The Procedural Nature of Gothic Architecture and Microarchitecture[*]

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This study delves into the procedural nature of Gothic architecture and microarchitecture, focusing on the dynamic processes that define architectural forms rather than treating them as static objects. The analysis emphasizes the role of algorithmic approaches in procedural design, where the creation process, rather than the resultant geometry, becomes the primary subject of study. Key sections of the work include original contributions by the author, such as an analysis of historical design methodologies and their application to contemporary computational practices. Through case studies and interpretive modelling, the author presents a unique perspective on how historical processes can inform modern architectural paradigms. The discussed method – Simulated Morphogenesis – connects historical insights with contemporary procedural techniques.

Keywords: Gothic; Architecture; Košice; Micro-architecture; Eastern Slovakia; Geometry; Grasshopper; Algorithmic modelling.

1 Introduction

The essence of the procedural approach is to examine objects, buildings and cities not as standalone entities, but as the result of a process. The subject of an analysis and interpretive modelling based on such analysis is thus not the objects, buildings and cities themselves, but the processes that create them. Accordingly, in the case of the procedural design method, the object of design is not geometry itself, but the process that creates it, which is essentially an algorithm. The question of procedural design is placed in a historical context in Mario Carpo's 2011 book *The Alphabet and the Algorithm*.[1] Following Bernard Cache and Gilles Deleuze, Carpo uses the expression "objectile": "the objectile is not an object but an algorithm – a parametric function which may determine an infinite variety of objects, all different (one for each set of parameters) yet all similar (as the underlying function is the same for all)".[2]

In his book, as well as in a related earlier article,[3] Carpo makes and develops the seemingly surprising but well-founded claim that for centuries before the standardization of mechanically reproduced images, we lived in a world that was algorithmic and normative, not visual and repetitive. Before the advent of printing, drawings were not precisely reproducible, so the description of the process played a greater role than precisely reproducible drawings. The author argues that the

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[1] CARPO, *The Alphabet and the Algorithm*.

[2] Ibidem, 40.

[3] CARPO, *Pattern Recognition*.

current architectural practice – which we now take for granted – only dates back to the Renaissance, and its theoretical basis is largely found in the writings of Leon Battista Alberti.[4] Two important changes occurred in Alberti's time: architectural design became allographic (the plans being made by someone other than the person who builds them),[5] and lost its procedural character.

In contrast, the characteristics of procedural design are clearly recognizable in the architecture of the period immediately preceding Alberti: Gothic architecture. According to Robert Bork, Gothic architecture was governed by procedural conventions rather than fixed proportional canons. The Gothic tradition thus treated the finished building as a physical imprint of a dynamic design process, whose internal logic was more important than the shape of the final product.[6] Referring to Herbert A. Simon, he argues that for complex systems, process description is more important than state description. "Process descriptions allow for open-ended growth and elaboration, while state descriptions are inherently closed and finite."[7] In the case of Gothic, moreover, the procedural rules could be interpreted on multiple levels, from small reliquaries to giant Gothic towers.

A procedural algorithm consists of the definition of input data, constants (*c*), parameters (*p*), and variables (*v*); as well as the relationships between them. Quoting Douglas Hofstadter, "any of them can vary (including the 'constant'); however, there is a kind of hierarchy of variability. […] *c* establishes a global condition; *p* establishes some less global condition which can vary while *c* is held fixed; and finally, *v* can run around while *c* and *p* are held fixed."[8] By modifying the parameters and variables, a single algorithm can generate a large number of outputs, which can be considered non-standard series.[9]

2 Some Basic Examples

2.1 The "Fialenbüchlein" of Matthäus Roritzer

One of the most important of the few printed architectural booklets of the Gothic period is Matthäus Roritzer's (Mathes Roriczer) *Büchlein von der Fialen Gerechtigkeit*, published in 1486.[10] The booklet describes the steps involved in the construction of a single architectural element, the pinnacle: essentially an algorithm that takes one from a simple square to a pinnacle. His writing is firsthand evidence of the procedural nature of Gothic design, even if only for a single building element (Figure 1).

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[4] CARPO, *The Alphabet and the Algorithm*, 51–79.

[5] Ibidem, 16.

[6] BORK, *The Geometry of Creation*, 2.

[7] Ibidem, 419.

[8] HOFSTADTER, *Gödel, Escher, Bach: An Eternal Golden Braid*, 643–644.

[9] CARPO, *Pattern Recognition*.

[10] SHELBY, *Gothic Design Techniques*; SÓDOR, *Matthes Roriczer 1486-ban megjelent fiatorony könyve.*

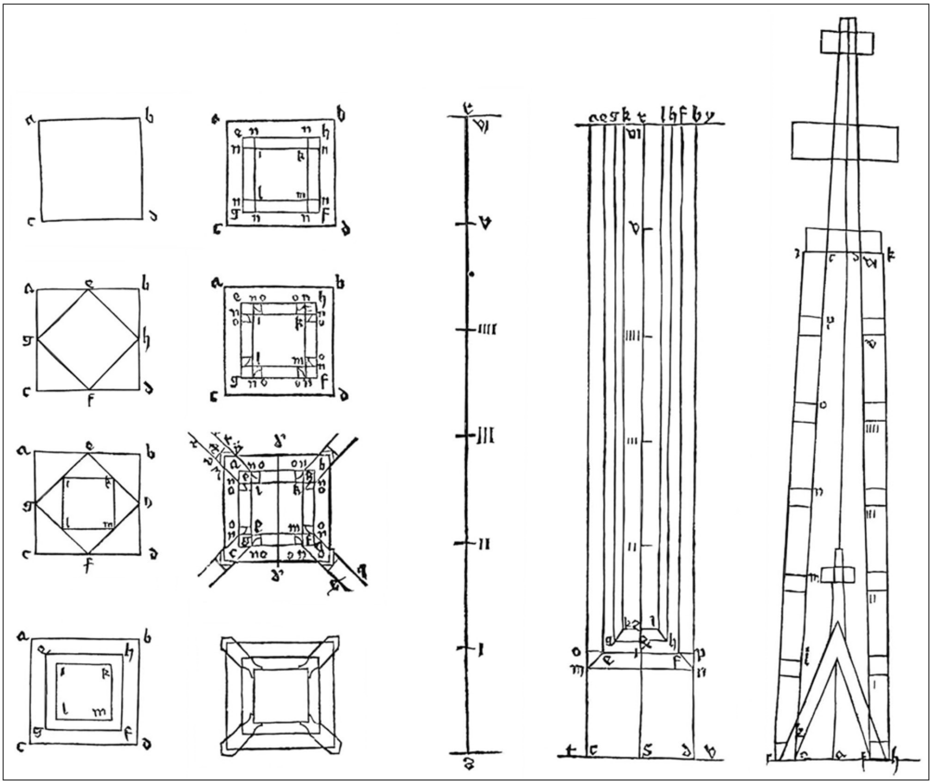

Figure 1: Steps of the construction of a pinnacle from Roritzer's book, edited together by Robert Bork.[11]

The input here is a square, defined by its four corner points. All dimensions of the pinnacle, both horizontal and vertical, are developed from this square, step by step. Some values can be considered parameters, such as the height of the body of the pinnacle being an integer multiple of the side length of the square. (The value of this parameter is six for Roritzer, but of course there are both slimmer and less slender pinnacles in Gothic architecture.)

The description is so precise and detailed (actually an analogue algorithm) that it is quite difficult to read as text, but by following it, a pinnacle can actually be constructed in 3D (Figure 2).

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[11] BORK, *Dynamic Unfolding and the Conventions of Procedure*, 2.

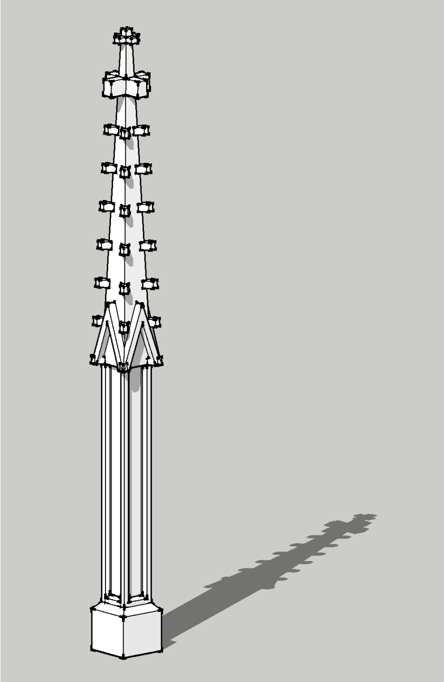

Figure 2: 3D model of a pinnacle based on Roritzer's description. Source: Author's work; Software used: Sketchup.

It is no coincidence that Roritzer chose to publish the construction of the pinnacle. This is an architectural element that was not used either before or after the Gothic period, but is found on Gothic buildings in all sizes, from small baldachins to the giant pinnacles of large towers, sometimes 25–30 metres high (Figures 3–4). Interestingly, the construction of pinnacles is based on similar rules in all sizes, and larger pinnacles are often surrounded by smaller ones, which in turn may be surrounded by even smaller ones. Pinnacles also appear not only in architecture, but also on sacrament houses (tabernacles), architectural altarpieces and metalwork; the construction is similar regardless of the material (stone, wood, metal) (Figures 5–7).

Figure 3: Pinnacles around a baldachin. St John's Chapel, Franciscan monastery, Bratislava, Slovakia. Source: author's photo.

Figure 4: Giant pinnacles around the octagon storey of the south tower of the Stephanskirche, Vienna, Austria. Source: Author's photo.

Figure 5: Stone pinnacles around a sacrament house. St Michael's Chapel, Košice, Slovakia. Source: Author's photo.

Figure 6: Wooden pinnacles on an altarpiece superstructure. St Martin’s Church, Spišská Kapitula, Slovakia. Source: Author’s photo.

Figure 7: Metal pinnacles on a shrine. National Gallery, Budapest, Hungary. Source: Author's photo.

This article will later demonstrate how a single pinnacle can be used to recursively grow a giant, complex Gothic tower using a recursive definition.

2.2 Procedural Modelling of Gothic Cross Vaults

Ribbed pointed cross vaults are also a very characteristic Gothic architectural element. As in the case of pinnacles, there are several different variants of these vaults found in Gothic buildings. Werner Müller presents a relatively simple construction method in his book based on Carl Alexander von Heideloff.[12] The solution shown in Figure 8 follows this description. The figure is a screenshot: the software used for modelling is Rhinoceros 3D developed by McNeel, and its parametric surface, Grasshopper. All algorithmic examples in the article are made with this software. The input for the cross vault generating algorithm is constituted of three parameters: the length of the vaulting bay (type: positive number), the width of the vaulting bay (type: positive number) and the profile of the vault rib (type: closed planar curve).

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[12] MÜLLER, *Grundlagen gotischer Bautechnik*, 149.

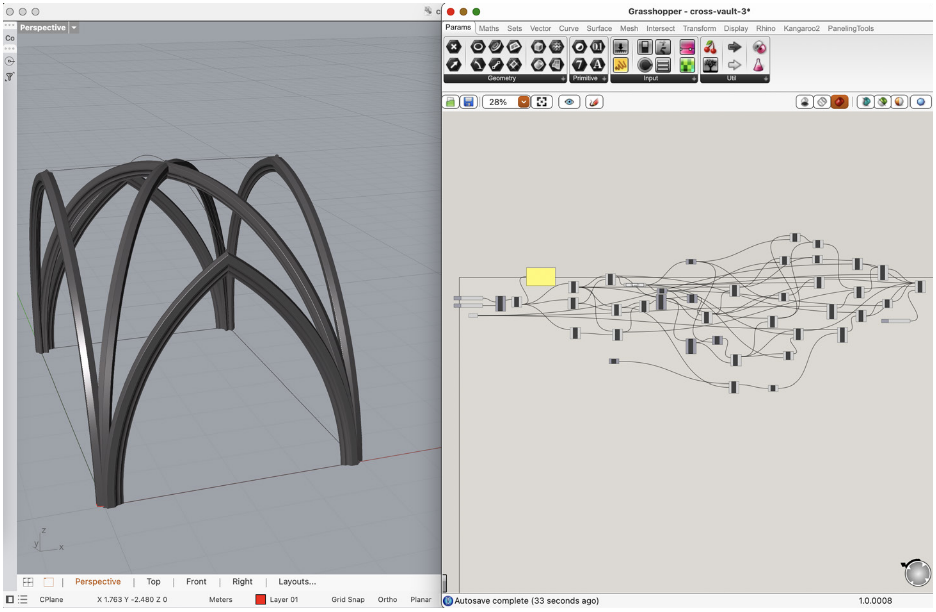

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Figure 8: Algorithmically generated Gothic cross vault. The dimensions of the vaulting bay and the rib profile are input parameters. The generated geometry is shown on the left side of the figure, and the data flow of the algorithm that generates it is shown on the right side. Source: Author's work.

Following Heideloff's description, the guiding curves of the diagonal ribs are regular semicircles, in a vertical plane, with vertical tangency at their springings. Their diameter is equal to the length of the diagonal of the rectangle that forms the vaulting bay. The guiding curves of the transverse ribs and the wall ribs use the same radius, and the curves start from the springings with vertical tangency on both sides, and then intersect in the middle. Since the sides of a rectangle are shorter than its diagonals, these arches will necessarily be pointed arches. The vaulting of the nave is created by a simple linear array of these elements (the number of vault fields being another adjustable parameter), while the vaulting of the aisles is created by adjusting the input parameters (vaulting bay width, length) and then again by array. In Gothic architecture, the width of the aisle is generally half the width of the nave,[13] so the parameters defining the size of the vaulting bay in the aisles can be automatically obtained from the parameters of the nave vault: the length of the aisle vault bay will be equal to the length of the nave vault bay, and the width will be half the width of the nave bay. In the example shown in Figure 9, the size of the vaulting bay in the nave is 4 × 4 m, and the length of the nave is 4 vaulting bays; in the example shown in Figure 10, the size of the vaulting bay in the nave is 10 × 5 m, and the length of the nave is 4 vaulting bays. In these examples, for the sake of comparability, the curve of the rib

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[13] This rule is also mentioned in medieval sources, e.g. COENEN, *Die Spätgotischen Werkmeisterbücher in Deutschland*, 146.

profile is constant, but it is technically straightforward to make the width of the rib profile a function of the input parameter of the width of the nave, as can be read from contemporary sources.[14] The vault shoulders are at the same height in both examples, so the figures show the system of a hall church.

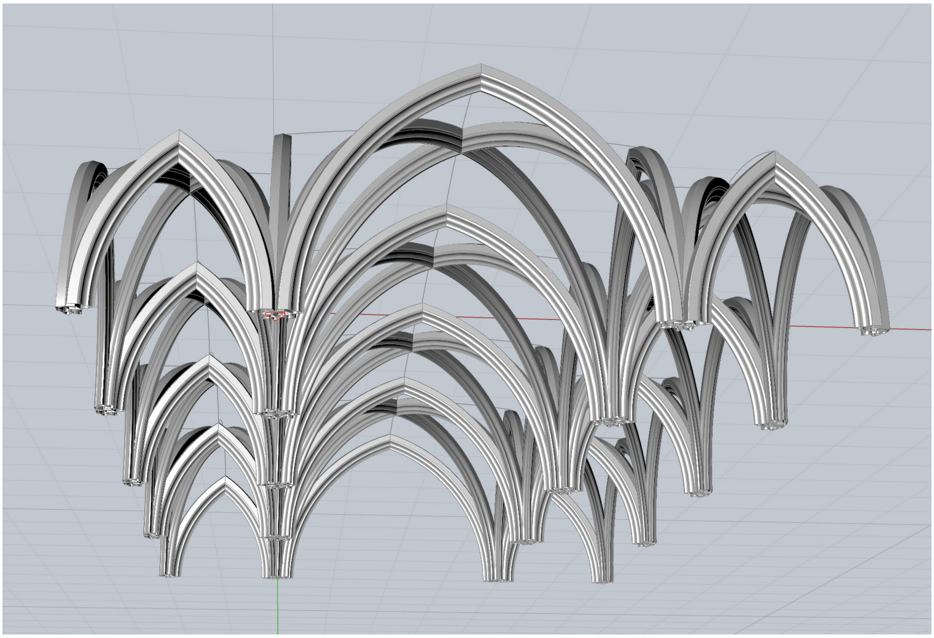

Figure 9: Algorithmically generated Gothic nave and side aisles. The dimensions of the nave vaulting bay (4 × 4 m), the rib profile and the number of vaulting bays are input parameters. Source: Author’s work.

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[14] E.g. Hans Hammer: Zeichnungen, fol. 17r. Wolfenbüttel, Herzog August Bibliothek, Cod. Guelf. 114.1 extrav.

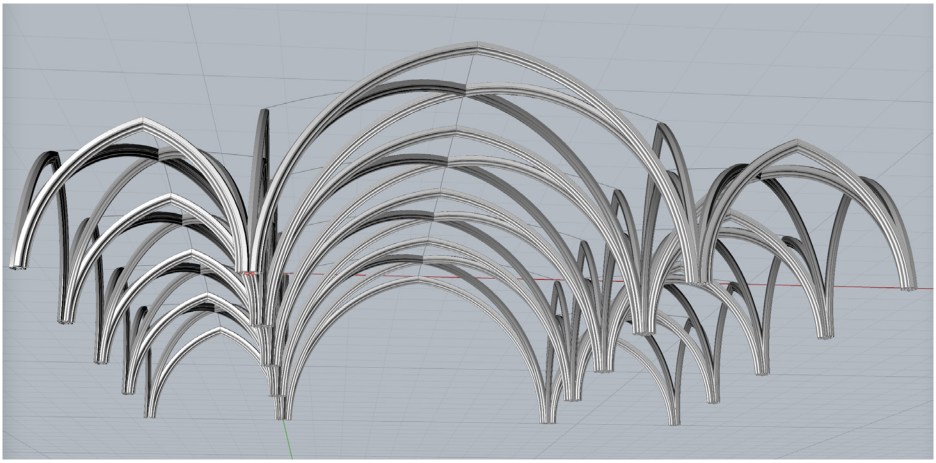

Figure 10: Algorithmically generated Gothic nave and side aisles. The dimensions of the nave vaulting bay (10 × 5 m), the rib profile and the number of vaulting bays are input parameters. Source: Author's work.

2.3 The Example of the Avas Church, Miskolc: Lorenz Lechler's Rules with Different Parameters

A comparison of the ground plan of the late Gothic Avas Church in Miskolc, Hungary with a contemporary source – Lorenz Lechler's *Unterweisungen*[15] – well illustrates how the parameters in a Gothic construction description, or recipe, can be changed in such a way that the logic of construction remains essentially the same, but the final result changes.[16]

The late Gothic reconstruction of Avas Church took place in the second half of the fifteenth century. The present-day external walls and the associated buttresses date from this period, while the position of the interior pillars can be reconstructed on the basis of fragments and excavations.[17]

The point cloud of the laser scanner survey of the church, or more precisely its horizontal section above the floor level, served as the basis for the geometric analysis (Figure 11). In Lechler's *Unterweisungen*, the main design module is the span of the sanctuary nave between the pillar axes, labelled as *a*. The width of the aisles is half of this, the thickness of the walls and the interior pillars is a tenth of this, and the height of the vault springings inside – both in the nave and in the aisles – is equal to the value of *a*.

These values also apply directly to Avas Church: the laser scanner floor plan shows that the wall thickness is 1/20 of the distance between the axes of the external walls, and the 10–wall-thickness (i.e. one–module-sized) nave fits the excavation plan; the surviving vault springings are at a height of one module. A surviving fragment of an

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[15] COENEN, *Die Spätgotischen Werkmeisterbücher in Deutschland*, 174–266.

[16] For details, see BERECZKI, *Order, Procedure, and Configuration in Gothic Architecture.*

[17] BERECZKI, *Régi kövek beszéde: az Avasi templom építéstörténete.*

original pillar shows that its thickness is equal to the thickness of the external wall, i.e. the value of *a*/10.

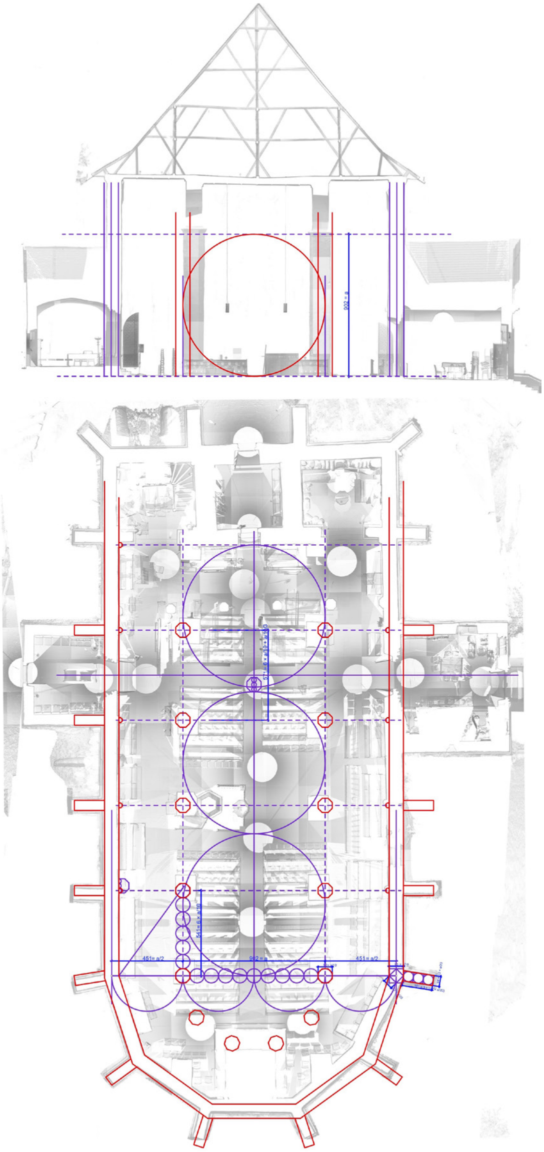

Figure 11: Laser scanner survey of Avas Church, Miskolc, Hungary (horizontal section directly above the floor level, vertical section) with reconstructed geometry and reconstructed position of the medieval pillars. North is towards the right side of the page. Source: Author's work, using laser scanner survey data (Photo.metric Kft., Hadas Építész Kft.).

The rules of Lechler do not apply directly to the length of the vault bays, the total length of the church and the size of the buttresses: these are examples of how parameters and connections are changed in a process description.

One of Lechler's rules for the length of a vault bay is 4 × *a*/10, i.e. four times the thickness of the wall. In the case of the Avas church, the length of the vault bay can be expressed as follows from the already known dimensions: 6 × *a*/10. Thus, although the parameters are different, the logic of construction – the length of the vault bay being an integer multiple of the wall thickness – is the same.

The total length of the nave in Lechler is four times the width of the nave (4*a*). In Miskolc this value is 3*a*, plus one-third of the wall thickness.[18] The logic of construction is therefore basically the same (the length of the nave being an integer multiple of the width of the nave), but the parameter used is again different.

Lechler describes two methods for the size of the buttresses, one of which has a width of √2 × *a*/10 and a length of twice the width. The buttresses of Avas Church can be constructed with a similar logic but with different parameters: the width of the buttress is √2 × *a*/20, and its length is three times its width. Although the proportions are not the same as in the *Unterweisungen*, the procedure is very similar, using square roots and multiplication. The relationship √2 × *a*/20 is a very common construction procedure in Gothic architecture based on the rotation of squares, called quadrature, and was featured by Roritzer too.

3 Delving Deeper: Simulated Morphogenesis

If the input parameters for creating the objectile are extracted from the analysis of existing structures, then structures similar to the existing structures in the desired aspects but not identical to them can be generated. Using a biological analogy, the extracted data can be considered the genotype, the generated structures the phenotype, and the process itself, according to my own definition, simulated morphogenesis.[19] Simulated morphogenesis deliberately avoids the use of artificial intelligence: both analytical methods and generative algorithms are based on human intuition, pattern recognition and design decisions. This is because in the case of generative artificial intelligence, the result, the goal, is the generated image, while the process remains hidden. In the case of simulated morphogenesis, on the other hand, the goal is precisely to understand and model the process, i.e. the process description.

The main steps of simulated morphogenesis are broadly similar to what Bill Hillier and Julienne Hanson describe in their method for the configurational analysis of cities:[20]

1. Observing of forms
2. Analysis of relations between their different parts
3. Creating interpretive generative algorithmic models and experimenting with them

In addition to the careful observation of forms, it is therefore important to analyse the relationships between the elements, i.e. the configuration. When creating

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[18] For a possible reason for the one-third wall thickness difference, see BERECZKI, *Order, Procedure, and Configuration in Gothic Architecture*, 678.

[19] In my previous publications, I used the term "artificial morphogenesis": BERECZKI – LOVRA, *Healing Urban Tissue Damages Using Artificial Morphogenesis*; BERECZKI, *The Procedural Turn: Artificial Morphogenesis in Urban Design*.

[20] http://otp.spacesyntax.net/overview-2

algorithmic models, it is then an important question how to achieve great formal richness with the simplest possible algorithms, just as in nature a single genotype generates an infinite number of phenotypes. Great diversity can be achieved with algorithms by using randomly generated variables with precisely defined limits in the right places and/or by turning on and off parts of the code according to predefined rules based on Boolean algebra, similar to how certain gene sequences are activated or deactivated in nature.[21]

For the first step (observation of forms), on-site examination, 3D scanning and analysing of Gothic buildings and micro-architectural works were carried out. During the data collection, I conducted (or contributed to) full or partial scans of the following Gothic buildings in the territory of present-day Northeast Hungary and Eastern Slovakia:

- Avas Church, Miskolc, Hungary
- Roman Catholic parish church, Abaújszántó, Hungary
- Calvinist church, Korlát, Hungary
- Church of the Exaltation of the Holy Cross, Bodrogkeresztúr, Hungary
- St Martin's Church, Spišská Kapitula, Slovakia
- Church of St James, Levoča, Slovakia

At the level of microarchitecture, I conducted full or partial 3D scans of the following Gothic works, also in the two adjacent regions mentioned above, with the exception of the Bratislava example:

- statue baldachins, St John's Chapel, Franciscan monastery, Bratislava, Slovakia
- altar superstructure, Coronation of Mary altar, St Martin's Church, Spišská Kapitula, Slovakia
- Gothic altar superstructures in the Church of St James, Levoča, Slovakia
- baldachin above the Relief of St Elisabeth, Church of St Elizabeth, Košice, Slovakia
- sacrament house, Roman Catholic parish church, Abaújszántó, Hungary
- sacrament house, Church of St Elizabeth, Košice, Slovakia
- sacrament house, Chapel of St Michael, Košice, Slovakia
- sacrament house, Church of St James, Levoča, Slovakia
- sacrament house, cathedral (former parish church) of Rožnava, Slovakia
- sacrament house, Roman Catholic parish church, Gelnica, Slovakia
- unfinished(?) sacrament house, St John the Baptist Church, Spišské Vlachy, Slovakia

For the second step (analysis of the relationships between different parts), the analysis of the 3D models of the surveyed-examined buildings-building elements and the survey drawings made on their basis in a CAD software provides information, similar to the way Robert Bork and other researchers of Gothic geometry develop the construction methods of Gothic buildings.[22]

The third step (creation and experimentation with interpretive, generative algorithmic models) is an experiment to create the desired forms and their variations by defining constants, parameters and variables, also incorporating randomness.

In addition to understanding Gothic geometry and Gothic design logic, one possible practical significance of the method is that due to the destruction of the medieval buildings on the territory of present-day Hungary, theoretical reconstructions are of great importance, but these often belong to the realm of fantasy. By procedurally

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[21] BERECZKI, *Generative Interpretations of Late Gothic Architectural Forms*, 283–284

[22] Eg. BORK, *The Geometry of Creation*; VELTE, *Die Anwendung der Quadratur und Triangulatur.*

analysing the available building elements and analogies, the theoretical reconstruction of the destroyed parts can be carried out through the exploration and modelling of the process, with simulated morphogenesis.

3.1 Key Concepts

In the study and modelling of the procedural nature of Gothic architecture and micro-architecture, three concepts stand out for their exceptional significance: **semilattice**, **recursiveness**, and **open-endedness**.

3.1.1 Semilattice

The concept of semilattice was introduced to the field of architecture in the context of natural cities by Christopher Alexander in 1965.[23] In this work, the author demonstrates the distinction between artificial and natural cities through a simple mathematical model. The mathematical model of artificial cities is a tree structure, where each element has one and only one parent. This strict hierarchy is characteristic of the urban planning principles of modernism and is most evident in the zoning of single-function districts (rigid, inflexible zoning) and the strict hierarchy of traffic. In contrast, the mathematical model of natural cities is a semilattice, where an element can be associated with multiple parents, resulting in a much more complex system of relationships, allowing for overlaps and a significantly greater number of possible variations (Figure 12).

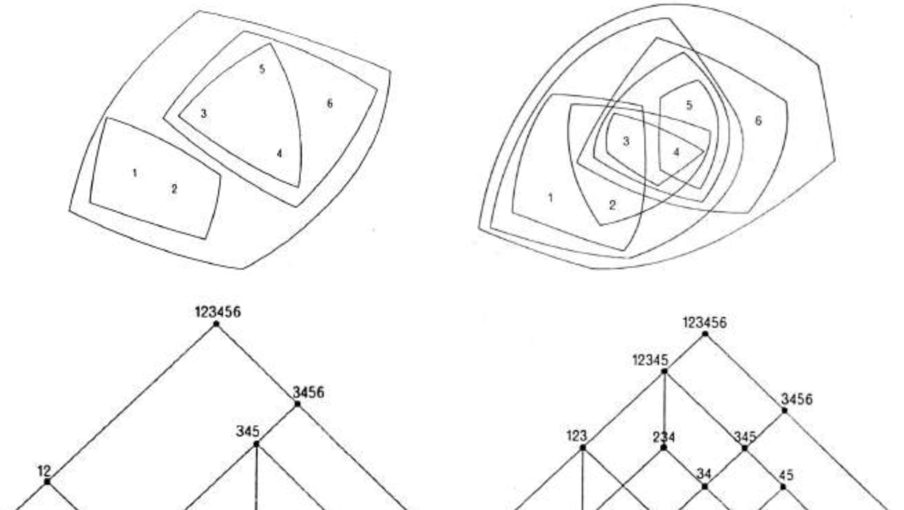

Figure 12: Tree and semilattice, based on Alexander.[24]

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[23] . ALEXANDER, *A City is Not a Tree.*

[24] ALEXANDER, *A City is Not a Tree: 50th Anniversary*, 6., 7.

3.1.1.1 Semilattice in Action: Procedural Modelling of the Altar Superstructures of Spišská Kapitula

Late Gothic altars in Spiš and its surroundings are also significant from the point of view of European Gothic art.[25] The Spišská Kapitula church is particularly important in this respect because the altars were not taken away to museums, as was the case with those in Sabinov (Slovakia), for example. Researchers are mostly interested in the works of figurative art that can be seen on the late Gothic altars – paintings and sculptures – and they assign a subordinate role to the architectural framework. However, these, primarily the altar superstructures (altar crowns), are among the most complex creations of Gothic geometry. In addition, a characteristic of the late Gothic period is that the boundaries between materials also dissolve: smaller metalworks, carved wooden altar crowns and carved stone sacrament houses were all made according to the same geometric principles.[26] From this point of view, the study of carved wooden altar crowns is also relevant to Gothic architecture.

Five late Gothic altars have been preserved in Spišská Kapitula Church: the high altar, the Altar of the Death of Mary, the Altar of the Archangel Michael, the Altar of the Three Magi, and the Altar of the Coronation of Mary. Of these, only the latter has retained its original altar crown. The crown of the Altar of the Death of Mary is known from a nineteenth-century, low-detail survey drawing,[27] and the crown of the main altar also appears on this drawing, but it had already lost much of its Gothic character by then: in the Baroque period it was extensively rebuilt, in such a way that some parts of the original altar crown were used for the Baroque altar.

During the project, I created a detailed photogrammetric survey of each altar (Figure 13), and then performed a geometric analysis and procedural model of the crown of the Coronation of Mary altar based on the survey. Based on the photos, I generated the 3D model with the Agisoft Metashape software, and the procedural model with the aforementioned Grasshopper by McNeel.

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[25] TÖRÖK, *Gótikus szárnyasoltárok a középkori Magyarországon*.

[26] For more details, see e.g. KAVALER, *Renaissance Gothic* 165–197; MAROSI, *Zum Prinzip Des "Pars Pro Toto" in Der Architektur Des Mittelalters*; BORK, *Turmhelme und Kleinkunst der Parlerzeit*.

[27] Archives of the National Monument Protection Office (now Hungarian Museum of Architecture and Monument Protection Documentation Centre), plan no. K 7704.

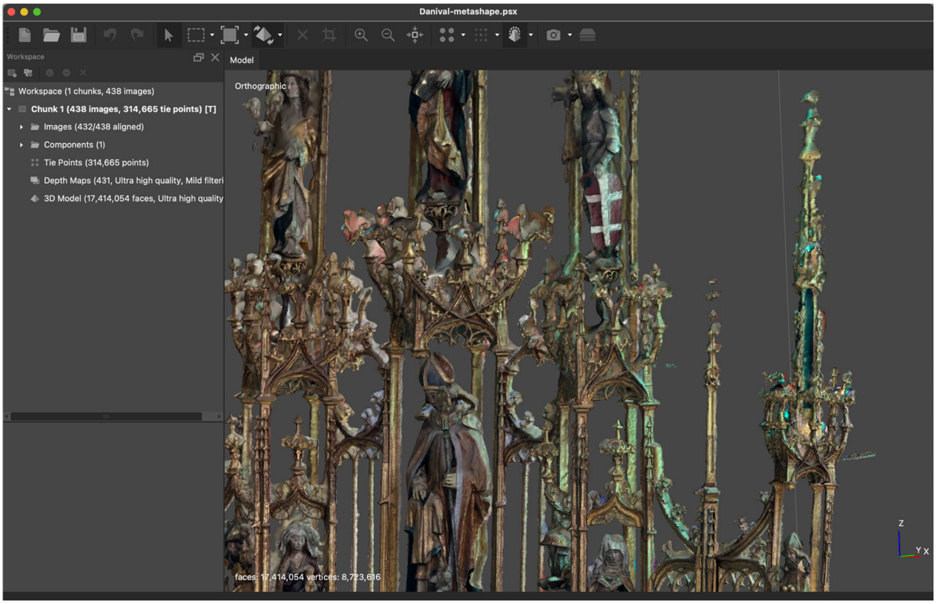

Figure 13: 3D model of the crown of the Altar of the Coronation of Mary at Spišská Kapitula, based on photogrammetric survey. Source: Author's work.

The configuration of the altar crown is essentially composed of two elements and their semilattice-like, recursive (more on this later) connections. These two elements are the baldachin and the geometry connecting the baldachins, which I have called a bridge. In Grasshopper, recursive definitions can be created using so-called clusters. A cluster can have inputs and outputs defined. An instance of a cluster can inherit its inputs from other elements (even from another instance of the same cluster) and pass its outputs to other elements (even to another instance of the same cluster). Three clusters had to be defined for the procedural model of the altar crown: a baldachin, a bridge and a pinnacle. The pinnacle is then embedded in the baldachin cluster and the bridge cluster, but there are also instances of it outside of these: the connections do not follow a tree structure, but rather an interconnected, semilattice structure.

In the configuration, the main element of the structure is the baldachin located at the lower level of the central baldachin tower. The distance between the columns of this baldachin is the main module, from which all other elements' horizontal and vertical dimensions are derived, sometimes in multiple steps similar to those of Roritzer's description (e.g., the heights of the baldachins and spires always being integer multiples of the module, regardless of the specific horizontal dimensions of the element).

Both instances of the baldachin cluster and instances of the bridge cluster inherit their input parameters from multiple elements (semilattice-nature) and then (in some cases) pass them on. The procedural model of the altar crown can be seen in Figure 14. Even this screenshot is suitable for illustrating the semilattice-like nature of the connections.

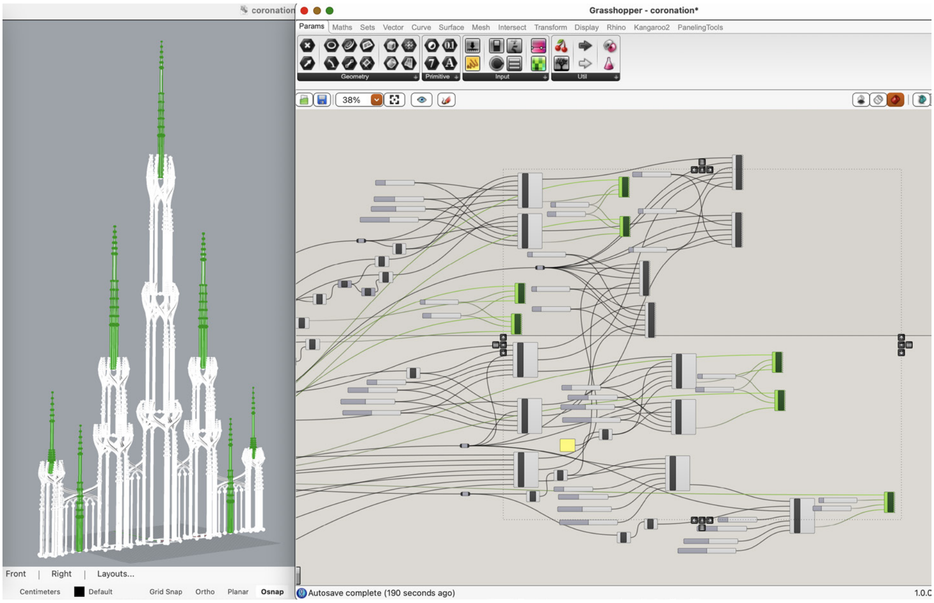

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Figure 14: Procedural model of the crown of the Altar of the Coronation of Mary, with pinnacle clusters highlighted that are not embedded in other clusters. Source: Author's work.

The level of detail on the nineteenth-century survey drawing is sufficient to establish that the crowns of the Altar of the Death of Mary and the high altar were made with similar geometric regularities, using similar elements, but with a different configuration and different dimensions. The algorithm of the procedural model created for the Coronation altar can be used to generate the crowns of the Death of Mary altar and the high altar; for this, the relationships between the elements must be rearranged and the parameters adjusted; and in the case of the high altar, the destroyed parts must be "regrown" using simulated morphogenesis. This also validates the algorithm. In Figure 15, the algorithmically generated model of the crown of the Death of Mary altar is shown together with a photo-scanned 3D model of the altar, and a photo-scanned model of a surviving medieval sculpture group is also inserted. Below are traditionally created 3D models of the predella and altar table. Figure 16 shows the model of the high altar in a similar way. The relationships and parameters for the generative theoretical reconstruction of the destroyed elements were provided by the geometric and configurational regularities observed on the surviving altar.

Figure 15: Algorithmically generated model of the destroyed crown of the Altar of the Death of Mary, with the photo-scanned 3D model of the altar. The algorithm is based on the crown of the surviving Altar of the Coronation of Mary, and the parameter values are based on the nineteenth-century survey drawing. Source: Author's work.

Figure 16: Algorithmically generated model of the destroyed crown of the high altar in Spišská Kapitula, with the photo-scanned 3D model of the altar. The algorithm is based on the crown of the surviving Altar of the Coronation of Mary, and the parameter values are based on the nineteenth-century survey drawing and the regularities observed on the Altar of the Coronation of Mary. The models of the predella and altar table were modelled with traditional method. Source: Author's work.

3.1.2 Recursive Definitions

The essence of a recursive definition is that the same description is repeated multiple times, at multiple hierarchical levels, seemingly embedded within itself. In this case, the elements at the lower level inherit their input parameters from the element(s) at the higher level, and pass their output to the element(s) at the lower level. If any instance of the description is modified, all other instances are also modified. Fractals are recursive definitions.

3.1.2.1 Recursive Definition in Action: Complexity as a Function of Size

Using recursion, a single pinnacle-generating algorithm can be used to generate large, complex Gothic towers.[28] This is made possible by the aforementioned scalability of Gothic structures: spire-like structures are similar at all scales. The basic form is most commonly a polygonal (usually square) body at the bottom, topped by a steep pyramid spire. This is essentially the Roritzer pinnacle structure. Between the two there is usually an intermediate, usually octagonal level, both in towers and, often, in micro-architectural works (Figures 3–7).

In the model presented here, the complexity of the structure is a function of its size. It uses three input parameters: the position of the structure (type: point in three-dimensional space), the height of the structure (type: positive number), and the slenderness of the structure (type: positive number between 0 and 1). A single variable, the height of the structure, controls the complexity. The starting position follows the most common shape of tower-like structures: a square-based prism at the bottom, an octagonal-based prism above it, and a pyramid above that. As the height increases, the code sections that control the details are gradually activated based on Boolean logic when a pre-defined height is reached: first, crockets appear on the edges of the pyramid (the spire), and then a gable crown appears around the pyramid (spire). This is the basic algorithm, in which the insertion points of the following recursive levels are also defined. The appearance of the different elements and recursive levels is controlled by logical "if/then" gates as a function of height. The main elements appear in the following order, when a pre-defined height is reached (Figure 17):

0. Basic form (square prism, octagonal prism, pyramid)
1. Crockets on the edges of the spire (pyramid)
2. Gables around the spire

The next step (the appearance of pinnacles around the octagonal storey) is the first recursive embedding. The pinnacles around the octagonal storey are generated by another instance of the same basic algorithm (cluster) that generates the tower itself (Figure 18). Since the pinnacles are still smaller in this phase, they are correspondingly simple, but they become complex according to exactly the same rules as the main structure (crockets appearing on their reaching the same height, etc.).

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[28] For details, see BERECZKI, *Generative Interpretations of Late Gothic Architectural Forms*, 287–292.

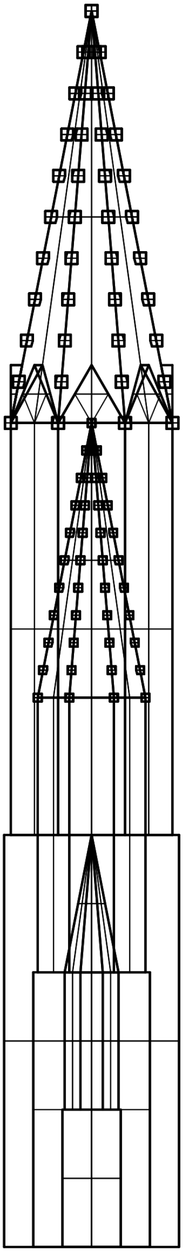

Figure 17: Steps in the growth of the tower-generating algorithm based on recursion, until the appearance of the first recursive level. Source: Author's work.

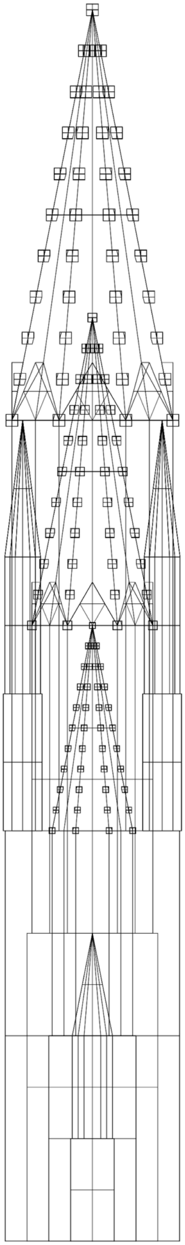

Figure 18: The step in the tower's growth when the first recursive level appears in the form of pinnacles around the octagon storey. Source: Author's work.

Every recursive level's insertion point is already defined in the base algorithm, but the recursive levels are only activated by "if/then" gates when a certain height is reached. The predefined insertion points are as follows:

3. Position of the giant pinnacles around the octagonal storey
4. Position of the buttresses around the tower shaft
5. Position of the pinnacles above the buttresses
6. Position of the pinnacles around the spire
7. Position of the pinnacles on the edges of the spire

In the model, each pinnacle is defined by the same base algorithm that defines the main tower itself, but their proportions vary according to their position. As the tower grows to an enormous size, more and more recursive levels appear (e.g. small pinnacles around the giant pinnacles, and so on). The entire tower grows out of a single recursive definition and is open-ended, theoretically able to grow to infinity. Along with the growth, its complexity also increases, as the complexity is a function of height according to the same rules at each recursive level. Self-embedding appears at three recursive levels in Figure 19. The state shown in Figure 18 is visible at the bottom of the figure. At the highest hierarchical level all elements are already active.

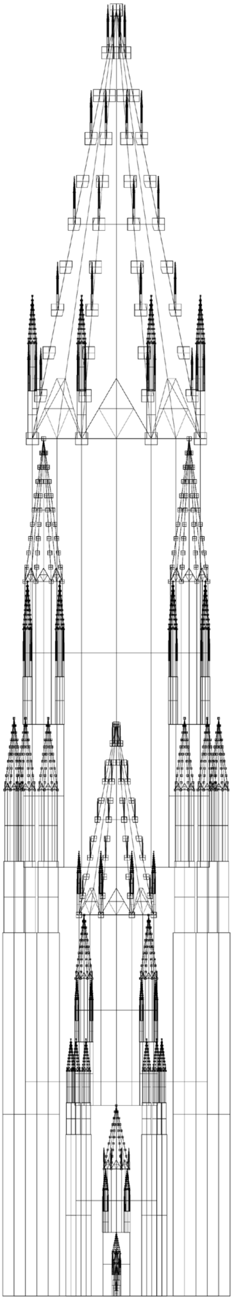

Figure 19: A stage in the growth of the recursion-based tower structure when three recursive levels are present. Source: Author’s work.

3.1.3 Open-Ended Systems

I use the term "open-ended systems" in the sense that both the procedure itself can be dynamically extended and modified, and the parameters and variables within it can be changed – by their very nature – to generate different geometries. If the relationships between the elements in a semilattice are changed, the result will also change significantly. If a single element is changed in a recursive system, the whole system is transformed. The system can also be extended at any time, existing outputs can serve as inputs for new elements, either in a semilattice or in a recursive manner, but most often in a combination of the two. A true Gothic building was rarely built in one phase and is never "finished" (in this way differing from its neo-Gothic counterparts), just as a natural city is never finished (in this way differing from the "ideal", planned cities of the Renaissance and Modernism).

The open-endedness of the altar crown algorithm presented above is best demonstrated by the fact that by rearranging the relationships and adjusting the parameters, the smaller, less element-rich crown of the Coronation of Mary altar has been transformed into the larger and more complex crown of the high altar. However, by further adjusting the parameters, possibly randomizing them within limits, and rearranging the relationships, it would be possible to create a practically infinite number of series, all of which would be related to each other.

The tower model is also open-ended, in part because the growth could theoretically continue to infinity. In addition, in the presented model, the proportions between the different parts (including the recursions) are pre-defined as constants, as is the number of crockets and the steepness of the spire and gables. However, these constants can also be treated as variables and randomized within controlled limits. Thus, near-infinite variation can be achieved with the same algorithm.

4 Better Understanding Tree- and Semilattice-Like Structures: High Gothic and Late Gothic

The question of the tree structure and semilattice structure is most evident in the relationship between Gothic and Late Gothic. For example while in classical Gothic it is completely clear where the boundaries of a vault bay lie, as Erwin Panofsky formulates, "Late Gothic permitted, even delighted in, flowing transitions and interpenetrations, and loved to defy the rule of correlation".[29] Using the vault as an example, while the cross-vaults used in classical Gothic have their bays clearly separated by transverse ribs (procedurally speaking, the input to the vault being provided by the parameters of a single bay; Figure 8), Late Gothic net vaults cannot be divided into such discrete parts, the structure inheriting its input from several places (Figure 20). The hierarchy of classical Gothic is mostly tree-like, while that of Late Gothic is mostly semilattice-like.

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[29] PANOFSKY, *Gothic Architecture and Scholasticism*, 50.

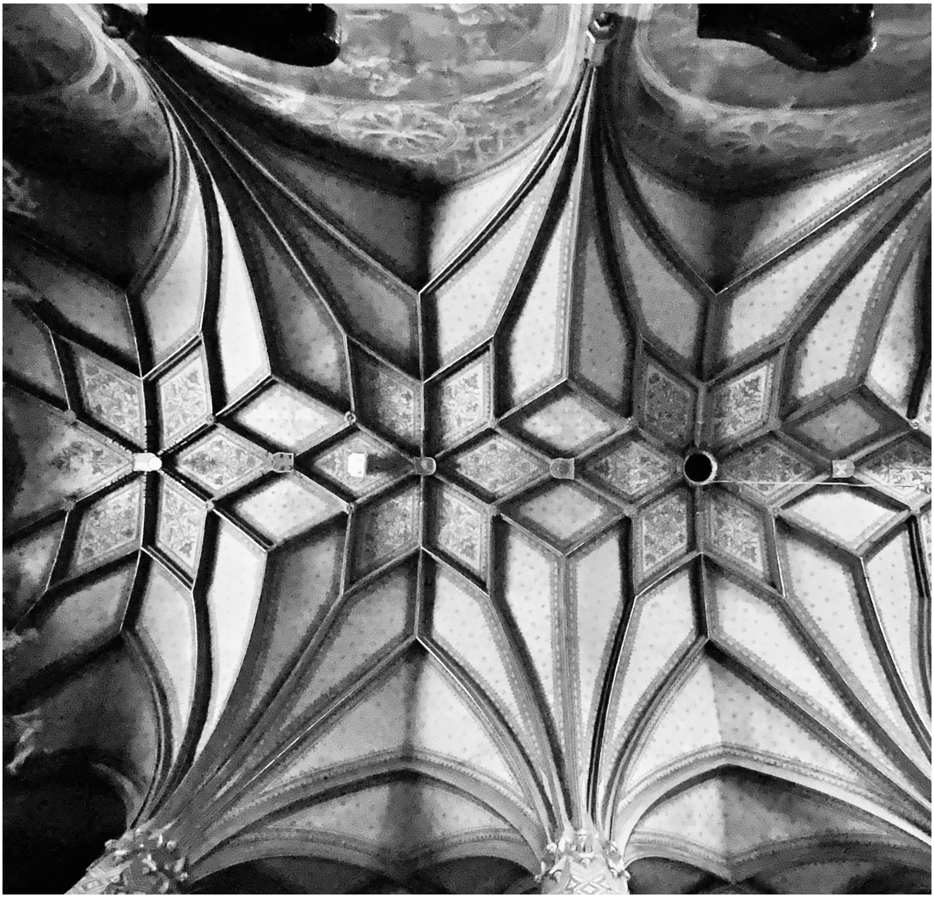

Figure 20: Late Gothic net vault of the cathedral (former parish church) of Rožňava, Slovakia. Source: Author's photo.

The difference between the way of thinking of High Gothic, Late Gothic and then neo-Gothic is beautifully illustrated by the case of St Elizabeth's Church in Košice.[30] The present external walls with its buttresses are the result of a first building period. However, the designer of the second period brought a new concept, in which the system of internal pillars and thus vaults is different from what his predecessor had planned. The new system is more complex than the presumably original plan and shows semilattice characteristics.

The new designer created a north–south axis at least as important as the east–west axis usual in medieval churches, and redesigned the internal supporting system so that the nave (from the western entrance to the triumphal arch) and the transept form

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[30] For its construction history see JUCKES, *The Parish and Pilgrimage Church of St Elizabeth in Kosice*.

a cross with equal arms in width and practically the same length in each direction. (The two arms of the longitudinal axis and the two arms of the transverse axis are of equal length; Figure 21.) The lower spaces of the aisles are located between the arms of this cross. Their shape is square, with a side length equal to the width of the nave and transept. The squareness appears clearly only on the western side; on the eastern side the diagonal chapels slightly distort it. In the middle of each of the squares of the aisles is a pillar, dividing each of them further into 4 spatial sections. The whole system is central, as the four-part spaces of the aisles with the central pillar are also central, thus giving the entire spatial system a fractal-like character (Figure 22). This spatial system (nave, transept, aisles) will be referred to as the main body in the following.

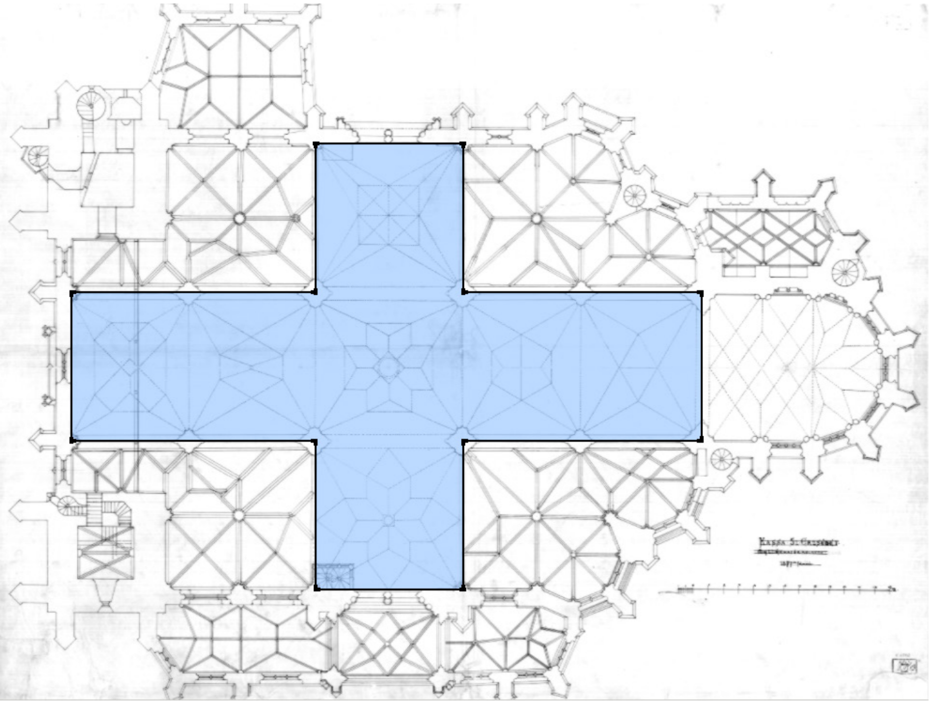

Figure 21: Ground plan of the Church of St Elizabeth in Košice before the nineteenth century reconstruction, highlighting the high spaces (nave, transept). Source: Author’s work, based on drawing K.6903 in the collection of the Hungarian Museum of Architecture and Monument Protection Documentation Centre.

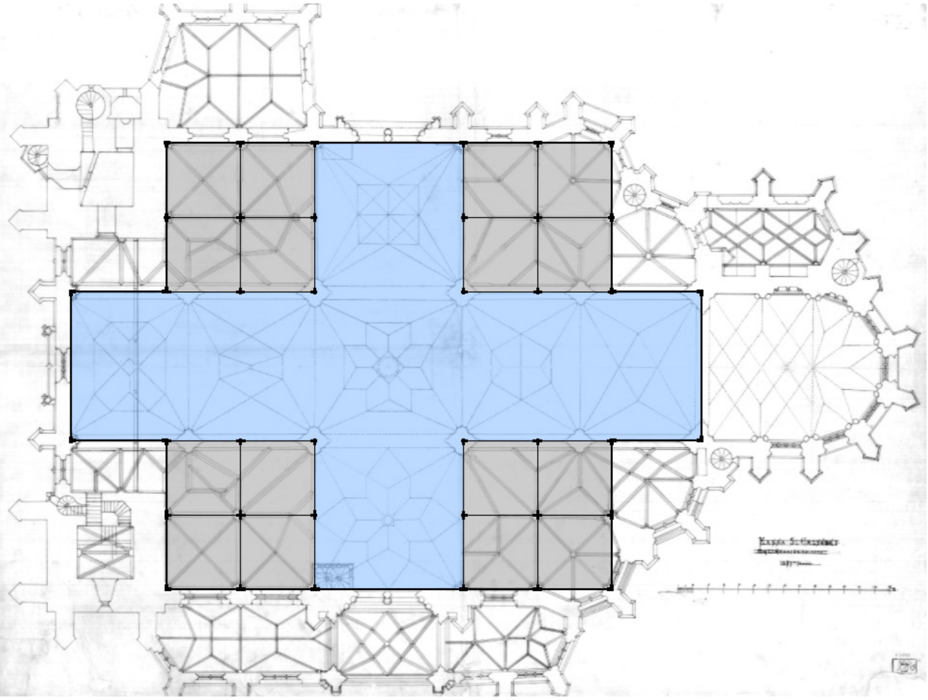

Figure 22: Ground plan of the Church of St Elizabeth in Košice before the nineteenth century reconstruction, highlighting the high spaces with blue (nave, transept) and the lower spaces (grey) between their arms. Source: Author's work, based on drawing K.6903 in the collection of the Hungarian Museum of Architecture and Monument Protection Documentation Centre.

The eastern part of the church (the sanctuary and the four eastern diagonal chapels with the nave section between them, which I will refer to collectively as the sanctuary ensemble) can also be considered an independent unit, all the more so because this type of sanctuary ensemble design is found elsewhere too (e.g. Braine, Xanten, Trier) (Figure 23). Viollet-le-Duc even illustrates architectural symmetry in his dictionary with the eastern part of the Braine church, which is similar to the Košice church.[31] The other peculiarity of the spatial layout becomes visible here: these two spatial systems (the main body and the sanctuary ensemble), which could probably be interpreted separately in reality, form an interconnection, which is a characteristic of semilattice structures. In this way, the space of the two eastern radiating chapels and the nave between them hierarchically belongs both to the main body and sanctuary ensemble (Figure 23).

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[31] VIOLLET-LE-DUC, *Dictionnaire Raisonné de l'architecture Française Du XIe Au XVIe Siècle*, 515.

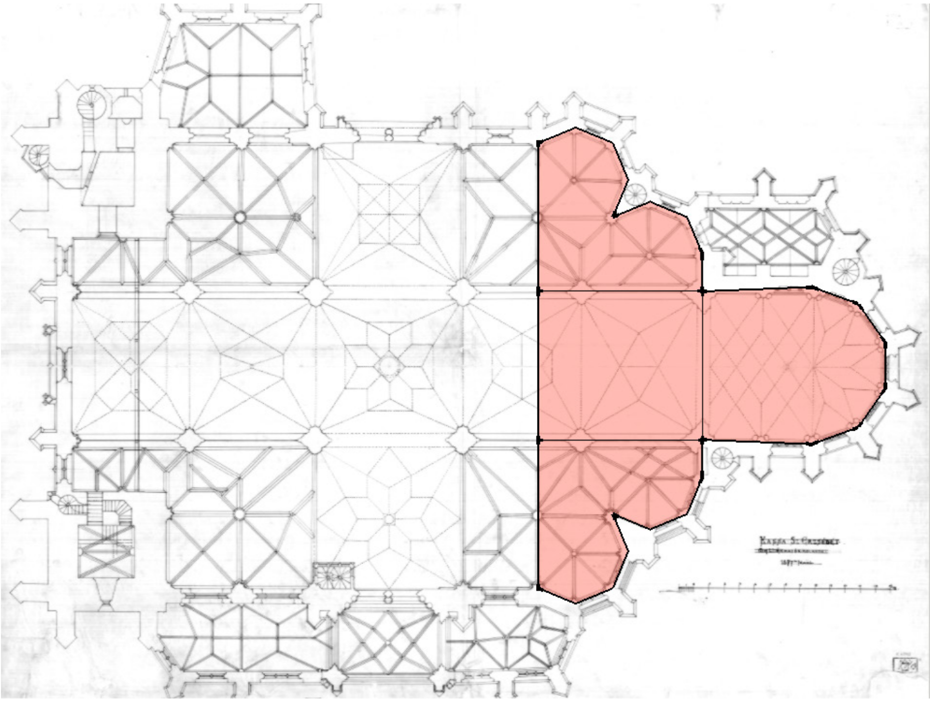

Figure 23: Ground plan of the Church of St Elizabeth in Košice before the nineteenth century reconstruction, highlighting the sanctuary ensemble. Source: Author's work, based on drawing K.6903 in the collection of the Hungarian Museum of Architecture and Monument Protection Documentation Centre.

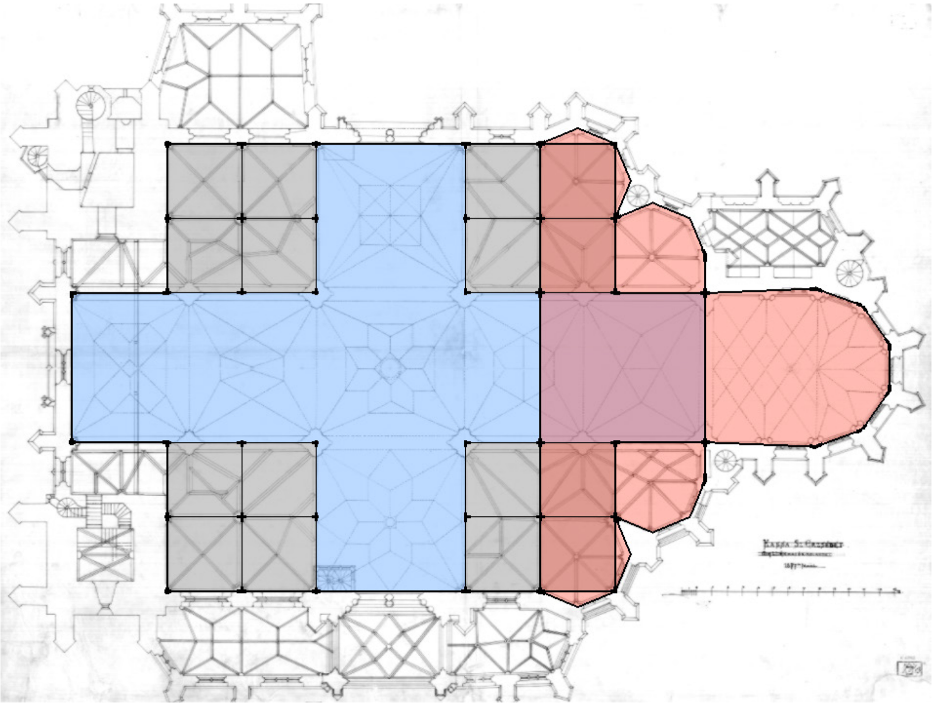

Figure 24: Ground plan of the Church of St Elizabeth in Košice before the nineteenth century reconstruction, highlighting the main body (nave, transept, aisles) and the sanctuary ensemble, and their intersection. Source: Author's work, based on drawing K.6903 in the collection of the Hungarian Museum of Architecture and Monument Protection Documentation Centre.

By adapting his own system to an existing structure (open-endedness), the designer of the second period introduced certain "irregularities" in some places, especially in the aisles, where the vaults had to adapt to both systems (the external buttress system defined in the first period and the internal buttress system defined in the second period). This and other "irregularities" (e.g., the transept being open to the aisles) that deviated from classical Gothic led to the disappearance of the church's unique yet characteristically late Gothic spatial system when it was "corrected" and a regular five-aisled longitudinal space created in its place during the Imre Steindl reconstruction at the end of the nineteenth century.

5 Closing Remarks: Towards a New Architecture?

The above discussed procedural thinking and recursive, semilattice-like structures disappeared from architecture after the Middle Ages. The Renaissance preferred the much simpler flat order – the fixed canons of proportion[32] – in architecture and in the "ideal" cities too. According to Mario Carpo, procedural methods are now about to

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[32] BORK, *The Geometry of Creation*, 2.

return: "the post-Albertian architecture of our digital future will have something in common with the pre-Albertian architecture of our artisanal past, but this does not mean that digital architecture might or should look Gothic – nor any other style. […] Similar processes do not necessarily beget similar shapes. Understanding these processes, on the contrary, will help us shape better things."[33] Robert Bork has a similar opinion: "The complex and procedurally based formal order of Gothic architecture, in fact, offers a highly sophisticated alternative to the classical tradition, one with real relevance for present-day architectural practice. […] Geometrical analysis of Gothic design thus has the potential to enrich architectural practice in the twenty-first century, in much the same way that formal and archaeological analysis of Gothic buildings enriched architecture in the nineteenth and early twentieth centuries."[34]

This new architecture does not exist yet, but similar currents can be found in contemporary music. For example, in some of his works György Ligeti created contemporary music based on medieval patterns. In these compositions, it is not the melodies themselves but the underlying logic of the system which is derived. The complexity is highly developed, yet intentionally imperceptible.[35] The result is not medieval but indeed very contemporary.

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[33] CARPO, *The Alphabet and the Algorithm*, 128.

[34] BORK, *Dynamic Unfolding and the Conventions of Procedure*, 1–2.

[35] STEINITZ, *Ligeti György*, eg. 126, 128, 224.

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