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The Mathematical Model of the Endocytosis

Endocytosis is one of the most peculiar types of the biological transport processes, which is understood as taking of colloid size particles by the cell. Two parts of endocytoses are known: phagocytosis and pinocytosis. The first one is used to describe the transport of the solid state substances, the other one, however, is for the characterization of the liquid droplets. Regarding the fact that in biological systems distinction was made between the two states of the matter, so recently rather endocytosis denomination is used.

Since 1931 it is known from Lewis' researches that certain cells are able to swallow liquid droplets similarly to phagocytosis and this phenomenon is named pinocytosis. On the surface of the cell membrane small bulges can be observed, the central part of them has small depression and the locally swallowed liquid droplets get into the cytoplasm. From the bottom of the craters the swallowed liquid droplets are detached in the form of small spheres, called vesicle.

The most diverse cells show pinocytosis. E.g. macrophages, the endothelial cells, cells of the wall of the capillaries, the Schwann-cells in the envelop of the neurons,- etc. According to certain suppositions pinocytosis is the one of the most frequent active mechanisms in the mechanism of transportation of the macromolecules. The phenomenon itself takes place in many steps: first the macromolecule is adsorbed onto the surface of the cell membrane, then the cell membrane embraces the macromolecule so creating the pinocytosis channels, then follows swallowing and they will be driven inside into the interior of the cell. The most modern electron microscopic pictures support this description of the mechanism of pinocytosis.

The model of pinocytosis *(Vincze 1972)* takes into consideration its statistical character, i.e. the probability of the formation of vesicle. Pinocytosis is inducible if to the culture of amoebae any protein is supplemented in colloid dissolved form. Glucose normally does not enter into the amoebae cell by normal conditions but if pinocytosis is induced by protein solution glucose also will enter the cytoplasm together with the protein. Let there be maximal number of vesicle M on the dS surface area of the cell membrane and c_K the concentration of the substance to be transported outside in the medium and c_b that inside the cell, than time dependent charge of the probability of coming into existence of the n-th vesicle may be expressed with the following equation:

$$K\frac{dP_n(t)}{d} = c_k\left[P_{n-1}(t) - P_n(t)\right] + c_b\left[P_{n+1}(t) - P_n(t)\right] \quad ; \quad 1 \le n \le M$$

The n-th vesicle may come about in two ways: either an (n-l)-th vesicle is already existing and as a consequence of the formation of the n-th vesicle or n+1 vesicle exist in the cell membrane from where one vesicle detaches itself toward the cytoplasm and so remains finally only n vesicle. The two phenomena are independent processes from each other from the point of view of probability, therefore, we receive the sum of them in the formula. At the same time probability of vesicle formation at either the inner or the outer surface of the cell membrane is a function of the concentration of the given substance; this is the reason why we must take into consideration the concentrations as multiplication factors. K is only a dimensional constant. This model is a first order differential equation system for which n=l case results in the following if the limit conditions are known.

$$K\frac{dP_1\ t}{dt} = c_k\left[P_0(t) - P_1(t)\right] + c_b\left[P_2(t) - P_1(t)\right]$$

We may speak about pinocytosis only in that case if at least one vesicle is given, therefore, $C_k.P_0(t)= 0$ and the same applies to $C_b.P_1(t)$ as well. If therefore, n=1, then these two members drop out. The equation, satisfying n = M limit condition is as follows:

$$K\frac{dP_M(t)}{dt} = c_k\left[P_{M-1}(t) - P_M(t)\right] + c_b\left[P_{M+1}(t) - P_M(t)\right]$$

from where, taking into account the maximum member of the vesicle, two members become equal to zero. Until the dynamics of pinocytosis is extremely difficult to follow experimentally, this model makes possible dynamic description of the phenomenon and to draw certain conclusions.

Formation of pinocytotic vesicle exhibits statistical distribution. The above equation makes possible quantitative characterization of pinocytosis, it does not depend on the velocity of the phenomenon i.e. from the velocity of formation of the vesicle, because this is included in the explanation of the equation. Exocytosis is transport of the macromolecules from the cells into the extracellular room. Such phenomena take place in every cell performing excretion or secretion. All these conclusions are in strict harmony with Palade's (1961) results who established through his electronmicroscopic investigations that vesicle cannot be considered as cross sections of pores. This also follows mathematically because the life span of the vesicle is only a few second *(Vincze 1985)*.

The statistical model described for pinocytosis is equally applicable to phagocytosis as well, without any constrains. About endocytosis, which is a specifically active mechanism, it is supposed that it is an energy consuming process and ATP being the source of this energy requirement in this case too. The energy liberated from the hydrolysis of the macroerg phosphate bond, in this case is probably necessary for the regeneration of the membrane structure, or to the formation of new membrane *(Vincze-Vincze 1987)*.

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