



AKADÉMIAI KIADÓ



UNIVERSITY of
DEBRECEN

International Review of
Applied Sciences and
Engineering

13 (2022) 3, 285-297

DOI:
10.1556/1848.2021.00383
© 2021 The Author(s)

ORIGINAL RESEARCH
PAPER



Geometric design, modelling and finite element analysis of spur gear pairs having normal teeth based on the modification of the pressure angle

Sándor Bodzás*

Department of Mechanical Engineering, University of Debrecen, Debrecen, Ótmető str. 2-4, H-4028, Debrecen, Hungary

Received: September 19, 2021 • Accepted: December 31, 2021
Published online: March 24, 2022

ABSTRACT

The aim of the thesis is the geometric design, CAD modelling and TCA of spur gear pairs having normal teeth based on the modification of the pressure angle. The first task is the geometric design and CAD modelling of the gear pairs (5 pairs are designed) where only the pressure angle is modified beside the constancy of the other initial parameters. The second task is to analyse the comparison possibility and the accuracy similarity of the 2D and the 3D models by the Hertz (equivalent) stress analysis. Finally, I give analysis of the maximum equivalent stress, normal stress and contact pressure for each pair while three teeth are rolling down on each other.

KEYWORDS

spur gear, CAD, TCA, pressure angle

1. INTRODUCTION

The TCA is a special sub area of the FEM analysis where the mechanical parameters are analysed on different types of toothed gear pairs on the tooth contact zone due to the loads [8].

The main property of the spur gear pairs having normal teeth is the application of the addendum modification that is why the tooth connection is established on the rolling circles [4–8, 11, 12]. Consequently, the base angle (α_p) is different from the pressure angle (α_w), which is interpreted between the common tangent line of the rolling circles and the common tangent line of the base circles (Fig. 1). The common tangent line of the base circles, which goes through the C main point, is called line of action. This line and the common tangent line of the rolling circles (r_{w1} , r_{w2}) always form an α_w angle that is the pressure angle (Fig. 1). Due to the modification of the centre distance (a) this angle will be also modified based on Fig. 1. The centre distance can be calculated by the following formula [4–8, 11, 12]:

$$a = r_{w1} + r_{w2} = r_{p1} \cdot \frac{\cos \alpha_p}{\cos \alpha_w} + r_{p2} \cdot \frac{\cos \alpha_p}{\cos \alpha_w} = a_0 \cdot \frac{\cos \alpha_p}{\cos \alpha_w} \quad (1)$$

where

$$r_b = r_p \cdot \cos \alpha_p = r_w \cdot \cos \alpha_w \quad (2)$$

The basic rack profile contains the base parameters of the normal section (circular pitch, whole depth, base profile angle and clearance). This profile has infinite number of teeth along a line. The base profile of an involute gear is standardized (Fig. 2).

The phenomenon when the tool center line and the tool reference line are not same is called gear having addendum modification. This process is also called addendum modification. This parameter can be calculated by the following formula [4–8, 11, 12].

*Corresponding author.
E-mail: bodzassandor@eng.unideb.hu

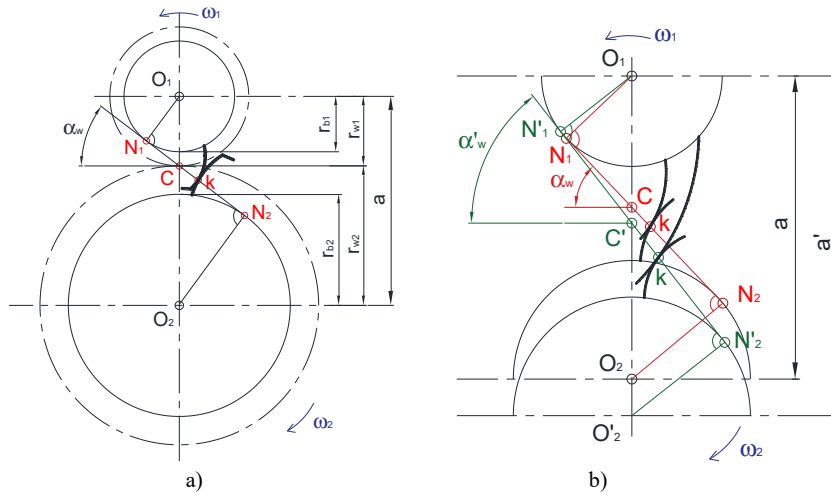


Fig. 1. Connection of the involute curves in case of different centre distances

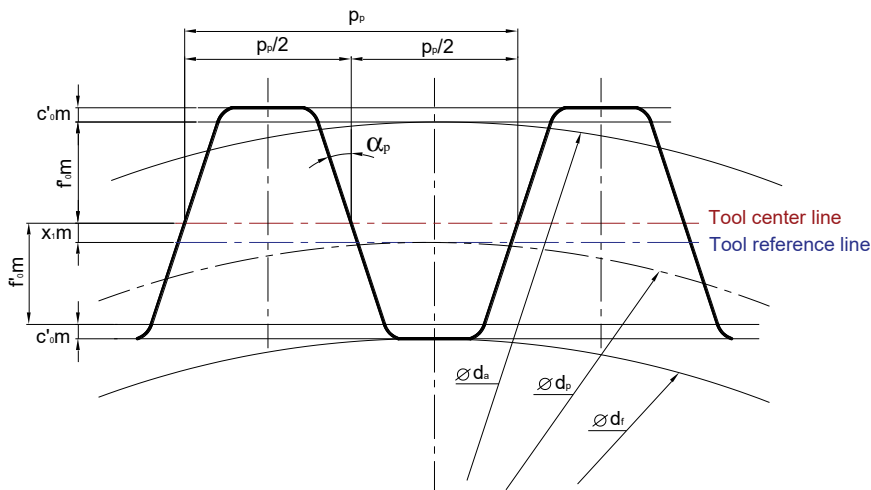


Fig. 2. Tool base profile in case of involute gear having normal teeth

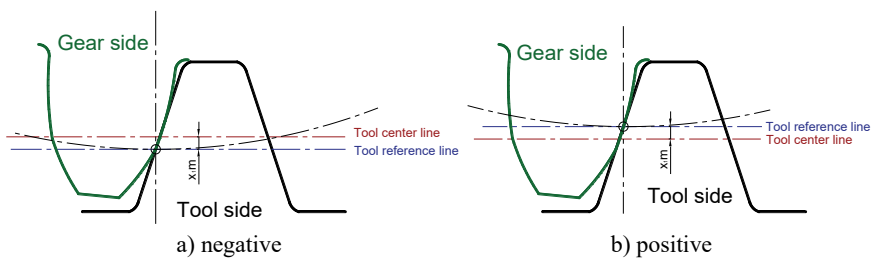


Fig. 3. The connection possibilities of the tool base profile and the gear profile

$$x \cdot m \tag{3}$$

The x can be positive when the basic profile is moved from the gear axis (Fig. 3b). The x can be negative when the basic profile is moved to the gear axis (Fig. 3a). If $x = 0$, the tool center line and the tool reference line are the same. Such type of gear pair is

called spur gear drive with no addendum modification [4-8, 11, 12, 13].

In a general way the basic theorem of the gear connection is the multiplication of the relative velocity vector and the normal vector is zero on the contact points of the gears [7, 8]:

$$\vec{n}_{1R} \cdot \vec{v}_{1R} = \vec{n}_{2R} \cdot \vec{v}_{2R} = 0 \tag{4}$$



2. GEOMETRIC DESIGN AND THE MODELLING OF TOOTHED GEAR PAIRS

The geometric establishment of the spur gear drive having normal teeth can be seen in Fig. 4. Addendum modification is used, thus the tooth connection occurs on the rolling circle diameters ($d_p \neq d_w$) [4-8, 11, 12]. The elementary and the normal center distances are not equal ($a_0 \neq a$) [4-8, 11, 12]. The initial parameters for the design process are $z_1, z_2, c'_0, l, \alpha_p$ and α_w . Gear I is the pinion that is assembled on the input shaft. This element drives Gear II, which is called gear (Fig. 4).

Based on the geometric formulas of the literature I have developed a computer software in Matlab programming language to determine the geometric parameters and the involute profiles of the pinion and the gear. The program can save the profile points of the gear pairs into a *txt* file.

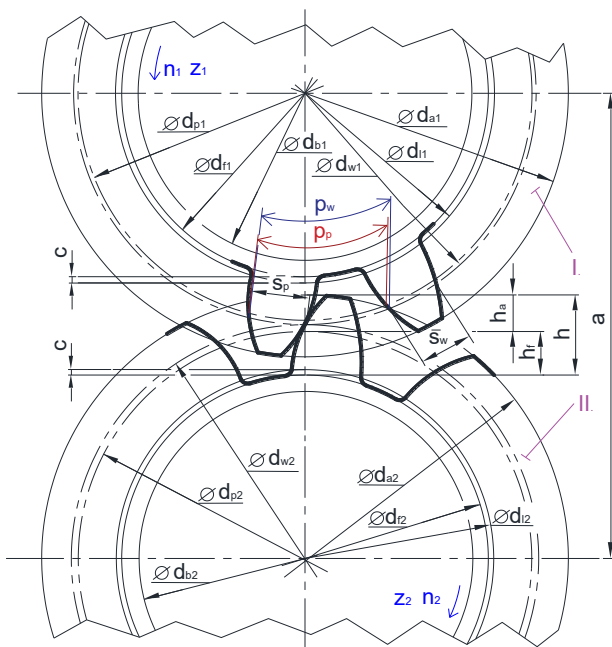


Fig. 4. The geometric establishment of the spur gear drive having normal teeth

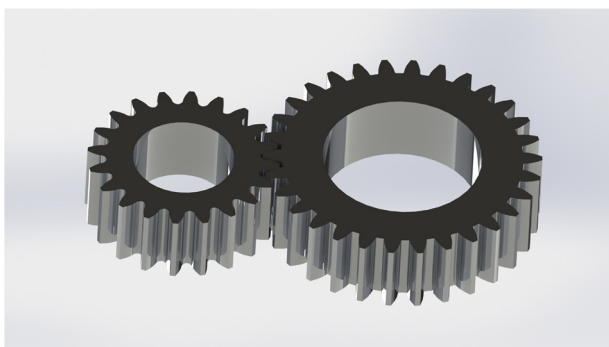


Fig. 5. The CAD model of the gear pair ($z_1 = 20, z_2 = 30, m = 6 \text{ mm}, \alpha_w = 21^\circ$)

This file can be imported into the SolidWorks software where an interpolated B-spline can be fit on these points [1-3]. Considering the received geometric results and the profile curves, the computer aided (CAD) models of the elements can be built up. Finally, the connection-correct gear assembly can be made (Fig. 5) [1-3].

I have designed five gear pairs. The initial geometric parameters can be seen in Table 1. I have only modified the α_w pressure angle beside the constancy of the other initial geometric parameters.

The shape of the involute profile is unchanged since the base circle diameters are constant. The profile curve is generated from this circle. In spite of that, the shapes of the teeth are different because of the geometric parameters. The tooth connections take place on different d_w rolling circle diameters.

3. COMPARATIVE FINITE ELEMENT ANALYSIS OF 2D AND 3D MODELS

The aim of the analysis is to compare the accuracy similarity between the 2D and the 3D models beside the constancy of the load and boundary conditions by static structural analysis. The analysed model is the Gear drive III in Table 1. The material type can be seen in Table 2. Frictionless contact is defined between the connecting surfaces of the teeth. The load moment is 80 Nm, which can be selected by experience.

I have created the 2D model by SpaceClaim software, where the thickness ($l = 50 \text{ mm}$ tooth length) is considered for the analysis (Fig. 6).

Coordinate systems are adapted into the middle points of the gears while local coordinate systems are defined in the contact point of the teeth.

Table 1. The initial geometric parameters of the designed toothed gear pairs

Geometric parameters	Gear drive I	Gear drive II	Gear drive III	Gear drive IV	Gear drive V
α_w [°]	20	21	22	23	24
m [mm]	6				
z_1	20				
z_2		30			
α_p [°]	20				
c'_0	0.2				
u	1.5				
l [mm]	50				
c [mm]	1.2				

Table 2. The applied material

Name	Steel
Linear Elastic	Isotropic Elasticity
Young's Modulus (E)	$210 \cdot 10^3 \text{ MPa}$
Poisson's Ratio (ν)	0.3



F: Copy of Sikmodell haromszögek
 Solution
 Time: 1. s
 2021.01.20. 16:38

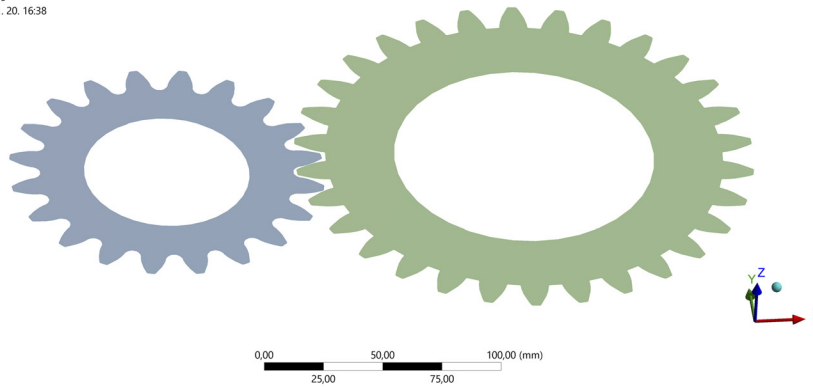


Fig. 6. The 2D model for the FEM analysis

The main problem of the comparison is the mesh type and the element size. It is well-known that the smaller the mesh size is, the longer the calculation process takes. In addition, if we use different element sizes for the two models the results will be different [9, 10]. It is known that a much smoother mesh can be useable in the case of the 2D model than in the case of the 3D model. Because of the computer's capacity, the mesh of the 3D model can be smoothed until a certain limit, which is bigger than in the case of the 2D model. Otherwise, the computer could not calculate the FEM problem. That is why I have to find an optimized mesh where the FEM results of the 2D and 3D models are comparable.

3.1. The analytic determination of the contact stress

The maximum Hertz stress (Equivalent stress) can be calculated in the case of toothed gear pairs by the following formula [11]:

$$\sigma_H^2 = 0.35 \cdot \frac{F_n \cdot \frac{1}{\rho_1} + \frac{1}{\rho_2}}{l \cdot \frac{1}{E_1} + \frac{1}{E_2}} \tag{5}$$

Consequently, the characteristic of the involute curve is that the common normal line of the connecting tooth curves touches the base circle diameters (d_{b1} , d_{b2}) from which the involute curve can be generated (Fig. 6).

The Pythagorean theorem can be useable for the O_1N_1C and the O_2N_2C rectangular triangles to determine the ρ_1 and ρ_2 involute curvatures on the common contact point which is the common tangent point of the rolling circles (Fig. 7):

$$\begin{aligned} \left(\frac{d_{w1,2}}{2}\right)^2 &= \rho_{1,2}^2 + \left(\frac{d_{b1,2}}{2}\right)^2 \rightarrow \rho_{1,2} \\ &= \sqrt{\left(\frac{d_{w1,2}}{2}\right)^2 - \left(\frac{d_{b1,2}}{2}\right)^2} \end{aligned} \tag{6}$$

$$\rho_1 = 22.778 \text{ mm}, \quad \rho_2 = 34.169 \text{ mm}$$

The load moment from the pinion affects the gear along the perimeter of the rolling circle diameter of the pinion. The circumferential force can be calculated by the following formula:

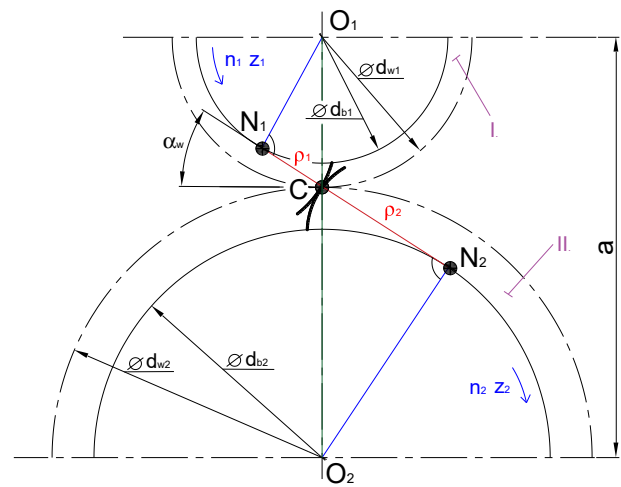


Fig. 7. The connection of involute curves

$$M = F_c \cdot \frac{d_{w1}}{2} \rightarrow F_c = \frac{2 \cdot M}{d_{w1}} = \dots = 1315.594 \text{ N} \tag{7}$$

Based on Fig. 8 the normal force is

$$F_n = \frac{F_c}{\cos \alpha_w} = \dots = 1418.914 \text{ N} \tag{8}$$

Substituting the calculated results into (5) and extracting of square root from (5), the calculated maximum Hertz stress on the contact zone is $\sigma_H = 276.239 \text{ MPa}$.

3.2. Approach by Normal Lagrange formulation

I calculated the simulation result by Augmented Lagrange formulation. When I compared the calculated results for the 2D and 3D models I got different values. Having checked the received reaction force in the software and the calculated results, I found that they are identical. I applied a parametric study for generalized plain strain 2D behaviour. I got much lower results than what I had calculated manually.

That is the reason why I have to approach the problem by Normal Lagrange formulation. The Normal Lagrange



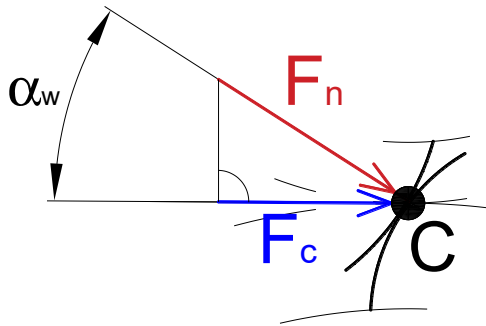


Fig. 8. The relation between the normal force and the circumferential force

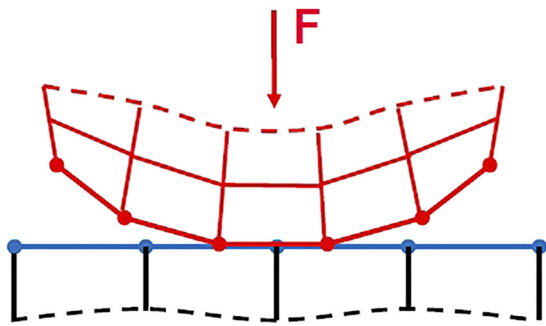


Fig. 9. The concept of the Normal Lagrange formulation

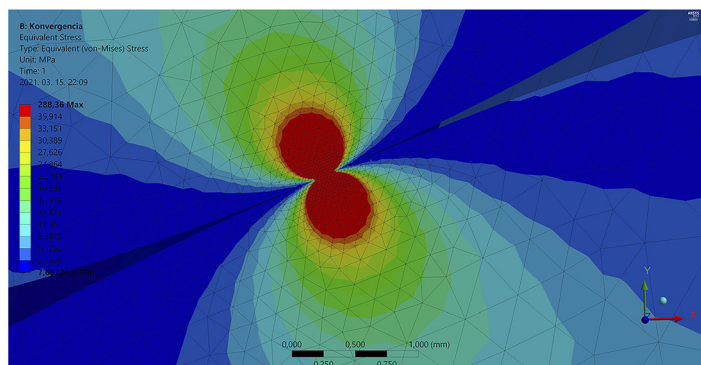
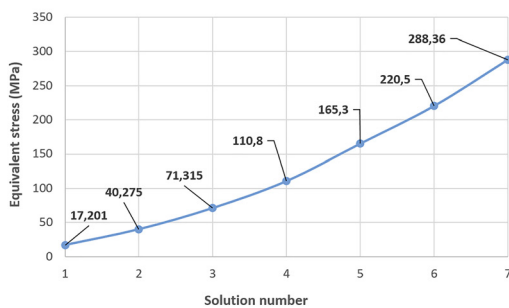


Fig. 10. The results of the convergence analysis using Normal Lagrange

Table of Design Points					
	A	B	C	D	E
1	Name	P1 - Face Sizing Sphere Radius	P2 - Equivalent Stress Maximum	<input type="checkbox"/> Retain	Retained Data
2	Units	mm	MPa		
3	DP 0 (Current)	1,1	296,97	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
4	DP 1	1	301,17	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
5	DP 2	0,9	293,92	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
6	DP 3	0,8	279,75	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
7	DP 4	0,7	276,24	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
8	DP 5	0,6	281,75	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
9	DP 6	1,2	290,86	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
*				<input type="checkbox"/>	

Fig. 11. The results of the parametric study on the 2D model using Normal Lagrange in case of plain stress behaviour

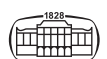
formulation adds an extra degree of freedom (contact pressure) to satisfy contact compatibility. Consequently, instead of resolving contact as contact stiffness and penetration, contact force (contact pressure) is solved explicitly as an extra degree of freedom. This process can enforce zero/nearly-zero penetration. Normal contact stiffness is not required. It requires Direct Solver, which can be more computationally expensive (Fig. 9) [14].

I also run the convergence analysis to decide the applicable element size, and to estimate the sphere radius for meshing. I cannot get significant changes after the 7th step. The result can be seen in Fig. 10. The maximum equivalent stress is 288.36 MPa. The 2D behaviour is plain stress.

Based on Fig. 10, I can measure the element size (0.025 mm) in the red contact zone for the other analysis.

After that, I make parametric study on the 2D model to optimize the element size and the sphere radius for the meshing. The results can be seen in Fig. 11.

I select the 7th line on Fig. 11 for setting it on the 3D model for meshing since I got the same result as with the calculation. The length is reduced to one tenth of the total length, that is why the load moment is 8 Nm. The element size inside the sphere is 0.025 mm. The number of divisions is 15 pieces. The results can be seen in Fig. 12. I got a lower result than the calculated and the received ones on the 2D model version.



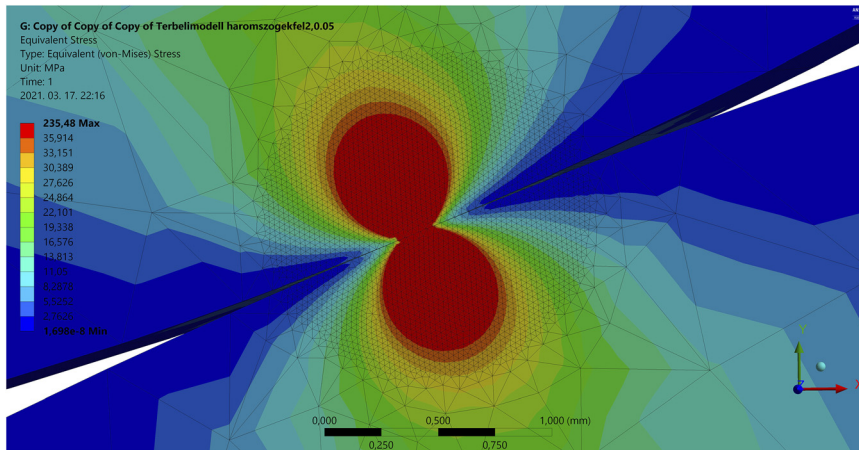


Fig. 12. The results on the 3D model using Normal Lagrange (7th line)

I select another line, which is the 3rd line (Figs 10 and 13). The initial parameters are the same as in the previous case except for the sphere radius (1.1 mm). The results can be seen in Fig. 13. In the engineering practice, 10–12% error

tolerance is acceptable. The results of the 2D model (Fig. 14) and the 3D model (Fig. 13) are within this acceptable zone.

I had the reaction forces calculated by the software (Fig. 15). This result and the calculated result are identical. Consequently,

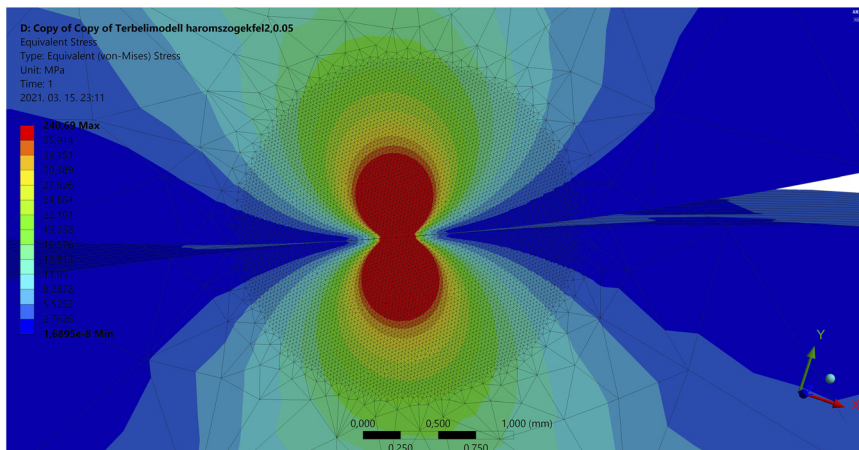


Fig. 13. The results on the 3D model using Normal Lagrange (1st line)

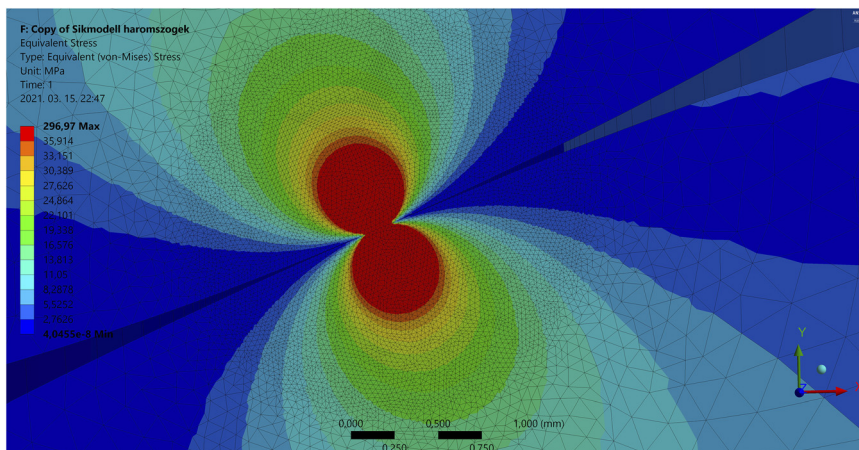


Fig. 14. The results on the 2D model using Normal Lagrange (1st line)



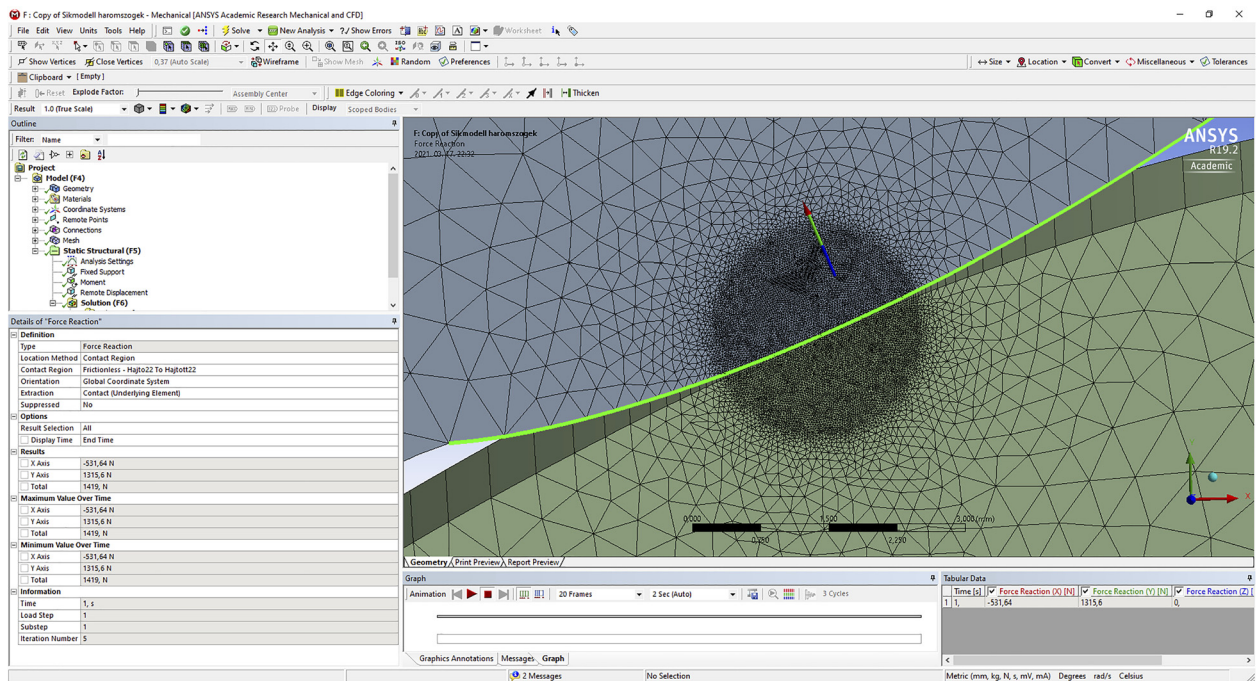


Fig. 15. The reaction forces on the 2D model

the key for the comparative simulation is the optimization of the element size to compare the results of the 2D and the 3D models. The software can calculate the reaction forces correctly, regardless of the meshing density, since the simulation force

result and the calculation force result are the same. It had the same result in the case of Augmented Lagrange formulation too.

The stress distributions on the contact surfaces on the pinion and the gear can be seen in Fig. 16.

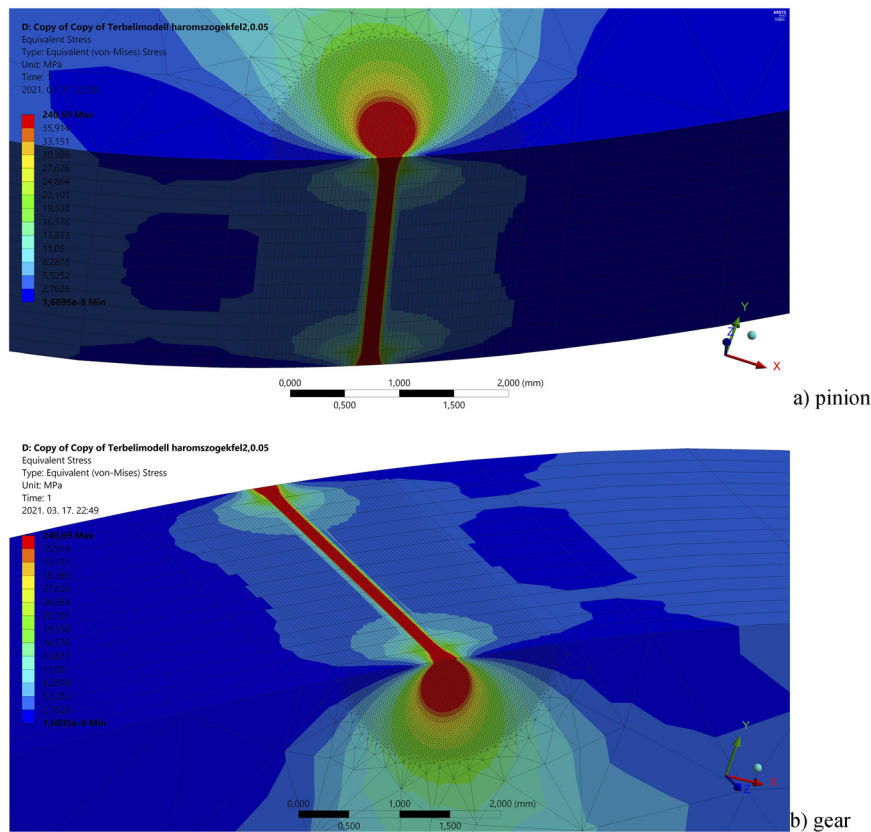


Fig. 16. The stress distributions on the contact surfaces

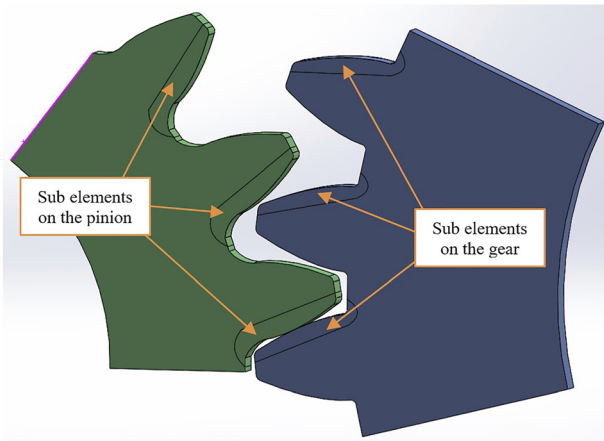


Fig. 17. The CAD models of the connecting elements (Gear drive III)

4. COMPARATIVE TCA IN THE FUNCTION OF THE PRESSURE ANGLE

4.1. Making the CAD models

Three-three teeth connections are analysed on all of the pairs according to Table 1. In order to make the calculation easier, the CAD models have to be simplified on each pairs (Fig. 17). I created sub elements, which are inserted into the teeth to generate dense meshing on the contact surfaces.

4.2. Setting the mesh, the load and the boundary conditions

The applied material is selected based on Table 2. The analysis type is static structural. The half tooth length is used

for the reduction of the calculation time. The applied load moment is 40 Nm with which the pinion is loaded.

Augmented Lagrange formulation is used for the analysis. The setting of the contacts can be seen in Fig. 18. Frictionless contact type is selected.

I have defined cylindrical joints for the pinion and the gear. Only the rotations around the axis of rotations are enabled. The other degrees of freedom are fixed.

Dense meshing (Fig. 19) is used on the sub elements (element size 0.2 mm). The number of divisions is 5 along the tooth length for the sweep method. The free face mesh type is quadratic and triangular combined. Automatic meshing is used on the outside areas.

The number of both the initial sub steps and the minimum sub steps are set for 100. The maximum sub steps are set for 1,000 steps. Frictionless support is used on the head surfaces. Only the rotations around the axis of rotations are

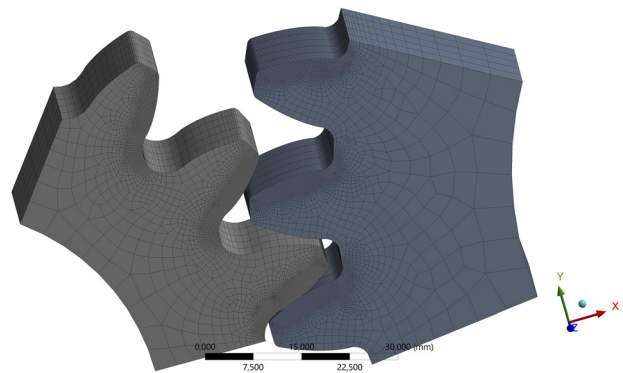


Fig. 19. The establishment of the mesh

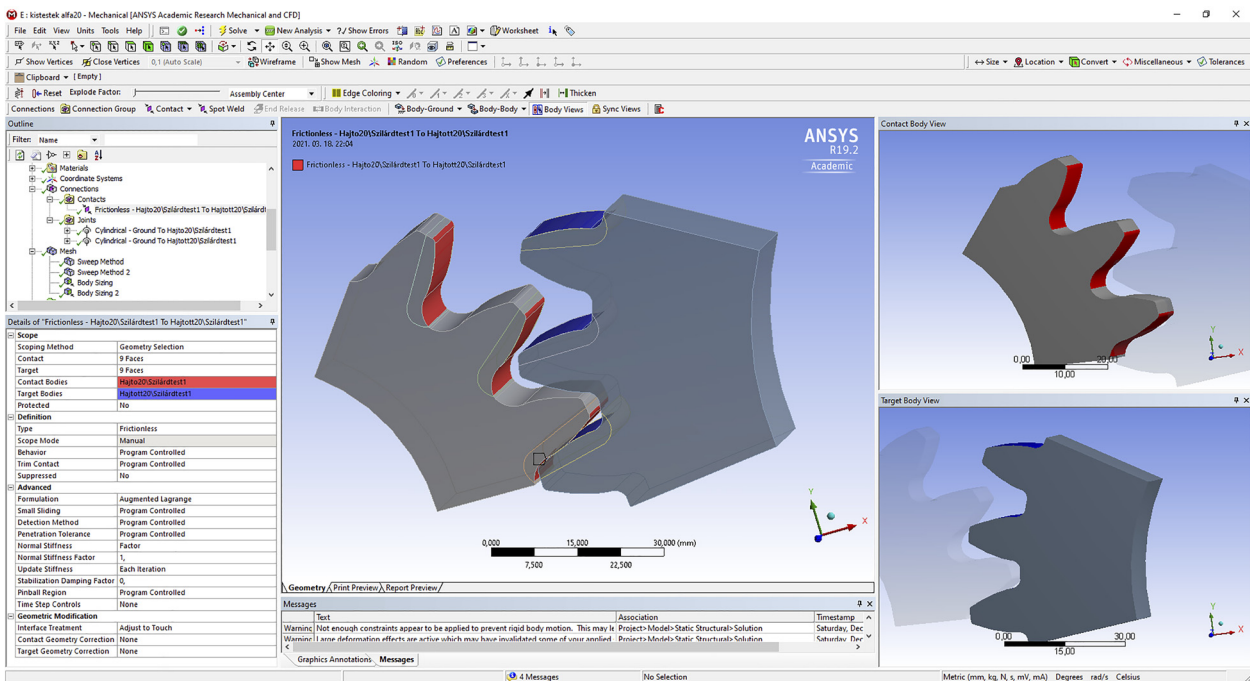


Fig. 18. The setting of the contacts

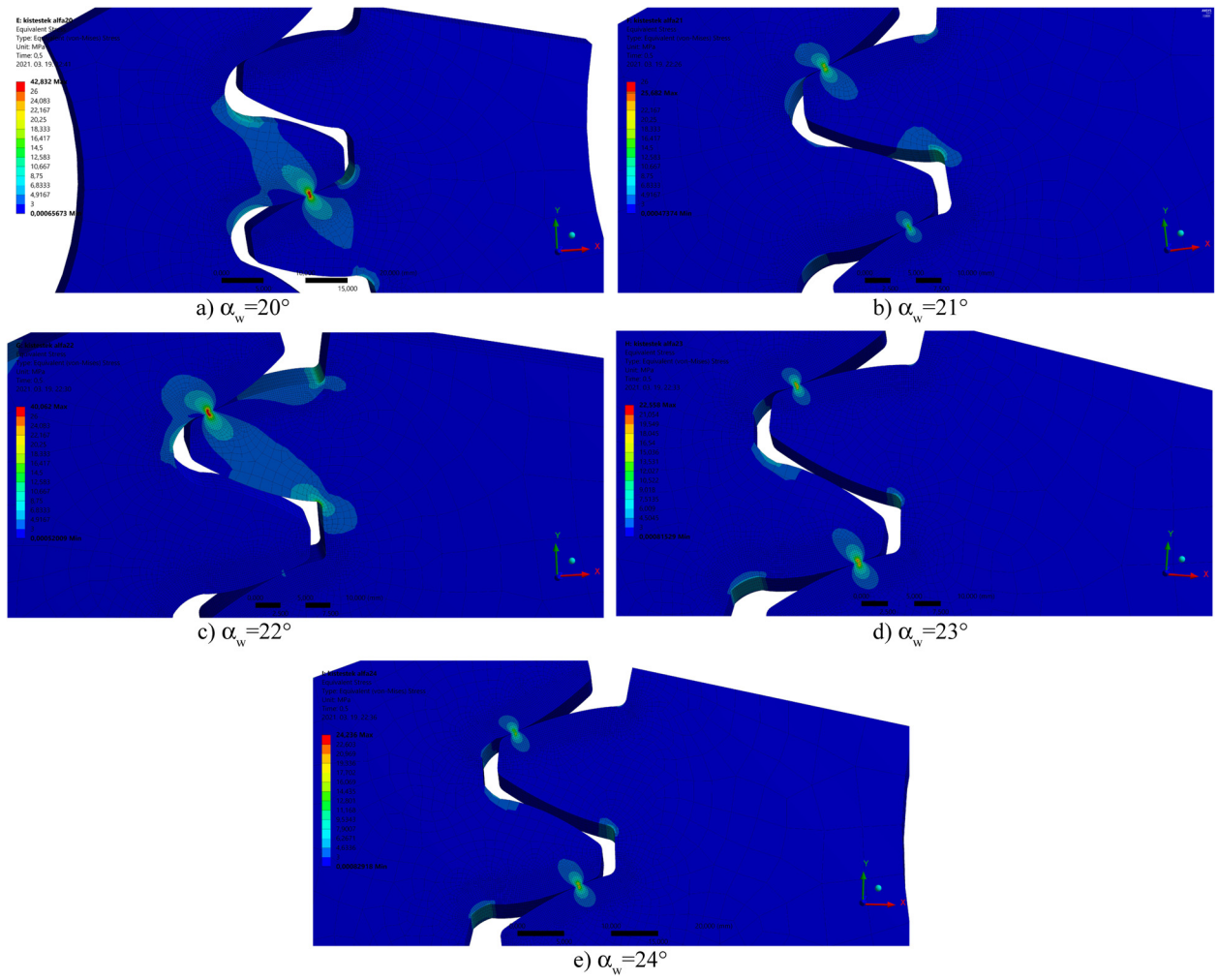


Fig. 20. The equivalent stress results ($t = 0.5$ s)

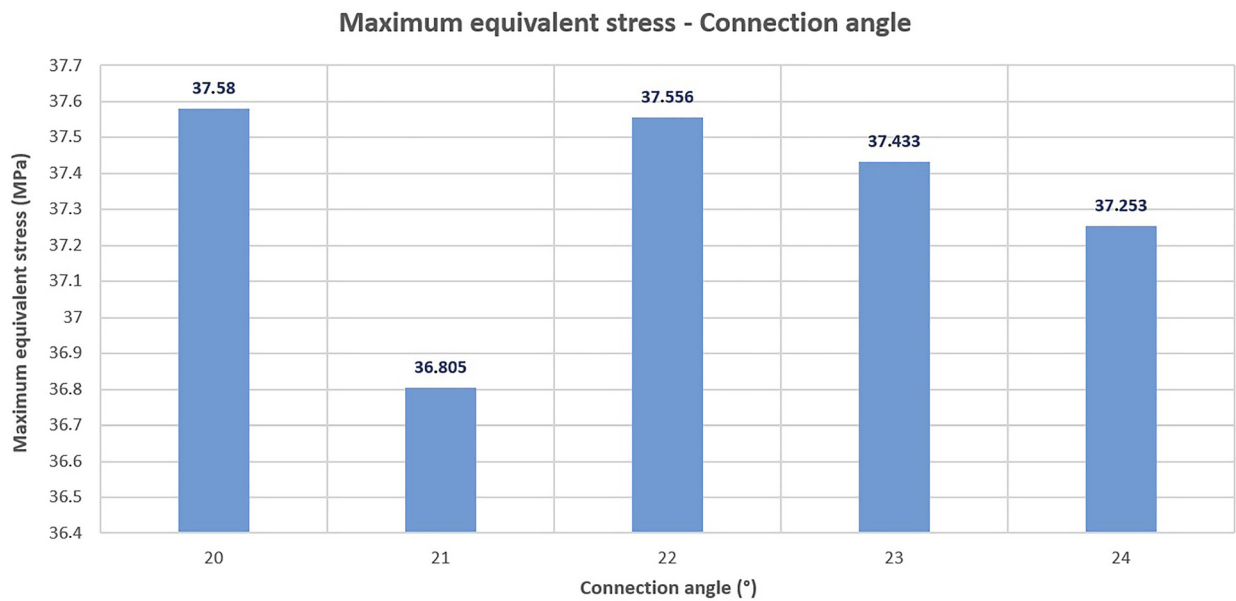
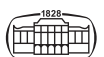


Fig. 21. The average results of the maximum equivalent stresses



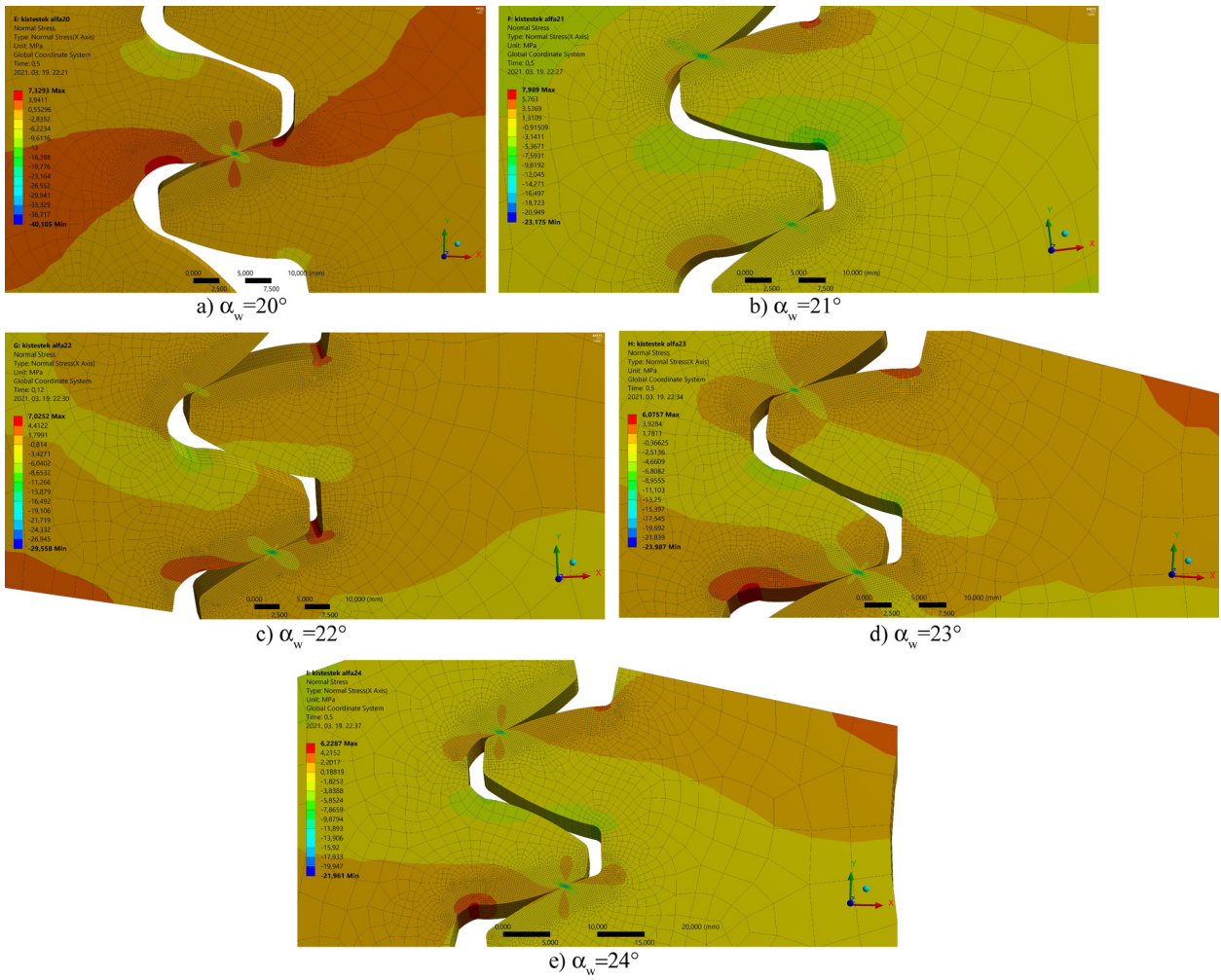


Fig. 22. The distribution of the maximum normal stress ($t = 0.5$ s)

Maximum normal stress - Connection angle

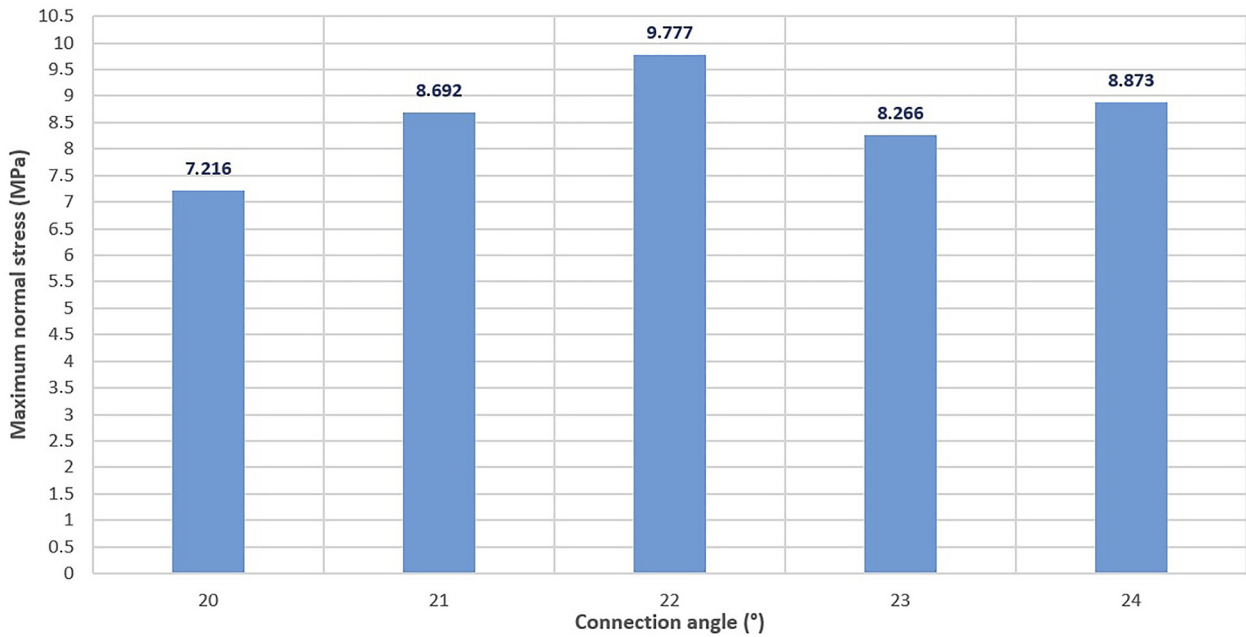


Fig. 23. The average results of the maximum normal stresses



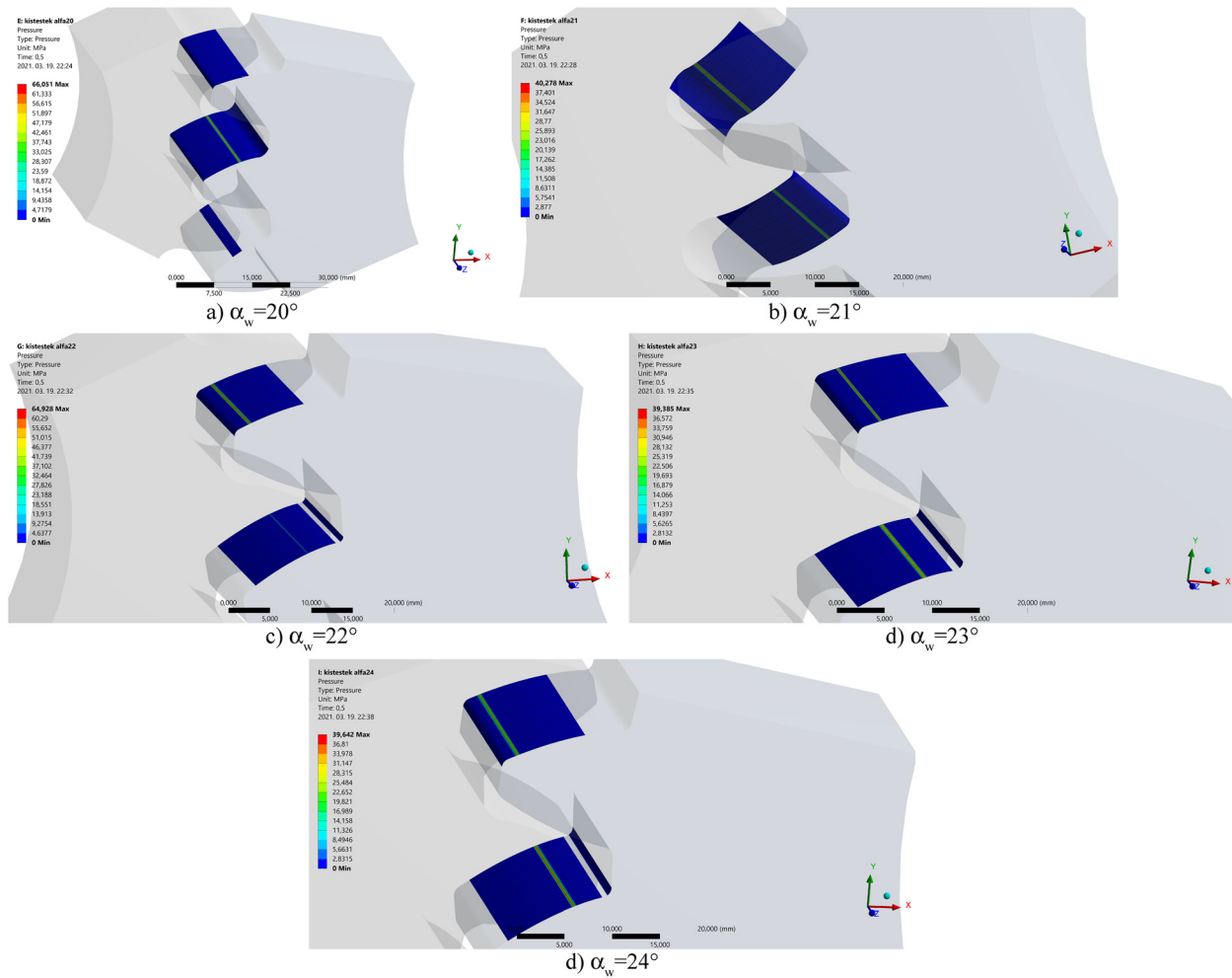


Fig. 24. The distribution of the maximum contact pressure ($t = 0.5$ s)

Maximum contact pressure - Connection angle

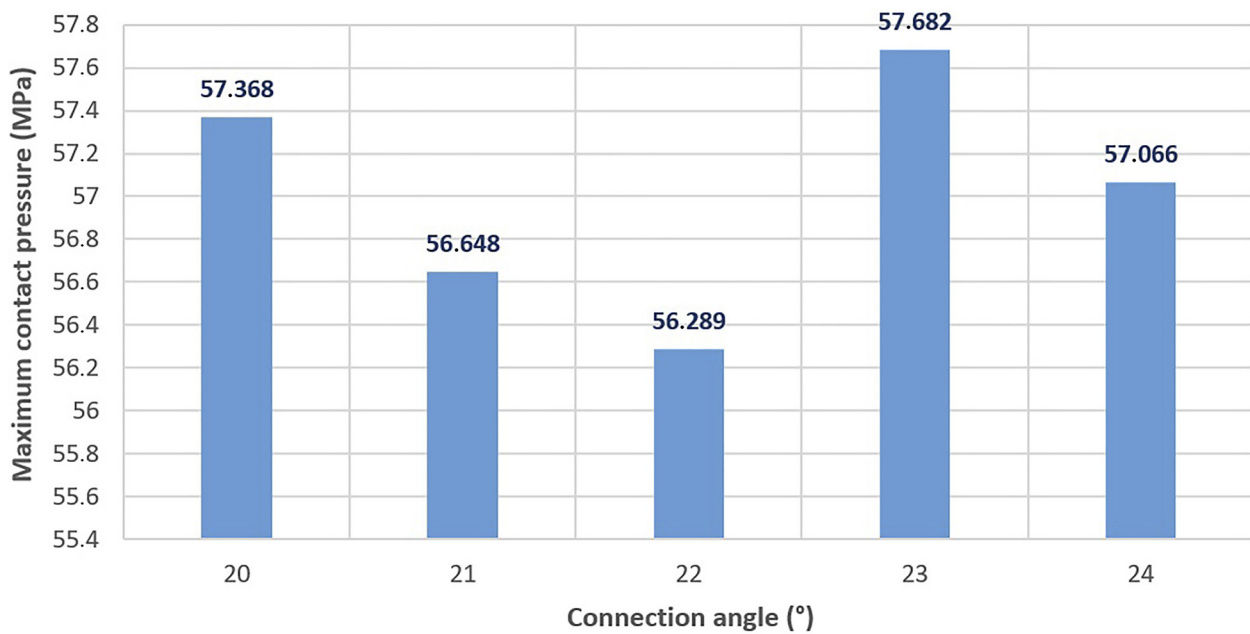


Fig. 25. The average results of the maximum contact pressure



enabled. During the analysis, the three teeth are continuously rolling down on each other.

4.3. Evaluation of the results

The equivalent and the normal stress, as well as the contact pressure are calculated by the software for each pair during the rolling down. The simulations are calculated in the global coordinate system. The results of the equivalent stress can be seen in Fig. 20 on an intermediary status ($t = 0.5$ s). I have to use average values due to the limitations of the computer capacity and the meshing. The average results can be seen in Fig. 21. The lowest result is received in the case of $\alpha_w = 21^\circ$. The highest result is in the case of $\alpha_w = 20^\circ$ (Fig. 21).

The results of the normal stress can be seen in Fig. 22 in an intermediary status ($t = 0.5$ s). The average results can be seen in Fig. 23. The lowest result is received in the case of $\alpha_w = 20^\circ$. The highest result is in the case of $\alpha_w = 22^\circ$ (Fig. 23).

The results of the contact pressure can be seen in Fig. 24 in an intermediary status ($t = 0.5$ s). The average results can be seen in Fig. 25. The lowest result is received in the case of $\alpha_w = 22^\circ$. The highest result is in the case of $\alpha_w = 23^\circ$ (Fig. 25).

5. CONCLUSION

This study looks into the field of CAD and TCA of spur gear pairs. The TCA is a subfield of the FEM. This area is the comprehensive research field of the connection analysis of various types of toothed gears where the tooth contact zone is analysed in mechanical and dynamical aspects based on different loads.

I have designed five types of spur gear pairs having normal teeth where only the pressure angle was different beside the constancy of the other initial parameters. I created a computer software in Matlab language to simplify the calculation and the designing process. Knowing the geometric parameters and the involute profile curves, the CAD models can be constructed. After that, the assembly and the beat examinations can be done by SolidWorks software to check the geometric accuracy.

Plenty of simulations can be performed by the Ansys software. Based on the references I worked on the Static Structural option which is mainly used in the field of connection analysis of toothed gear pairs.

I did comparative FEM analysis for a given geometry ($\alpha_w = 22^\circ$) among the Hertz (equivalent) stresses of the 2D and 3D models and the manual calculation. I approached the problem by Augmented Lagrange and Normal Lagrange formulations. The Normal Lagrange formulation proved better since the results were closer to the manual result. During the analysis I did parametric studies and convergence analyses to determine the comparative meshing possibility and the element size. Finally, I managed to get equal stress results, which are within 10–12% tolerance limit for the 2D and 3D models compared to the manual result. I also checked the similarity of the reaction forces on the FEM

models and the manual calculation. I received the same results.

Considering the five types of gear pairs, I compared the effect of modification of the pressure angle on the mechanical parameters (maximum equivalent and normal stress, and maximum contact pressure). I applied three connecting teeth for each pairs. I divided the 1 s duration of rolling down for 100 pieces for which I got the results calculated. I made diagrams to evaluate the average results by MS Excel program.

I created a unique FEM model and analysis process to compare these gear pairs as a function of the modification of the pressure angle. By refining the meshing better results can be achieved but a stronger computer would be required for this purpose. The used computer capacity was the following: Processor: Intel(R) Core(TM) i5-8300H CPU @ 2.30 GHz, Memory (RAM) size: 16,0 GB, System type: 64 bit operation system, Windows version: Windows 10 Enterprise. It is well-known that by smoothing the element size of the meshing, the calculation time is increasing exponentially, which would require more developed computer apparatus too.

ACKNOWLEDGEMENT

The work is supported by the EFOP-3.6.1-16-2016-00022 project. The project is co-financed by the European Union and the European Social Fund.

I appreciate the help and the simulation knowledge of assistant lecturer Dávid Huri.

I thank for the help of Péter Szurdoki, who is an English teacher. He reviewed the publication regarding the English grammar.

I would like to thank Professor Illés Dudás and Professor Alfonso Fuentes, who introduced this interesting TCA research field to me.

REFERENCES

- [1] S. Bodzás, "Computer aided designing and modelling of spur gear pairs having normal and modified straight teeth," *Int. Rev. Appl. Sci. Eng.*, vol. 10, no. 2, pp. 157–63, 2019, 7 p. <https://doi.org/10.1556/1848.2019.0019>.
- [2] S. Bodzás, "Analysis of the effect of the addendum modification coefficient for contact surfaces of spur gear," *Strojnický Casopis/J. Mech. Eng.*, vol. 69, no. 1, pp. 5–16, 2019, 12 p. <https://sciendo.com/doi/10.2478/scjme-2019-0001>.
- [3] S. Bodzás, "Tooth contact analysis of spur gear pairs having normal straight teeth in the function of the module," *Acta Facultatis Technicae Zvolen*, vol. 23, no. 2, pp. 91–103, 2018, 13 p. https://ft.tuzvo.sk/sites/default/files/aft_2_2018_journal_0.pdf.
- [4] J. Drobni, *Gépelemek III*. National Publisher, University of Miskolc, 1993, p. 231.
- [5] Gy. Erney, *Fogaskerekek*. Budapest: Technical Publisher, 1983, p. 460.



[6] V. Goldfarb, E. Trubachev, and N. Barmina, *Advanced Gear Engineering*. Springer, 2018, p. 197, ISBN 978-3-319-60398-8.

[7] F. L. Litvin, *A fogaskerékkapcsolás elmélete*. Budapest: Technical Publisher, 1972.

[8] F. L. Litvin and A. Fuentes, *Gear Geometry and Applied Theory*. Cambridge University Press, 2004, p. 800, ISBN 978 0 521 81517 8.

[9] T. Mankovits and D. Huri, *Modellezés és szimuláció (A lineáris rugalmasságtan és a végeelem módszer)*. University of Debrecen, lecture note, 2015, p. 183 [Online]. Available: https://mecheng.unideb.hu/sites/default/files/upload_documents/modellezesszimulacio_mankovits-huri.pdf. Accessed: Nov. 15, 2021.

[10] S. Moaveni, *Finite Element Analysis, Theory and Application with ANSYS*. Pearson Education Limited, 2015, p. 928, ISBN 10: 0-273-77430-1.

[11] Z. Terplán, *Gépelemek IV*. Budapest: Book Publisher, 1975, p. 220.

[12] I. Vörös, *Gépelemek III., Fogaskerekek*, 2nd Edition. Budapest: Book Publisher, 1958, p. 515.

[13] R. Golebski and A. Szarek, "Diagnosis of the operational gear wheel wear," *Tech. Gaz.*, vol. 24, no. 4, pp. 658-61, 2017, ISSN 1330-3651 (Print), ISSN 1848-6339 [Online]. Available: <https://doi.org/10.17559/TV-20180321171428>. Accessed: Nov. 16, 2021.

[14] Ansys Lecture 3: *Introduction to Contact*, Ansys Mechanical Structural Nonlinearities. Costumer Training Material, 2010 [Online]. Available: https://www.researchgate.net/profile/Musavir_Bashir/post/how_can_I_solve_such_type_of_problem_in_ansys_CFD/attachment/5ac9a4be4cde260d15d76e9f/AS%3A613009869258752%401523164350093/download/ansys+mech+structural.pdf. Accessed: Nov. 16, 2021.

NOMENCLATURE

Symbol	Unit	Parameter
ρ_1	[mm]	Curvature radius of the involute curve on the pinion
ρ_2	[mm]	Curvature radius of the involute curve on the gear
σ_H	[MPa]	Hertz (Equivalent) stress
α_p	[°]	Base profile angle ($\alpha_p = 20^\circ$)
α_w	[°]	Pressure angle
c'_0		Clearance factor ($c'_0 = 0.25$)
a	[mm]	Normal center distance
a₀	[mm]	Elementary center distance
CAD		Computer Aided Design
d_{a1}	[mm]	Outside circle diameter of the pinion

(continued)

Continued

Symbol	Unit	Parameter
d_{a2}	[mm]	Outside circle diameter of the gear
d_{b1}	[mm]	Base circle diameter of the pinion
d_{b2}	[mm]	Base circle diameter of the gear
d_{f1}	[mm]	Root circle diameter of the pinion
d_{f2}	[mm]	Root circle diameter of the gear
d_{p1}	[mm]	Pitch circle diameter of the pinion
d_{p2}	[mm]	Pitch circle diameter of the gear
d_{w1}	[mm]	Rolling circle diameter of the pinion
d_{w2}	[mm]	Rolling circle diameter of the gear
E₁	[MPa]	Young modulus of the pinion
E₂	[MPa]	Young modulus of the gear
F_c	[N]	Circumferential force
FEM		Finite Element Analysis
F_n	[N]	Normal force
h	[mm]	Whole depth
h_a	[mm]	Addendum
h_f	[mm]	Dedendum
l	[mm]	Tooth length
m	[mm]	Transverse module
p_p	[mm]	Circular pitch on the pitch circle
p_w	[mm]	Circular pitch on the rolling circle
r_{p1}	[mm]	Pitch circle radius of the pinion
r_{p2}	[mm]	Pitch circle radius of the gear
r_{w1}	[mm]	Rolling circle radius of the pinion
r_{w2}	[mm]	Rolling circle radius of the gear
s_{p1}	[mm]	Tooth (arc) thickness of the pinion on the pitch circle
s_{p2}	[mm]	Tooth (arc) thickness of the gear on the pitch circle
s_{w1}	[mm]	Tooth (arc) thickness of the pinion on the rolling circle
s_{w2}	[mm]	Tooth (arc) thickness of the gear on the rolling circle
TCA		Tooth Contact Analysis
u		Tooth ratio
x		Addendum modification coefficient
x₁		Addendum modification coefficient of the pinion
x₂		Addendum modification coefficient of the gear
z₁		Number of teeth of the pinion
z₂		Number of teeth of the gear
ω_1	[1/s]	Angular velocity on the pinion
\vec{n}_{2R}		Normal vector of the gear
\vec{v}_{2R}		Relative velocity vector of the gear
\vec{n}_{1R}		Normal vector of the pinion
\vec{v}_{1R}		Relative velocity vector of the pinion

Open Access. This is an open-access article distributed under the terms of the Creative Commons Attribution 4.0 International License (<https://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited, a link to the CC License is provided, and changes - if any - are indicated. (SID_1)

