

Infrared fixed point in quantum Einstein gravity

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ABSTRACT: We performed the renormalization group analysis of the quantum Einstein gravity in the deep infrared regime for different types of extensions of the model. It is shown that an attractive infrared point exists in the broken symmetric phase of the model. It is also shown that due to the Gaussian fixed point the IR critical exponent ν of the correlation length is $1/2$. However, there exists a certain extension of the model which gives finite correlation length in the broken symmetric phase. It typically appears in case of models possessing a first order phase transitions as is demonstrated on the example of the scalar field theory with a Coleman-Weinberg potential.

KEYWORDS: Models of Quantum Gravity, Renormalization Group

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1 Introduction

The renormalization group (RG) method can create a bridge between theories with different energy scales [1, 2]. Generally, one starts with the high-energy ultraviolet (UV) Lagrangian of the model, which describes well the short-range interaction of the elementary excitations. Furthermore, the UV Lagrangian usually has a very simple form with a small number of couplings due to a number of symmetry restrictions. On the other hand, the low-energy infrared (IR) limit of the models is usually very complicated, where the long-range interactions may induce infinitely many new couplings, non-localities, or global condensates breaking some symmetries. The IR theory can be obtained by the successive elimination of the degrees of freedom with the help of the RG technique. Similar problems appear in various field-theoretic quantum gravity models if one goes beyond the UV regime [3–5]. The quantum Einstein gravity (QEG) seems to be non-renormalizable according to perturbative considerations. However it was shown, that the model has an UV fixed point, which makes QEG renormalizable and provides the asymptotic safety [3, 6–13]. The phase space contains a non-Gaussian UV fixed point (NGFP) and a trivial Gaussian fixed point (GFP). The former is an UV attractive, i.e. the NGFP attracts the RG trajectories when the UV cutoff is removed to infinity. It is a focal-type fixed point characterized by complex exponents. The GFP is sometimes referred an IR one, nevertheless it is a hyperbolic or saddle point type fixed point, therefore it is rather a crossover type point. The fixed points can be characterized by universal scaling laws of the corresponding couplings with the appropriate critical exponents. The scalings, and the exponent in the UV NGFP was extensively studied so far [14–19].

Our goal is to investigate the low-energy IR limit of QEG. Recently it has been shown that an attractive IR fixed point exists in quantum gravity [20]. The existence of the attractive IR fixed point was also investigated in [21, 22] for scalar models, and the IR physics of QEG was also investigated in detail [23–28] from several aspects.

It was shown that the broken symmetric phase of the d -dimensional $O(N)$ scalar model [29] and the 2-dimensional (2d) sine-Gordon (SG) model [30, 31] also possesses an attractive IR fixed point. In these models one can define the correlation length ξ as the reciprocal of the scale k_c where the RG evolution stops enabling us to determine the corresponding critical exponent ν of ξ in the vicinity of the IR fixed point. This method enabled us to determine the critical exponents and the type of the phase transition appearing in scalar models, e.g. it was shown in the $O(N)$ model that the exponent ν at the IR fixed point equals to one that is obtained in the vicinity of the crossover Wilson-Fisher (WF) fixed point [29]. Furthermore, the infinite nature of the phase transition in the SG model was also uncovered by this method [30], where the IR fixed point has the same exponent ν of ξ as the Kosterlitz-Thouless (KT) type fixed point being a crossover point too. These examples suggest that the attractive IR fixed point inherits the value of the exponent ν from the value taken at the fixed point passed by the RG trajectory previously, which is typically a hyperbolic type crossover singular point. This comes from the fact that the broken symmetric phase is characterized by a condensate built up by the bulk amount of soft modes, and its global nature appears along the whole flow in the broken symmetric phase.

This argument implies that in quantum gravity the IR fixed point of the broken symmetric phase may be strongly affected by the GFP. We show numerically that the critical exponent $\nu = 1/2$ has the same value at the GFP and at the IR one. Approaching the IR fixed point the scale parameter k of the RG flows tends to zero, its reciprocal determines a diverging correlation length which signals a continuous phase transition.

The IR fixed point is situated at finite dimensionless cosmological constant, so the dimensionful coupling should tend to zero. We note, however, that the correlation length does not diverge in the neighborhood of the critical point when the phase transition is of a first order type [32]. If there is a first order phase transition in the IR limit, then the dimensionful cosmological constant remains finite, which may provide a possibility to handle the ‘famous’ cosmological constant problem.

In this article we show that certain extensions of the QEG may show such scaling behavior in the IR limit, which is typical in models having a first order phase transition. In order to demonstrate the first order type scaling, we investigate the scalar model with Coleman-Weinberg (CW) potential which possesses a $U(2) \times U(2)$ symmetry [33–40]. One can consider the CW model as a prototype of the first order transition. Certain limit of the model shows an $O(8)$ symmetry giving second order phase transition and having a crossover type Wilson-Fisher and an IR fixed points. In this case the degeneracy induced scaling can give us a diverging correlation length. When the general situation takes place in the CW model, the first order of the phase transition is signaled by the appearing of a non-trivial minimum of the potential during the RG evolution. To detect this, one has to determine the evolution of the whole potential [40]. It mathematically means that one should solve a partial differential equation (or its discretized system of ordinary differential

equations version), which solution is quite involved, and can be unstable especially in the broken symmetric phase [41]. Our method gives an alternative identification for the first order phase transition, which can be handled numerically in an extremely easy way. The dynamically generated degeneracy scale appears in the CW model, too. However, as the RG trajectory approaches the critical point the degeneracy scale does not tend to zero, but it takes a finite value implying a finite correlation length. For vanishing value of the coupling λ_2 of the CW model there is a continuous phase transition, while for nonzero values of λ_2 the correlation length remains finite in the IR limit and a first order phase transition is shown up. In this manner the identification of the order of the phase transition can be reduced to a fine tuning of the couplings to their critical values. The other advantage of this method is that it is not necessary to know where the fixed point is situated in the phase space.

As a rule, various extensions of QEG do not change the type of the scaling in the IR limit, although it was shown [3], that the IR flows strongly depend on the choice of the extension. Usually a second order phase transition appears in the IR limit, but a special extension of QEG is also known in which a first order phase transition takes place.

It seems to contradict to the fact that a continuous phase transition appears in the UV regime. Let us note, however, that there are several models which show up phase transitions of different orders in the UV and in the IR regimes. For example, in the case of the 2d massive SG model [42, 43] the UV scaling predicts a KT-type, infinite order transition, while a complete analysis, which takes into account all the quantum fluctuations shows that a second order phase transition takes place in the model [42–44].

The paper is organized as follows. In section 2 we give a short summary of the RG method. In section 3 the RG treatment of the CW model is presented, and the scaling properties of a first order phase transition are discussed. The IR fixed point of the phase space of QEG is mapped out and the values of the critical exponent ν belonging to the correlation length are determined in section 4 for various dimensions. In section 5 we determined the value of ν for various types of extensions of QEG, and put much emphasis on finding such an extension where ξ of QEG does not diverge suggesting a first order phase transition in its IR limit. Finally, in section 6 the conclusions are drawn up.

2 Renormalization

We consider the RG treatment of the Euclidean QEG. We note that the Lorentzian form of the model is also investigated thoroughly [45]. The Wetterich RG evolution equation for the effective average action Γ_k reads as

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{\partial_t R_k}{\Gamma_k' + R_k} \tag{2.1}$$

with the ‘RG time’ $t = \ln k$, the prime denotes the differentiation with respect to the field variable, and the trace Tr denotes the integration over all momenta and the summation over the internal indices. Eq. (2.1) is valid for scalar, fermionic or gauge fields, too. The function R_k plays the role of the IR regulator. We usually use the optimized one [46–48] of the form

$$R_k = (k^2 - q^2)\theta(k^2 - q^2) \tag{2.2}$$

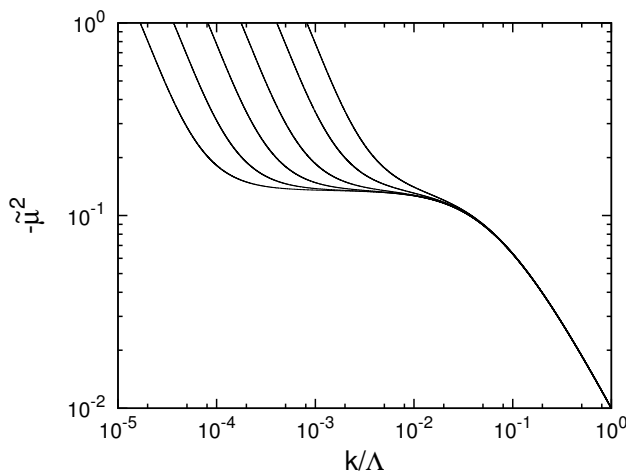


Figure 1. The scaling of the coupling $\tilde{\mu}$ in the broken symmetric phase for the CW model is presented. The curves correspond to evolutions with different initial conditions. The momentum scale that corresponds to the degeneracy tends to zero as the the reduced temperature approaches zero.

which provides fast convergence in the flows and analytic formulae for the evolution equations. We impose a certain functional ansatz for the effective average action in the local potential approximation which contains couplings. From the Wetterich RG equation in eq. (2.1) one can get a coupled system of evolution equations for the dimensionless couplings which is solved numerically.

The momentum scale k covers the momentum interval from the UV cutoff Λ to zero, unless a singularity occurs at some finite critical momentum scale k_c . During the numerical calculations we set $\Lambda = 1$.

3 First order phase transition

The CW model exhibits a $U(2) \times U(2)$ symmetry. We summarized the derivation presented in [40] in appendix A. It leads to the following RG equations for the dimensionless couplings,

$$\begin{aligned}
 \partial_t \tilde{\mu}^2 &= -2\tilde{\mu}^2 - \frac{5\tilde{\lambda}_1 + 18\tilde{\lambda}_2}{9\pi^2(1 + \tilde{\mu}^2)^2}, \\
 \partial_t \tilde{\lambda}_1 &= -\tilde{\lambda}_1 + \frac{8(2\tilde{\lambda}_1^2 + 9\tilde{\lambda}_1\tilde{\lambda}_2 + 54\tilde{\lambda}_2^2)}{9\pi^2(1 + \tilde{\mu}^2)^3}, \\
 \partial_t \tilde{\lambda}_2 &= -\tilde{\lambda}_2 - \frac{4(\tilde{\lambda}_1\tilde{\lambda}_2 + 2\tilde{\lambda}_2^2)}{3\pi^2(1 + \tilde{\mu}^2)^3},
 \end{aligned}
 \tag{3.1}$$

where a second order phase transition appears when λ_2 does not evolve. Then in the broken symmetric phase the correlation length diverges as the reduced temperature t tends to zero. It is defined as $t \sim \tilde{\lambda}_{1\Lambda}^* - \tilde{\lambda}_{1\Lambda}$, where the lower index Λ refers to the value at the UV scale Λ and the upper index $*$ denotes the corresponding UV critical value. We numerically determined the divergence of ξ and show the obtained results in figure 1. The RG equations in eq. (3.1) become degenerate, which means that the expression of the form $1 + \tilde{\mu}^2$ in the denominator tends to zero. As the temperature approaches its critical value,

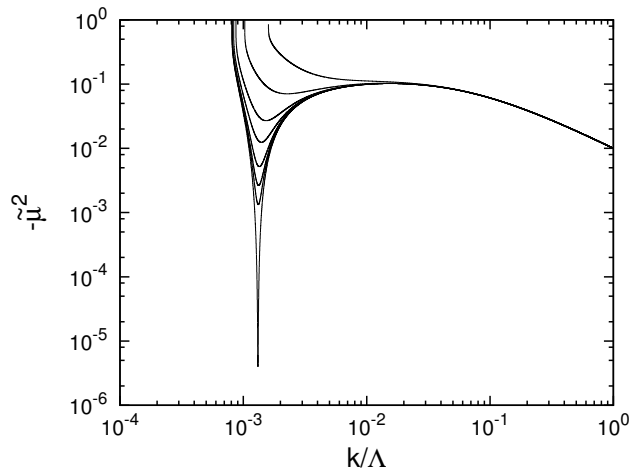


Figure 2. The scaling of the coupling $\tilde{\mu}$ in the broken symmetric phase is shown. The curves correspond to evolutions with different initial conditions. The degeneracy momentum scale tends to a finite value as the reduced temperature approaches zero.

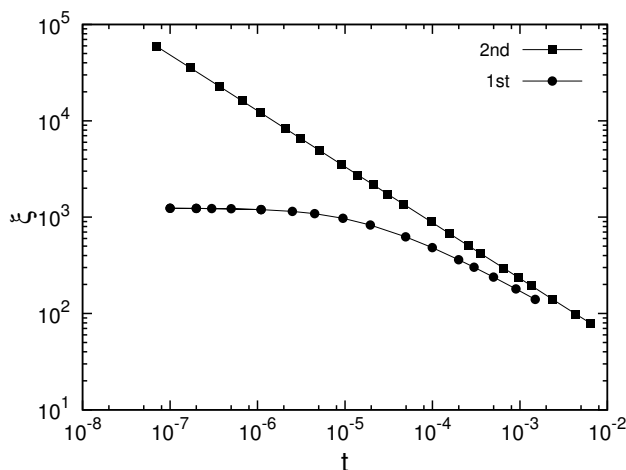


Figure 3. The dependence of the correlation length on the reduced temperature is presented. For the second order phase transition ξ diverges as $t \rightarrow 0$, while in the case of the first order phase transition it tends to a constant value for $t \rightarrow 0$.

the momentum scale of the degeneracy tends to zero and the correlation length diverges signaling a continuous phase transition in the model.

When the coupling λ_2 evolves then the order of the phase transition changes from a second to a first order one. This can also be caught by the flow of $\tilde{\mu}^2$ in figure 2. As the temperature approaches its critical value, the negative coupling $-\tilde{\mu}^2$ does not grow monotonically but it tries to change its sign. The change appears when $t = 0$. The degeneracy also appears but its corresponding momentum scale k does not tend to zero as $t \rightarrow 0$ but the flows stop at the same critical value of k . We plotted the scaling of the correlation length ξ as the function of the reduced temperature t in figure 3. The results show that ξ diverges as a power law $\xi \sim t^{-1/2}$ in the case of the second order phase transition, while ξ tends to a constant value for a first order phase transition.

Thus, the degeneracy induced IR scaling of the correlation length can account for the type of the phase transition in a very simple way.

4 Quantum Einstein gravity

The QEG effective action is defined as

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^d x \sqrt{\det g_{\mu\nu}} (-R + 2\Lambda_k) \tag{4.1}$$

with the metrics $g_{\mu\nu}$, the Ricci scalar R and with the couplings, i.e. with the dimensionful Newton constant G_k and the cosmological constant Λ_k . Eq. (2.1) provides the evolution equations for the dimensionless couplings $g = k^{d-2}G_k$ and $\lambda = k^{-2}\Lambda_k$. The phase space of the QEG is spanned by the evolving parameters g and λ . By using the optimized RG regulator one obtains the following evolution equations [49],

$$\begin{aligned} \partial_t \lambda &= -2\lambda + \frac{g}{2}d(d+2)(d+5) \\ &\quad - d(d+2)\frac{g}{2} \frac{(d-1)g + \frac{1-4\lambda(1-1/d)}{d-2}}{g-g_b}, \\ \partial_t g &= (d-2)g + \frac{(d+2)g^2}{g-g_b} \end{aligned} \tag{4.2}$$

with

$$g_b = \frac{(1-2\lambda)^2}{2(d-2)}. \tag{4.3}$$

One can introduce the gravitation anomalous dimension

$$\eta = \frac{(d+2)g}{g-g_b}. \tag{4.4}$$

In case of $d = 4$ the RG flow equations in eq. (4.2) possess two fixed points, which can be analytically identified, these are shown in figure 4 (the points UV and G). There is a UV non-trivial fixed point (NGFP) at finite values of $g_{UV}^* = 1/64$ and $\lambda_{UV}^* = 1/4$. Here the eigenvalues of the linearized RG transformation are complex, and the RG trajectories run away with decreasing scale parameter k from the UV fixed point as if it were a repulsive focal point, or otherwise one can say, that one has to do with a UV attractive fixed point. Furthermore, there is a trivial, saddle point-like Gaussian fixed point at the origin of the phase space $g_G^* = 0$ and $\lambda_G^* = 0$. There the linearized RG transformation has the eigenvalues $s_1 = -2$ and $s_2 = 2$. The negative reciprocal of the negative eigenvalue gives the exponent $\nu = 1/2$. The GFP can be considered as a crossover (CO) fixed point between the UV and the IR scaling regimes. The particular trajectory running from the UV fixed point into the Gaussian one, the separatrix splits the phase space into two regions according to the two phases of the model (see figure 4). The trajectories approaching the separatrix from the left give negative values for the cosmological constant and vanishing Newton constant in the IR limit, the corresponding phase can be called the strong-coupling or symmetric one [3, 50]. There the scale k has a well-defined limit $k \rightarrow 0$. Other trajectories

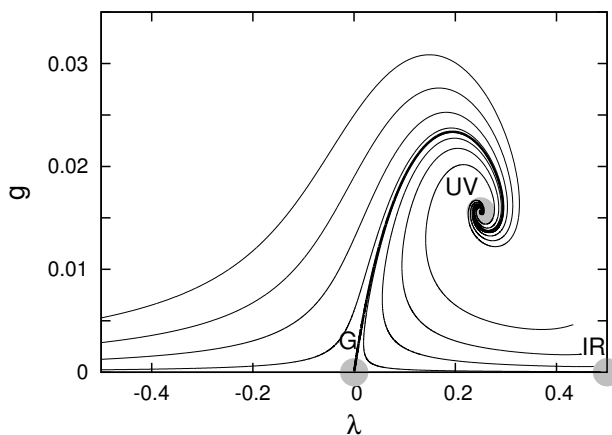


Figure 4. The phase structure of quantum Einstein gravity. There is an UV non-Gaussian, a crossover Gaussian and an IR fixed point. The thick line represents the separatrix.

getting around the separatrix from the right give large positive values of λ and small Newton constant if the RG flows tend to the IR regime. This phase can be called the weak-coupling or broken symmetric phase. There are several models known which have similar phase structure. The non-linear sigma model for $d > 2$ exhibits a symmetric and a broken symmetric phase, furthermore the model has a NGFP in the UV [51–53]. The 2d SG model also has two phases and an UV NGFP [31]. There the UV limit leads to singularity of the RG equations. A similar behavior can be found in the IR behavior of the broken symmetric phase. The latter seems to be typical, since there are many examples where the RG flows tend to a singular regime in the broken symmetric phase of the model. We argue that this degeneracy induced singularity is a consequence of the appearing IR fixed point [29–31, 54]. We show that the IR fixed point also exists in QEG in the positive cosmological constant regime.

The RG flow shown in figure 4 indicates that there is an attractive IR fixed point in the broken symmetric phase of the model at $g_{\text{IR}}^* = 0$ and $\lambda_{\text{IR}}^* = 1/2$. Although the IR fixed point does not seem to satisfy the eqs. in (4.2), since they give expressions like $0/0$, a suitable reparametrization of the couplings enables one to uncover this fixed point [21]. The singularity of the RG flows seems to be a numerical artifact of the strong truncation of the functional ansatz for the effective action in eq. (4.1), but it was shown for other models in [29] that such a singularity possesses a specific scaling behavior induced by the IR fixed point, therefore it has significant physical importance. This takes place in QEG, too. We introduce the new couplings according to $\chi = 1 - 2\lambda$, $\omega = 4g - (1 - 2\lambda)^2$ and the new 'RG time' $\partial_\tau = \omega \partial_t$. We note that the idea of redefining the 'RG time' was already used in QEG [58]. Then in the case of $d = 4$ the evolution equations can be written as

$$\begin{aligned}
 \partial_\tau \chi &= -4\omega + 2\chi\omega(8 + 21\chi) \\
 &\quad + 24\omega^2 + 6\chi^2(3\chi(\chi + 1) - 1), \\
 \partial_\tau \omega &= 8\omega^2(1 - 6\chi) - 2\chi(42\chi^2 + 9\chi - 4) \\
 &\quad - 6\chi^3(\chi(6\chi + 5) - 2).
 \end{aligned}
 \tag{4.5}$$

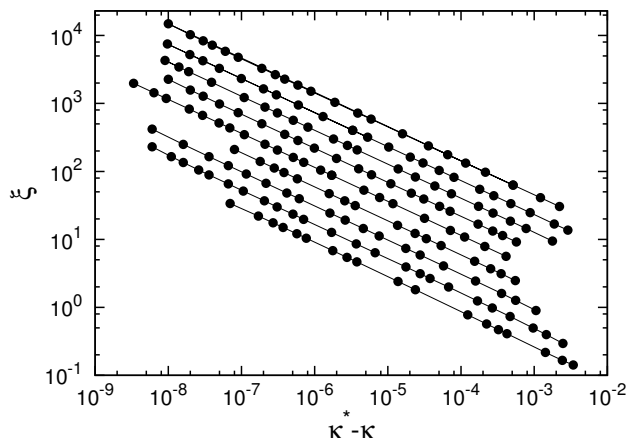


Figure 5. The scaling of the correlation length, giving $\nu = 1/2$ for the exponent. For simplicity we fixed the UV value of g_Λ .

These flow equations have three different fixed points. The Gaussian fixed point appears at $\omega_G^* = -1$ and $\chi_G^* = 1$ with hyperbolic nature. The focal-type UV NGFP can be identified by the fixed point $\omega_{UV}^* = -3/16$ and $\chi_{UV}^* = 1/2$. However a new fixed point appears at $\omega_{IR}^* = 0$ and $\chi_{IR}^* = 0$ that can be identified with the IR fixed point and corresponds to $g_{IR}^* = 0$ and $\lambda_{IR}^* = 1/2$. The IR fixed point shows very slight, marginal attractivity. The above reparametrization of the original couplings could successfully uncover the IR physics of the d -dimensional $O(N)$ model [29], even in the 2d $O(2)$ case, where an essential scaling signalling an infinite order phase transition appears, furthermore this simple trick was capable of describing the phase structure and the order of the phase transition in periodic models [30, 31, 54].

The gravitation anomalous dimension η shows singular behavior in the IR limit of the weak coupling phase as $g \rightarrow g_b$ at a finite value of k_c . It shows that there are no modes in the theory which can be characterized by the scale where $k < k_c$. However, if we introduced the RG scale $k' = k_c - k$, then the IR scaling behavior would become visible just as in [29]. The new RG scale does not cause any trouble, since it is just a bookkeeping device to enumerate the modes to be integrated out. The scale k_c can be identified by the reciprocal of the correlation length, i.e. $\xi \sim 1/k_c$. The singularity appears due to the bulk amount of soft modes close to the instability of the RG flows [50, 55–57]. As the UV value of the couplings g and λ approach their critical value, the flow reaches smaller and smaller value of k_c , giving diverging ξ . We determined the values of ξ as the function of $\kappa = g_\Lambda \lambda_\Lambda$ measured from its UV critical value κ^* , and plotted it in figure 5. The results show that ξ scales in the IR scaling regime according to a power law function

$$\xi \sim (\kappa - \kappa^*)^{-\nu}, \tag{4.6}$$

with exponent $\nu = 1/2$. We considered the problem with varying dimension d , and plotted the scaling of ξ for $d = 4 \dots 11$. We obtained that the IR scaling behavior and the value of ν is insensitive to the dimension.

The value of ν obtained in the IR is in contrast to the UV result $\nu = 1/3$ [14, 15]. Furthermore the UV exponent ν can also be related to the imaginary part θ'' of the scaling exponent in the UV fixed point as $\nu = 1/\theta''$ [16–19].

The apparent discrepancy can be easily resolved with the following argument. The fixed points for QEG determine well separated scaling regimes in the phase space, and the critical exponents should be calculated in each regions one by one, therefore it is not surprising that we obtain different values for ν . Around the UV NGFP one obtains $\nu_{UV} = 1/3$, around the crossover GFP one has $\nu_{CO} = 1/2$, which shows the mean field nature of the model there. In the IR regime we have $\nu_{IR} = 1/2$. The latter two values coincide. It is not evident, since other exponents in the CO and in the IR regime may differ. The value of η is approximately zero in the CO, while in the IR it diverges. Similar behavior appeared in the 2d SG model [30]. The coincidence of ν in the CO and in the IR may come from the fact that the condensate can affect the CO regime due to its global property.

There is an obvious similarity between the phase structures of the QEG model and scalar models. They usually contain a spontaneously broken and a symmetric phase and it seems that the broken phase contains an IR fixed point. In the $O(N)$ model the WF fixed point, in the 2d SG model the Kosterlitz-Thouless (KT) fixed point plays the role of the CO one. These are the analogues of the crossover Gaussian fixed point in the QEG model. The $O(N)$ model, the 2d SG model, and the QEG bears an IR fixed point and the IR value of the exponent ν of ξ equals the the value obtained in the corresponding CO fixed point.

The coincidence may suggest that the IR fixed point analysis is unnecessary, since the CO fixed point has all the information on ν . We note however, that there are models where there are no CO fixed points but the IR one exists, which provides the only possibility to get the value of ν and get the order of the phase transition [54]. On the other hand it is much easier to calculate numerically the value of ν in the IR than in the CO, since the former needs a fine tuning of (usually) one coupling without any knowledge of the fixed point, while in the latter case we have to find the exact place of the fixed point which is a difficult mathematical task.

5 Extensions of QEG

Using different forms for the IR cutoff to suppress the low momentum modes of the graviton and the ghost fields [59–62] one can get evolution equation of qualitatively same form including equations which can be also singular at certain values of the cosmological constant. Further extension of QEG can be obtained via e.g. including matter field [63–69] or higher order terms in the curvature scalar [70–74]. Considering one possibility of the IR regularization [3, 58] one obtains e.g.

$$\begin{aligned} \partial_t \lambda &= -2\lambda + \frac{g}{6\pi} \frac{3 - 4\lambda - 12\lambda^2 - 56\lambda^3 + \frac{107-20\lambda}{12\pi}g}{(1 - 2\lambda)^2 - \frac{1-10\lambda}{12\pi}g}, \\ \partial_t g &= 2g - \frac{g^2}{3\pi} \frac{11 - 18\lambda + 28\lambda^2}{(1 - 2\lambda)^2 - \frac{1-10\lambda}{12\pi}g} \end{aligned} \tag{5.1}$$

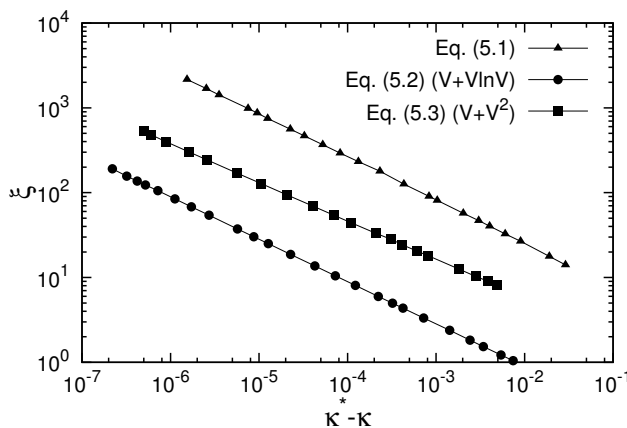


Figure 6. The scaling of the correlation length for various extensions of QEG. The scaling shows second order phase transition with exponent $\nu = 1/2$.

for the couplings. Our numerical results showed that the IR scaling is similar to the previous ones obtained from eqs.in (3.1). We got a similar second-order phase transition, with the same exponent $\nu = 1/2$, as is shown in figure 6 (indicated by triangles).

Another possible extension of QEG can be achieved by introducing terms to its effective action containing the functions of the Euclidean spacetime volume $V = \int d^d x \sqrt{\det g_{\mu\nu}}$. These terms introduce new couplings, which increases the dimension of the phase space.

When the new term has the form of $V + V \ln V$, with its coupling u , then the extended evolution equations become

$$\begin{aligned} \partial_t \lambda &= -2\lambda + \frac{g}{\pi} \left(\frac{5}{1 - 2\lambda - u} - 4 \right), \\ \partial_t g &= (d - 2)g, \\ \partial_t u &= -2u + \frac{10}{\pi} \frac{gu}{(1 - 2\lambda - u)^2} \end{aligned} \tag{5.2}$$

with the choice of the optimized cutoff. If we use the term of the form $V + V^2$ with the corresponding coupling w we obtain

$$\begin{aligned} \partial_t \lambda &= -2\lambda + \frac{g}{\pi} \left(\frac{5}{1 - 2\lambda} - 4 \right) + \frac{32gw}{1 - 2\lambda}, \\ \partial_t g &= (d - 2)g, \\ \partial_t w &= -6w + \frac{5gw}{\pi(1 - 2\lambda)^2} + \frac{1024\pi gw^2}{(1 - 2\lambda)^3}. \end{aligned} \tag{5.3}$$

The scaling of ξ is shown in figure 6 for different types of extensions, too. The results show, that the extended models exhibit a second order phase transition with the same critical exponent $\nu = 1/2$. This IR scaling is also driven by the GFP similarly to the previous results.

One can learn from these examples that if the hyperbolic-type GFP exists then it results in a continuous phase transition in the IR limit with a diverging correlation length.

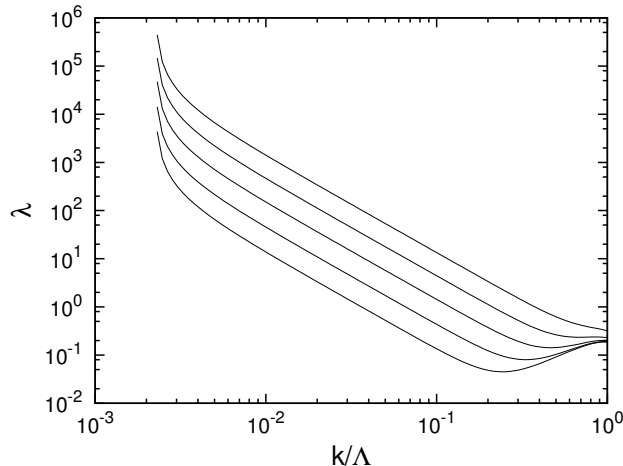


Figure 7. The flow of the cosmological constant for the extension $V + \sqrt{V}$ for various initial values of λ_Λ , $g_\Lambda = 1$ and $v_\Lambda \approx 5$. There is a finite momentum scale k_c where the evolution stops.

The extensions of QEG can introduce new couplings and increase the dimension of the phase space. There can appear further relevant and irrelevant couplings which can modify the value of the exponent ν from its mean field value ($\nu = 1/2$) to its physical one just as in the case of the $O(N)$ models [29]. Other extensions may give new fixed points or shift the GFP from the origin [75] which might strongly affect the IR scaling behavior.

However, if we choose the $V + \sqrt{V}$ as an additive term to the effective action with the coupling v then the evolution equations are [28]

$$\begin{aligned} \partial_t \lambda &= -2\lambda + 8\pi g \left[-\frac{1}{2\pi^2} + \frac{1 - 2\lambda}{v^2} \right], \\ \partial_t g &= (d - 2)g, \\ \partial_t v &= \frac{8\pi g}{v}. \end{aligned} \tag{5.4}$$

We note that the equations lose their validity when $v \rightarrow 0$. It is apparent that the GFP does not exist in this type of extension. Since the continuous-type phase transition is strongly related to the existing hyperbolic-type GFP, we do not necessarily expect a second order phase transition in the IR with diverging correlation length ξ , in principle any type of phase transition might appear [54].

There exists an analytic solution of the flow equation in eqs. (5.4) which reads as

$$\lambda = -\frac{8G^2(2k^6 - 3k^4\Lambda^2 + \Lambda^6) + 6G(\Lambda^4 - k^4) - 3\lambda v_\Lambda^2}{3(v_\Lambda^2 - 8\pi G(\Lambda^2 - k^2))}. \tag{5.5}$$

The flows of the cosmological constant are plotted in figure 7. It shows that a dynamical momentum scale k_c appears during the flow, where the evolution of λ becomes singular and stops. It means that in the broken symmetric phase one cannot go to an arbitrary small scale k , implying that the correlation length does not diverge. According to the example of the CW model one suggests that there is a first order phase transition between the weak- and the strong-coupling phases. This extension of QEG seems to make the originally second order phase transition into a first order one, due to the vanishing GFP there.

This result also implies that the dimensionful cosmological constant remains finite in the IR, which is needed to explain the cosmological constant problem.

6 Summary

We investigated the infrared behavior of the quantum Einstein gravity with the help of the renormalization group method. We showed that there exists an attractive IR fixed point in the broken symmetric phase of the model. It is generally characterized by a diverging correlation length with the corresponding exponent $\nu = 1/2$, showing the mean field nature of the continuous phase transition which is inherited by the hyperbolic crossover Gaussian fixed point.

This property holds for any dimension, but not for any type of extension for the effective action. We showed that there exists such type of extension, where the IR degeneracy defined correlation length does not diverge implying a first order phase transition in the IR limit. It seems to be the consequence of the disappearing GFP in this extension. The mechanism is demonstrated via the Coleman-Weinberg model, which is a typical example for first order transitions.

The appearance of the first order phase transition gives a finite but small value of IR dimensionful cosmological constant, which may suggest a possible explanation of the cosmological constant problem.

Acknowledgments

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A The Coleman-Weinberg model

The Lagrangian of the $U(2) \times U(2)$ symmetric scalar model is given by [40]

$$\mathcal{L} = \frac{1}{2} \text{tr}[\partial_\mu \Phi \partial^\mu \Phi^\dagger] + \frac{1}{2} \mu^2 [\Phi \Phi^\dagger] - g_1 (\text{tr}[\Phi \Phi^\dagger])^2 - g_2 \text{tr}[\Phi \Phi^\dagger \Phi \Phi^\dagger], \quad (\text{A.1})$$

with the dimensionful couplings μ^2 , g_1 , g_2 and the 2×2 matrix field variable which is parametrized as

$$\Phi = \Sigma + i\Pi = \sum_{\mu} t_{\mu} (\sigma_{\mu} + i\pi_{\mu}), \quad (\text{A.2})$$

where t_{μ} are the $U(2)$ generators, $\text{tr}[t_{\mu} t_{\nu}] = \delta_{\mu\nu}$, and

$$\begin{aligned} \Sigma &= \begin{pmatrix} \frac{1}{\sqrt{2}}(a^0 + \sigma) & a^+ \\ a^- & \frac{1}{\sqrt{2}}(-a^0 + \sigma) \end{pmatrix}, \\ \Pi &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + \eta) & \pi^+ \\ \pi^- & \frac{1}{\sqrt{2}}(-\pi^0 + \eta) \end{pmatrix}, \end{aligned} \quad (\text{A.3})$$

where $a^0 = \sigma_3$, $a^\pm = (a^1 \mp ia^2)/\sqrt{2} = (\sigma_1 \mp i\sigma_2)/\sqrt{2}$, $\sigma = \sigma_0$, $\pi^0 = \pi_3$, $\pi^\pm = (\pi_1 \mp i\pi_2)/\sqrt{2}$ and $\eta = \pi_0$. The potential of the model becomes

$$U = V + W\xi, \tag{A.4}$$

where

$$\begin{aligned} V &= \frac{1}{2}\mu^2\phi^2 + \frac{1}{24}\lambda_1\phi^2, \\ W &= \lambda_2, \end{aligned} \tag{A.5}$$

with

$$\begin{aligned} \varphi &= \sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{a}^2, \\ \xi &= (\sigma^2 + \vec{\pi}^2)(\eta^2 + a^2) - (\sigma\eta - \vec{\pi}\vec{a})^2, \\ \lambda_1 &= 24\left(g_1 + \frac{1}{2}g_2\right), \\ \lambda_2 &= 2g_2. \end{aligned} \tag{A.6}$$

By using the eq. (2.1) one arrives at the functional equations in dimension $d = 3$,

$$\partial_k V = \frac{k^4}{6\pi^2} \left(\frac{1}{E_\sigma^2} + \frac{4}{E_{\pi\eta}^2} + \frac{3}{E_a^2} \right), \tag{A.7}$$

with

$$\begin{aligned} E_\sigma^2 &= k^2 + 2V' + 4V''\varphi, \\ E_{\pi\eta}^2 &= k^2 + 2V', \\ E_a^2 &= k^2 + 2V' + 2W\varphi \end{aligned} \tag{A.8}$$

and for W ,

$$\begin{aligned} \partial_k W &= -\frac{k^4}{6\pi^2} \left\{ \frac{8W' + 4W\varphi^{-1}}{E_{\pi\eta}^4} - \frac{8W^2}{E_{\pi\eta}^6} \right. \\ &\quad + \left[-4V''\varphi^{-1} + 4W''\varphi + 10W' + 2W\varphi^{-1} \right. \\ &\quad \quad \left. + \frac{8(V'' + W + W'\varphi)^2}{2V'' - W}\varphi^{-1} \right] \frac{1}{E_\sigma^4} \\ &\quad + \left[4V''\varphi^{-1} + 14W' - 6W\varphi^{-1} \right. \\ &\quad \quad \left. - \frac{8(V'' + W + W'\varphi)^2}{2V'' - W}\varphi^{-1} \right] \frac{1}{E_a^4} \left. \right\}, \end{aligned} \tag{A.9}$$

from which the flow equations for the couplings can be derived.

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