frac{16}{1.06^8}}{\frac{1}{0.06}\cdot(1-\frac{1}{1.06^8})} = \frac{PV_{NEW2}}{\frac{1}{0.06}\cdot(1-\frac{1}{1.06^8})}.$$

$$C_{NEW2} = \frac{199.36}{\frac{1}{0.06}\cdot(1-\frac{1}{1.06^8})} = \frac{199.36}{6.21} = 32.103.$$

The result of the equivalent annual costs for NEW2 is less than for NEW1, therefore it may be advisable to purchase the NEW2 machine. This means that the annual cost that is determined by taking into account the time value of money provides a more appropriate choice than when using the net present value.

This does not mean we have discovered a shortcoming in the present value or the net present value calculation, because the present value calculation is the basis of the derivation of the equivalent annual cost.

> You can think of the equivalent annual cost of NEW1 or NEW2 as an annual rental charge. Suppose the financial manager is asked to rent the NEW1 machine to the plant manager currently in charge of production. There will be six equal rental payments starting in year 1. The six payments must recover both the original cost of NEW1 in year 0 and the cost of running it in years 1 to 6. Therefore the financial manager has to make sure that the rental payments are worth 178.84 million HUF, the total PV (costs) of the NEW1 machine. You can see that the financial manager would calculate a fair rental payment equal to NEW1's equivalent annual cost. ***Our rule for choosing between plant and equipment with different economic lines is, therefore, to select the asset with the lowest fair rental charge, that is, the lowest equivalent annual cost.***

So far, inflation and technological change have been disregarded. Inflation and technological change influence costs and revenues through the change in prices. If inflation is higher, the increase in prices is greater too. However, if we take into account technological change, we should assume a decrease in prices. If modern machinery is bought, the purchase price and the operating cost can be lower year by year due to the improvement in efficiency. If we suppose that the costs decrease each year due to technological improvement, this means that the real rental cost decreases too. Let us examine the following example.

Example 4.15:

Suppose that the rental charges of our NEW1 machine (examined in example 4.14) decreases, because the purchase price and the operating costs decrease due to technological development. Naturally, if the operating costs fall, the rental charges will follow the change in costs. Suppose that the rental costs of our new machine decrease by 10 percent per year. The following question arises: How much will it cost to rent the machine? The following information is given about the new machine.

Year	NEW1 (million HUF)
0	110
1	14
2	14
3	14
4	14
5	14
6	14
PV	178.84

The rental costs must cover the present value of total costs:

$$PV_{NEW1} = 110 + \frac{14}{1.06^1} + \frac{14}{1.06^2} + \frac{14}{1.06^3} + \frac{14}{1.06^4} + \frac{14}{1.06^5} + \frac{14}{1.06^6} = 178.84,$$

$$PV_{NEW1} = \frac{rent_1}{1.06^1} + \frac{rent_2}{1.06^2} + \frac{rent_3}{1.06^3} + \frac{rent_4}{1.06^4} + \frac{rent_5}{1.06^5} + \frac{rent_6}{1.06^6} = 178.84.$$

We suppose that the rental cost decreases by 10 percent annually:

$$PV_{NEW1} = \frac{rent_1}{1.06^1} + \frac{0.9 \cdot rent_1}{1.06^2} + \frac{0.9^2 \cdot rent_1}{1.06^3} + \frac{0.9^3 \cdot rent_1}{1.06^4} + \frac{0.9^4 \cdot rent_1}{1.06^5} + \frac{0.9^5 \cdot rent_1}{1.06^6} = 178.84,$$

$$\frac{rent_1}{1.06^1} \cdot \frac{\left(\frac{0.9}{1.06}\right)^6 - 1}{\frac{0.9}{1.06} - 1} = 178.84,$$

$$rent_1 = 45.76.$$

The real rent is 45.76 million HUF in the first year. If we take inflation into account, we should calculate increasing prices in a similar way to our calculation of the decreasing prices resulting from technological improvement.

In the following subsection we will examine the problem of choosing investment opportunities when resources are limited.

4.6.3. The third case: Choosing capital investments when resources are limited

The value of the firm may increase if the firm accepts all projects that have a positive net present value. The increase in the firm's value depends on the opportunity cost of capital taken into account in the net present value calculation. However, a firm is not able to accept all projects that have a positive net present value, because its resources are limited. In this case, what are the decision criteria? How can we choose among the different investment opportunities that yield profit?

Picture 10.

A portfolio of the available investment opportunities should be chosen in order to maximize the net present value of the combination of investments. Let us take a closer look at the following example.

Example 4.16:
Suppose the budgetary constraint is 80 million HUF. The opportunity cost of capital is 5%. The company may choose among the following investment opportunities:

Year	Project1 (million HUF)	Project2 (million HUF)	Project3 (million HUF)
0	-40	-40	-80
1	5	5	40
2	25	30	40
3	40	40	50

The company may invest in either Project1 and Project2 together, or only in Project3, within the available budgetary constraints. The firm cannot invest in all projects at the same time with only 80 million HUF capital available.
In the first step, determine the net present value for each project (the opportunity cost of capital is 5%).

Year	Project1 (million HUF)	Project2 (million HUF)	Project3 (million HUF)
0	-40	-40	-80
1	5	5	40
2	25	30	40
3	40	40	50
NPV	21.99	26.53	37.57

Project3 has the highest net present value (37.57 million HUF), and Project2 has the second highest value. Project1 has the lowest benefit since its net present value is the lowest among the three projects. If the firm is faced with a budgetary limit, it should choose the best combination of the projects to realize the highest profit. However, the combination of Project1 and Project2 has a higher net present value than Project3:

$$21.99 + 26.53 = 48.52 > 37.57.$$

When resources are limited, the best decision to make is to choose the project which has the highest (net) present value per unit of initial investment. The (net) present value per unit of initial outlay for a project is the highest when the profitability index is the highest too. In subsection 4.2, we defined profitability as the following:

The profitability index (PI) shows the present value for only one unit of the required initial investment (C_0).

$$\mathrm{PI} = \frac{\mathrm{PV}}{|C_0|}.$$

Calculate the profitability index of the three projects.

Year	Project1 (million HUF)	Project2 (million HUF)	Project3 (million HUF)
0	-40	-40	-80
1	5	5	40
2	25	30	40
3	40	40	50
NPV	21.99	26.53	37.57
PV	61.99	66.53	117.57
PI	1.55	1.66	1.47

Project1 and Project2 have the largest net benefit per unit of initial investment costs; consequently the combination of the two projects should be accepted.

4.7. Terms and Questions

capital budgeting decision,
discount rate (constant, changing)
discounted payback period,
discounted payback period rule,
equivalent annual cost,
expansion decision,
independent decision,
initial investment cost,
internal rate of return,
internal rate of return rule,
limited resources,
modified internal rate of return,
modified profitability index,
multiple rates of return,
mutually exclusive projects,
net present value of an investment,
net present value rule,
rental charges,
replacement decision,
opportunity cost of capital
payback period,
payback period rule,
present value of an investment,
profitability index,
profitability index rule
shortcomings of internal rate of return,
shortcomings of discounted payback period method,
shortcomings of payback period method,
terminal value.

Theoretical questions

1. What is the difference between the profitability index and the modified profitability index?

2. Explain the net present value rule?

3. How can we calculate the internal rate of return of an investment?

4. Give the definition of the capital budgeting decision.

5. What is the difference between an expansion and a replacement decision?

6. Explain how the profitability index is calculated.

7. How can we choose the appropriate discount rate to evaluate an investment possibility?

8. Give the definition of the opportunity cost of capital.

9. How can the net present value of an investment opportunity be affected if the opportunity cost of capital increases?

10. How can we derive the present value of an investment?

11. Give the shortcomings of the payback period.

12. What is the difference between a payback period calculation and a discounted payback calculation?

13. Explain how to derive the discounted payback period.

14. Give the internal rate of return rule.

15. Explain how to evaluate mutually exclusive projects.

16. How can we determine the rental cost of machinery?

17. Explain how we can decide on capital investments when resources are limited.

18. What is the difference between mutually exclusive projects and independent projects?

19. Explain the net present value rule.

20. Explain the shortcomings of the calculation of the internal rate of return for an investment opportunity.

21. How can we calculate the modified internal rate of return for an investment?

22. Describe the relationship between present value and the profitability index calculation.

23. Explain the modified internal rate of return rule.

24. Give the definition of mutually exclusive investments.

25. How can we calculate the equivalent annual cost of an investment?

Calculation exercises

1.

You have the following information about an investment opportunity:

Year	Initial expenditure (million HUF)	Revenue-Expenditure (million HUF)
0	48	
1		12.6
2		14.2
3		16.5
4		18
5		24

The annual opportunity cost of capital is 3.5 percent.

a) Calculate the present value of the investment opportunity.
b) Calculate the net present value of the investment opportunity.
c) Would you accept the investment offer? Why (not)?

2.

Consider the cash-flow streams of the following two projects:

Year	Project I (million HUF)	Project II (million HUF)
0	-25	-30
1	15	15
2	15	18
3	16	20
4	18	20

The annual opportunity cost of capital is 8 percent.

a) Calculate the present value for each investment opportunity.
b) Calculate the net present value for each investment opportunity.
c) Which one is the more profitable investment opportunity? Why?
d) Calculate the profitability index for each investment opportunity.
e) Determine the modified profitability index.

3.

Project Best has an initial cost of 60 000 EUR, and the future net cash flows are 10 000 EUR per year for six years. Suppose that the required rate of return is 15 percent.

a) Determine the net present value for the project.
b) Determine the profitability index for the project.
c) What is the payback period? Give the shortcomings of the payback period.
d) What is the discounted payback period for the project?
e) Determine the internal rate of return of Project Best.

4.

According to the forecast made by your firm's financial manager, the design and construction of new machinery costs 50 million EUR. According to the precise forecast, it will produce an inflow after operating costs of 18.6 million HUF in year 1, 20.4 million HUF in year 2, and 24.2 million HUF in year 3. The opportunity cost of capital is equal to 12 percent.

a) Give the cash outflow and inflows streams in a table (similar to the table in which data are presented in problem 2).
b) Calculate the net present value of the investment opportunity.
c) Calculate the profitability index of the investment opportunity. Apply the profitability index rule.
d) Would you accept the investment offer? Why?

5.

The expected cash flows of two projects are shown in the following table:

Year	Project I (million HUF)	Project II (million HUF)
0	-25	-45
1	0	0
2	0	0
3	0	0
4	51.84	93.312

a) Calculate the internal rate of return for Project I and Project II.
b) Which project has the greater internal rate of return?
c) At what level of required rate of return is it worth accepting Project I and Project II?

6.

Determine the MIRR of the following investment possibility if the firm's required rate of return is 8 percent:

Year	Investment (million HUF)
0	-10 000
1	80 000
2	50 000

7.

Consider the cash-flow streams of the following two investment opportunities:

Year	Investment A (million HUF)	Investment B (million HUF)
0	-200	-160
1	100	60
2	120	80
3	140	100
4		120

The annual opportunity cost of capital is 14.5 percent.

a) Calculate the payback period for each investment opportunity.
b) Calculate the discounted payback period for each investment opportunity.
c) Give the deficiencies of the discounted payback period and payback period calculation.
d) Which investment proves to be more profitable? Why?

8.

The design and construction of a new factory costs 20 million EUR. According to your financial manager's forecast, it will produce an inflow after operating costs of 22 million EUR in year 1, 40 million EUR in year 2, and 46.8 million EUR in year 3. The opportunity cost of capital is equal to 6.5 percent.

a) Give the cash-flow streams in a table.
b) Calculate the net present value of the investment opportunity.
c) Would you accept the investment offer? Why?
d) Calculate the profitability index of the investment opportunity. Apply the profitability index rule.
e) Calculate the internal rate of return for the investment opportunity. Give the shortcomings of the calculation of the internal rate of return.

9.

Consider the cash-flow streams of the following two projects:

Year	Investment A (million HUF)	Investment B (million HUF)
0	-800	-680
1	150	120
2	150	120
3	150	120
4	200	200
5	200	200
6	200	

The annual opportunity cost of capital is 8 percent. Taxes and inflation are ignored.

a) Calculate the payback period for each investment opportunity.
b) Calculate the discounted payback period for each investment opportunity.
c) Calculate the profitability index for each payment.
d) Which investment is the more profitable opportunity? Why?
e) Determine the equivalent annual benefit for the two investment opportunities.

10.

You would like to introduce a new production method at your Fun Company. You can choose between two alternative methods. The revenues from using the two methods are the same. The costs related to the introduction of the new production management method are given in the following table:

Year	Production method I (million HUF)	Production method II (million HUF)
0	50	60
1	15	20
2	15	20
3	15	20
4	15	

a) Determine the equivalent annual cost for each production management method, if the required rate of return is 6 percent.
b) Which method should you introduce?

11.
According to the loan agreement, we have to pay 2.4 million HUF at present and ten subsequent payments of 1.2 million HUF annually over eight years. The annual opportunity cost of capital is 6 percent.

a) Calculate the present value of the payments.
b) Determine the present value of the cash flows if the opportunity cost of capital is 12 percent.
c) Determine the present value of the cash flows if the opportunity cost of capital is 16 percent.
d) Explain the relationship between the present value of cash flows and the opportunity cost of capital.

12.
You have an investment opportunity. According to the feasibility study, you would have to pay 1 200 000 HUF immediately, and the investment will yield 400 000 HUF benefit in year 1, 600 000 HUF in year 2, and 800 000 and 800 000 in years 3 and 4, respectively. The annual opportunity cost of capital is 4.5 percent. Taxes and inflation are ignored.

a) Calculate the present value of this investment opportunity.
b) Calculate the net present value of this investment opportunity.
c) Would you accept the investment offer? Why?
d) Calculate the profitability index of this investment possibility.
e) Calculate the modified profitability index.
f) Calculate the internal rate of return of this investment possibility.
g) Calculate the modified internal rate of return.
h) Calculate the payback period for this investment possibility.
i) Calculate the discounted payback period for this investment possibility.
j) Draw the net present value curve as a function of the discount rate.

k) Indicate the internal rate of return on the net present value curve.

13.

You are saving to purchase a car at the end of four years. You can earn 6 percent annually on your savings. The price of your chosen car is 4 800 000 HUF.

a) How much do you need to put aside at the end of each year over the four year period?
b) How much do you need to put aside at the end of each year over the four year period if the interest rate changes by 2.5 percentage points?

14.

You have agreed to pay 20 000 HUF per month for a modern computer over 2 years.

a) What is the equivalent price of this computer, if the interest rate is 8 percent annually?
b) What is the equivalent price of this computer, if the interest rate is 2 percent monthly?

15.

Suppose that the DEB Company can choose from among three investment opportunities. The cash flow streams of the three investments opportunities are shown in the following table:

Year	MAC1 (million HUF)	MAC2 (million HUF)	MAC3 (million HUF)
0	-45	-60	-50
1	15	20	20
2	25	35	20
3	30	45	50

The opportunity cost of capital is 10 percent. Taxes, inflation and depreciation are ignored.

a) Calculate the net present value of this investment opportunity.
b) Calculate the profitability index of this investment possibility.
c) Calculate the internal rate of return of this investment possibility.
d) Calculate the payback period for this investment possibility.
e) Calculate the discounted payback period for this investment possibility.
f) Would you agree to the purchase? Why (not)?
g) Calculate the equivalent annual benefit of each machine.
h) Suppose the budget limit is 100 million HUF. Which machine should the company purchase? Give reasons for your answer.

16.

A bond will pay cash flows of 400 000 HUF per year for four years. What is the present value of the cash flow streams,

a) if the discount rate is 10 percent annually?
b) if the discount rate is 15 percent annually?
c) What is the relationship between the discount rate and the present value of the cash flow streams?

17.

Suppose that your company is considering expanding production by acquiring a new production line in order to produce products EX1. The price of the new technical equipment is 3 million EUR. The purchasing price could be depreciated straight-line for tax purposes over 10 years. You would have to rent a warehouse owned by the neighbouring firm. The rental cost is 500 EUR per month in the first year, and thereafter the rental fee increases by 2 percent a year. The total costs, which include the operation costs and overhead costs, are 1000 EUR per month. We can disregard other costs. The revenues from the sale of the products EX1 are expected to be 20 000 EUR in year 1, and thereafter the revenues are forecast to grow by 4 percent a year. The tax on income is 12 percent. The opportunity cost of capital is 8 percent. Taxes, inflation and depreciation are ignored.

a) Give the cash flow stream if the expected lifetime of the production line is 10 years.
b) Calculate the net present value of this investment opportunity.
c) Mention costs that may be incurred in relation to production, and are disregarded in this calculation problem.

18.

You have two investment possibilities. In the case of the first investment you would have to pay 6 200 000 HUF immediately, and the investment would yield a benefit of 2 500 000 HUF in year 1, 1 800 000 HUF in year 2, and 2 500 000 - 2 500 000 in years 3 and 4. On the basis of the second investment offer, you would have to pay 8 500 000 HUF up front, and the investment would yield a benefit of 3 500 000 HUF in year 1, 2 800 000 HUF in year 2, 2 600 000 in year 3 and 4 000 000 in year 4. The annual opportunity cost of capital is 16 percent. Taxes, inflation and depreciation are ignored.

a) Calculate the present value of this investment opportunity.
b) Calculate the net present value of this investment opportunity.
c) Would you accept the investment offer? Why (not)?
d) Calculate the profitability index of this investment possibility.
e) Determine the modified profitability index.
f) Calculate the internal rate of return of this investment possibility.
g) Calculate the payback period for this investment possibility.
h) Calculate the discounted payback period for this investment possibility.

i) Draw the net present value curve as a function of the discount rate.
j) Indicate the internal rate of return on the net present value curve.

19.

The BESTPRODUCT Company should select a machine from two machines (BEST I and BEST II) in order to introduce a new good. Suppose that the rate of return required by the owners is 12 percent. Consider the following information about the acquisition costs and the annual operating costs.

Year	**Machine BEST I (million HUF)**	**Machine BEST II (million HUF)**
1	18.5	16.8
2	3.2	4
3	3.2	4
4	3.2	4
5	3.2	4
6	3.2	4
7	3.2	4
8	3.2	

The Revenues are deemed to be same from production by machines BEST I and Best II. Taxes, inflation and depreciation are ignored.

a) Calculate the net present value of the two machines.
b) Calculate the equivalent annual cost of machines BEST I and BEST II.
c) Which machine should the company purchase? Give reasons for your answer.
d) What is the fair annual rental charge for each machine?

20.

The Pick-Pack Company has to choose between two machines (PICK I and PACK II). The two machines do the same job, and cost the same (120 million HUF), but they have different life-terms. Suppose the required rate of return is 14 percent. Consider the following information about the annual net benefit.

Year	**Machine PICK (million HUF)**	**Machine PACK (million HUF)**
0	**-120**	**-120**
1	**45**	**60**
2	**45**	**60**
3	**45**	**60**
4	**45**	**60**

5	**45**	**60**
6	**45**	
7	**45**	
8	**45**	

a) Calculate the net present value of each machine.
b) Would you accept the investment offer? Why (not)?
c) In this situation does a decision made on the basis of the net present values rule have any disadvantages?
d) Calculate the profitability index of this investment possibility.
e) Calculate the internal rate of return of this investment possibility.
f) Are there any disadvantages of the internal rate of return calculation?
g) Calculate the discounted payback period for this investment possibility.
h) Calculate the equivalent annual net benefit of the PICK and PACK machines.
i) Which machine should the company purchase? Give reasons for your answer.
j) How is the equivalent annual benefit affected if the owners reduce the required rate of return by 4 percentage points?

21.
The cash flow streams of four machines are shown in the following table:

Year	PROJECT I (million HUF)	PROJECT II (million HUF)	PROJECT III (million HUF)	PROJECT IV (million HUF)
0	-40	-56	-48	-60
1	20	25	22	30
2	20	25	22	30
3	20	25	22	30
4	25	30	30	30
5	25			

The machines do the same job, but their prices and life-terms are different. Suppose the annual opportunity cost of capital is 15 percent.

a) Suppose you have to choose between PROJECT II, PROJECT III, and PROJECT IV. Which one is the better investment possibility? Calculate the net present value of the investment opportunities.
b) Calculate the internal rate of return for the three investment possibilities (PROJECT II, PROJECT III, and the PROJECT IV). Which one would you choose on the basis of your internal rate of return calculation?
c) Explain the shortcomings of the internal rate of return calculation?

d) Calculate the discounted payback period for each investment possibility (PROJECT I, II, III, and IV). Which project would you choose if you used the discounted payback period rule and the cut-off period was two years?
e) Calculate the equivalent annual net benefit of each investment possibility.
f) Calculate the profitability index of each project.
g) Which investment possibility should you accept if your budget limit to achieve investment possibilities was 120 million HUF? Give the available portfolio of the projects. Give reasons for your answer.

22.
The Intelligent Software Systems company is considering an investment of 200 million HUF. The net cash flow streams are given in the following table:

Year	Investment (million HUF)
0	62
1	62
2	80
3	80
4	80

a) Calculate the net present value of the investment based on a zero discount rate.
b) Calculate the net present value of the investment based on a five percent discount rate.
c) Calculate the net present value of the investment based on a ten percent discount rate.
d) Calculate the net present value of the investment based on a fifty percent discount rate.
e) Suppose that the required rate of return is 20 percent. Would you accept the investment offer? Why (not)?
f) Determine the modified internal rate of return for the investment.

5. Valuation of Bonds

So far, we have mainly evaluated investments in real assets. We have tried to determine the present value and net present value of an investment, and we have applied different financial indicators to evaluate investment opportunities. We can distinguish between real assets and financial assets. A real asset is a tangible, and is generally used to produce goods and services. Real assets includes inter alia: a factory, machinery, a production line, human capital, real estate. Real asset, as physical capital, is tangible and visible.

A financial asset is an intangible asset, and generally its value is determined by a contractual claim. A financial asset is a promise to pay predetermined future cash flows to the owner of the asset. Financial assets are more liquid than real assets. Financial assets include, inter alia: bonds, stocks, bank deposits, derivative securities.

In this chapter we will examine how we can determine the value of a stock. We will analyse the different types of bonds.

Picture 11.

A ***bond*** is a long-term contract under which a borrower agrees to make payments of interest and the original principal (***face value***) on predetermined dates to the owner of the bond. The interest payments are determined by the ***coupon rate*** and the principal amount.

The *face value* (par value) of a bond is the price that the bondholder pays for the bond when it is issued. The *coupon rate* of a bond shows the interest paid each year; it is determined as a percentage of the face value, or of the amount of the debt.

For example, suppose that John purchases a bond for 1 000 EUR which pays 10 percent interest a year. In this case the face value of the bond is 1 000 EUR, and the coupon rate is 10 percent. This means that John gets 100 EUR each year to end of the maturity of the bond.

The bond is a debt security, and is issued in order to borrow money. The issuers borrow money from investors, and guarantee to pay predetermined interest rates and repay the principal amount during the life of the bond. If you buy a bond, you lend money to the issuer.

Face value: The principal amount of a bond that is repaid during the life of the bond, or at the end of the term.
Coupon: The initial interest payments of a bond.
Coupon rate: The annual coupon divided by the face value of a bond.
Maturity: The specified date at which the principal amount of a bond is paid.
Yield to maturity: The rate required in the market on a bond.

Different types of bonds can be distinguished, according to who issues them. The issuer can be a:

- government,
- local government,

- state,
- corporation.

Picture 12.

Corporate bonds are debt securities issued by corporations. Government bonds are issued by the government, or a state or local government.

Suppose your BEST Manufacturing Company wants to borrow 100 000 EUR for 15 years in order to finance its future plans. You, as financial manager, borrow 100 000 EUR by selling 100 individual bonds for 1 000 EUR each. You get 100 000 EUR and you guarantee to pay to the holders 120 EUR per year as the interest on each bond, and repay the principal amount at the end of 15 years.

In this case, the maturity is 15 years, and the face value is 1 000 EUR for each bond. The coupon rate of your bond is 12 percent. This means that you have to pay 12 000 EUR altogether as interest each year to the holder of the bonds.

5.1. Perpetuities

The issuer of a perpetuity pays constant cash flows for each year, and repeats the payments forever. The stream of constant cash flows last forever, because the bond is inheritable.

Example 5.1

Suppose that John offers you a perpetuity that pays the same amount of interest - 500 EUR - each year. How much do you need to pay for this bond? Assume that the interest rate is constant and is equal to 10 percent, and the first payment occurs at the end of the first year.

The value of this bond is equal to the present value of its future cash flows:

$$\mathrm{PV} = \frac{500}{1.1^1} + \frac{500}{1.1^2} + \frac{500}{1.1^3} + \frac{500}{1.1^4} + \cdots + \frac{500}{1.1^n} + \cdots = \sum_{t=1}^{\infty} \frac{500}{1.1^t}. \quad (5.1)$$

Determine the sum of the first 30 elements of the stream of cash flows (EUR):

$$\mathrm{PV} = \frac{500}{1.1^1} + \frac{500}{1.1^2} + \frac{500}{1.1^3} + \frac{500}{1.1^4} + \cdots + \frac{500}{1.1^{30}} = \frac{500}{0.1} \cdot \left(1 - \frac{1}{1.1^{30}}\right) = 4713.46.$$

The sum of the first 50 elements of the stream of cash flows (EUR):

$$\mathrm{PV} = \frac{500}{1.1^1} + \frac{500}{1.1^2} + \frac{500}{1.1^3} + \frac{500}{1.1^4} + \cdots + \frac{500}{1.1^{50}} = \frac{500}{0.1} \cdot \left(1 - \frac{1}{1.1^{50}}\right) = 4957.41.$$

The sum of the first 80 elements:

$$\mathrm{PV} = \frac{500}{1.1^1} + \frac{500}{1.1^2} + \frac{500}{1.1^3} + \frac{500}{1.1^4} + \cdots + \frac{500}{1.1^{80}} = \frac{500}{0.1} \cdot \left(1 - \frac{1}{1.1^{80}}\right) = 4997.56.$$

Finally, the sum of the first 100 elements of the stream of cash flows is the following (EUR):

$$PV = \frac{500}{1.1^1} + \frac{500}{1.1^2} + \frac{500}{1.1^3} + \frac{500}{1.1^4} + \cdots + \frac{500}{1.1^{100}} = \frac{500}{0.1} \cdot \left(1 - \frac{1}{1.1^{100}}\right) = 4999.64.$$

We can see that the sum converges to 5000 EUR, as the number of terms in the sum (5.1) increases. In this case the present value can be written as the following:

$$PV = \frac{500}{0.1} = 5000 \text{ EUR}.$$

How can we determine the general formula to evaluate a perpetuity? In the first step, let us write the general formula:

$$PV = \frac{C_1}{(1+r_1)^1} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \cdots + \cdots = \sum_{t=1}^{\infty} \frac{C_t}{(1+r_t)^t},$$

where the payments are constant:

$$C_1 = C_2 = C_3 = \cdots = C_n = \cdots C,$$

and the interest rate is constant, too:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \cdots = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}. \quad (5.2)$$

Let us try to determine the sum of the constant cash-flow streams, which lasts forever. The addend terms of the present value in (5.2) represent the elements of a geometrical series. The sum of a finite geometric series (S_n) can be written as the following:

$$S_n = a_1 \cdot \frac{q^n - 1}{q - 1},$$

where a_1 is the first element of the geometric series and q is the quotient of the series. Substitute the suitable elements into the summation equation:

$$PV = \frac{C}{(1+r)^1} \cdot \frac{\left(\frac{1}{1+r}\right)^n - 1}{\frac{1}{1+r} - 1} = \frac{C}{(1+r)^1} \cdot \frac{\left(\frac{1}{1+r}\right)^n - 1}{\frac{-r}{1+r}} = \frac{C}{r} \cdot \left(1 - \frac{1}{(1+r)^n}\right). \quad (5.3)$$

Using the sum formula of the geometric series, we have supposed that the number of elements in the geometric series is finite. We need to correct the result given in equation (5.3), because the number of terms in equation (5.2) is infinite, so we suppose that n goes to infinity:

$$n \to \infty.$$

We may write equation (5.3) in a simplified form, because the limit of $\frac{1}{(1+r)^n}$ is zero as n goes to infinity:

$$\frac{1}{(1+r)^n} \to 0 \text{ as } n \to \infty$$

$$PV = \lim_{n\to\infty}\left[\frac{C}{r} \cdot \left(1 - \frac{1}{(1+r)^n}\right)\right] = \frac{C}{r}.$$

Addendum 5.1

We can determine the present value of constant cash flows that occurs at the end of each year in other ways:

$$PV = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \cdots = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}.$$

$$PV = C \cdot \left(\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots\right). \tag{5.4}$$

Multiply both sides of the equation (5.4) by $\frac{1}{(1+r)^1}$, we have:

$$PV \cdot \frac{1}{(1+r)^1} = C \cdot \left(\frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{1}{(1+r)^4} + \cdots\right). \tag{5.5}$$

Subtract the equation (5.5) from the equation (5.4); we have:

$$PV - PV \cdot \frac{1}{(1+r)^1} = C \cdot \left(\frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots\right) - C \cdot \left(\frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{1}{(1+r)^4} + \cdots\right),$$

$$PV \cdot \left(1 - \frac{1}{(1+r)^1}\right) = \frac{C}{(1+r)},$$

$$PV \cdot \frac{r}{(1+r)^1} = \frac{C}{(1+r)}.$$

Multiplying both sides by (1+r), we have:

$$PV \cdot r = C,$$

The present value of a perpetuity can be expressed as the following:

$$PV = \frac{C}{r}.$$

5.2. Growing perpetuities

In the case of a perpetuity, we have supposed that the cash flow streams are constant and the interest rate remains the same forever. A growing perpetuity is an asset in which the stream of cash flows grows by a constant rate (percent).

Example 5.2

Suppose your uncle Peter makes you a present of a growing perpetuity that pays 50 EUR next year, and grows at a rate of 5 percent. What is the value of your growing perpetuity if the interest rate is 10 percent? From another perspective: Suppose that you would like to support a social club, so you give 50 EUR in the first year, and the amount of your subsidies increases by 5 percent each year. What is the current value of your subsidization, if the interest rate is 10 percent? First, give the current value of the stream of cash flows:

$$PV = \frac{C}{(1+r)^1} + \frac{C \cdot (1+g)^1}{(1+r)^2} + \frac{C \cdot (1+g)^2}{(1+r)^3} + \cdots + \cdots = \sum_{t=1}^{\infty} \frac{C \cdot (1+g)^{t-1}}{(1+r)^t}.$$

Let us try to determine the sum of this geometrical series in a similar way to the perpetuity examined in the previous chapter. We use the sum formula of the geometric series:

$$S_n = a_1 \cdot \frac{q^n - 1}{q-1},$$

where the first element is $a_1 = \frac{C}{(1+r)^1}$, and the quotient of the geometric series is equal to $\frac{1+g}{1+r}$:

$$PV = \frac{C}{(1+r)^1} \cdot \frac{(\frac{1+g}{1+r})^n - 1}{\frac{1+g}{1+r} - 1} = \frac{C}{(1+r)^1} \cdot \frac{(\frac{1+g}{1+r})^n - 1}{\frac{g-r}{1+r}}. \qquad (5.6)$$

We assume that g is less than r and n goes to infinity:

$$g \ll r.$$

$$n \to \infty.$$

Let us determine the limit of $\frac{C}{r-g} \cdot (1 - (\frac{1+g}{1+r})^n)$ in equation (5.6), as n goes to infinity:

$$(\frac{1+g}{1+r})^n \to 0 \text{ as } n \to \infty \text{ and } g \ll r.$$

$$PV = \lim_{n\to\infty}[\frac{C}{r-g} \cdot (1 - (\frac{1+g}{1+r})^n)] = \frac{C}{r-g}.$$

The present value of a growing perpetuity is the following:

$$PV = \frac{C}{r-g}.$$

Let us return to Example 5.2. The present value of your growing annuity in Example 5.2 can be written as the following:

$$PV = \frac{C}{r-g} = \frac{50}{0.1-0.05} = \frac{50}{0.05} = 1000 \text{ EUR}.$$

Addendum 5.2

We can determine the present value of cash flows growing at a rate of g percent which occurs at the end of each year in a similar way to the method illustrated in Addendum 5.1:

$$PV = \frac{C}{(1+r)^1} + \frac{C \cdot (1+g)^1}{(1+r)^2} + \frac{C \cdot (1+g)^2}{(1+r)^3} + \cdots + \cdots = \sum_{t=1}^{\infty} \frac{C \cdot (1+g)^{t-1}}{(1+r)^t}.$$

$$PV = C \cdot (\frac{1}{(1+r)^1} + \frac{(1+g)^1}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \cdots). \qquad (5.7)$$

Multiplying both sides of the equation (5.7) by $\frac{1+g}{1+r}$, we have:

$$PV \cdot \frac{1+g}{1+r} = C \cdot (\frac{(1+g)^1}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \frac{(1+g)^3}{(1+r)^4} + \cdots). \qquad (5.8)$$

Subtract equation (5.8) from equation (5.7); we have:

$$PV - PV \cdot \frac{1+g}{1+r} = C \cdot \left(\frac{1}{(1+r)^1} + \frac{(1+g)^1}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \cdots\right) - C \cdot \left(\frac{(1+g)^1}{(1+r)^2} + \frac{(1+g)^2}{(1+r)^3} + \frac{(1+g)^3}{(1+r)^4} + \cdots\right),$$

$$PV \cdot \left(1 - \frac{1+g}{1+r}\right) = \frac{C}{(1+r)},$$

$$PV \cdot \frac{r-g}{(1+r)^1} = \frac{C}{(1+r)}.$$

Multiplying both sides by (1+r), we have:

$$PV \cdot (r-g) = C,$$

The present value of a growing perpetuity can be expressed as the following:

$$PV = \frac{C}{r-g}.$$

5.3. The present value of bonds

The value of a financial asset can be determined by the present value of its future cash flows. If you have a bond, the value of your bond is equal to the present value of the future stream of cash flows.

Example 5.3

Assume that you buy a 1 000 EUR face value bond at 5 percent, maturing in five years. Let us write the payments of the bond and determine its value if the opportunity cost of capital is 10 percent. The bond provides an annuity stream of 50 EUR payments and a 1 000 EUR principal payment at the end of the fifth year.

Year	Payments (EUR)
1	50
2	50
3	50
4	50
5	50 + 1 000

The value of the bond is equal to the present value of the future payments (EUR):

$$PV = \frac{50}{1.1^1} + \frac{50}{1.1^2} + \frac{50}{1.1^3} + \frac{50}{1.1^4} + \frac{1\,050}{1.1^5} = \frac{50}{0.1} \cdot \left(1 - \frac{1}{1.1^5}\right) + \frac{1\,000}{1.1^5} = 810.46.$$

The price of a bond is equal to the sum of the present value of interest payments and the present value of the principal amount. If the principal is paid at maturity, the interest payments represent an annuity, because the interest payment is a given constant percent of the principal amount. The present value of a bond can be written as the following:

$$PV = \sum_{t=1}^{n} \frac{\text{interest payments}}{(1+r)^t} + \frac{P_t}{(1+r)^n},$$

where r is the required rate of return, P_t is the principal payment at maturity, n is the number of periods. If the principal is paid in a single instalment at maturity, the interest payments are the same, so we can calculate the price of a bond by using the present value of the annuity:

$$PV = \frac{\text{interest payments}}{r} \cdot \left(1 - \frac{1}{(1+r)^n}\right) + \frac{P_t}{(1+r)^n}.$$

If the principal is paid in several instalments, the interest payments are not constant; it changes according to the principal not paid. In this case, the present value of a bond can be written as the following:

$$PV = \sum_{t=1}^{n} \frac{\text{interest payments}_t}{(1+r)^t} + \frac{P_t}{(1+r)^n}.$$

Example 5.4
Assume that your friend purchases a 10 000 EUR face value bond at a coupon rate of 10 percent maturing in four years. The principal is paid in two equal instalments in the last two years of the maturity period. Let us write the payments of the bond and determine its value if the opportunity cost of capital is 15 percent.

Year	**Payments (EUR)**
1	**1 000**
2	**1 000**
3	**1 000 + 5 000**
4	**500 + 5 000**

The value of the bond is equal to the present value of the interest payments and the present value of the principal payments (EUR):

$$PV = \frac{1\,000}{1.15^1} + \frac{1\,000}{1.15^2} + \frac{6\,000}{1.15^3} + \frac{5\,500}{1.15^4} = 8\,715.45.$$

As we can see, the interest payments depend on the amount of the face value not paid, because the issuer pays interest on the principal debts. In the last year of the maturity the principal debt is 5 000 EUR, so the issuer pays only 10 percent of this amount (500 EUR).

5.4. The relationship between the price of a bond and the interest rate

The yield to maturity is the required rate of return by the holders of bond. The required rate of return can be used as a discount rate to determine the price of a bond. As we have mentioned, the price of a bond is equal to the present value of its future payments. In this section we analyse the relationship between the price of a bond, the interest rate and the coupon rate of the bond.

Example 5.5

Suppose that you have a bond that has a coupon rate of 8 percent and a face value of 20 000 EUR. The issuer of the bond pays interest each year and the principal amount is paid in a lump-sum at maturity. Let us write the future payments of the bond and try to determine the price of the bond if the interest rate is 10 percent and the bond matures in 5 years. The future payments are shown in the following table:

Year	Payments (EUR)
1	1 600
2	1 600
3	1 600
4	1 600
5	1 600 + 20 000

If the discount rate is 10 percent, the present value of the future payments can be given as the following (expressed in euros):

$$\mathrm{PV} = \frac{1\,600}{1.1^1} + \frac{1\,600}{1.1^2} + \frac{1\,600}{1.1^3} + \frac{1\,600}{1.1^4} + \frac{1\,600+20\,000}{1.1^5} = 18\,483.69. \tag{5.9}$$

According to the result in equation (5.9), we can see that the actual price of the bond is lower than the face value. We have learnt that if the discount rate increases, the present value of payments decreases. This means that the price of a bond decreases if the required rate of return by bondholders increases. The price changes in opposite directions relative to the interest rate. Let us examine the price of the bond at two different interest rates. In the second step let the interest rate be lower than the coupon rate. Suppose that the interest rate decreases by two percentage points, so it is equal to 6 percent. In this case, the present value of the interest payments and the principal amount paid at maturity is the following (expressed in euros):

$$\mathrm{PV} = \frac{1\,600}{1.06^1} + \frac{1\,600}{1.06^2} + \frac{1\,600}{1.06^3} + \frac{1\,600}{1.06^4} + \frac{1\,600+20\,000}{1.06^5} = 21\,684.95. \tag{5.10}$$

We have found what we expected, namely that the present value of the future payments is greater if the interest rate is lower. Finally, we determine the present value of the future payments of the bond at 8 percent interest rate (expressed in euros).

$$\mathrm{PV} = \frac{1\,600}{1.08^1} + \frac{1\,600}{1.08^2} + \frac{1\,600}{1.08^3} + \frac{1\,600}{1.08^4} + \frac{1\,600+20\,000}{1.08^5} = 20\,000. \tag{5.11}$$

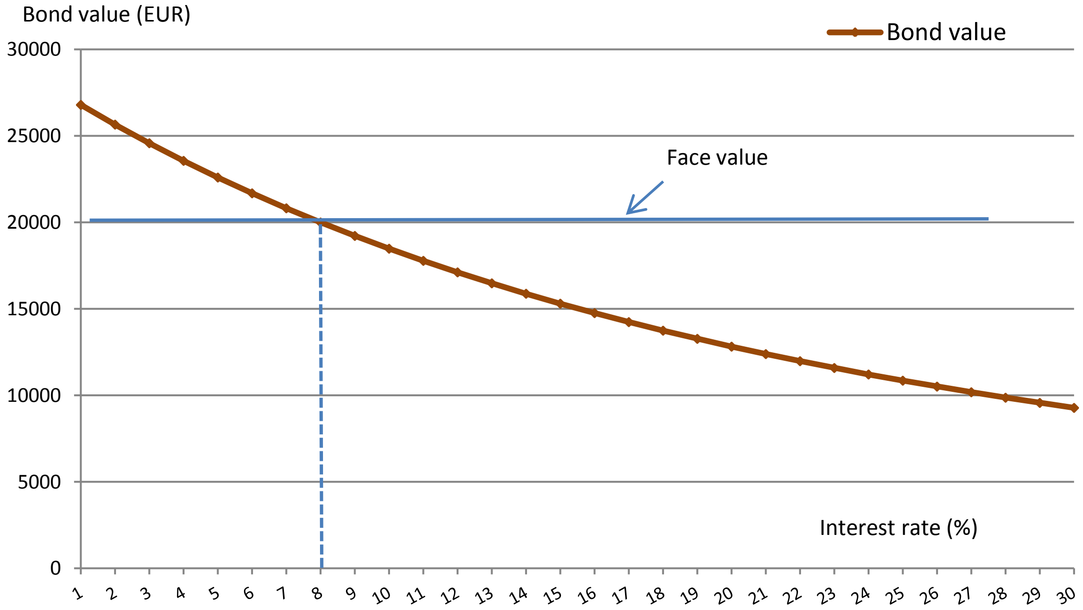
Let us summarize the results of the previous calculations in Table 5.1.

Table 5.1 The relationship between the price of a bond and the discounted rate

Discount rate (%)	Present value of payments (EUR)
6	21 684.95
8	20 000
10	18 483.69

All other things being equal, the greater the interest rate, the lower the present value of the future payments of a bond (Table 5.1). The price of a bond will decline when the interest rate rises. The price of a bond is less than its face value when the interest rate is greater than the coupon rate (Table 5.1; Figure 5.1). All bonds that sell below their face value are called discount bonds.

Figure 5.1 Bond value as a function of the interest rate

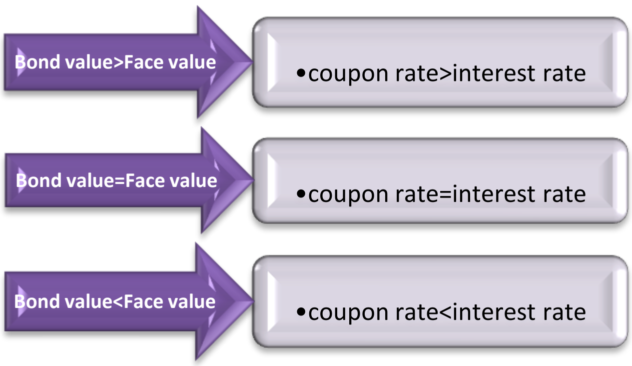


According to the results of the present value calculations ((5.9) – (5.10)), the price of a bond is equal to its face value when the interest rate is equal to the coupon rate, all other things being equal. However, if the interest rate is less than the coupon rate, the price of a bond exceeds its face value, if all other influencing factors remain the same (Table 5.1; Figure 5.1, 5.2). Bonds that sell at a higher price than their face value are called premium bonds.

Discount bond: *"A bond that sells below its par value. This occurs whenever the going rate of interest rises above the coupon rate.*

Premium bond: *A bond that sells above its par value. This occurs whenever the going rate of interest falls below the coupon rate".*

(Besley – Brigham, 2015)



5.5. The relationship between the time to maturity of a bond and the price of a bond

As we have discussed in the previous section, there is a relationship between the price of a bond and the discount rate used in the evaluation of the bond. A bond owner can receive benefit or realize loss due to changes in the interest rate. The present value of future interest payments and the present value of the principal amount paid at maturity changes in the opposite direction in relation to the interest rate. The sensitivity of the change depends on the time to maturity and the coupon rate. In this subsection, we will show the relationship between the price of a bond and the time to maturity.

Example 5.6

Suppose that a bond has a 12 percent coupon rate and a 10 000 EUR face value. Interest is paid annually and the bond has a 10 year maturity. Determine the current value of the bond if the bondholder's required rate of return is 10 percent. The future stream of cash flows of the bond is the following:

Year	Payments (EUR)	Year	Payments (EUR)
1	1 200	**6**	1 200
2	1 200	**7**	1 200
3	1 200	**8**	1 200
4	1 200	**9**	1 200
5	1 200	**10**	1 200 + 10 000

The bond value can be calculated as the following (expressed in euros):

$$\mathrm{PV} = \frac{1\,200}{1.1^1} + \frac{1\,200}{1.1^2} + \cdots + \frac{1\,200}{1.1^{10}} + \frac{10\,000}{1.1^{10}} = \frac{1\,200}{0.1} \cdot \left(1 - \frac{1}{1.1^{10}}\right) + \frac{10\,000}{1.1^{10}} = 11\,228.91.$$

The coupon rate is greater than the discount rate and consequently the present value of the future inflows of the bond is greater than its face value. Examine how the current value of the bond changes as the time to maturity decreases. Calculate again the current value of the bond at 5 and 1 periods to maturity, when all other things are equal. The present value of the 5-year bond is the following (in euros):

$$PV = \frac{1\,200}{1.1^1} + \frac{1\,200}{1.1^2} + \cdots + \frac{1\,200}{1.1^5} + \frac{10\,000}{1.1^5} = \frac{1\,200}{0.1} \cdot \left(1 - \frac{1}{1.1^5}\right) + \frac{10\,000}{1.1^5} = 10\,758.16,$$

and the 1-year bond value (in euros) is:

$$PV = \frac{11\,200}{1.1^1} = 10\,181.82.$$

Repeat the calculation for the 10, 5 and 1 year bonds at a 5 percent discount rate when all other influencing factors remain the same.

10-year bond:

$$PV = \frac{1\,200}{1.05^1} + \frac{1\,200}{1.05^2} + \cdots + \frac{1\,200}{1.05^{10}} + \frac{10\,000}{1.05^{10}} = \frac{1\,200}{0.05} \cdot \left(1 - \frac{1}{1.05^{10}}\right) + \frac{10\,000}{1.05^{10}} = 15\,405.21 \text{ EUR},$$

5-year bond:

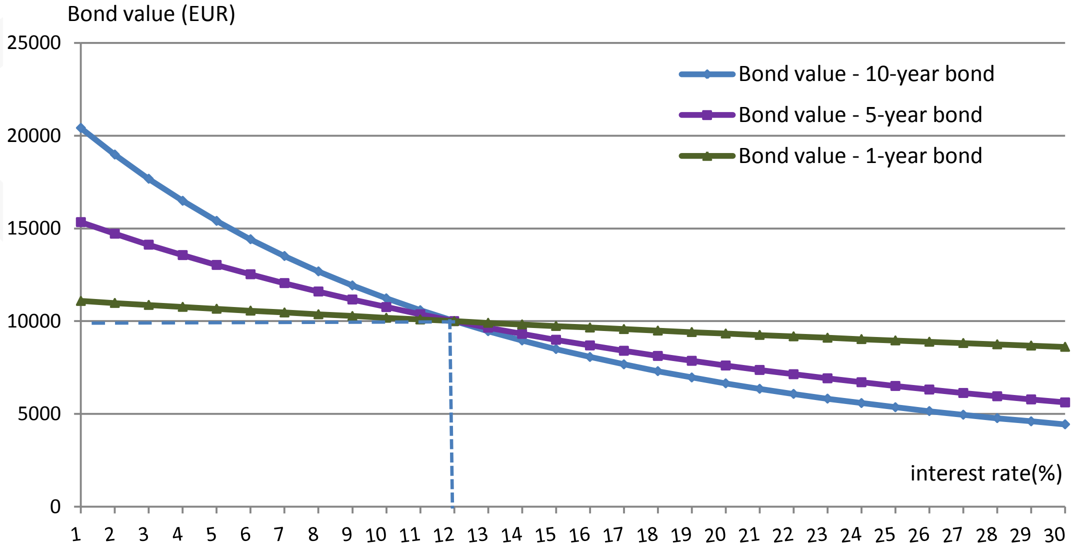
$$PV = \frac{1\,200}{1.05^1} + \frac{1\,200}{1.05^2} + \cdots + \frac{1\,200}{1.05^5} + \frac{10\,000}{1.05^5} = \frac{1\,200}{0.05} \cdot \left(1 - \frac{1}{1.05^5}\right) + \frac{10\,000}{1.05^{10}} = 13\,030.63 \text{ EUR},$$

1-year bond:

$$PV = \frac{11\,200}{1.05^1} = 10\,666.67 \text{ EUR}.$$

The change in the bond's value is the greatest for the 10-year bond and is smallest for the 1-year bond. A bond with a longer time to maturity is more sensitive to the change in interest rates than a bond with a shorter time to maturity (Figure 5.3).

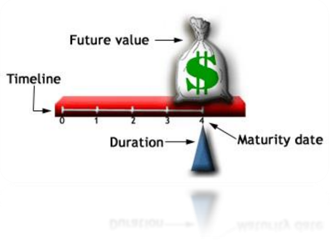
Figure 5.3 The relationship between the time to maturity of a bond and the price of the bond



One unit change in the interest rate results in a different change in the bond value, depending on the time to maturity. A relatively small change in the interest rate can cause a larger change in the value of a bond with longer maturity than in a short-term bond. The longer the time to maturity, the greater the interest rate risk, when all other things remain the same.

5.6. Yield to maturity (YTM) and yield to call (YTC)

Yield to maturity of a bond



The yield to maturity (YTM) is the average rate of return that a bond holder will earn on a bond if it is held to maturity. If you have a bond, and you know the current price of your bond, the yield to maturity can be computed.

Picture 13

Example 5.7

Suppose you have a 10-year, 5 percent coupon, 25 000 EUR par value bond, and its current price is 19 967.44 EUR. What rate of interest would you earn on this bond? First, let us give the future payments of this bond.

Year	Payments (EUR)	Year	Payments (EUR)
1	1 250	**6**	1 250
2	1 250	**7**	1 250
3	1 250	**8**	1 250
4	1 250	**9**	1 250
5	1 250	**10**	1 250 + 25 000

The bond holder gets 1 250 EUR interest at the end of each year for ten years, and 25 000 EUR as the principal payment at maturity. The bond's value currently is 19 967.44 EUR. The current value can be written as the following:

$$\text{PV} = \frac{1\,250}{(1+\text{YTM})^1} + \frac{1\,250}{(1+\text{YTM})^2} + \cdots + \frac{1\,250}{(1+\text{YTM})^{10}} + \frac{25\,000}{(1+\text{YTM})^{10}} = \frac{1\,250}{\text{YTM}} \cdot \left(1 - \frac{1}{(1+\text{YTM})^{10}}\right) + \frac{25\,000}{(1+\text{YTM})^{10}} = 19\,967.44 \text{ EUR.} \qquad (5.11)$$

Using a financial calculator or Excel we can solve the equation (5.11) for yield to maturity. The yield to maturity is equal to 8 percent. The general formula for calculating the yield to maturity is the following:

$$\frac{\text{INT}}{(1+\text{YTM})^1}+\frac{\text{INT}}{(1+\text{YTM})^2}+\cdots+\frac{\text{INT}}{(1+\text{YTM})^{10}}+\frac{\text{Principal payment}}{(1+\text{YTM})^{\text{N}}}=\frac{\text{INT}}{\text{YTM}}\cdot\left(1-\frac{1}{(1+\text{YTM})^{\text{N}}}\right)+$$
$$+\frac{\text{Principal payment}}{(1+\text{YTM})^{\text{N}}}=\text{Price of the bond}, \tag{5.12}$$

where INT is the annual interest paid to the bond holder, and N is the number of years to maturity.

As an alternative, we can give an approximation for the yield to maturity by applying the following equation (Besley – Brigham, 2015):

$$\text{approximate yield to maturity}=\frac{\text{Annual interest+Accured annual capital gains}}{\text{Average value of the bond during its life}},$$

$$\text{approximate yield to maturity}=\frac{\text{INT}+\frac{\text{M}-\text{V}_\text{d}}{\text{N}}}{\frac{2\cdot\text{V}_\text{d}+\text{M}}{3}}, \tag{5.13}$$

where INT is the annual interest, M is the par value, V_d is the current value of the bond, and N is the number of years until maturity. The approximate value of the yield to maturity using equation (5.13) is the following:

$$\text{approximate yield to maturity}=\frac{1\,250+\frac{25\,000-19\,967.44}{10}}{\frac{2\cdot 19\,967.44+25\,000}{3}}=\frac{1\,753.256}{21\,644.96}=0.08.$$

Yield to call of a bond

Bonds known as callable bonds can be recalled prior to maturity. In this case, investors do not realize the yield to maturity, because the bond is recalled before maturity. Instead of the yield to maturity, we can calculate the yield to call, which shows the average rate of return that a bond holder will earn on a bond if it is held until the bond is called. The yield to call is equal to the average rate of return earned on a bond if it is held until the bond is first called. The calculation of the yield to call is similar to the yield to maturity. Instead of the current price of the bond we have to substitute the call price in equation 5.12:

$$\frac{\text{INT}}{(1+\text{YTC})^1}+\frac{\text{INT}}{(1+\text{YTC})^2}+\cdots+\frac{\text{INT}}{(1+\text{YTC})^{\text{Nc}}}+\frac{\text{Call price of the bond}}{(1+\text{YTC})^{\text{Nc}}}=\frac{\text{INT}}{\text{YTC}}\cdot\left(1-\frac{1}{(1+\text{YTC})^{\text{Nc}}}\right)+$$
$$+\frac{\text{Call price of the bond}}{(1+\text{YTC})^{\text{Nc}}}=\text{Price of the bond}, \tag{5.14}$$

where Nc is the number of years until the bond is called.

Example 5.8

Determine the yield to call of the bond analysed in Example (5.7). Assume that the bond is called at the end of the ninth year, the call price is 28 000 EUR. The interest is 1 250 EUR in each year for ten years and the bond's price is currently 19 967.44 EUR. The equation to determine the yield to call can be written as the following:

$$\frac{\mathrm{INT}}{(1+\mathrm{YTC})^1}+\frac{\mathrm{INT}}{(1+\mathrm{YTC})^2}+\cdots+\frac{\mathrm{INT}}{(1+\mathrm{YTC})^{\mathrm{Nc}}}+\frac{\text{Call price of the bond}}{(1+\mathrm{YTC})^{\mathrm{Nc}}}=\frac{\mathrm{INT}}{\mathrm{YTC}}\cdot\left(1-\frac{1}{(1+\mathrm{YTC})^{\mathrm{Nc}}}\right)+$$
$$+\frac{\text{Call price of the bond}}{(1+\mathrm{YTC})^{\mathrm{Nc}}}=\text{Price of the bond},$$

where Nc is the number of years until the bond is called.

$$\mathrm{PV}=\frac{1\,250}{(1+\mathrm{YTC})^1}+\frac{1\,250}{(1+\mathrm{YTC})^2}+\cdots+\frac{1\,250}{(1+\mathrm{YTC})^9}+\frac{28\,000}{(1+\mathrm{YTC})^9}=\frac{1\,250}{\mathrm{YTC}}\cdot\left(1-\frac{1}{(1+\mathrm{YTC})^9}\right)+\frac{28\,000}{(1+\mathrm{YTC})^9}=$$
$$19\,967.44\ \mathrm{EUR}.$$

The yield to call (YTC) is equal to 9.31 percent.

5.7. Duration (DUR) and Volatility

Duration

Duration (DUR) is the average maturity of the bond payments. Duration is one of the characteristics of a bond; it shows the weighted average of time to each cash payment. The weights of each cash flow are equal to the present value of the cash flows divided by the price of the bond (the sum of the present value of all the bond payments):

$$\mathrm{DUR}=1\cdot\frac{\mathrm{PV}(C_1)}{\mathrm{PV}}+2\cdot\frac{\mathrm{PV}(C_2)}{\mathrm{PV}}+3\cdot\frac{\mathrm{PV}(C_3)}{\mathrm{PV}}+\cdots+n\cdot\frac{\mathrm{PV}(C_n)}{\mathrm{PV}},$$

where $\mathrm{PV}(C_i)$ is the present value of the payment in year i, and PV is the present value of all payments. The general formula of the duration can be written as the following:

$$\mathrm{DUR}=\sum_{t=1}^{n}\frac{\frac{C_t}{(1+r)^t}\cdot t}{\sum_t^n\frac{C_t}{(1+r)^t}}=\sum_{t=1}^{n}\frac{\frac{C_t}{(1+r)^t}\cdot t}{P},$$

where P is the price of the bond today.

Example 5.9

Consider the following cash flows of securities A and B. The interest rate is 6%.

Year	A (EUR)	B (EUR)
1	10	4
2	10	4
3	10	4
4	10	4
5	110	104

Calculate the durations of securities A and B.

The duration of security A:

$$\mathrm{DUR}=\frac{1\cdot\frac{10}{1.06}+2\cdot\frac{10}{1.06^2}+3\cdot\frac{10}{1.06^3}+4\cdot\frac{10}{1.06^4}+5\cdot\frac{110}{1.06^5}}{\frac{10}{1.06}+\frac{10}{1.06^2}+\frac{10}{1.06^3}+\frac{10}{1.06^4}+\frac{110}{1.06^5}}=\frac{495.1}{116.85}=4.24\ years.$$

The duration of security B:

$$\text{DUR} = \frac{1\cdot\frac{4}{1.06}+2\cdot\frac{4}{1.06^2}+3\cdot\frac{4}{1.06^3}+4\cdot\frac{4}{1.06^4}+5\cdot\frac{104}{1.06^5}}{\frac{4}{1.06}+\frac{4}{1.06^2}+\frac{4}{1.06^3}+\frac{4}{1.06^4}+\frac{104}{1.06^5}} = \frac{422.22}{91.58} = 4.61\ years.$$

The duration of bond A is shorter, because the greater percentage of the payments is paid earlier. In the first three years, more than 22% of the future cash flow is paid for bond A, while in the case of bond B only 11.67 % of all the payments are paid in the first three years.

Table 5.2 Calculation of payments made as a percentage of the bond's total value

Year	A (%)	Payments made as a percentage of total value (%)	B (%)	Payments made as a percentage of total value (%)
1	8.07	8.07	4.12	4.12
2	7.62	15.69	3.89	8.01
3	7.19	22.87	3.67	11.67
4	6.78	29.65	3.46	15.13
5	70.35	100.00	84.86	100.00
Total	100		100	

Volatility – Modified duration

Examine how the price of Bond A changes as the interest rate changes.

Year	A (EUR)
1	**10**
2	**10**
3	**10**
4	**10**
5	**110**

Determine the present value of the future payments of Bond A if the interest rate is equal to 6%, 10% and 12%.
The present value of the future payments at 6 percent interest is the following:

$$\text{PV} = \frac{10}{1.06} + \frac{10}{1.06^2} + \frac{10}{1.06^3} + \frac{10}{1.06^4} + \frac{110}{1.06^5} = 116.85\ \text{EUR}.$$

The present value of the future payments at 10 percent interest is the following:

$$\text{PV} = \frac{10}{1.1} + \frac{10}{1.1^2} + \frac{10}{1.1^3} + \frac{10}{1.1^4} + \frac{110}{1.1^5} = 100\ \text{EUR}.$$

The present value of the future payments at 12 percent interest is the following:

$$\text{PV} = \frac{10}{1.12} + \frac{10}{1.12^2} + \frac{10}{1.12^3} + \frac{10}{1.12^4} + \frac{110}{1.12^5} = 92.79\ \text{EUR}.$$

Interest rate (%)	Price of Bond A (EUR)
6	116.85
10	100
12	92.79

One measure of the interest rate risk is the volatility of a bond. Volatility (modified duration) shows the percentage change in the bond price when the interest rate (yield to maturity) changes by 1 percent. Volatility is equal to the duration of a bond divided by 1 plus the interest rate (yield to maturity):

$$\text{Volatility} = \frac{\text{Duration}}{1 + \text{r}}.$$

The volatility of Bond A at a 6 percent interest rate is the following:

$$\text{Volatility} = \frac{4.24}{1.06} = 4\%.$$

The modified duration of bond A is 4%, which means that a 1 percentage point change in the interest rate results in a 4 percent change in the bond price. The calculated percent change in the bond price is only a strong approximation. We have seen in chapter 5.5 that the bond value as a function of the interest rate is not a linear function, so the percentage change cannot be the same at each interest rate (Figure 5.3).

5.8. Terms and Questions

bond,
bond value,
corporate bond,
coupon,
coupon rate,
duration,
discount bond,
face value,
government bond,
growing perpetuity,
interest payments,
maturity,
modified duration,
par value,
perpetuity,
principal amount,
yield to call,
yield to maturity,
volatility.

Theoretical questions

1. What is the difference between the yield to maturity and the yield to call?

2. Explain the relationship between the bond's value and the interest rate.

3. How can we calculate the value of a perpetuity?

4. Give the definition of a discount bond.

5. What is the difference between a discount bond and a premium bond?

6. Explain how the value of a growing perpetuity is calculated.

7. How can we choose the appropriate discount rate to evaluate an investment possibility?

8. Give the definition of face value.

9. How can the value of a bond be affected if the condition of the repayment of the principal amount changes?

10. How can we derive the yield to call of a bond?

11. Give the definition of the coupon rate.

12. What is the difference between the coupon rate and the yield to maturity?

13. Explain how we can derive the yield to maturity.

14. Give the definition of a premium bond.

15. Explain how to determine the yield to maturity of a growing perpetuity.

16. What is the relationship between the time to maturity and the price of a bond?

17. Give the definition of a bond.

18. What is the definition of the time to maturity?

19. Give the definition of the volatility of a bond.

20. How can the duration of a bond be calculated?

Calculation exercises

1.

The 10 000 EUR face value *Delta bond* has a coupon rate of 6% (interest paid annually), and matures in 5 years. The discount rate is 12%.

a) Give the future payments of the bond.
b) What is the present value of the *Delta bond*?

2.

Consider a 5-year bond with a 20 000 EUR face value, a yield to maturity of 6%, a 10% coupon rate and an annual coupon frequency.

a) Give the future payments of the bond.
b) What is the present value of the bond?
c) Compute the present value for yields to maturity of 5% and 12%.
d) What is the relationship between the present value of the bond and the face value?

3.

The 18 000 EUR face value *Gamma bond* has a coupon rate of 15% (interest paid annually), and matures in 10 years.

a) What is the *Gamma bond's* value if the discount rate is 5%?
b) What is the *Gamma bond's* value if the discount rate is 15%?
c) What is the *Gamma bond's* value if the discount rate is 20%?

4.

Consider the following prices of four bonds with 5-year maturities.

Year	Price (%)	Bond Coupon (%)
A	85.6	4
B	98	6
C	118.1	8
D	148.6	10

a) Determine the yields to maturity.
b) Which bond had the highest yield to maturity?

5.

Suppose that the one-year spot rate of interest is 3% and the two-year spot rate is 5%.

a) Determine the forward rate of interest for year 2.

6.

Suppose you have a bond that has a face value equal to 5 000 EUR and a 12 percent coupon rate of interest. The bond matures in 5 years and its current price is 62 00 EUR. The interest is paid yearly.

a) Give the future payments of your bond.
b) Determine the bond's yield to maturity.

7.

Assume that a bond's face value is 20 000 HUF and the interest is paid annually at a 6 percent coupon rate. The bond matures in 10 years, and is callable at 21 200 HUF in three years. The current price of the bond is 21 000 HUF.

a) Give the future payments of your bond.
b) Determine the bond's yield to maturity.

8.

Consider a 15-year bond with a 2 000 EUR face value, a yield to maturity of 8%, a 6% coupon rate and an annual coupon frequency.

a) What is the present value of the bond?
b) Compute the present value for a yield to maturity of 4% and one of 10%.
c) If the yield to maturity rises, does the bond price fall or rise?

9.

Consider the following prices of four bonds with 10-year maturities.

Year	Price (%)	Bond Coupon (%)
A	**96.4**	**3**
B	**100.6**	**4**
C	**120**	**6**
D	**126.8**	**8**

a) Which bond had the highest yield to maturity?

10.

Suppose that the one-year spot rate of interest is 4% and the two-year spot rate is 6%.

a) Determine the forward rate of interest for year 2.

11.

An *Alpha Corp.* bond has a 15% coupon rate and a 100 000 HUF face value. The bond has 5 years to maturity. The required rate of return is 16 percent. Interest is paid annually and the principal amount will be paid equally in the two last years of the maturity.

a) Give the future payments of the bond.
b) What is the present value of the bond in the case of annual coupon payments?
c) Compute the present value if the bond pays semi-annual coupons and the yield is a semi-annually compounded rate.
d) Which bond has the highest yield to maturity?

12.

Consider the following spot rates.

Year	Spot rate (r_i %)
1.	5
2.	5.8
3.	6.2
4.	5

a) Determine the discount factors for each year.
b) Determine the forward rates for each year.
c) Determine the present value of a 4%, three-year bond (annual coupons, face value is 12 000 HUF).

13.

The BEST Natural OIL Company has a 2 000 EUR face value bond that has a coupon rate of 10% (interest paid annually), and the required rate of return is 12%.

a) Determine the price of the bond if the maturity date is 20 years.
b) Determine the price of the bond if the maturity date is 10 years.
c) Determine the price of the bond if the maturity date is 5 years.
d) Determine the price of the bond if the maturity date is 1 year.

14.

Consider a 4-year coupon bond with a face value of $500 and a coupon rate of 12% per year payable semi-annually and a yield to maturity of 15% per year compounded semi-annually.

a) Give the future payments of the bond.
b) What is the duration of the bond?

15.
The Horizon Corporation has 500 EUR face value bonds that have a coupon rate of 12 percent. The bond has 5 years to maturity and the principal amount will be paid in equal amounts over the five years. Interest is paid annually.

a) Give the interest and principal payments in each year.
b) Determine the price of the bond if the market interest rate is 15 percent.
c) Determine the price of the bond if the market interest rate is 5 percent.
d) Determine the price of the bond if the market interest rate is 12 percent.

16.
Consider the following prices of four bonds with 5-year maturities.

Year	Price (%)	Bond Coupon (%)
A	84.5	4
B	92	5
C	114.6	6
D	120	8

a) Which bond had the highest yield to maturity?
b) Determine the duration of each bond.

17.
Consider the following cash flows of securities. The interest rate is 8%.

Year	A	B	C
1.	50	16	20
2.	50	16	20
3.	50	16	20
4.	250	16	20
5.		116	220

a) Give the coupon interest of bonds A, B, and C.
b) Calculate the durations of securities A, B, and C.
c) Calculate the volatilities of securities A, B, and C.

6. Valuation of Stock

In this chapter, we examine the different types of stocks and what kind of factors influence the value of a stock. Generally the value of an asset is determined by the present value of expected future cash flow streams. It is no different in the case of stocks, so this approach works for stocks, too.


Picture 14.

The future cash flow stream consists of two components:

- the dividends (DIV) expected, which are paid annually,
- the price of the stock (P) when the stock is sold. This price is the price expected by the investors.

In the first step, we give the general formula used to determine the value of a stock, and continue our examination with different types of stocks. In our analysis, three cases can be distinguished:

- There is no growth in dividends; dividends remain the same.
- Dividends grow at a constant rate (g).
- Dividends grow at variable rates.

6.1. Stock Prices

As we have mentioned, the value of a stock is determined by the present value of the stock's future payments. If you have a stock, your expected return depends on its expected dividend per share and the capital gains (or losses). Suppose that the current price of your share is P_0 and the price in year 1 is P_1. The amount of the capital gain or loss can be expressed as the following:

$$P_1 - P_0.$$

However, the ***capital gain*** can be expressed as a percentage of the original price (P_0). The capital gain is equal to the change in price ($P_1 - P_0$) during a given year, divided by the price (P_0) at the starting year:

$$\frac{P_1 - P_0}{P_0}.$$

In this case, the expected return is equal to the sum of the dividend per share (DIV_1) and the expected increase (or decrease) in the price ($P_1 - P_0$), all given as a percentage of the current price (P_0):

$$r = \frac{DIV_1 + P_1 - P_0}{P_0}. \qquad (6.1)$$

The ***expected rate of return*** is the return on the stock that an investor expects if he/she purchases a stock. The expected rate of return has two components:

- expected capital gain and
- expected dividend yield.

The ***expected dividend yield*** shows the expected dividend per share divided by the current price of the stock:

$$\frac{DIV_1}{P_0}.$$

To express the current price of the stock from equation (6.1):

$$P_0 = \frac{DIV_1 + P_1}{1+r}, \qquad (6.2)$$

According to equation (6.2), the current price of a stock can be determined from the expected dividend per share and the price of the stock in year 1. The expected return (r) is called the market capitalization rate, or the cost of equity capital. We suppose that the price of a stock in year 1 can be determined in similar way to the current price of a stock, so it is influenced by the expected dividend per share in year 2 and the expected price in year 2:

$$P_1 = \frac{DIV_2 + P_2}{1+r}.$$

We can express the current price of the stock in terms of the expected dividend per share in year 1 and year 2, and the expected price at the end of the second year:

$$P_0 = \frac{DIV_1}{1+r} + \frac{P_1}{1+r} = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}.$$

To continue the valuation of a stock, we can write a formula for the second year price:

$$P_2 = \frac{DIV_3 + P_3}{1+r}.$$

The current price can be written as the following:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{P_2}{(1+r)^2} = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \frac{P_3}{(1+r)^3}.$$

If we continue the determination of P_0 using the future dividends and future price of the stock in the final period (n), we get the general formula for the current price of the stock. The general formula for the valuation of a stock with the assumption of a finite period is:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \cdots + \frac{DIV_n}{(1+r)^n} + \frac{P_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{DIV_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}. \qquad (6.3)$$

However, the current value of a stock given in equation (6.3) must be corrected by the infinity life term of the stock. A stock is an asset without maturity. In this case (n), equation (6.3) goes to infinity:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_2}{(1+r)^2} + \frac{DIV_3}{(1+r)^3} + \cdots + \frac{DIV_n}{(1+r)^n} + \cdots = \sum_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t}.$$

6.2. Valuation of stock with the assumption of no growth in dividends

In this subsection, we determine the value of a stock whose dividends remain the same. In this case there is no growth in dividends, so we suppose that the growth rate (g) of dividends is equal to zero:

$$g = 0,$$

$$DIV_1 = DIV_2 = DIV_3 = \cdots = DIV_\infty.$$

The present value of the future cash flows can be written as the following:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_1}{(1+r)^2} + \frac{DIV_1}{(1+r)^3} + \cdots + \frac{DIV_1}{(1+r)^n} + \cdots = \sum_{t=1}^{\infty} \frac{DIV_1}{(1+r)^t}.$$

The future cash flows of a zero growth stock are very similar to a perpetuity which pays constant cash flows forever. In chapter 5.1, we gave the general formula of a perpetuity:

$$P = \frac{C}{r}. \qquad (6.4)$$

Applying equation (6.4) to a zero growth stock, we get the general formula of the stock:

$$P_0 = \frac{DIV_1}{r}. \qquad (6.5)$$

Example 6.1

Suppose you prefer the stock of the *Bestever Corporation*. It pays an annual dividend of 50 EUR. The company plans to maintain the dividend at this level in the future. The required rate of return is 10 percent. Determine the price of the preferred stock.

The company pays a constant dividend:

$$DIV_1 = DIV_2 = DIV_3 = \cdots = DIV_\infty = 50\ EUR,$$

the current price of the stock can be given using the equation (6.5):

$$P_0 = \frac{DIV_1}{r} = \frac{50\ EUR}{0.1} = 500\ EUR.$$

6.3. Valuation of stock with the assumption of constant growth in dividends – the constant growth model

Generally, investors count on the dividends increasing each year. Suppose that a firm increases its dividends at a constant rate, let us try to determine the value of the stock. Let the constant growth rate be g. In the first step, determine the future payments of a stock with constant growth dividends:

Table 6.1 The stream of future payments for a stock with constant growth in dividends

Year	Future payments
1	DIV_1
2	$DIV_1 \cdot (1+g)$
3	$DIV_1 \cdot (1+g)^2$
4	$DIV_1 \cdot (1+g)^3$
5	$DIV_1 \cdot (1+g)^4$
. . .	. . .
n	$DIV_1 \cdot (1+g)^{n-1}$
. . ∞	. . $DIV_1 \cdot (1+g)^{\infty}$

The present value of the stream of dividends can be written as the following:

$$P_0 = \frac{DIV_1}{1+r} + \frac{DIV_1 \cdot (1+g)}{(1+r)^2} + \frac{DIV_1 \cdot (1+g)^2}{(1+r)^3} + \cdots + \frac{DIV_1 \cdot (1+g)^{n-1}}{(1+r)^n} + \cdots = \sum_{t=1}^{\infty} \frac{DIV_1 \cdot (1+g)^{t-1}}{(1+r)^t}.$$

In chapter 5.2, we defined and analysed growing perpetuities. The definition of a growing perpetuity is the following:

A growing perpetuity is an asset in which a stream of cash flows grows by a constant rate (percent).

Actually, a stock with constant growth in dividends complies with the requirements for a growing perpetuity examined in Chapter 5.2. This means that the present value formula of a growing perpetuity can be adapted to evaluate a stock with a constant growth rate. The present value of a growing perpetuity is the following:

$$\text{PV} = \frac{\text{C}}{\text{r}-\text{g}}. \qquad (6.6)$$

We apply equation (6.6) to give the value of a stock with a constant growth rate, and we get the constant growth model (or the Gordon model):

$$P_0 = \frac{DIV_1}{r-g}. \qquad (6.7)$$

The formula (6.7) can be used when the expected growth rate is less than the discount rate (r):

$$g \ll r.$$

Example 6.2

Suppose you prefer the stock of the *Study Corporation*. The firm pays a dividend of 50 EUR in year 1 and investors expect a 5 percent growth rate. The company plans to maintain the constant growth in dividends at this level in the future. The required rate of return is 10 percent. Determine the current value of the preferred stock.

The current price of the stock can be given using equation (6.7):

$$P_0 = \frac{DIV_1}{r-g} = \frac{50\ EUR}{0.1-0.05} = 1\ 000\ EUR.$$

The expected growth rate is different according to the industry or company. Normal (constant growth is expected to continue into the foreseeable future at about the same rate as that of the economy as a whole (i.e. in line with the growth of the nominal gross national product (GNP)) (Besley - Brigham, 2015).

Example 6.3

Suppose you prefer the stock of the *Engine Corporation*. The firm have just paid a dividend of 100 EUR, and investors expect an 8 percent growth rate. The company plans to maintain the constant growth in dividends at this level in the future. The required rate of return is 10 percent. Determine the current value of the preferred stock.

First, we should determine the estimated dividend in year 1, when investors expect an 8 percent growth rate:

$$DIV_1 = DIV_0 \cdot (1+g),$$

$$DIV_1 = 100 \cdot 1.08 = 108\ EUR.$$

The current price of the stock can be given using equation (6.7):

$$P_0 = \frac{DIV_1}{r-g} = \frac{108}{0.1-0.08} = 5\ 400\ EUR.$$

6.4. Expected rate of return on a stock with the assumption of constant growth in dividends

In this subsection, we analyse how the expected rate of return can be determined and what the main factors influencing the expected rate of return are. Let us start with the constant growth model:

$$P_0 = \frac{DIV_1}{r-g}. \tag{6.8}$$

Express the expected rate of return from equation (6.8):

$$r \cdot P_0 - g \cdot P_0 = DIV_1,$$

$$r \cdot P_0 = DIV_1 + g \cdot P_0,$$

$$r = \frac{DIV_1}{P_0} + g. \tag{6.9}$$

We can see that the expected rate of return is equal to the sum of the dividend yield ($\frac{DIV_1}{P_0}$) and the expected rate of growth (g) in dividends:

$$r = Dividend\ yield + Expected\ rate\ of\ growth.$$

Example 6.4

Suppose you have a stock whose current price is 220 EUR. You expect the stock to pay 19.8 EUR dividends in year 1 and the expected growth rate of the dividend is 6 percent. Determine the expected dividend yield and the expected rate of return.

The dividend yield is the following:

$$\frac{DIV_1}{P_0} = \frac{19.8}{220} = 0.09.$$

The dividend yield is 9 percent. Applying the equation (6.9), we get the expected rate of return:

$$r = \frac{DIV_1}{P_0} + g,$$

$$r = \frac{19.8}{220} + 0.06 = 0.09 + 0.06 = 0.15.$$

The required rate of return is 15 percent.

The following question can arise. What is the percentage change in the stock value when the dividend grows by a constant g percent?

Example 6.5

Suppose you are strongly considering selling your stock (analysed in example 6.4), so you want to determine the estimated value of your stock at the end of the next year. Determine the value of the stock in year 1 and the percentage change in the price of the stock.

Using the Gordon growth formula for the price of the stock in year 1, we get the value of the stock (in EUR):

$$P_0 = \frac{DIV_1}{r-g},$$

$$P_1 = \frac{DIV_2}{r-g} = \frac{DIV_1 \cdot (1+g)}{r-g},$$

$$P_1 = \frac{19.8 \cdot 1.06}{0.15-0.06} = \frac{20.988}{0.09} = 233.2\ EUR.$$

The percentage change in the stock's value can be written as the following:

$$\frac{P_1 - P_0}{P_0} = \frac{233.2-220}{220} = \frac{13.2}{220} = 0.06.$$

According to the result, the value of the stock increases by 6 percent over one year, if the growth rate is equal to 6 percent.

Let us give the percentage change in the price of the stock in a general formula:

$$\frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1 = \frac{\frac{DIV_2}{r-g}}{\frac{DIV_1}{r-g}} - 1 = \frac{DIV_2}{r-g} \cdot \frac{r-g}{DIV_1} - 1,$$

$$\frac{P_1 - P_0}{P_0} = \frac{DIV_2}{DIV_1} - 1 = \frac{DIV_1 \cdot (1+g)}{DIV_1} - 1 = g.$$

The stock price is expected to increase by g percent when the dividend is expected to grow forever at a constant rate g.

6.5. Profitability and measures of market value

In this subsection we analyse different financial indicators in relation to stocks. Market value ratios measure the firm's stock price in relation to its earnings and book value per share (Besley - Brigham, 2015).

Market value ratios reflect what investors think about the firm's future prospects and how investors regard the firm, based on its past performance (Besley - Brigham, 2015).

Picture 15.

Price/Earnings Ratio

The price/earnings ratio measures how much investors are willing to pay per euro (dollar) of current earnings. It can be calculated as the ratio of the market price per share and the earnings per share:

$$P/E\ \ ratio = \frac{Price\ per\ share}{Earnings\ per\ share}.$$

Earnings per share are equal to the ratio of the net income of stockholders and the shares outstanding:

$$EPS = \frac{Net\ income\ available\ to\ stockholders}{Number\ of\ shares\ outstanding}.$$

P/E ratios are higher for firms which have high growth prospects for the future. P/E ratios are lower for riskier firms.

Market/Book ratio

The market to book ratio is the relationship between a stock's market price and its book value:

$$Market/book\ value\ ratio = \frac{Market\ price\ per\ share}{Book\ value\ per\ share}.$$

Book value per share is the total equity divided by the number of shares outstanding:

$$Book\ value\ per\ share = \frac{Common\ equity}{Number\ of\ shares\ outstanding}.$$

If the market book ratio is less than 1, this could mean that the firm is not generally successful in creating value for its stockholders (Ross et al., 1993).

Return on Equity (ROE)
The return to equity (ROE) is the ratio of net income and common equity:

$$ROE = \frac{Net\ income\ available\ to\ stockholders}{Common\ equity}.$$

In effect, the return on equity measures the rate of return on stockholders' investment.

Return on Total Assets (ROA)
The return on assets is the ratio of the net income and total assets:

$$ROA = \frac{Net\ income}{Total\ assets}.$$

The return on assets shows the profit per euro (dollar) of assets.

The ***plowback ratio*** measures the amount of earnings that have been retained after dividends have been paid out to stockholders:

$$Plowback\ ratio = 1 - payout\ ratio.$$

The ***payout ratio*** is the ratio of the dividends and earnings per share (EPS):

$$Payout\ ratio = \frac{DIV}{EPS}.$$

The payout ratio measures the proportion of earnings paid out as dividends to stockholders. Applying the payout ratio formula we get:

$$Plowback\ ratio = 1 - \frac{DIV}{EPS}.$$

The ***dividend growth rate*** is the result of multiplying the plowback ratio and the return on equity:

$$g = Plowback\ ratio \cdot ROE = (1 - Payout\ ratio) \cdot ROE.$$

Example 6.6
Suppose that a company pays 40% of earnings per share as dividends to its stockholders, and the current market value of the stock is 50 EUR. According to the forecast, the dividend payments for the next year are expected to be 2 EUR per share. The return on equity per share is 12 percent.
Determine the plowback and payout ratio, and give the market estimate of the market capitalization.

$$Payout\ ratio = \frac{DIV}{EPS} = 0.4,$$

$$Plowback\ ratio = 1 - \frac{DIV}{EPS} = 1 - 0.4 = 0.6,$$

The growth rate of dividends can be calculated as the following:

$$g = Plowback\ ratio \cdot ROE = 0.6 \cdot 0.12 = 0.072.$$

Finally, the market capitalization is the sum of the dividend yield and the growth rate:

$$r = \frac{DIV_1}{P_0} + g,$$

$$r = \frac{2}{50} + 0.072 = 0.04 + 0.072 = 0.112.$$

6.6. Terms and Questions

capital yield,
constant growth model,
dividend,
dividend growth rate,
dividend yield,
Earnings per share
expected rate of return,
Gordon model,
growth rate,
Market/Book ratio,
payout ratio,
plowback ratio,
Price/Earnings ratio,
price of a stock,
required rate of return,
Return on Equity (ROE),
Return on Total Assets (ROA),
stock growth rate,
stock with no growth in dividends,
stock with constant growth in dividends.

Theoretical questions

1. What is the difference between the capital yield and the dividend yield?

2. Explain the relationship between a stock with no growth in dividends and a perpetuity.

3. How can we calculate the value of a stock?

4. What is the Gordon model?

5. Explain how the value of a stock with a constant growth in dividends is calculated.

6. What are the main factors influencing the expected rate of return?

7. How can the value of a stock be affected if the condition of the dividend payment changes?

8. How can we derive the capital yield of a stock?

9. How can the dividend yield of a bond be determined?

10. What is the difference between a growing perpetuity and a stock with a constant growth in dividends?

11. What is the relationship between the expected rate of return and the price of a stock?

12. What are the future payments of a stock?

13. What is the difference between the required rate of return and the expected rate of return of a stock?

14. Explain how the value of a stock with zero growth in dividends is calculated.

15. Give the definition of the payout ratio.

16. What is the difference between the payout ratio and the plowback ratio?

17. How can the return on equity of a stock be calculated?

18. What are the main factors influencing dividend growth?

19. Give the definition of the plowback ratio.

20. How can the return on total assets be determined?

Calculation exercises

1.

Suppose that the ***Alfa Company*** pays an end-of-year dividend of 25 EUR per share, and the company sells its stock at 200 EUR after the dividend.

a) What is the current price of the stock if the market capitalization rate is 5%?
b) What is the current price of the stock if the market capitalization rate is 8%?
c) What is the current price of the stock if the market capitalization rate is 10%?

2.

Suppose that a stock is expected to provide a dividend of 10 EUR per share today. The market capitalization rate is 12%.

a) What is the current price of the stock if dividends are expected to remain the same forever?
b) What is the current price of the stock if dividends are expected to increase at a rate of 4% per year, forever?
c) What is the current price of the stock if dividends are expected to decrease at a rate of 2% per year, forever?

3.

Suppose that a stock is expected to provide a dividend of 18 EUR per share next year. The market capitalization rate is 10%.

a) What is the current price of the stock if the firm pays a dividend of 18 EUR, forever?
b) What is the current price of the stock if the expected rate of growth in dividends is 4%?

4.

A firm pays 12.5 EUR dividends at the end of the next year, and the current price of the firm's stock is 120 EUR. Dividends grow at a constant rate of 8 percent.

a) Determine the expected rate of return.
b) Determine the expected rate of return if the expected dividend growth rate is 4%.

5.

Suppose that a stock is expected to provide a dividend of 15 EUR per share today. The market capitalization rate is 14%.

a) What is the current price of the stock if dividends are expected to remain the same, forever?

b) What is the current price of the stock if dividends are expected to increase at a rate of 2% per year, forever?
c) What will the price of the stock be next year if dividends are expected to increase at a rate of 2% per year, forever?
d) What is the current price of the stock if dividends are expected to decrease at a rate of 3% per year, forever?

6.

The current price of the Peter Company's stock is 40 EUR and the current dividend is 5.6 EUR. The required rate of return is 8 percent. Dividends are expected to grow at a constant rate of 10 percent per year into perpetuity.

a) Determine the current price of the stock.
b) Determine the expected price of the stock 8 years from now.

7.

Suppose that a stock is expected to pay a dividend of 20 EUR per share next year. The market capitalization rate is 8%.

a) What is the current price of the stock if dividends are expected to increase at a rate of 6% per year for six years and subsequently remain the same forever?
b) What is the current price of the stock if dividends are expected to increase at a rate of 6% per year and there are 10 years of dividends to be paid?

8.

According to the forecast of your firm's financial manager, the expected dividends are the following:

Year	**Dividend per share**
1.	**15**
2.	**15**
3.	**18**
4.	**18**
5.	**20**

Suppose that the current price of the stock is 2 000 EUR. The market capitalization rate is 5%.

a) Determine the expected price in year 3.
b) Determine the expected price in year 5.

9.

Big Forest Communications have paid an 8 EUR dividend today. The dividend is expected to grow at a 5 percent constant rate over the next 6 years. The dividend will remain the same forever after 6 years. The required rate of return is 12 percent.

a) Determine the amount of dividend expected in year 6.
b) Determine the current price of the stock.
c) Determine the expected price at the beginning of year 7.

10.

The MECHAT Company pays an end-of-year dividend of 80 EUR a share, and the company sells its stock at 1 200 EUR today.

c) Determine the expected rate of return, if the expected dividend growth rate is 4%.
d) Determine the return on equity if the plowback ratio is 50%
e) What are the earnings per share?

11.

The BEST GROUPCOM Company's return on equity was 10.5 percent, the earnings per share were 25 EUR. The market capitalization rate is 5%.

a) Determine the Company's retention ratio (a dividend of 20 EUR per share).
b) Determine the dividend growth rate.

12.

Suppose that a stock is expected to provide a dividend of 30 EUR a share today. The market capitalization rate is 15%.

a) What is the current price of the stock if dividends are expected to remain the same forever?
b) What is the current price of the stock if dividends are expected to increase at a rate of 2.5% per year, forever?
c) What is the current price of the stock if dividends are expected to decrease at a rate of 1.5% per year, forever?

13.

Suppose you bought the ***BESTTEST Company's*** stock for 25 EUR one year ago today. Today, you received a 4 EUR dividend from the company and you immediately sold the stock for 30 EUR.

a) What is the total return that you have earned?

b) What is the dividend yield?
c) What is the capital gain?

14.

The BESTCLASS Company pays a dividend of 150 EUR a share today, which is expected to increase at a rate of 5% for 5 years, and thereafter the dividend growth rate will be greater by 2 percentage points. The discount rate is 10%.

a) What is the current price of the stock?
b) What is the current price of the stock if dividends are expected to remain the same after 5 years?

7. Introduction to Options

Suppose you have an investment opportunity, which has been evaluated by your financial manager. He/she has concluded that the NPV of the investment is positive. This means that if you undertake the investment, you will realize a profit. However, according to your investment counsellor, it is better to delay the investment, because later it will yield a greater benefit. You have a choice: to make the investment today, or wait until later. This choice is an option.

Generally ***an option*** is a contract that gives its owner the right to purchase or sell an asset at a fixed price on a given date, or before a given date. An option is a special type of financial contract, because the option gives the buyer the right without any obligation, and the seller receives the option fee and has the obligation to sell or to buy the asset at the strike price.

Picture 16.

There are two main types of option:

- call options and
- put options.

Call options give the owner the right to buy an asset at a specified price on or before a predetermined date. In this case the seller of the option has the obligation to sell the given asset at the strike price if the option holder wants it.

Put options give the owner the right to sell an asset at a specified price on or before a predetermined date. The seller of the put option has the obligation to buy the given asset at the exercise price if the option owner wants to sell it.

The specified price of the option is called the ***exercise price or strike price***.
The ***exercise price or strike price*** at which the option owner has the right to buy or sell the asset is specified in the option contract. With a call option the option holder has the right to buy the underlying asset at the strike price.

Exercising the option means purchasing or selling the given asset in accordance with the option contract.

The ***expiration date*** is the last date on which the option can be exercised. According to the exercise period of the option, we can distinguish two types of options:

- European options and
- American options.

If the option can be exercised only on the expiration date, it is called a ***European*** option. However, if the option can be exercised on the expiration date or any time before the expiration date, the option is called an ***American option***.

In the following sections, we will only examine European options, and the time value of money is ignored.

7.1. Call options

European Call options give the option holder the right to buy (i.e. to call in) an asset at a specified price on a given date.

Let a stock be the subject of the option.

Exercise 7.1

Suppose your company buys one call on the stock of the ***Alfa*** company at an exercise price of 200 EUR. You have to pay 10 EUR for the call option. The price of the stock on the expiration date is 120 EUR. What will you do at maturity?

Picture 17.

First, let us study the position diagram and the profit diagram. The position diagram shows the payoff to the owner of the option as a function of the asset price.

When we illustrate the position diagram, we disregard the option fee.

The option fee is paid by the buyer as a transaction cost. The option fee represents the payment of a premium to the option writer. The call option buyer expects that the price of the stock will increase; however, the option writer expects the opposite, i.e. that the price of the stock will fall. The option buyer's gain can be written as the following:

$$\text{Gain} = \max\{P_0 - P_s, 0\}, \qquad (7.1)$$

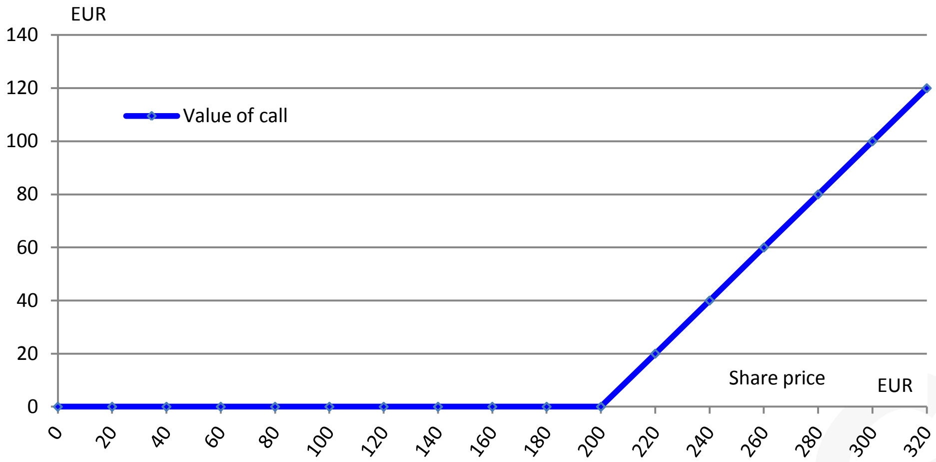
$$\text{option buyer gain} = \begin{cases} 0\,, & \text{if } P_0 \le P_s \\ P_0 - P_s, & \text{if } P_0 > P_s \end{cases},$$

where P_0 is the market price of the share and P_s is the strike price of the share. If the market price is less than the strike price at the exercise date, the buyer does not exercise the option, so the buyer's gain is zero, disregarding the option transaction cost. The value of the option is shown in Figure 7.1.

Try to draw the Alfa stock owner's position diagram.

According to the option buyer's position diagram, it is not worth exercising the call below the strike price. However if the price of the Alfa stock is greater than the strike price, the buyer can realize a profit.

Figure 7.1 The position diagram for the buyer of the call option



The profit diagram shows the amount of the profit as a function of the asset price (Figure 7.2) In the case of the position diagram, the option fee is not included in the value of the call. If we want to illustrate the net payoffs when the option is exercised, we have to take into account the cost of buying the call option. This is paid by the buyer to the option seller.

The stock buyer's gain, taking into account the transaction cost, can be written as the following:

$$\text{Gain} = \max\{P_0 - P_s, 0\} - d, \qquad (7.2)$$

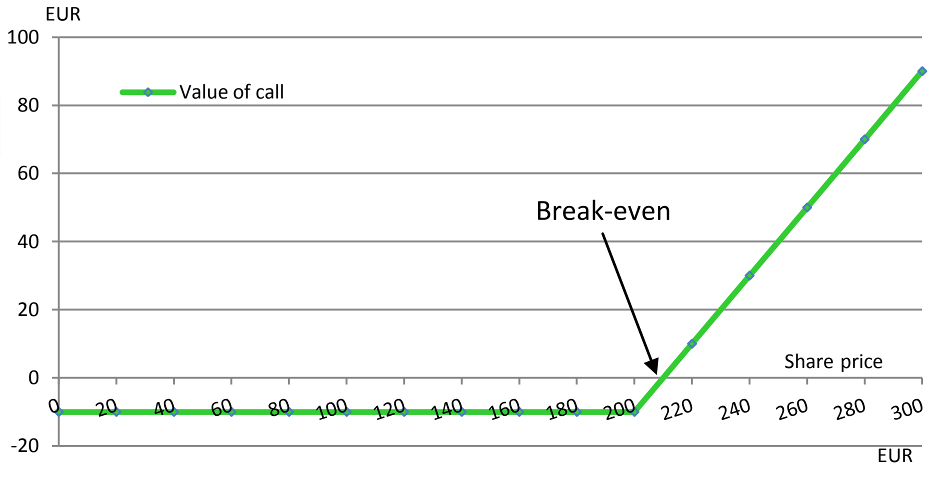
$$\text{option buyer gain} = \begin{cases} -d\,, & \text{if } P_0 \le P_s \\ P_0 - P_s - d, & \text{if } P_0 > P_s \end{cases}$$

where P_0 is the market price of the share, P_s is the strike price of the share, and d is the option fee received by the option writer.

According to the position diagram, the value of a call option is equal to zero if the stock price is less or equal to the exercise price (Figure 7.1). If we take into account the option fee paid by the option holder, the value of the option is negative if the stock price is less than the strike price plus option fee (Figure 7.2).

The break-even point is the point where the call option value is equal to zero (Figure 7.2).

Figure 7.2 The profit diagram for the buyer of the call option



The price at which the break-even is achieved: 210 EUR
The break-even point is the point at which the buyer's gain is equal to zero.
Generally, the break-even point of a call option can be determined as the following:

$$\text{Break} - \text{even} = \text{strike price} + \text{ option fee},$$

$$\text{Break} - \text{even} = P_s + d.$$

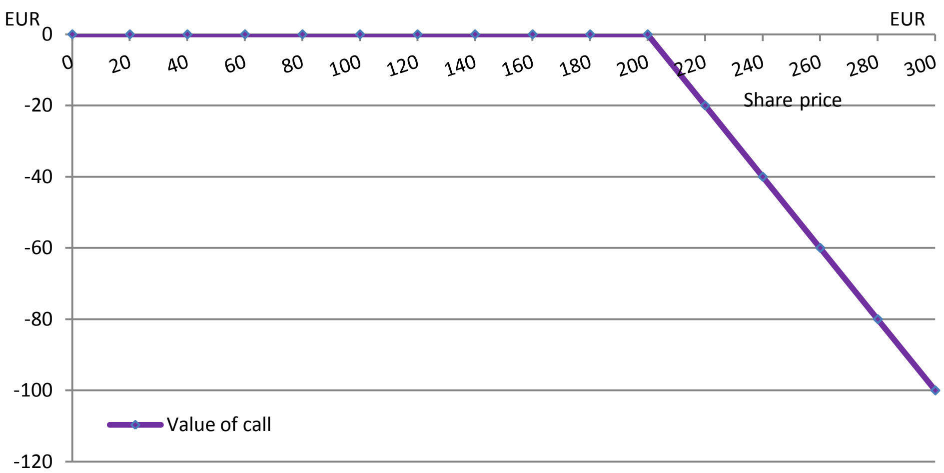
An option is an asymmetric transaction because only the buyer has the right, while only the seller has the obligation. The seller of the stock has a position diagram and a profit diagram. The buyer's profit is the seller's loss, and the buyer's loss is equal to the seller's profit.

The option writer's gain without the transaction cost can be expressed from equation (7.1) as the following:

$$\text{option writer gain} = \begin{cases} 0, & \text{if } P_0 \leq P_s \\ P_s - P_0, & \text{if } P_0 > P_s \end{cases}. \quad (7.3)$$

Let us illustrate the Alfa stock seller's position diagram and profit diagram.

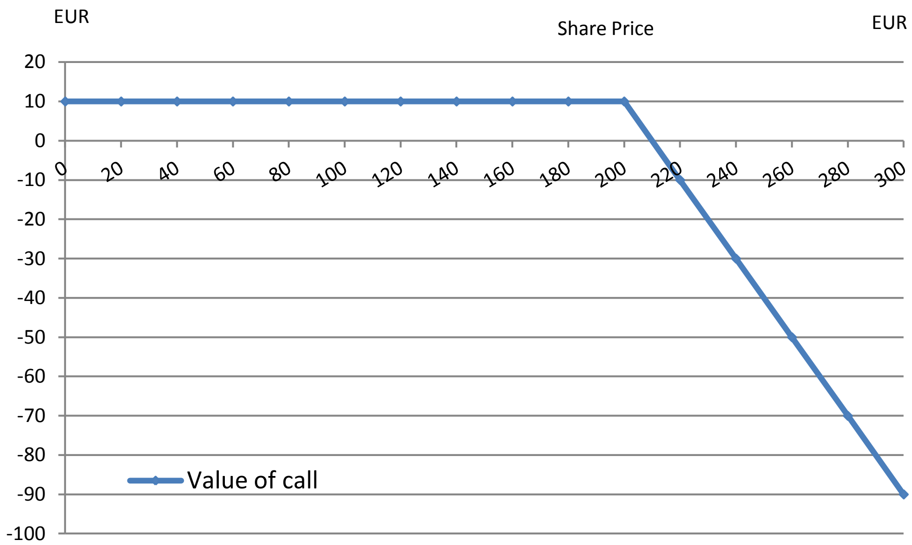
Figure 7.3 The position diagram for the seller of the call option



The call option writer gains if the market price of the share is less than the strike price plus the transaction cost (Figure 7.4). The option writer's gain with the transaction cost can be expressed as the following:

$$\text{option writer gain} = \begin{cases} d, & \text{if } P_0 \leq P_s \\ P_s + d - P_0, & \text{if } P_0 > P_s \end{cases} \quad (7.4)$$

Figure 7.4 The profit diagram for the seller of the call option



Continuing with Example 7.1, the owner of the call option will exercise the option at the maturity date if the market price exceeds the exercise price. In this case, the owner's gain is equal to the difference between the market price of the share at the expiration date and the sum of the strike price and the option fee:

$$P_0 - P_s - d.$$

Suppose the market price of the share at the expiration date is 250 EUR. In this case you have the right to buy the share at a strike price that is lower than the current market price. You will exercise the option, and gain (expressed in euros):

$$250 - 200 - 10 = 40.$$

However, if the market price is only 150 EUR, you will not exercise the option and your loss is the transaction cost of the option that is paid in advance:

$$\text{Loss} = 10.$$

7.2. Put options

A put option is the opposite of a call option. The holder of a put option has the right to sell securities (to put them) for a specified price on (or before) a fixed date. The buyer has the obligation to pay the fee but has no other obligation later. The put option writer has the obligation to buy the asset involved in the option contract when the put option owner exercises the option.

Example 7.2

Suppose you buy one put on ***Beta*** stock with an exercise price of 150 EUR. You have to pay 5 EUR for the call option. The price of the stock on the expiration date is 120 EUR. What will you do at maturity?

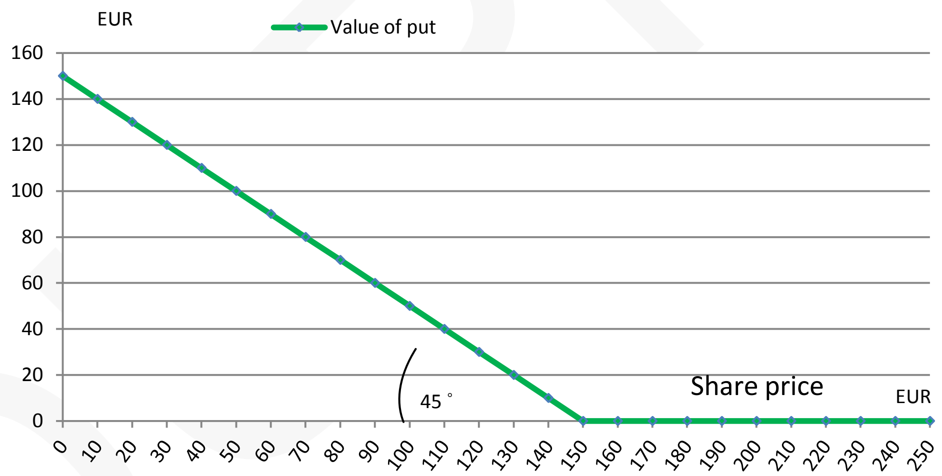
Determine your gain as a function of the market price of the share, and illustrate the position diagram and the profit diagram.
As we have mentioned, the position diagram shows the payoffs to the owner of the option (the value of the option) as a function of the asset market price, when the transaction cost of the option is disregarded.
The option fee represents the payment of the premium to the option writer. The put option buyer expects that the price of the stock will decrease, while the option seller expects the opposite, i.e. that the price of the stock will increase. The gain (disregarding the option fee) of the put buyer can be written as the following:

$$\text{Gain} = \max\{P_s - P_0, 0\}, \qquad (7.5)$$

$$\text{option buyer gain} = \begin{cases} 0\,, & \text{if } P_0 \geq P_s \\ P_s - P_0, & \text{if } P_0 < P_s \end{cases},$$

where P_0 is the market price of the share and P_s is the strike price of the share. If the market price is less than the strike price at the exercise date, the buyer will exercise the put option, so the buyer will sell the share at the strike price. In this case the buyer's gain is equal to the difference between the exercise price and the market price of the share. In other cases, the gain is equal to zero if the option fee is disregarded. The value of the put option is shown in Figure 7.5 (the option fee is not included).

Figure 7.5 The position diagram for the buyer of the put option



The profit diagram shows the value of the put option as a function of the asset price (Figure 7.6). If we want to illustrate the net payoffs when the option is exercised, we have to take into account the cost of buying the put option. The option fee is paid by the buyer to the option seller, so it can be regarded as the option writer's premium.
The stock buyer's gain, taking into account writer's premium, can be written as the following:

$$\text{Gain} = \max\{P_s - P_0, 0\} - d, \qquad (7.6)$$

$$\text{option buyer gain} = \begin{cases} -d, & \text{if } P_0 \geq P_s \\ P_s - P_0 - d, & \text{if } P_0 < P_s \end{cases},$$

where P_0 is the market price of the share, P_s is the strike price of the share, and d is the option fee received by the option writer. The break-even point can be found where the option buyer's gain is equal to zero. The break-even point is the following, from the equation (7.6):

$$P_s - P_0 - d = 0,$$

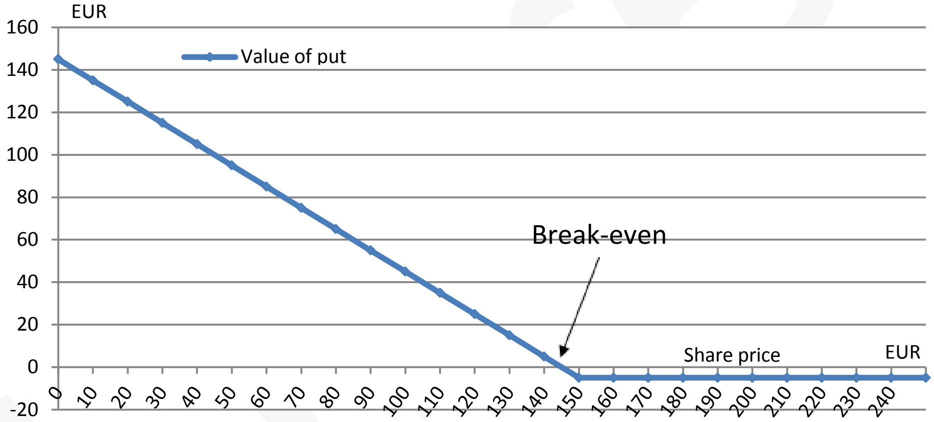
$$P_0 = P_s - d,$$

$$\text{Break} - \text{even point} = P_s - d.$$

The break-even point in our Example (7.2) is the following (expressed in EUR):

$$\text{Break} - \text{even point} = P_s - d = 150 - 5 = 145.$$

Figure 7.6 The profit diagram for the buyer of the put option

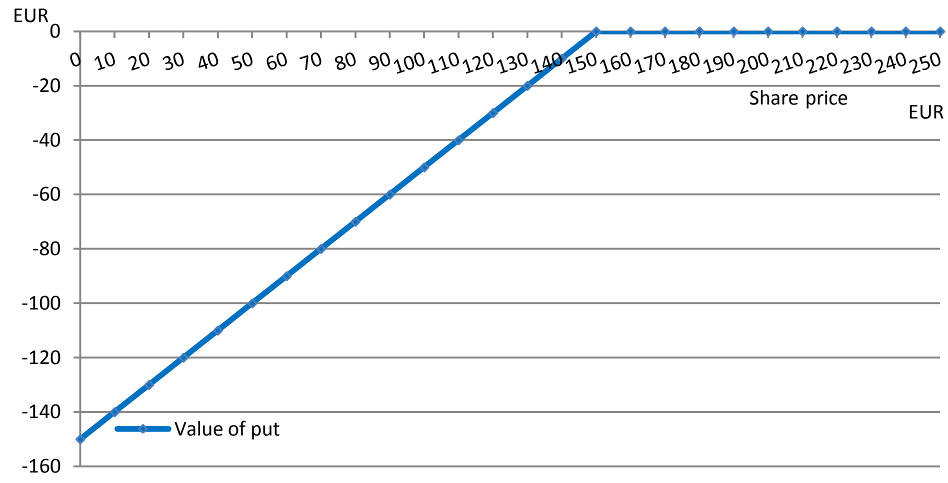


In the following step, we illustrate the option writer's position and profit diagram. As we have mentioned, the buyer's gain is equal to the seller's loss, and conversely, the buyer's loss is the seller's gain. This means that we should mirror the buyer's position diagram on the horizontal axis to obtain the seller's position diagram (Figure 7.7).
The option writer's payoff without the transaction cost can be expressed from equation (7.5) as the following:

$$\text{option writer gain (loss)} = \begin{cases} 0, & \text{if } P_0 \geq P_s \\ P_0 - P_s, & \text{if } P_0 < P_s \end{cases}. \quad (7.7)$$

When the market price of the share is less than the strike price the option writer makes a loss, if we disregard the option fee (Figure 7.7).

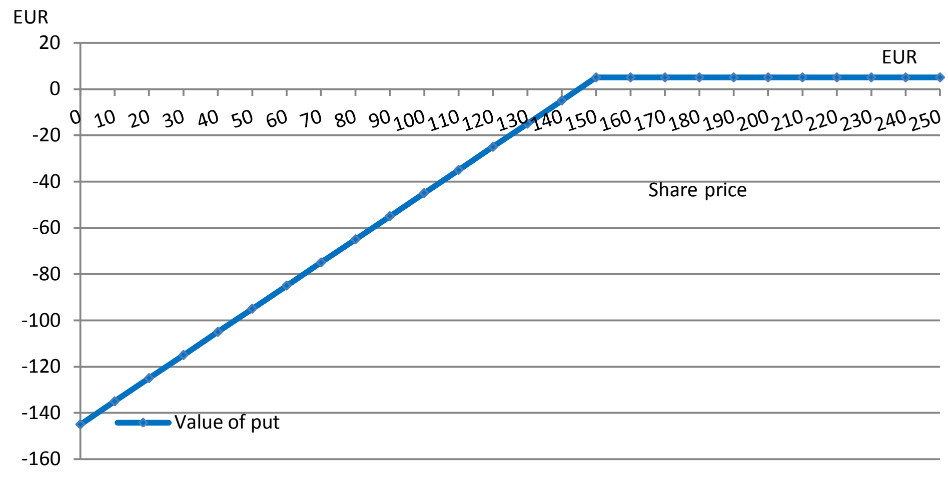
Figure 7.7 The position diagram for the seller of the put option



The profit diagram can be obtained (Figure 7.8) in a similar way to the position diagram. The put option writer gains if the market price of the share is greater than the strike price minus the transaction cost (Figure 7.8). The option writer's gains, with the transaction cost, can be expressed as the following:

$$\text{option writer gain} = \begin{cases} d\,, \text{ if } P_0 \geq P_s \\ P_0 + d - P_s, \text{ if } P_0 < P_s \end{cases} \tag{7.7}$$

Figure 7.8 The profit diagram for the seller of the put option



Go back to Example 7.2. The put option owner will exercise the option at the maturity date if the market price is lower than the exercise price. In this case, the owner's gain is equal to the difference between the strike price of the share and the sum of the market price at the expiration date and the option fee:

$$P_s - (P_0 + d).$$

According to Example 7.2, the market price of the share at the expiration date is 120 EUR. In this case the option holder exercises his/her right, and sells the share at a strike price that is higher than the current market price. This means that the put option holder has made a gain:

$$150 - 120 - 5 = 25 \text{ EUR}.$$

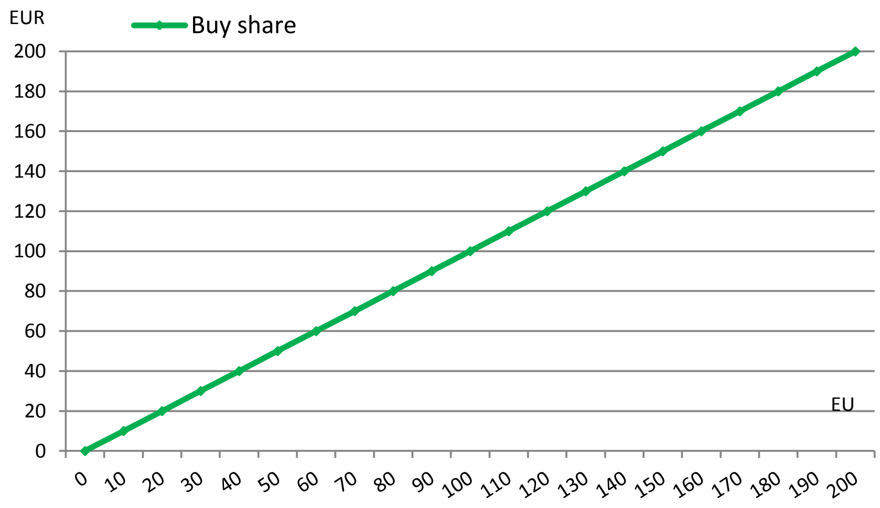
when the market price is greater than 145 EUR, it is not worth exercising the option right, i.e. the option holder has made a loss that equals the value of the option fee.

7.3. Value of option – put-call parity

Let us examine two option strategies. The two strategies protect against a decrease in the price of the stock.

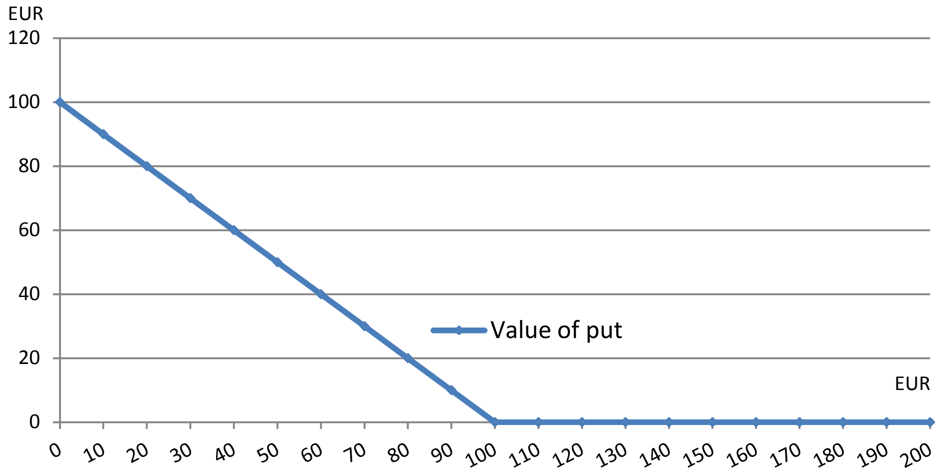
Example 7.3
First strategy:
Suppose you buy a share of *Pear stock* and buy a put option for the *Pear stock.* The exercise price is 100 EUR. Let us illustrate the payoff of your purchasing decisions.
The payoff from buying Pear stock increases as the price of the share increases above the purchasing price. If the market price of the Pear stock decreases below the purchasing price, you make a loss (Figure 7.9).

Figure 7.9 Payoff from buying a share



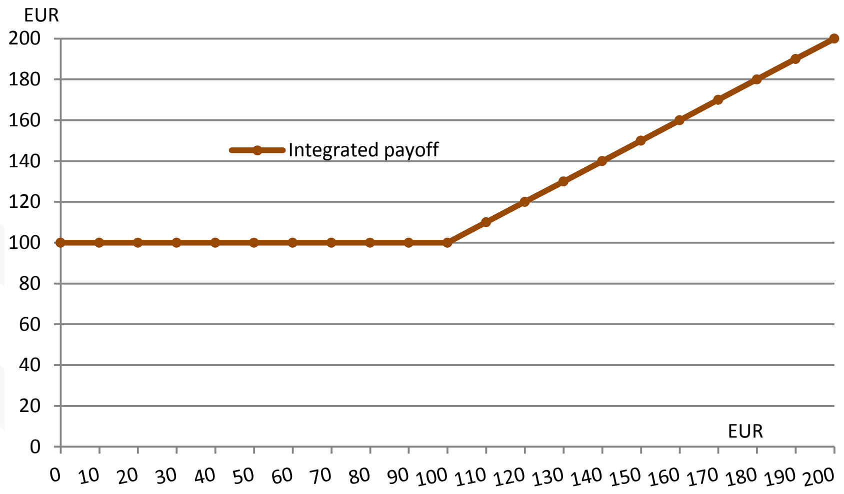
The payoff of the Pear put option is positive when the price is below the strike price (Figure 7.10).

Figure 7.10 Payoff from buying a put option



Let us illustrate the integrated position diagram. We have to sum vertically the position diagram of buying a share and the position diagram of a put option. The payoff of the strategy is constant when the market share price is less than the strike price (100 EUR). However, if the Pear market price increases above the strike price, the holder's gain increases as the market share price increases.

Figure 7.11 Integrated payoff of the put option and the share purchase



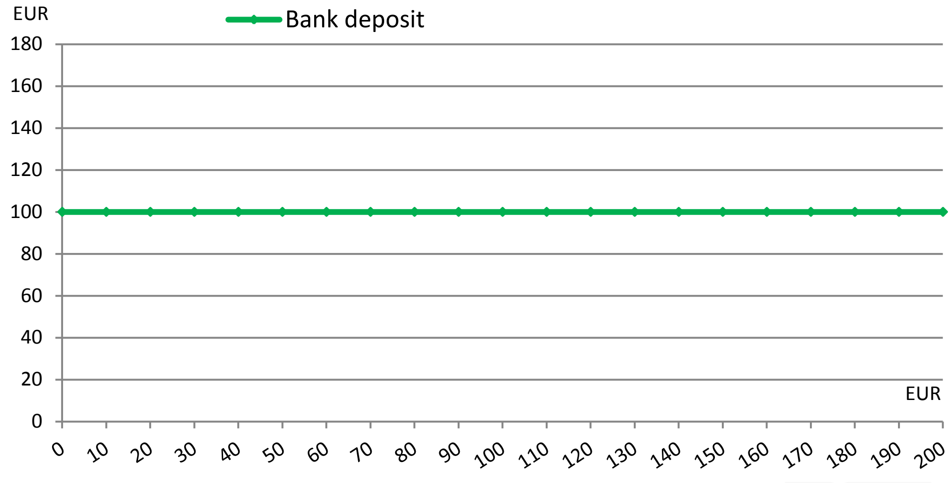
The second strategy is the following:

Suppose you deposit 100 EUR in your bank account, and buy a call option of Pear stock.

Bank deposit

There is no risk in your financial decision, so your bank will pay 100 EUR. The change in the price of the Pear stock does not affect the payoff from your bank deposit. The position diagram of the bank deposit is shown in Figure 7.12.

Figure 7.12 Payoff of the bank deposit of 100 EUR



If the market price of the Pear stock increases, you exercise your call option, so you purchase the Pear stock. However, if the Pear share price falls below the strike price, you will not exercise your right, because your option is valueless (Figure 7.13).

Let us add the payoff of the call option to the bank deposit and determine and illustrate the integrated payoff.
When the market price of the stock falls below the strike price, you have 100 EUR in your bank account and you do not exercise your call option. Your payoff below the strike price is constant, and is equal to 100 EUR. When the market stock price increases and exceeds the strike price, your call option is more valuable, as the stock price is greater (Figure 7.14).

Figure 7.13 Payoff of the call option

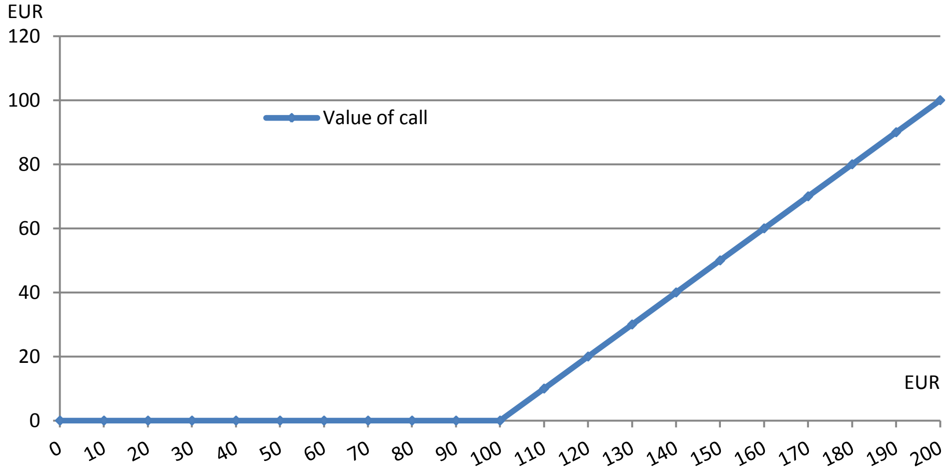
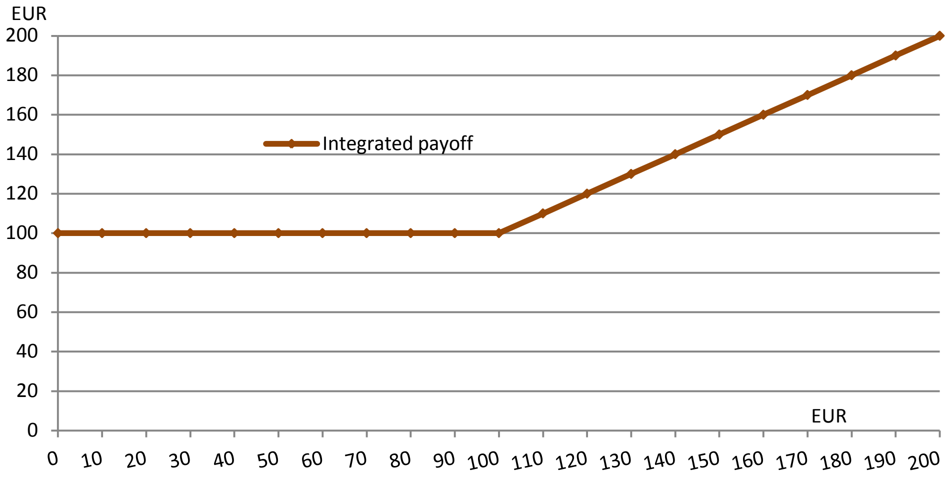


Figure 7.14 Integrated payoff of the call option and the bank deposit



Comparing the two strategies, we find that both strategies have the same payoff (Figure 7.11 and 7.14). If you buy a share and a put option for the purchased share, your payoff is identical to buying a call option and depositing an amount equal to the strike price. The relationship between the call (European) option and the put (European) option can be written as the following (Brealey – Myers, 2014):

$$\text{Value of call option} + \text{present value of exercise price} = \text{Value of put option} + \text{share price},$$

$$\text{Payoff from buy call and investing present value of exercise price in safe asset} \\ = \text{Payoff from buy put option and buy share.}$$

The relationship among share price, call option and put option values, and the present value of the exercise price is called the put – call parity (Brealey – Myers, 2014). The put and call options have identical strike prices and expiratory dates.

7.4. Terms and Questions

American option,
break-even point,
buyer of the option,
call option,
European option,
exercise price,
exercise an option,
expiration date,
option,
option fee,
option writer,
option premium payment,
position diagram,
profit diagram,
put – call parity,
put option,
seller of the option,
strike price,
transaction cost,
value of call
value of put.

Theoretical questions

1. What is the difference between an American and a European option?

2. What are the main factors influencing an option's value?

3. How can we calculate the value of an option?

4. What is the put–call parity?

5. Explain how the value of a call can be determined.

6. How can the value of a put option be affected if the market price of the share changes?

7. What is the strike price?

8. What is the difference between call and put options?

9. What is the difference between the position diagram and the profit diagram of an option?

10. How can we determine the put option writer's gain?

11. What is the expiration date of an option?

12. What is the relationship between the option buyer's gain and the seller's gain?

13. Explain how the break-even point of a call can be determined.

14. Give an example of the object of an option.

15. What is the difference between selling a put option and buying a put option?

Calculation exercises

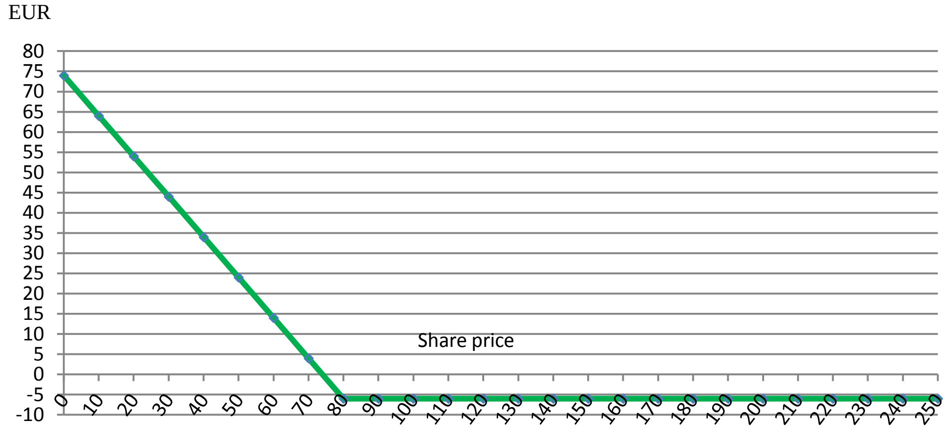
1.

Suppose that you buy a one-year European call option on the "ENG share" with an exercise price of 180 EUR. The price of the call option is 10 EUR. (Ignore the time value of money)

a) Draw the position diagram for the buyer of the call option.
b) Draw the profit diagram for the buyer of the call option.
c) Draw the position diagram for the seller of the call option.
d) What will be your profit or loss if the share's market price is 220 EUR at its maturity date?

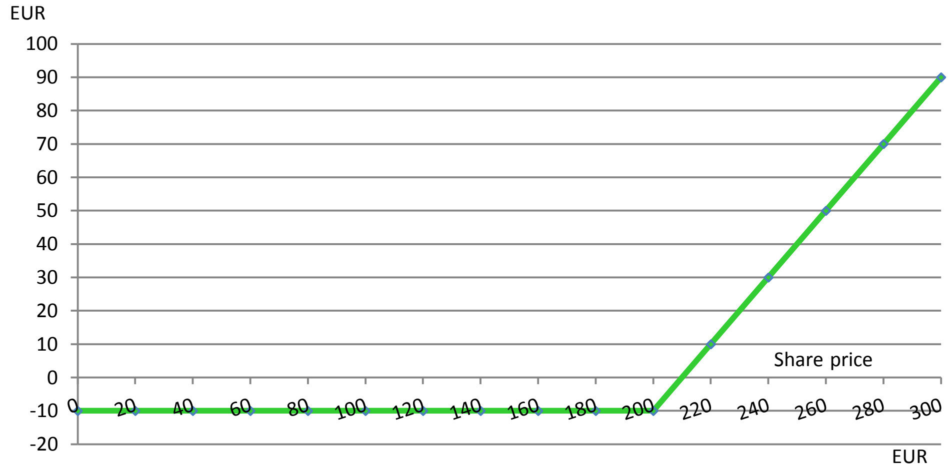
2.

The profit diagram of an option is illustrated in the following graph.



a) Give the type of the option.
b) Determine the strike price of the option.
c) Determine the transaction cost of the option.
d) Draw the position diagram for the buyer of the option.
e) Draw the profit diagram for the seller of the option.
f) Give the break-even point.

3.
The profit diagram of an option is illustrated in the following graph.



a) Give the type of the option.
b) Determine the strike price of the option.
c) Determine the transaction cost of the option.
d) Draw the position diagram for the buyer of the option.
e) Draw the profit diagram for the seller of the option.
f) Give the break-even point.

4.

Suppose that you buy a one-year European put option on the “MECH share” with an exercise price of 50 EUR. The price of the put option is 2 EUR. (Ignore the time value of money)

a) Draw the position diagram for the buyer of the put option.
b) Draw the profit diagram for the buyer of the put option.
c) Draw the position diagram for the seller of the put option.
d) Draw the profit diagram for the seller of the put option.
e) What will your profit or loss be if the share is selling for 100 EUR at its maturity date?

5.

Suppose that you buy a one-year European call option on the “RING share” with an exercise price of 220 EUR. The price of the call option is 10 EUR.

a) Draw the position diagram for the buyer of the call option.
b) Draw the profit diagram for the buyer of the call option.
c) At what price is break-even achieved?
d) What will your profit or loss be if the share is selling for 200 EUR at its maturity date?
e) What will your profit or loss be if the share is selling for 220 EUR at its maturity date?
f) What will your profit or loss be if the share is selling for 250 EUR at its maturity date?

6.

Suppose Uncle John buys a put option at a price of 5 EUR with an exercise price of 120 EUR.

a) At what price can the break-even point be found?
b) Illustrate the break-even point in a profit diagram.

7.

Suppose that your friend buys a one-year European call option on the “OPT share” with an exercise price of 80 EUR and a put option on the same share with an exercise price of 80 EUR. The price of the call option is 8 EUR and the price of the put option is 5 EUR. (Disregard the time value of money)

a) Draw the position diagram for the buyer of the call option.
b) Draw the position diagram for the buyer of the put option.
c) Draw the position diagram for the buyer of the composite position.
d) What is the price where the break-even point is reached?
e) What will the investor’s profit or loss be if the share is selling for 100 EUR at maturity date?
f) At what range of price does your friend experience a loss?

8.

Suppose that an investor buys a one-year European call option on the “GOODINVEST” share with an exercise price of 150 EUR and sells a call option on the “GOODINVEST” share with an exercise price of 100 EUR. The price of the call option is 8 EUR. (Disregard the time value of money)

a) Draw the position diagram for the buyer of the call option.
b) Draw the position diagram for the seller of the call option (sold) with an exercise price of 150 EUR.
c) Draw the position diagram of the composite position.
d) At what price is the break-even achieved?
e) What will the investor’s profit or loss be if the share is selling for 140 EUR at its maturity date?
f) At what range of price does the investor experience a loss?

9.

Suppose that an investor buys a one-year European put option on the “Cica” share with an exercise price of 20 000 HUF and buys a call option on the “Cica” share with the same exercise price. The price of the call option is 1 200 HUF and the price of the put option is 1 300 HUF. (Disregard the time value of money)

a) Draw the position diagram for the buyer of the put option.
b) Draw the profit diagram for the buyer of the call option.
c) Draw the position diagram of the composite position.
d) What will the investor’s profit or loss be if the share is selling for 18 000 HUF at maturity date?
e) What will the investor’s profit or loss be if the share is selling for 20 000 HUF at maturity date?
f) Determine the break-even point of the option strategy.

10.

Suppose you purchase a call option with an exercise price of 12 000 HUF at a cost of 800 HUF, sell two call options with an exercise price of 14 000 EUR at a cost of 1 000 HUF, and buy a call option with an exercise price of 16 000 HUF at a cost of 1 200 HUF. (Disregard the time value of money)

a) Draw the position diagram of the composite option.
b) Draw the profit diagram of the composite option.
c) What is the breakeven stock price?
d) What will your profit or loss be if the share price is 15 000 HUF at maturity date?

11.

Suppose you sell a put option with an exercise price of 18 000 HUF at a cost of 600 HUF, purchase two call options with an exercise price of 20 000 HUF at a cost of 800 HUF, and buy a put option with an exercise price of 22 000 HUF at a cost of 1 000 HUF. (Disregard the time value of money)

a) Draw the position diagram of the two call options.
b) Draw the position diagram of the composite option.
c) Draw the profit diagram of the composite option.
d) What is the breakeven stock price?
e) What will your profit or loss be if the share price is 15 000 HUF at maturity date?
f) What will your profit or loss be if the share price is 21 000 HUF at maturity date?
g) What will your profit or loss be if the share price is 24 000 HUF at maturity date?

8. Introduction to international financial decisions - exchange rates

A corporation that operates internationally has to make international financial decisions. These decisions are influenced by many financial and other economic factors, such as foreign tax rates, interest rates, exchange rates, political risk, and the change in currencies. In this chapter we give an introduction to the foreign exchange market.

The ***foreign exchange market*** is the market where one country's currency is traded for another country's currency. This market operates electronically.

8.1. The nominal exchange rate

We can distinguish two types of exchange rates.

- nominal exchange rates and
- real exchange rates.

Picture 18.

Suppose a Japanese firm imports goods from Austria and the USA. In this case the Japanese firm needs euros and U.S. dollars to pay for the Austrian and American products. However, if the Japanese firm exports goods and services to Austria, the firm gets euros and sells them in exchange for yen. The manager of the Japanese firm and the manager of the Austrian firm are the actors in the foreign exchange market, and they have to be informed about the relevant exchange rates.

The ***nominal exchange rate*** gives the number of units of one country's currency that can purchase a unit of another country's currency. In other words, the nominal exchange rate is the price of one country's currency in terms of the other country's currency:

$$exchange\ rate = \frac{a\ given\ country's\ currency}{another\ country's\ currency}.$$

For example, if the exchange rate is 0.9033 EUR/USD, you can get 0.9033 EUR for one U.S. dollar (Table 8.1). The euro is quoted at 0.9033. The reciprocal of the EUR/USD exchange rate gives the USD/EUR exchange rate. This rate is equal to 1.071, so the U.S. dollar is quoted at 1.071 (Table 8.1).

According to Table 8.1, the exchange rate in terms of yen per dollar is 124.387. A decrease in the nominal exchange rate means a nominal appreciation of the given currency. If the nominal exchange rate between the yen and the U.S. dollar is $110\,\frac{\text{JPY}}{\text{USD}}$, this means that 1 dollar buys 110 yen.

Example 8.1

Suppose that the Japanese firm exports goods to the U.S. and the American firm pays 50 000 U. S. dollars for the Japanese products.

This means that the Japanese firm has:

$$50\ 000\mathrm{USD} = 50\ 000\mathrm{USD} \cdot 124.387 \frac{\mathrm{JPY}}{\mathrm{USD}} = 6\ 219\ 350\mathrm{JPY}.$$

Suppose that the exchange rate changes and the new rate is the following:

$$1\ \mathrm{USD} = 150\ \mathrm{JPY},$$

so we can buy 150 yen for 1 U.S. dollar.

In this case the Japanese firm realizes a greater revenue, because the value of the yen has increased:

$$50\ 000\mathrm{USD} = 50\ 000\mathrm{USD} \cdot 150 \frac{\mathrm{JPY}}{\mathrm{USD}} = 7\ 500\ 000\mathrm{JPY}.$$

Table 8.1 Exchange rates versus USD (19$^{\text{th}}$ Aug. 2015)

Currency Code	Currency	Level	Units
EUR	Euro	0.9033	EUR per USD
JPY	Japanese Yen	124.3870	JPY per USD
GBP	Pound Sterling	0.6396	GBP per USD
CHF	Swiss Franc	0.9775	CHF per USD
CAD	Canadian Dollar	1.3097	CAD per USD
AUD	Australian Dollar	1.3589	AUD per USD
NZD	New Zealand Dollar	1.5218	NZD per USD
SEK	Swedish Krona	8.5280	SEK per USD
NOK	Norwegian Krone	8.2433	NOK per USD
BRL	Brazilian Real	3.4806	BRL per USD
CNY	Chinese Yuan Renminbi	6.3961	CNY per USD
RUB	Russian Rouble	65.5980	RUB per USD
INR	Indian Rupee	65.3724	INR per USD
TRY	New Turkish Lira	2.8689	TRY per USD
THB	Thai Baht	35.5029	THB per USD
IDR	Indonesian Rupiah	13,830.3000	IDR per USD

Currency Code	Currency	Level	Units
MYR	Malaysian Ringgit	4.0949	MYR per USD
MXN	Mexican New Peso	16.4309	MXN per USD

Table 8.1 Cont. Exchange rates versus USD (19th Aug. 2015)

Currency Code	Currency	Level	Units
ARS	Argentinian Peso	9.2534	ARS per USD
DKK	Danish Krone	6.7417	DKK per USD
ILS	Israeli New Sheqel	3.8262	ILS per USD
PHP	Philippine Peso	46.3108	PHP per USD

Source: Quandl (2016)

Suppose that the nominal exchange rate of the yen to the U.S dollar increases from $124.387\,\frac{\text{JPY}}{\text{USD}}$ to $150\,\frac{\text{JPY}}{\text{USD}}$. In this case, it takes more yen to purchase a dollar, so the value of the Japanese yen decreases, i.e. the yen depreciates.

If the nominal exchange rate of a given country's currency to another country's currency increases, the given country's currency depreciates.

If the nominal exchange rate of a given country's currency to another country's currency decreases, the given country's currency appreciates. The value of a given country's currency and its exchange rate are inversely related.

8.2. The real exchange rate

As we have examined in the previous subsection (Chapter 8.1), the nominal exchange rate measures the relative price of two countries' currency. The real exchange rate measures the purchasing power of a given country's currency relative to another country's currency at current exchange rates and prices. The real exchange rate can be calculated as the following:

$$\text{real exchange rate} = \text{nominal exchange rate} \cdot \frac{P^f}{P^h},$$

where P^h is the overall price level in the domestic country and P^f is the overall price level in a foreign country.

Nominal exchange rate is the rate at which two countries' currencies trade against each other.

Real exchange rate is the rate at which two countries' goods trade against each other.

(Reinert, 2012)

For example, the real exchange rate between the yen and the U.S. dollar measures the amount of Japanese goods that trade against U.S. goods. If the price level in the U.S. increases, the real value of the Japanese yen falls if all other influencing factors remain the same. However, if the price level in Japan rises, the real value of the Japanese yen rises too, because it takes fewer Japanese goods to purchase U.S. goods. If the nominal exchange rate rises with all other factors unchanged, the real exchange rate moves in the same direction, so it rises, too. In this case, it takes more Japanese yen to buy a U.S. dollar, so more Japanese goods to buy U.S. goods. The real value of the yen falls. We can therefore conclude that the real exchange rate rises when the overall price level in a foreign country rises, or the domestic price level falls, or the nominal exchange real increases. An increase in the real exchange rate is appreciation, while a decrease is called depreciation.

The ***effective exchange rate (EER)*** is the weighted average rate of the exchange rates between the domestic currency and other countries' currency, where the weights are the different countries' shares in the domestic country's trade. The effective exchange rate shows the measure of the overall value of a currency against the basket of given currencies. Therefore the effective exchange rate is called the multilateral exchange rate.
The effective exchange rate can be calculated in the following way:

$$\text{effective exchange rate}_t = \prod_{i=1}^{n} (\text{er}_{i,\text{given country}})_t^{\ w_i}, \qquad (8.1)$$

where n is the number of trading partner countries, w_i is the trade weight assigned to the currency of trading partner i, and $\text{er}_{i,\text{given country}}$ is the average exchange rate of the currency of partner i vis-à-vis the given country in period t.

We can distinguish between the:
- nominal effective exchange rate and
- the real effective exchange rate.

When the nominal exchange rate is included in the equation (8.1) of the effective exchange rate, we can calculate the nominal effective exchange rate (NEER). However, if there are real exchange rates in the equation (8.1), the formula can be applied to the calculation of the real effective exchange rate (REER).

8.3. Interest rates and exchange rates

Example 8.2
Suppose that you plan to invest 20 000 U.S. dollars for one year. Your *'MEGA'* bank offers you two investment possibilities. According to the first opportunity, the bank will pay 6 percent interest rate per year for your 20 000 U.S. dollar deposit. However, if you put your money in a yen deposit, the bank will pay 8 percent annually. Which alternative should you choose? How much will you have after a year?

U.S. deposit
1 year:

$$20\ 000\ USD => 20\ 000\ USD \cdot 1.06 = 21\ 200\ USD.$$

Yen deposit
1 year:
First, it is necessary to buy yen for your dollars. According to the exchange rate (Table 8.1), you can buy:

$$20\ 000\ USD = 20\ 000\ USD \cdot 124.387\,\frac{\text{JPY}}{\text{USD}} = 2\ 487\ 740\ \text{YPN}.$$

Your bank pays 8 percent interest annually on yen deposits, so at the end of the first year you will have:

$$2\ 487\ 740\ \text{YPN} => 2\ 487\ 740\ \text{YPN} \cdot 1.08 = 2\ 686\ 759.2\ \text{YPN}.$$

Suppose that the exchange rate does not change over a one year period. In this case, the future value of your money in dollar terms is the following:

$$2\ 686\ 759.2\ \text{YPN} => \frac{2\ 686\ 759.2\ \text{YPN}}{124.387\frac{\text{JPY}}{\text{USD}}} = 21\ 600\ \text{USD}.$$

However, if the exchange rate changes over a year, your revenue may increase or decrease, too. If the value of the yen falls, the future value of your money expressed in dollars decreases. Suppose that the nominal exchange rate has increased by 20 percent by the end of the first year. In this case, at the end of the first year you will have:

$$2\ 686\ 759.2\ \text{YPN} => \frac{2\ 686\ 759.2\ \text{YPN}}{124.387\frac{\text{JPY}}{\text{USD}} \cdot 1.2} = 18\ 000\ \text{USD}.$$

When you make a U.S. dollar deposit of 20 000 USD, your gain is 1 200 U.S at a 6 percent interest rate. However if you make a Japanese yen deposit (with the nominal exchange rate of) $149.2644\,\frac{\text{JPY}}{\text{USD}}$, you will have made a \$2 000 loss.

At what future level of the nominal exchange rate can we realize a 1 200 USD benefit?

To determine the future exchange rate, we have to solve the following simple equation for *x*:

$$2\ 686\ 759.2\ \text{YPN} => \frac{2\ 686\ 759.2\ \text{YPN}}{\text{x}} = 21\ 200\ \text{USD},$$

$$\text{x} = 126.734\,\frac{\text{JPY}}{\text{USD}}.$$

According to the result, if the nominal exchange rate between the yen and the U.S dollar increases by 1.887 percent, we can realize a 1 200 USD benefit:

$$1.01887 = \frac{126.734\frac{\text{JPY}}{\text{USD}}}{124.387\frac{\text{JPY}}{\text{USD}}}.$$

Examine the ratio of the two interest rates offered by the *'MEGA'* bank:

$$\frac{1.08}{1.06} = 1.01887.$$

We find that the interest rate offered for the yen deposit is 1.887% greater. Interest rate parity theory says that the difference between the forward exchange rate and the spot exchange rate is equal to the difference in interest rates between two countries:

$$\frac{\text{forward exchange rate}}{\text{spot exchange rate}} = \frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}}.$$

Let the forward premium be equal to *fp*; in this case:

$$\frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}} = \frac{(1+\text{fp})\cdot\text{spot exchange rate}}{\text{spot exchange rate}} = 1 + \text{fp}. \qquad (8.2)$$

Appendix (8.1)

Determine the rate of return on an investment in foreign currency. Suppose that the initially invested amount of the home currency is C_{h0}, the future value of the home country deposit is C_{hf}, the spot rate in the home currency at which the foreign currency is purchased is S, the suitable forward exchange rate is F, and finally the interest rate on the home deposit and the foreign deposit is respectively r_h and r_f.

The rate of return from investment in foreign currency is the following:

$$r_h = \frac{C_{hf} - C_{ho}}{C_{ho}},$$

where

$$C_{hf} = \frac{C_{ho}}{S} \cdot (1 + r_f) \cdot F. \qquad (8.1a)$$

Applying equation (8.2) with the new symbol introduced in the Appendix (8.1), we get:

$$F = (1 + fp) \cdot S$$

$$C_{hf} = \frac{C_{ho}}{S} \cdot (1 + r_f) \cdot (1 + fp) \cdot S = C_{h0} \cdot (1 + r_f) \cdot (1 + fp) \qquad (8.2a)$$

Substituting equation (8.2a) into the equation of the rate of return:

$$r_h = \frac{C_{hf} - C_{ho}}{C_{ho}} = \frac{C_{ho} \cdot (1+r_f) \cdot (1+fp) - C_{ho}}{C_{ho}} = (1 + r_f) \cdot (1 + fp) - 1,$$

$$\frac{1+r_h}{1+r_f} = 1 + fp,$$

$$\frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}} = \frac{\text{forward exchange rate}}{\text{spot exchange rate}}.$$

8.4. Purchasing Power Parity (PPP)

The absolute form of purchasing power parity

We can distinguish between an absolute and a relative from of purchasing power parity. According to absolute purchasing power parity, the prices of the same consumer basket in different countries should be equal when measured in a common currency.

> ***Purchasing Power Hypothesis***: "The nominal exchange rate will adjust so that the purchasing power of a currency will be the same in every country."
>
> (Reinert, 2012)

The absolute form of purchasing power parity holds if there are no international barriers such as transportation costs, transaction costs, and tariffs. Consumers shift their demand to wherever prices are lower.

The overall price level and the purchasing power of a given country are inversely related. This means that if the price level increases in a given country, the purchasing power of the currency in this country falls. Suppose that the price level in a given country is $P_{country1}$, the purchasing power will be $\frac{1}{P_{country1}}$. When the price level in a given country increases, the purchasing power decreases. What can we say about the purchasing power of the domestic currency in another country? To give the answer, we should know the nominal exchange rate between the two countries and the purchasing power of the second country's currency. According to the purchasing power parity theory, the purchasing power of country$_1$'s currency is the following:

$$\frac{1}{P_{country1}} = \frac{1}{\text{nominal exchange rate}} \cdot \frac{1}{P_{country2}},$$

$$\text{Country}_2 \text{ price of goods in the Country}_2 = \frac{\text{Country}_1 \text{ price of goods in the Country}_1}{\text{nominal exchange rate } \left(\frac{\text{currency}_1}{\text{currency}_2}\right)}.$$

According to the theory of purchasing power parity, any difference in the rate of inflation will be offset by a change in the exchange rate. For example, if the price level increases by 10 and 15 percent respectively in Country$_1$ (Japan) and Country$_2$ (U.S.), the amount of yen that we can buy for 1 U.S. dollar rises by 4.545 percent:

$$\frac{1.15}{1.1} - 1 = 1.04545 - 1 = 0.04545.$$

In short, purchasing power parity theory says that the home currency price of a product in different countries, if converted into a common currency at the spot exchange rate, is the same across all the countries of the world.

The ***relative form of purchasing power parity***

When there are international barriers such as transportation costs, quotas, or transaction costs, the prices of the same consumer basket in different countries can be different when determined in a common currency. The relative form of purchasing power parity says that the percentage change in the exchange rate between two countries' currency is equal to the percentage change in the ratio of the price indices of the two countries. The relative percentage change in the price indices between the two countries in a given period of time should result in a change in the exchange rate between the two countries' currencies over the same period of time.

Suppose that the price index in the home country and the foreign country is P_h and P_f respectively, the inflation rate in the home country is I_h, and the inflation rate in the foreign country is indicated by I_f. It is assumed that the price levels in the two countries are equal.

If the price index and the exchange rate of the foreign currency change, the foreign price index from the home country's perspective is:

$$P_f \cdot (1 + I_f) \cdot (1 + e_f),$$

where e_f shows the percentage change in the value of the foreign country's currency. On the basis of the purchasing power parity, the percentage change in the foreign country's currency should maintain parity in the new price indexes of the two countries (Madura, 2008). Setting the formula for the new price index of the foreign country equal to the equation of the new price index of the home country, we get the following (Madura, 2008):

$$P_f \cdot (1 + I_f) \cdot (1 + e_f) = P_h \cdot (1 + I_h).$$

Express the percentage change in the value of the foreign currency (e_f):

$$1 + e_f = \frac{P_h \cdot (1+I_h)}{P_f \cdot (1+I_f)},$$

holding the initial assumption that P_h is equal to the P_f, we get:

$$e_f = \frac{1+I_h}{1+I_f} - 1.$$

8.5. International investment decisions – NPV

In Chapter 4, we examined how we can make investment decisions. We evaluated investment decisions by using different indicators, such as net present value, the profitability index, the discounted payback period and the internal rate of return. However, we did not discuss these investments in an international environment, so we disregarded exchange rates, foreign rates, and the foreign cost of capital. In this subsection we will extend our knowledge to evaluate international investments by using the net present value method.

Example 8.2

Suppose that you, as the general a manger of the Austrian DEMK Company, would like to extend your company's production internationally. The first target country is the United States. The financial manager has forecasted the future costs and revenues of the planned investment. The future cash flows of the investment can be seen in the following table.

Year	Investment (million U.S dollar)
0	-50
1	15
2	15
3	25
4	30
5	38
6	40

Determine the net present value of the investment. The following question can arise: Which currency should we apply to calculate the net present value of the planned investment?
Firstly, calculate the net present value in terms of U.S. dollars. To determine the net present value we have to apply the dollar cost of capital as a discount rate. The net present value of the investment is the following, assuming that the dollar discount rate is 10 percent:

$$\text{NPV} = -50 + \frac{15}{1.1^1} + \frac{15}{1.1^2} + \frac{25}{1.1^3} + \frac{30}{1.1^4} + \frac{38}{1.1^5} + \frac{40}{1.1^6} = 61.48 \text{ million U.S dollar.} \quad (8.3)$$

Determine the benefit in terms of the euro, so convert the result given in equation in (8.3) into euros. We need the spot rate of exchange. Suppose that the EUR/USD spot rate is 0.9033 EUR/USD according to Table 8.1, so the net present value in euros is:

$$\text{NPV} = 61.48 \text{ million U.S dollar} \cdot 0.9033 \frac{\text{EUR}}{\text{USD}} = 55.5 \text{ million EUR.} \quad (8.4)$$

What happens if the EUR/USD exchange rate changes? Our cash flows will occur in different years, and we can assume that the exchange rate does not remain the same in the following years. In this case, we can disregard the change in the exchange rate, because we can hedge the foreign exchange risk. For example, if we can buy and sell foreign currency forward, we can sell dollars forward. This means that it is not necessary to worry about the exchange rate risk.

In the second step, suppose we can sell dollars forward, and so hedge the exchange rate risk. We have to determine the forward exchange rate between the euro and the U.S dollar, which depends on the Austrian interest rate and American interest rate. Assume that the interest rate in Austria is 2 percent and the dollar interest rate is 5 percent. According to the interest rate parity theory examined in subsection 8.3, we can determine the one-year forward exchange rate as the following:

$$\frac{\text{forward exchange rate}}{\text{spot exchange rate}} = \frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}},$$

$$\text{one}-\text{year exchange rate}=\frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}}\cdot\text{spot exchange rate},$$

$$\text{one}-\text{year exchange rate}=\frac{1.02}{1.05}\cdot 0.9033=0.878,$$

$$\text{two year exchange rate}=\frac{(1+\text{interest rate}_{\text{given country}})^2}{(1+\text{interest rate}_{\text{another country}})^2}\cdot\text{spot exchange rate},$$

$$\text{two}-\text{year exchange rate}=\frac{1.02^2}{1.05^2}\cdot 0.9033=0.852,$$

Similarly, we can calculate the exchange rates for the following years:

Year	Investment (million U.S dollar)	Forward exchange rates (EUR/USD)
0	-50	0.9033
1	15	$\frac{1.02}{1.05}\cdot 0.9033=0.878$
2	15	$\frac{1.02^2}{1.05^2}\cdot 0.9033=0.852$
3	25	$\frac{1.02^3}{1.05^3}\cdot 0.9033=0.828$
4	30	$\frac{1.02^4}{1.05^4}\cdot 0.9033=0.804$
5	38	$\frac{1.02^5}{1.05^5}\cdot 0.9033=0.781$
6	40	$\frac{1.02^6}{1.05^6}\cdot 0.9033=0.759$

We can determine the future cash flows in terms of euros by using the forward exchange rates:

Year	Investment (million U.S dollar)	Forward exchange rates (EUR/USD)	Investment (million euro)
0	-50	0.9033	$-50\cdot 0.9033=-45.165$
1	15	0.878	$15\cdot 0.878=13.17$
2	15	0.852	$15\cdot 0.852=12.78$
3	25	0.828	$25\cdot 0.828=20.7$
4	30	0.804	$30\cdot 0.804=24.12$
5	38	0.781	$38\cdot 0.781=29.678$
6	40	0.759	$40\cdot 0.759=30.36$

Finally we need the risk adjusted euro discount rate to determine the net present value of the investment in terms of euros. The euro discount rate can be determined in the following way:

$$\frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}}=\frac{1+\text{required rate of return}_{\text{given country}}}{1+\text{required rate of return}_{\text{another country}}},$$

$$1 + \text{required rate of return}_{\text{given country}} = \frac{1+\text{interest rate}_{\text{given country}}}{1+\text{interest rate}_{\text{another country}}} \cdot (1 + \text{required rate of return}_{\text{another country}}),$$

$$1 + \text{required rate of return}_{\text{given country}} = \frac{1.02}{1.05} \cdot 1.1 = 1.0686.$$

We can see that the required euro return is less than the dollar rate of return, because the euro interest rate is less than the dollar interest rate. Using the risk adjusted euro discount rate, the net present value in terms of euros is the following:

$$\text{NPV} = -45.165 + \frac{13.17}{1.0686^1} + \frac{12.78}{1.0686^2} + \frac{20.7}{1.0686^3} + \frac{24.12}{1.0686^4} + \frac{29.678}{1.0686^5} + \frac{30.36}{1.0686^6} = 55.5 \text{ million EUR.}$$

This result is equal to the net present value obtained in equation (8.4). This means that the result of the net present value calculation discounting the dollar cost of capital and assuming no exchange rate risk is the same as the result of the net present value calculation discounting the euro discount rate and assuming that the exchange risk can be hedged.

International investment decisions

"The simplest way to calculate the NPV of an overseas investment is to forecast the cash flows in foreign currency and discount them at the foreign currency cost of capital.

The alternative is to calculate the cash flows that you would receive if you hedged the foreign currency risk. In this case you have to translate the foreign currency cash flows into your own currency using the forward exchange rate and then discount these domestic cash flows at the domestic cost of capital".

(Brealey - Myers, 2014)

8.6. Terms and Questions

absolute form of purchasing power parity,
effective exchange rate,
exchange rate,
foreign exchange market,
foreign exchange rate risk,
forward exchange rate,
forward premium,
inflation rate,
interest rate parity,
multilateral exchange rate,
nominal effective exchange rate,
nominal exchange rate,
purchasing power parity,
purchasing power parity hypothesis,
rate of return,
real effective exchange rate,
real exchange rate,
relative form of purchasing power parity,
spot exchange rate.

Theoretical questions

1. What is the difference between the nominal and the real exchange rate of a given country's currency to another country's currency?

2. What are the main factors influencing international financial decisions?

3. Explain the theory of interest rate parity.

4. What is the difference between the spot exchange rate and the forward exchange rate?

5. Give the main factors influencing the benefits of foreign financial investments.

6. What is the difference between the real exchange rate and the effective exchange rate?

7. How can we determine the nominal effective exchange rate?

8. Explain the theory of purchasing power parity.

9. What is the difference between the absolute form of purchasing power parity and the relative form of purchasing power parity?

10. How can we determine the real effective exchange rate?

Calculation exercises

1.

Suppose that annual interest rates in Australia are 3 percent, while interest rates in Germany are 5 percent.

a) Does interest rate parity exist?
b) What should the forward premium of the euro be on the base of the interest rate parity?
c) Suppose that the euro's spot rate is 1.6 AUD. What should the one-year forward rate of the euro be?

2.

Suppose that annual interest rates in Country1 are 6 percent, while interest rates in Country2 are 3 percent.

a) Does interest rate parity exist?
b) What should the forward premium of Country2's currency be on the basis of the interest rate parity?
c) Suppose that Country2's currency spot rate is 0.8 (in terms of Country1's currency). What should the one-year forward rate of the euro be?

3.

Suppose that annual interest rates in Country1 are 4 percent, while interest rates in Country2 are 8 percent.

a) Does interest rate parity exist?
b) What should the forward premium of Country2's currency be on the basis of the interest rate parity?

c) Suppose that the one-year forward rate of Country2's currency is 0.8 (in terms of Country1's currency). What should the spot rate of the euro be?

4.
Suppose that Country1's inflation rate is 2 percent, and the inflation rate in Country2 is 10 percent.

a) How would we expect the difference in the inflation rates to affect the value of Country1's currency?
b) Does the expected relationship always occur?

5.
Suppose that one-year risk free interest rate in Country1 is 12 percent, and the one-year risk free interest rate in Country2 is 4 percent. Country1's currency is the *silver dollar*, and country2's currency is the *gold dollar*. The spot rate of Country1's currency is 18 *gold dollars.*

a) What is the amount of the forward premium?
b) Determine the one-year forward rate of the silver dollar?
c) Suppose that the interest rate in Country1 decreases by 6 percentage points? What is the amount of the forward premium?
d) Determine the one-year forward rate of the silver dollar on the assumption that Country1's interest rate changes?

6.
Assume that Country1's currency is the *silver forint*, and Country2's currency is the *gold forint*. Suppose that the spot rate of Country1 is 20 *gold forints*. The inflation rate in Country2 over this year is expected to be 8 percent, and the inflation rate over this year is expected to be 6 percent. Your firm needs 50 million *silver forints*.

a) Give the expected amount of *gold forints* to be paid for the *silver forints* in one year?

7.
We have information about the currency cross rates for various European countries. The currency cross rates can be found in the following table:

Currencies	GBP £	EUR €	USD $	JPY ¥
Czech Republic 1.00 CZK	0.02802	0.03694	0.03989	4.74820
Denmark 1.00 DKK	0.10140	0.13403	0.14470	17.18740
European Union	0.75610	N/A	1.07950	128.16000

1.00 EUR				
Hungary 1.00 HUF	0.00243	0.00320	0.00347	0.41050
Norway 1.00 NOK	0.08020	0.10600	0.11442	13.58990
Poland 1.00 PLN	0.16972	0.22410	0.24213	28.73700
Russia 1.00 RUB	0.00896	0.01184	0.01280	1.51800
Sweden 1.00 SEK	0.08165	0.10790	0.11650	13.83980
Switzerland 1.00 CHF	0.68930	0.91110	0.98370	116.83000
Turkey 1.00 TRY	0.23358	0.30810	0.33260	39.50440
United Kingdom 1.00 GBP	N/A	1.32130	1.42660	169.44000

Source: Financial Times (2016)

a) How many Hungarian forints do you get for one euro?
b) What is the cross rate between the U.S. dollar and the Japanese yen?
c) How many Russian roubles do you get for one Swiss franc?
d) What is the cross rate between the Swedish Kroner and the Turkish lira?

8.

Suppose that the Italian *Best Coffee Company* would like extend its production internationally. The general manager is considering constructing a new plant in the United States. The future forecast cash flows of the investment (in millions of U.S. dollars) can be seen in the following table.

Year	Investment (millions U.S dollar)
0	-40
1	10
2	12
3	15
4	28
5	30
6	40

Suppose that the spot exchange rate is 0.9832 EUR/USD. According to the financial manager's information, the interest rate in Italy is 6 percent and the dollar interest rate is 4 percent.

a) Determine the net present value of the investment in terms of U.S. dollars if the dollar opportunity cost of capital is 8 percent.
b) Calculate the net present value of the investment in terms of euros.
c) Determine the forward exchange rates in each year.
d) Calculate the euro required rate of return.
e) Give the cash flows in terms of euros, if the company hedges the exchange rate risk.

f) According to the financial manager's forecast, the U.S. dollar will appreciate by 6 percent a year. What is the impact of the change in the U.S. dollar exchange rate for the investment?
g) According to the financial adviser's forecast, the U.S. dollar will depreciate by 4 percent a year. What is the impact of the change in the U.S. dollar exchange rate for the investment?
h) Determine the profitability index of the dollar cash flows from the investment.
i) Give the discounted payback period of the dollar cash flows from the investment.

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