PhD Thesis Summary

Some New Findings on Mixed Bertrand-Edgeworth Duopolies

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Contents

1 Contribution of the thesis and applied methodology

The dissertation aims at presenting equilibrial firm behavior in a mixed Bertrand-Edgeworth duopoly environment. Assuming partial public ownership on the market, we provide a thorough analysis concerning equilibrium prices and quantities, endogenous timing and social welfare effects. The contribution of the thesis to knowledge is broadening the theory of Bertrand-Edgeworth models for a better understanding of duopolistic markets with public ownership.

In general, modelling duopol markets by means of game theory has the following steps.

1. Setting the framework and assumptions.
2. Setting the participants (players), their decision sets (strategy sets) and their objective functions (that they want to maximize).
3. Solving the model (finding the equilibrial behavior of the players) using an equilibrium concept (most frequently the Nash equilibrium).
4. Discussion of the results.

We follow these steps in our analysis.[1] Because of its frequent use in the present dissertation, we give the formal definition of the pure-strategy Nash equilibrium as it can be found for example in Forgó, Pintér, Simonovits, and Solymosi [2005].

Definition 1.1. Let us denote by $G = \{1, ..., n; S_1, ..., S_n; f_1, ..., f_n\}$ a game with n players, where S_i denotes the strategy set of player i, while f_i stands for the payoff function of player i. A strategy profile s^* is a pure-strategy Nash equilibrium, if $f_i(s_i^*, s_{-i}^*) \geq f_i(s_i, s_{-i}^*)$ holds true for every $s_i \in S_i$ and every $i = 1, ..., n$.

[1] Among many others, we refer the reader to another interesting application of game theory by Balogh and Kormos [2014].

In other words, a strategy profile is a Nash equilibrium if and only if none of the players can increase her payoff by a unilateral strategy modification. The definition can be trivially restricted to the two-player case.

Clearly, the solution of a game depends strongly on the objective functions of the participants. Provided that there is a difference in the objective functions between the standard (with purely private firms) and the mixed (with appearance of public ownership) duopoly models, the equilibria of the two model versions will not remain the same.

The dissertation focuses on Bertrand-Edgeworth duopolies where the state is present on the market as the exclusive or partial owner of a firm. The overall aim of the thesis is to provide a thorough game theoretic analysis of such markets.

The main contribution of the thesis is summarized as follows:

1. The existence of a pure strategy Nash equilibrium in a mixed Bertrand-Edgeworth duopoly is analyzed. Furthermore, the equilibrium profiles are characterized for all the given subcases.

2. Provided that the ordering of firms' decisions is endogenous (i.e. we consider the timing game), the equilibrial ordering of moves is given for mixed and semi-mixed Bertrand-Edgeworth duopolies.

3. A social welfare analysis is carried out for the mixed Bertrand-Edgeworth duopoly, i.e. we characterize the difference the presence of a public firm makes in social welfare compared to the standard case with only private firms competing on the market.

2 Motivation

Oligopolies are present in our everyday life. Therefore, it is important to build sophisticated models that explain and predict oligopolist behavior. Every model that has valid assumptions in describing or helping in the description of a real life market structure contributes to knowledge on producer behavior.

To illustrate the importance of the topic, we provide evidence from practical life. We present the following incomplete list of famous duopolies and oligopolies on the global and on the Hungarian national markets:

- credit card market: competition between VISA and MasterCard (and American Express in the U.S.)
- airplane industry: competition between Airbus and Boeing
- market of computers' graphics processing units: competition between AMD and Nvidia
- sports equipment industry: competition between the leading Nike and Adidas
- oil industry: the competition between the Norwegian Statoil and the Russian Gazprom in certain countries
- commercial banks in New Zealand and Hungary: one of the competitors (Kivibank) is state-owned in New Zealand, while Budapest Bank has recently been purchased by the Hungarian state
- Hungarian mobile phone network supplier market: competition between Telekom, Telenor and Vodafone
- Hungarian petrol station suppliers: the partially state-owned MOL competes with OMV, Shell, Agip and a few smaller suppliers
- Hungarian hypermarket suppliers: Tesco, Auchan and InterSpar

A market can be considered a duopoly (or oligopoly) if two (or three, four, etc...) firms have large control over the whole market, even if there exist other smaller suppliers. Still, a game theoretic framework needs strict assumptions about the participants, strategies and objective functions.

As listed above, there exist oligopolies where the state is also present on the market as an owner. These oligopolies provide motivation for the analysis of mixed

oligopoly models. An initial building block of oligopoly theory is duopoly theory. There are many different ways of modelling a mixed duopoly. As we present in the dissertation, the Bertrand-Edgeworth framework has proved to grasp some important features of this market structure that appear in real life. Thus, adding new results to the theory of Bertrand-Edgeworth duopolies is a step towards a better understanding of duopolist behavior.

3 Key research questions

When doing research by means of game theory in general, one of the most important issues to be discussed is the existence of a pure-strategy equilibrium in the given game. If exactly one pure strategy equilibrium exists, then the game is theoretically easy to handle, rational players are expected to choose their one and only strategy that leads to equilibrium.[2] If there are multiple pure strategy equilibria, then several strategy profiles can emerge in equilibrium. If the number of pure strategy equilibria equals zero, then the concept of mixed equilibrium is needed to give a solution to the game. There are several ways to prove that a mixed Nash equilibrium profile exists for any game with some specific features. One can use for example a fixed point theorem, such as Kakutani's or Brouwer's (see e.g. Fudenberg and Tirole [1991]).

Shubik [1955] was the first to state that in the case of Bertrand-Edgeworth duopolies, the existence of a pure-strategy Nash equilibrium cannot be guaranteed.

In the related literature we find examples of two directions concerning the way of handling the lack of pure-strategy Nash equilibria. The first approach avoids the concept of Nash equilibrium and uses other concepts, such as ϵ-equilibrium or iterated elimination of strictly dominated strategies. For contributions we refer the reader to Dixon [1987] and Börgers [1992]. The second approach is the characterization of pure and/or mixed Nash equilibria, including the reconstruction of models so that the new model has a pure-strategy Nash equilibrium. Examples of this ap-

[2]We have to note that it cannot be guaranteed in practice that people really choose their equlibrial strategy.

proach are among others Deneckere and Kovenock [1992], Deneckere and Kovenock [1996], Reynolds and Wilson [2000], Chowdhury [2005], and Lepore [2008].

When observing this type of duopolies, the first research question refers to the existence of pure Nash equilibrium.

Research question 1. *Under what conditions does a pure strategy Nash equilibrium in a Bertrand-Edgeworth duopoly with public ownership exist?*

When solving the existence problem, the characterization of the pure strategy Nash equilibria also has to be dealt with. The goal of this analysis is stated in the next research question.

Research question 2. *Given entire or partial public ownership in one of the competitors in a Bertrand-Edgeworth duopoly and provided that a pure Nash equilibrium exists, what are the equilibrium prices and quantities of both firms for the simultaneous and the sequential versions of the game?*

As far as the timing of decisions is concerned, there exist simultaneous and sequential games. In the simultaneous case, all the players make their strategic decisions without knowing any other player's choice, while in the sequential case, players choose strategies in sequence, therefore, the latter players can observe the others' choice before making the decision.

Endogenous timing means that the ordering of firm decisions is not exogenously given. In this framework the two firms' equilibrium payoffs for different orderings will lead to an equilibrial timing of decisions. Of course, a lack of pure strategy equilibrium can also occur in the timing game.

The timing of decisions is often endogenized in the oligopoly literature. For seminal contributions in this field, we refer the reader to Hamilton and Slutsky [1990], Deneckere and Kovenock [1992] and Matsumura [1995] for standard oligopolies, and Pal [1998], Matsumura [2003], Jacques [2004], Anam, Basher, and Chiang [2007], Lu

[2007], Lu and Poddar [2009] and Tomaru and Kiyono [2010] for mixed oligopolies, among others.

The third research question involves the problem of endogenous timing.

Research question 3. *Which ordering of decisions emerges if a private and a purely or partially public firm compete on the market in a Bertrand-Edgeworth duopoly provided that timing is endogenous?*

The main objective of the state is assumed to maximize social welfare. In mixed oligopoly frameworks, the state is not assumed to be a regulator, but it is a market participant that tries to maximize welfare by its price and production decisions, see e.g. Merrill and Schneider [1966], Harris and Wiens [1980], or Brandao and Castro [2007]. When the state enters a duopoly market by acquiring some or all shares of one of the competing firms, the level of social welfare generated on the market may not remain the same, as there is a modification in one firm's objective function. Therefore, it is necessary to state a question concerning the change in social welfare the presence of a public firm may cause.

Research question 4. *What is the direction and magnitude of social welfare change the appearance of a purely or partially public firm generates in a Bertrand-Edgeworth duopoly framework?*

The four main research questions are linked to each other. The links are as follows. Firstly, we will give constructive proofs to the existence of pure strategy Nash equilibria, wherever it is possible. This means that when proving the existence, we present the equilibrium profile. Secondly, when analyzing the timing game, we strongly rely on the results of the exogenous orderings of moves. Thirdly, the social welfare effect of public ownership can be examined by comparing the public firm's equilibrium payoff (objective function value) in the mixed duopoly case to the calculated social welfare in the standard case.

4 Structure of the thesis

The dissertation consists of seven sections. We aim at providing a clear structure where background, related work, detailed analysis of the topic and own work is presented. The thesis is organized as follows.

Section 2 recalls relevant results from the field of duopoly theory. We present classical models from both price-setting and quantity-setting types. Furthermore, Bertrand-Edgeworth duopolies are introduced. We present some different approaches of the model. The key assumptions are introduced, the existence of a game theoretic equilibrium is investigated, and some interesting contributions are recalled.

Section 3 offers a survey on mixed oligopolies, i.e. models with a state-owned competitor. The difference in the assumptions of the standard and mixed versions of the model is highlighted. We also specify the models used in the main analysis. The survey follows a unified way of presenting the results: the main assumptions from some important contributions are collected, the outcomes of the models are presented and the differences from other models are highlighted.

Section 4 provides a detailed analysis of the mixed Bertrand-Edgeworth duopoly in the so-called production-to-order (PTO) framework. In this section the public firm is assumed to be fully owned by the state. The PTO setting assumes that production takes place after the amount of sales are fixed in a contract. This setting models among others the airplane industry. We give the pure-strategy Nash equilibrium for all the three possible orderings of moves, solve the timing game and discuss the public firm's social welfare effect.

Section 5 deals with the mixed Bertrand-Edgeworth duopoly in the production-in-advance (PIA) case. In the PIA case we assume that production takes place already before sales are realized. Markets of perishable goods can be modelled with this setting. Just like the previous section, this section also contains the results on the existence and characterization of pure-strategy Nash equilibria for all possible orderings of moves. Furthermore, the timing game and the social welfare issue is also discussed.

In Section 6, the assumption of pure public ownership is relaxed: we will assume that one of the firms is only partially owned by the state. The objective function of the firm with both public and private ownership changes compared to the case of pure public ownership. Its objective function becomes a weighted sum of total social welfare and its own profit, where the weights are the proportions of public and private ownership. This modification leads to new model outcomes. Within Section 6, we refer to the results on the production-to-order case and present the analysis of the production-in-advance case.

Finally, Section 7 concludes and restates the contribution of the dissertation. The main results are also collected in Section 7 of the dissertation.

5 Results and conclusion

The dissertation aimed at analyzing firm behavior in the mixed and semi-mixed Bertrand-Edgeworth duopolies by means of game theory. The main concept we used throughout the thesis was the most frequently used equilibrium concept, the Nash equilibrium.

Assuming partial public ownership on the market, we provided a thorough analysis concerning equilibrium prices and quantities, endogenous timing and social welfare effects. The contribution of the thesis to knowledge is broadening the theory of Bertrand-Edgeworth models for a better understanding of duopolistic markets with public ownership.

A mixed duopoly model considers a market situation, where one of the two competing firms is under pure or partial state ownership. Thus, the state does not act as an outside regulator on such markets, but as a market participant, it aims at driving the market to a socially better equilibrium. Therefore, when modelling mixed duopolies by means of game theory, social welfare appears in the public firm's payoff function.

There are several duopoly and oligopoly models with different strengths and focuses. The first duopol models - the Cournot- and Bertrand-duopolies - were born in

the 19th century, and since then, many directions in improving them have been investigated in the literature concerning both price-setting and quantity-setting frameworks. A model that tackles several critics that have been addressed to Cournot- and Bertrand-type models is Bertrand-Edgeworth duopoly. The main advantage of this model family is that it can handle the problem of unlimited capacities. On the other hand, if two private firms compete on the market, we have to face a lack of pure-strategy Nash equilibria for certain parameter settings, which makes practical applicability rather difficult.

The theory of duopolies with public ownership was analyzed first by Merrill and Schneider [1966]. Endogenizing the timing of decisions began later. Endogenous timing in mixed Bertrand-Edgeworth duopolies with capacity constraints was investigated in the present dissertation.

We analyzed all together four variants of the mixed Bertrand-Edgeworth model. The common features of the presented models are as follows:

- There are two firms competing on the market of a homogenous good.
- The decision variables of the firms are both price and quantity.
- The consumer side is given by a market demand function, which is monotone, strictly deceasing and twice continuously differentiable.
- The two firms cannot produce a higher amount than their respective capacity constraints.
- Both firms have constant and identical unit costs.
- One of the competing firms has pure private ownership, while there is a certain share of the state in the other one.
- All the parameters are common knowledge.

The four models can be differentiated either according to the share of the state in the public firm, or according to the timing of demand satisfaction. We consider

on the one hand, mixed and semi-mixed models, while, on the other hand, we assume both production-to-order (PTO) and production-in-advance (PIA) frameworks. PTO means that production takes place only after sales are realized, while in the PIA framework items are produced before they are sold. In the latter case there might emerge supplies that cannot be sold (think of the markets of perishable goods), while the PTO framework lets the game reduce to a price-setting game, as quantities are obtained by substituting the firms' price levels into the demand curve.

As far as the timing of decisions is concerned, the two decisions can be made simultaneously, or sequentially, where the latter variant consists of public leadership and private leadership. We considered all three possible orderings of moves.

The following Table 1 can be helpful in positioning the four models.

Table 1: Model variants

Models	**Pure public ownership**	**Limited public ownership**
PTO	mixed PTO	semi-mixed PTO
PIA	mixed PIA	semi-mixed PIA

Now we provide an overview of the contents of each section, afterwards we summarize the results based on the research questions we stated.

The introductory section stated the overall aim of the thesis as well as its contribution to knowledge. The game-theoretic methodology is clarified and we provided motivation by recalling real-life oligopolistic markets where public ownership is present. The key research questions were also presented in Section 1.

Section 2 provided an introduction to the most simple duopolistic models, the Cournot- and the Bertrand-duopolies. With the aim of reducing the shortcomings of these classical models, we introduced the Bertrand-Edgeworth competition. By recalling contributions from the relating literature, we discussed the question of rationing rules and the existence of pure-strategy Nash equilibria.

Section 3 offered a survey on mixed oligopolies. We introduced the production-

to-order and production-in-advance frameworks and put down the main assumptions of the four models we discussed.

The formal discussion began in Section 4, where we carried out the analysis of the mixed PTO model. We gave the formal assumptions of the model and characterized pure-strategy Nash equilibria for the so-called strong-private-firm and weak-private-firm cases, making a distinction in the private firm's capability of influencing equilibrial outcome. We also provided a numerical example to illustrate the results. The implicit solution of the timing game and the public firm's social welfare effect are also discussed. The results of Section 4 are published in Balogh and Tasnádi [2012].

Section 5 considered the mixed PIA framework. In this more complicated case we analyzed separately the strong-private-firm case, the weak-private-firm case and the high-unit-cost case for all three possible orderings of moves. Numerical examples are attached to each case. We also presented the solution of the timing game and analyzed the public firm's social welfare effect. The results of Section 5 can be found in Balogh and Tasnádi [2014].

Section 6 observed the two semi-mixed frameworks. The main distinction of this section compared to the previous ones is that we have not allowed for purely public ownership in the public firm. On the contrary, we considered a purely private firm and a so-called mixed firm with an exogenously given (less-than-one) ratio of public ownership. We recalled the results on equilibrial firm behavior for the PTO case (Tasnádi [2013]) and presented similar results for the PIA framework. The analyses in Section 6 have proved to be less complicated than those of Sections 4 and 5, however, the results were rather negative concerning the existence of pure-strategy Nash equilibrium profiles. Some results of Section 6 can also be found in Balogh [2014].

We turn to giving our answers to the research questions and stating the main results of the dissertation.

The first question referred to the existence of pure-strategy Nash equilibria: Under what conditions does a pure-strategy Nash equilbrium in a Bertrand-Edgeworth

duopoly with public ownership exist?

Answering this question is quite straightforward. In Sections 4 and 5 we showed that at least one pure-strategy Nash equilibrium exists even in the regions, where the standard version of the game does not have any. In other words: we proved that replacing one of the private firms to a purely public firm results in the appearance of at least one pure Nash equilibrium. For the semi-mixed variants the result is not so positive. We obtained that if the residual payoff-maximizing price levels (which can be directly calculated from the firms' capacity constraints and the market demand function) of any of the firms exceeds the unit cost level, then the game does not have any pure Nash equilibrium. This result matches that of the standard Bertrand-Edgeworth duopoly (with purely private firms), therefore the appearance of partial public ownership gives no remedy for the lack of pure equilibria.

Based on these arguments, we can state the first main result.

Main result 1. *Under quite general conditions for the demand function, there exists at least one pure-strategy Nash equilibrium in the mixed Bertrand-Edgeworth duopoly, given any parameter setting. The semi-mixed Bertrand-Edgeworth duopoly has a pure-strategy Nash equilibrium if and only if an extra condition is satisfied for the firms' capacities.*

The solutions of the existence problem were constructive throughout the dissertation, which means that we also characterized the pure-strategy Nash equilibrium points whenever they existed, and thus, answered the following second research question: Given entire or partial public ownership in one of the competitors in a Bertrand-Edgeworth duopoly and provided that a pure Nash equilibrium exists, what are the equilibrium prices and quantities of both firms for the simultaneous and the sequential versions of the game?

We have had several types of pure Nash equilibria in the previous sections. It is desirable to provide a brief and transparent summary of the pure equilibria of the observed models. As there are several model versions and there exist three timing variants to each setting, we present here only the simultaneous-moves-case

equilibria. The sequential-moves equilibria can be derived from the simultaneous equilibria in most of the cases. Whenever we obtained multiple equilibria, these were not interchangeable. All the equilibria are given in the propositions of Sections 4, 5 and 6. In the following list, when necessary, we recall the notations having been used throughout the dissertation.

1. **Mixed PTO.** In the *strong-private-firm case*, the price levels $p_1^* \leq p_2^* = p_2^d$ and $p_1^* \leq p_2^d$, $p_2^* = p_2^m$ lead to pure equilibria. This means that the private firm becomes either a monopolist on the residual demand curve, or stays on the original demand curve and sells its entire capacity. Additionally, under certain conditions (see page **??**) , the private firm can become a monopolist in equilibrium up to its capacity limit.
   In the *weak-private-firm case*, firms set the market clearing price in equilibrium.

2. **Mixed PIA.** In the *strong-private-firm case*, the price and quantity levels $p_1^* \leq p_2^d(q_1^*)$, $q_1^* \in [0, k_1]$, $p_2^* = p_2^m(q_1^*)$, $q_2^* = q_2^m(q_1^*)$ lead to a pure equilibrium under certain conditions, i.e. the private firm is a monopolist, but only on the residual demand curve. Additionally, also under certain extra conditions, the private firm can become a monopolist in equilibrium up to its capacity limit.
   In the *weak-private-firm case*, the private firm will choose in equilibrium the highest price level at which it can sell its entire capacity provided that the public firm has no incentive to undercut the private firm's price. A particular case of this equilibrium is clearing the market. Besides, under certain conditions, the private firm might become a monopolist up to its capacity limit.
   In the *high-unit-cost-case*, all kinds of equilibria mentioned in the previous two cases are possible under certain conditions.

3. **Semi-mixed PTO.** A pure (SP)NE exists if and only if $\max\{p_1^s; p_2^m\} \leq p^c$. Provided that the condition is satisfied, the only pure equilibrium is clearing the market, i.e. $p_1^* = p_2^* = p^c$.

4. **Semi-mixed PIA.** A pure (SP)NE exists if and only if $\max\{p_1^s(k_2); p_2^m(k_1)\} \leq p^c$. Provided that the condition is satisfied, the only pure equilibrium is clearing the market, i.e. $p_1^* = p_2^* = p^c; q_1^* = k_1; q_2^* = k_2$.

The results on the characterization of pure-strategy Nash equilibria are stated in the second main conclusion.

Main result 2. *The characterization of pure-strategy Nash equilibria depend strongly on the model assumptions. The five main types of Nash equilibria in the mixed PTO, mixed PIA, semi-mixed PTO and semi-mixed PIA Bertrand-Edgeworth duopolies are as follows: (1) the firms clear the market; (2) the private firm is a monopolist on the residual demand curve; (3) the private firm sells its entire capacity and earns as much as if it were a monopolist on the residual demand curve; (4) the private firm is a monopolist on the market demand curve up to its capacity limit; (5) the private firm sells its entire capacity at the highest price level, where it is still not worth for the public firm to undercut the private firm's price.*

Timing of decisions is often endogenized in the recent literature. This means that the firms play in fact a two-stage game. In the first stage they decide when to announce their price and production decisions (before the other firm, after the other firm, or at the same time). In the second stage, firms announce their price and production levels. The following, third research question focused on the problem of endogenous timing. Which ordering of decisions emerges if a private and a purely or partially public firm compete on the market in a Bertrand-Edgeworth duopoly provided that timing is endogenous?

We obtained different results in the observed models concerning endogenous timing. In the mixed PTO case all three orderings of moves were timing game equilibria. Considering the mixed PIA case, both firms were better off if they become the leader, therefore, the timing game equilibrium lies at simultaneous moves. Finally, in the semi-mixed cases, we could determine the equilibrium of the timing game only for parameter settings, where the price- and quantity-setting game had a pure equilib-

rium. For this case all three possible orderings were equilibria of the timing game, as the ordering of price and quantity decisions did not matter concerning payoffs.

The results on endogenous timing are presented in the third main conclusion.

Main result 3. *Timing of decisions does not matter, i.e. the timing game has multiple equilibria in the production-to-order mixed Bertrand-Edgeworth duopoly. The same is true for both the production-to-order and the production-in-advance cases of the semi-mixed Bertrand-Edgeworth duopoly for the parameter settings, where a pure-strategy Nash equilibrium exists in the price- and production-setting game. Finally, the timing game equilibrium of the production-in-advance mixed Bertrand-Edgeworth duopoly lies at simultaneous moves.*

When the state enters a duopoly market by acquiring partial or entire ownership in one of the competing firms, the level of social welfare generated on the market may not remain the same, as there is a modification in one firm's objective function. Our last research question referred to the change in social welfare the presence of a public firm may cause: What is the direction and magnitude of social welfare change the appearance of a purely or partially public firm generates in a Bertrand-Edgeworth duopoly framework?

Based on the most plausible pure-strategy Nash equilibria of the observed model, we could derive the public firm's social welfare effect for the parameter settings, where we had at least one pure Nash equilibrium. We pointed out that the mixed PTO case resulted in higher social welfare than the pure PTO case, i.e. the appearance of the public firm made the outcome more competitive, providing a surprising result. For the mixed PIA case we concluded that the result is less competitive than that of the mixed PTO case, i.e. the social welfare becomes lower in equilibrium. As far as the semi-mixed models are concerned, we could conclude for the favorable parameter-settings, where a pure strategy exists, that the social welfare remains the same as that of the standard (purely private) case. Consequently, socializing a certain proportion of a firm does not result in welfare growth, unless the firm gets under pure public ownership.

The following Table 2 provides a summary of the social welfare effects experienced in the different models wherever a direct comparison could be made.

Table 2: Social welfare effects of public ownership

	Public (or mixed) firm's social welfare effect
Mixed PTO	Positive (compared to standard PTO)
Mixed PIA	Negative (compared to mixed PTO)
Semi-mixed PTO	No effect (compared to standard PTO)
Semi-mixed PIA	No effect (compared to standard PIA)

The results concerning social welfare effects are stated in our last main conclusion.

Main result 4. *The production-to-order mixed Bertrand-Edgeworth duopoly environment leads to a higher social welfare in equilibrium than the standard version of the game. In the production-in-advance mixed Bertrand-Edgeworth duopoly the social welfare becomes lower than that of the production-to-order mixed duopoly. Finally, the appearance of a partially public firm generates no change in social welfare in equilibrium provided that there is a pure-strategy Nash equilibrium of the price- and quantity-setting game.*

To sum up the results, the dissertation contributed to the understanding of duopolies, where public ownership is present on the supplier side of the market.

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6 The author's scientific activity

Journal articles

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Tamas L. Balogh, Janos Kormos (2014): A computational model of outguessing in two-player non-cooperative games. Acta Univ. Sap. Informatica 6: 71-88.

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Working papers

Tamas L. Balogh, Christian Ewerhart (2013): On the origin of r-concavity. University of Zurich

Tamas L. Balogh, Attila Tasnadi (2014): Mixed duopolies with advance production. HAS "Lendület" Strategic Interactions Research Group

Conference talks

Balogh Tamás László, Bertók Kornél (2011): A gazdaságinformatikus képzés hallgatói megítélése a Debreceni Egyetemen. Conference title: Informatika a felsöoktatásban (Debrecen). Date: 24-27th August 2011

Tamas L. Balogh, Attila Tasnadi (2012): Effects of Advance Production on a Price Setting Mixed Duopoly. Conference title: SING 8 (Budapest). Date: 16-18th July 2012

Tamas L. Balogh, Janos Kormos (2014): A computational model of outguessing. Conference title: ICAI (Eger). Date: 29th January - 1st February 2014

Balogh Tamás László, Bánszki I., Beringer D, Varga L. (2014): A mozgás és gondolkodás együttes élménye: A Medve Szabadtéri Matekverseny. Conference title: Matematikatanárok Rátz László Vándorgyülése (Keszthely). Date: 3-6th July 2014

Other talks

University of Debrecen, Faculty of Economics and Business Administration, Research Forum: six talks between January 2011 and June 2014

University of Zurich: talk at the Microeconomics PhD Seminar, September 2012