

Numerical simulation of higher order chemical reactive flow of ternary hybrid nanofluid across an extending cylinder with heat generation and induction effects

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ABSTRACT

This study scrutinizes the influences of magnetic induction on the chemical reactive flow of trihybrid nanofluid with heat transfer in a boundary layer across an extended cylinder containing metallic nanoparticles. The trihybrid nanofluid is produced by dispersing silicon dioxide, cobalt ferrite nanoparticles in water and titanium dioxide, the base liquid. In energy conversion devices, thermal management systems, and chemical and petrochemical reactors where improved heat and mass transmission are essential, it is especially pertinent. Thermal conductivity is enhanced by the addition of ternary hybrid nanofluids, which makes the model applicable to cooling systems and nanocoating applications. The addition of chemical reactions, magnetic induction, and heat generation further increases its applicability to fields like nuclear engineering, biomedical devices, and smart manufacturing systems that need to precisely manage reactive transport phenomena. The basic fluid's thermophysical properties are considerably improved by the addition of ternary hybrid nanoparticles. The bvp4c method is used to confirm the results' authenticity and accuracy. Figures and Tables are used to present and analyze the results. It has been noted that the liquid flow is decreased and magnetic induction profile is improved by the magnetic Prandtl number.

1. Introduction

The magnetization has so many applications, it is one of the most important ideas in engineering and production. Magnetic fields and fluid nanoparticles have an effect on the performance of clutches, compressors, heat transfer, and other industrial items. The cooling rate of many

engineering devices can be adjusted and made available by magnetic fields. A metamaterial's strength and dispersion have an impact on the flow properties. Venkatesh et al. [1] investigated the numerical research of MHD Maxwell fluid with NPs (nanoparticles) across a surface: effects of heat radiation, CBC (convective boundary condition), and induced MF (magnetic field). Ilango et al. [2] investigate the induced MF and Dufour-Soret characteristics on the flow of Casson fluid through a

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Nomenclature

C	fluid phase concentration
(u, v)	Velocity fields ($m.s^{-1}$)
q_w	Heat flux (W/m^2)
q_m	Mass flux ($kg.m^2.s^{-1}$)
C_{fx}	Skin friction
σ^*	Stefan-Boltzmann constant ($W/m^2.k^4$)
Sh_x	Sherwood number
γ	curvature term
σ_f	Electrical Conductivity (sm^{-1})
k^*	Mean absorption coefficient (cm^{-1})
λ	inverse of magnetic Prandtl number
μ_f	Dynamic viscosity ($kg.m^{-1}.s^{-1}$)
σ_0	Surface tension
Nu_x	Nusselt number
ρ_f	Fluid density (kg/m^3)
σ	Surface tension (N/m)

β	Magnetic induction parameter
$\psi(x, y)$	Streams functions
k_f	Thermal conductivity ($W.m^{-1}.k^{-1}$)
C_p	Specific heat of the fluid ($J.kg^{-1}.k^{-1}$)
D_B	Mass diffusivity coefficient ($m^2.s^{-1}$)
B_0	Uniform magnetic field ($kg.s^{-2}.A^{-1}$)
Pr	Prandtl number
T	Fluid temperature (k)
Ma	Marangoni ratio parameter
γ_C	Surface tension coefficients for concentration
ν_f	Kinematic viscosity ($m^2.s^{-1}$)
γ_T	Surface tension coefficients N/m
Φ	Volume fraction of nanoparticles
NPs	Nanoparticles
THNF	Trihybrid nanofluid
MC	Marangoni convection
NF	Hybrid nanofluid

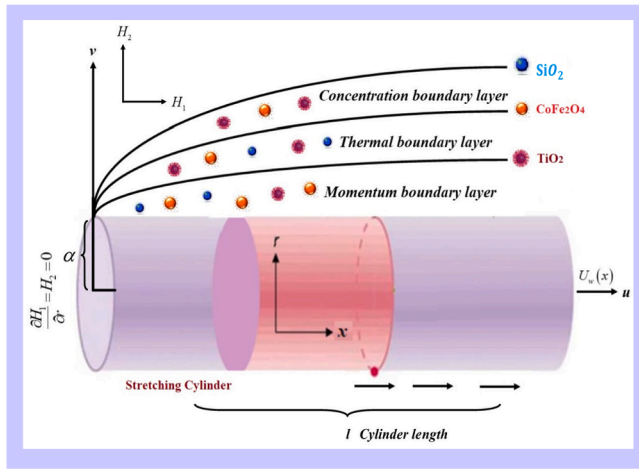


Fig. 1. Flow description.

porous media over a sheet. Das et al. [3] investigated MHD flow down a channel with an induced MF, and heat radiation. Shankar et al. [4] inspected the influence of induced MF on MHD flow in conducting vertical microchannel walls. Sakata et al. [5] investigated the impact of an intrinsic MF on ion escape from ancient Mars using MAESTRO multifluid MHD simulations. Some recent HNF literature can be found in Refs [6–9].

The fluid dynamics phenomena known as MC (Marangoni convection) are brought on by variations in a liquid's ST (surface tension). It occurs when distinct areas of a liquid surface have varying temperatures. A gradient generated by temperature variations in surface tension leads liquid to flow from locations with low surface tension to areas with high ST. This movement is identified as MC (Marangoni convection). ST variations in a liquid can cause the phenomenon known as MC (Marangoni convection). It has many important applications in materials processing, microfluidics, and surfactant behavior. By altering the temperature differential across a liquid surface, researchers may generate Marangoni flow, which can be utilized to drive liquid flow via microchannels, control crystal formation, and create stable foams and emulsions. Abbas et al. [10] investigated the LTNE characteristics on Marangoni convective flow of Casson fluid with CCFM (Cattaneo-Christov flux model) and elastic deformation. Okasha et al. [11] scrutinized the impact of the CCFM on the radiative flow of a tetra HNF with a

surface TG (tension gradient). Wang and Shi [12] explored the characteristics of substratum temperature on MC instabilities in a sessile drop dispersing at a constant contact line model. Lin et al. [13] investigated the MHD thermos-capillary MC heat transmission of power-law fluids with thermal gradients. Abbas et al. [14] investigated the surface TG (tension gradient) in dusty Ellis fluid flow across the Riga plate using the CCFM and Stephen blowing influence.

Ahammad [15] inspected the buoyancy-driven HF (hydromagnetic flow) and heat transmission in a partially heated square enclosure using NFs. Alanzi and Ahammad [16] investigated the heat transmission properties for solar energy on the flow of tangent hyperbolic HNF across a sensor wedge surface. Krishna et al. [17] inspected radiation absorption in MHD convective flow of NFs across a plate. Liaqat et al. [18] used an ANN to optimize the chemical reactive flow of a HNF with radiation and heat source for bio-nanomedicine uses. Abbas et al. [19] inspected the characteristics of ED on the CCFM flow of a Williamson-based HNF with LTNE conditions. Aoudia et al. [20] inspected the thermo-solutal convective flow of NF with MC using an ANN, taking into account radiation and thermophoresis characteristics. Alshammari et al. [21] inspected the consequence of thermal radiation and LTNE on a bio-convection HNF using modified and classical HCMs. Galal et al. [22] inspected the modelling and design issues for heat and mass transport using a Boger HNF in industrial uses. Aoudia et al. [23] conduct a computational analysis of the effect of thermal radiation on the chemical reactive flow of microbes in HNF over a disk.

The class of nanofluids known as THNF (ternary hybrid nanofluid) consists of three different classes of NPs (nanoparticles) dispersed throughout a base fluid. These nanoparticles, which usually consist of oxides, metals, or carbides, have sizes ranging from 1 to 100 nanometers. Because of the distinctive characteristics of the NPs they contain, ternary hybrid nanofluids differ from ordinary fluids. These features include greater thermal conductivity, enhanced stability, increased heat transmission, and increased surface area. This resulted to the use of TEHNF in a number of areas, such as energy, electronics, and biomedical engineering [24–33].

In simple terms, resonance, structural vibrations, a rise in mean drag, and lift oscillations are caused by vortex molting around a circular cylinder. Because it can effectively reduce flow separation suppression, drag, vibration noise, vortex shedding flow control is crucial for industrial uses [34]. For a considerable amount of time, researchers have examined flow management over a circular cylinder. The flow through a cylinder in a restricted region, like as a stream in a lateral trench, gets far more attention than flow inside a cylinder, which is used in many engineering activities. In many scenarios, such as blood flow around

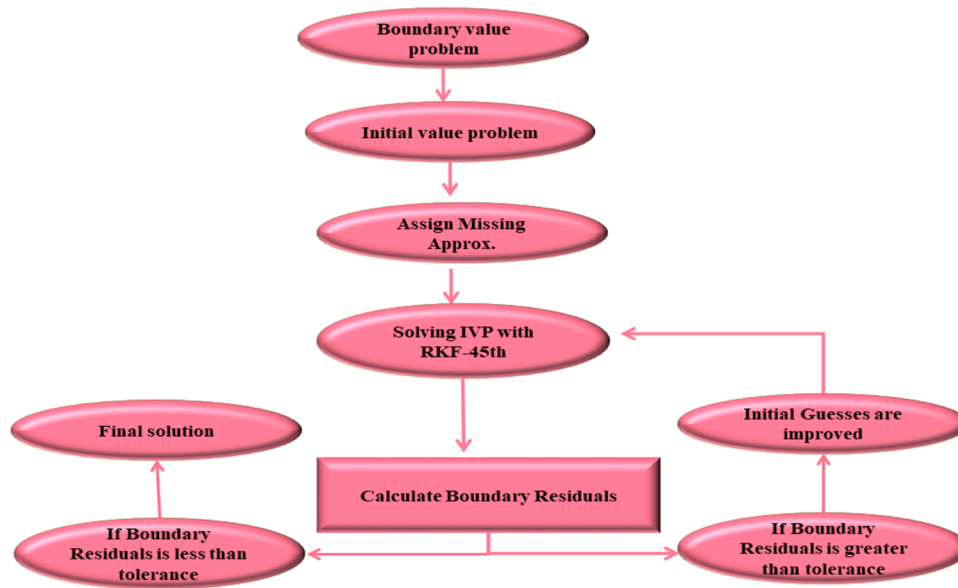
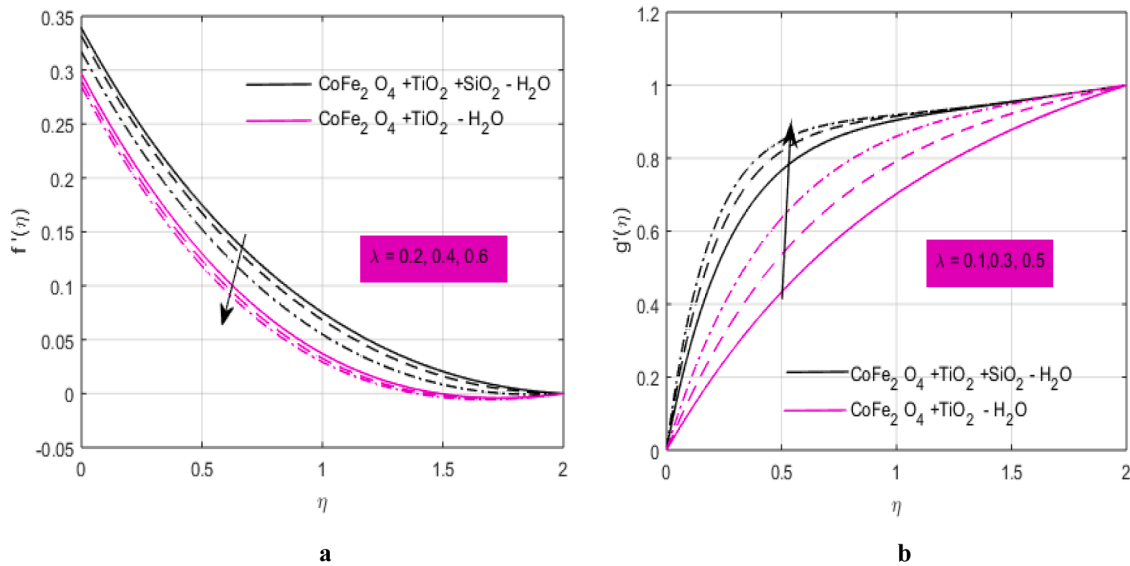


Fig. 1. (c) Flowchart of numerical solution.

Fig. 2. (a and b) Influence of λ on f' and g' .

cylindrical items near walls or crossways methods from active sources, wall effects must be considered when scaling problems. Furthermore, natural forms of random and unorganized exterior roughness have been scrutinized in addition to some types of ordered roughness. A 3D-printed solid with regular periodic grooves may develop a curved surface as a result of vibrations during the printing process [35]. Salahuddin et al. [36] investigation focused on the variously created nanostructures that, when heat is generated, impose stiff and sinusoidal barriers that impact the motorized presentation and flow effectiveness of the flow.

The numerical simulation of electromagnetic induction properties on chemical convective flow of a trihybrid nanofluids across an expanding cylinder with Marangoni convection is a novel effort in the field of mass and heat transfer. Additionally, there hasn't been much research done on how chemical reaction and electromagnetic induction work together to affect Marangoni convection in a situation of expanding cylinders. This study takes a novel and creative method to comprehend heat transmission in trihybrid nanofluid by including a heat source, which further complicates the simulation. A static-applied magnetic field is

used in this situation to analyze the horizontally extending cylinder. The dynamic model becomes an ODE system with sufficient far stream and wall conditions. The second objective is to boost thermal energy propagation's profitability and efficiency across a variety of biological and commercial domains.

2. Mathematical formulation

Consider the characteristics of heat generation on incompressible axisymmetric Marangoni convective flow of THNF across a cylinder with a higher-order chemical reaction. It has been proposed that the x -axis is oriented in the radial direction, whereas the x -axis is taken along the straight cylinder axis. When the Reynolds number is high, the ME rotates it, making the induced ME (magnetic effect) significant. In general, $H(H_1, H_2)$, the mutual limitation, enhances from the magnetic field. The H_2 (parallel component) is the induced MF of the vertical element that originates and dissipates at the surface of the cylinder. At the free stream, $H_e = H_0$ is the assumed value of H_1 . T_w and T_∞ are the

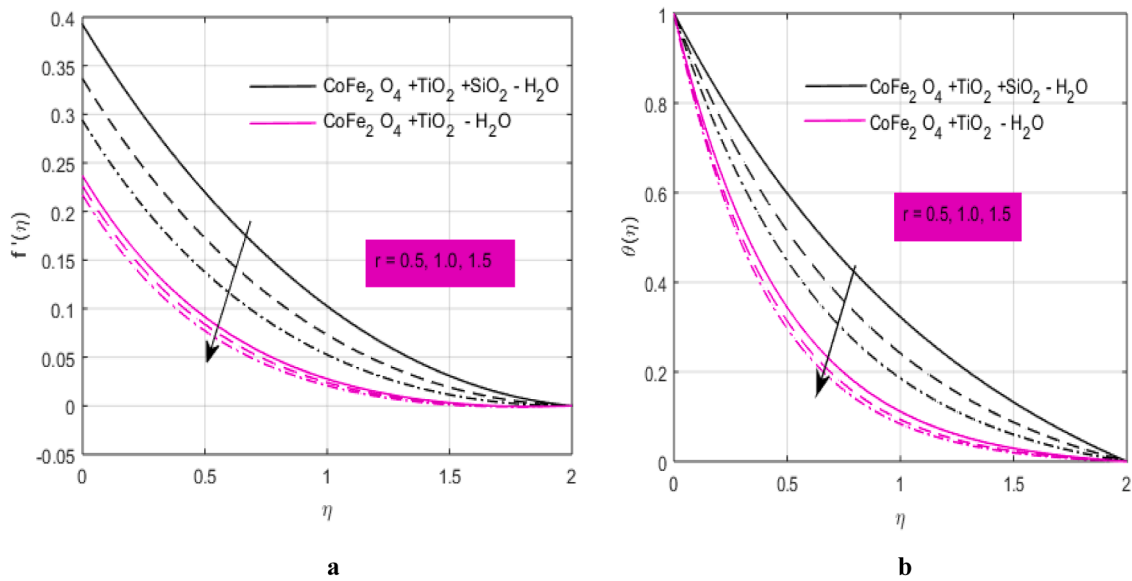


Fig. 3. (a and b) Outcome of r on $f'(\eta)$ and $\theta(\eta)$.

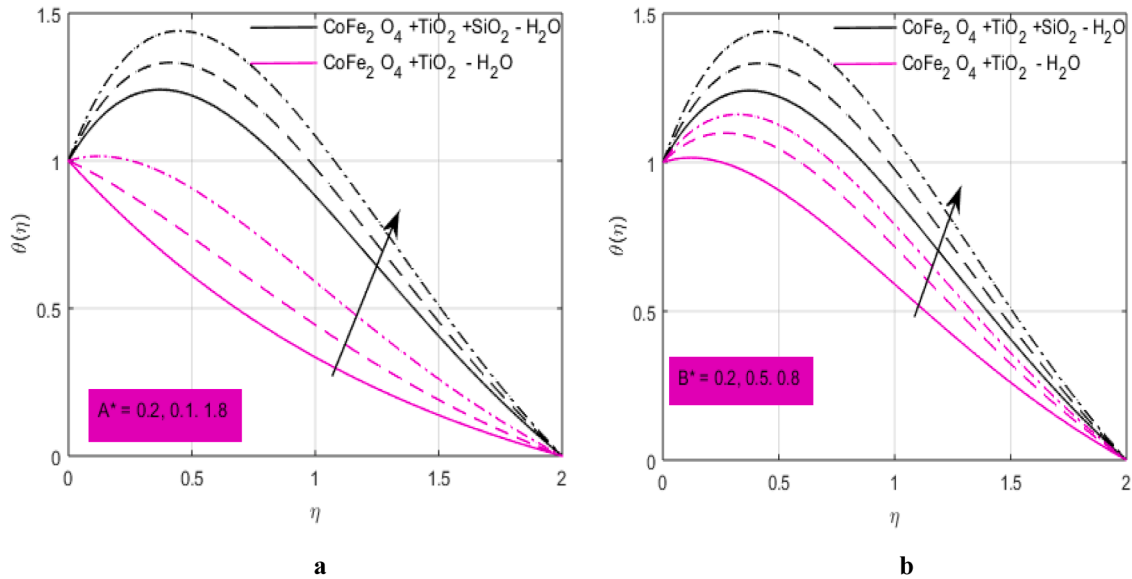


Fig. 4. (a and b) Outcome of A^* on $\theta(\eta)$.

specified values for the wall and far stream temperatures. In addition, it is assumed that the cylinder is extended towards the axial direction with stretching velocity $U_w = U_0(x/l)$, where l is the cylinder's length and U_0 is a constant, as seen in Fig. 1. The basic computations which regulate the flow of fluids are described as [37–41]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \quad (1)$$

$$\frac{\partial(rH_1)}{\partial x} + \frac{\partial(rH_2)}{\partial r} = 0, \quad (2)$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r}\right) \rho_{thnf} - \frac{\mu_e}{4\pi} \left(H_1 \frac{\partial H_1}{\partial x} + H_2 \frac{\partial H_2}{\partial r}\right) = \mu_{thnf} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}\right), \quad (3)$$

$$\left(u \frac{\partial H_1}{\partial x} + v \frac{\partial H_1}{\partial r}\right) - H_1 \frac{\partial u}{\partial x} - H_2 \frac{\partial u}{\partial r} = \eta_0 \left(\frac{\partial^2 H_1}{\partial r^2} + \frac{1}{r} \frac{\partial H_1}{\partial r}\right), \quad (4)$$

$$\left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r}\right) (\rho c_p)_{thnf} = (K_{thnf}) \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}\right) + \frac{K_{thnf} \rho_{thnf} U_0}{2\mu_{thnf} X} [A^*(T_w - T_\infty)f' + B^*(T - T_\infty)], \quad (5)$$

$$\left(u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r}\right) = D_B \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r}\right) - K_c(C - C_\infty)^n. \quad (6)$$

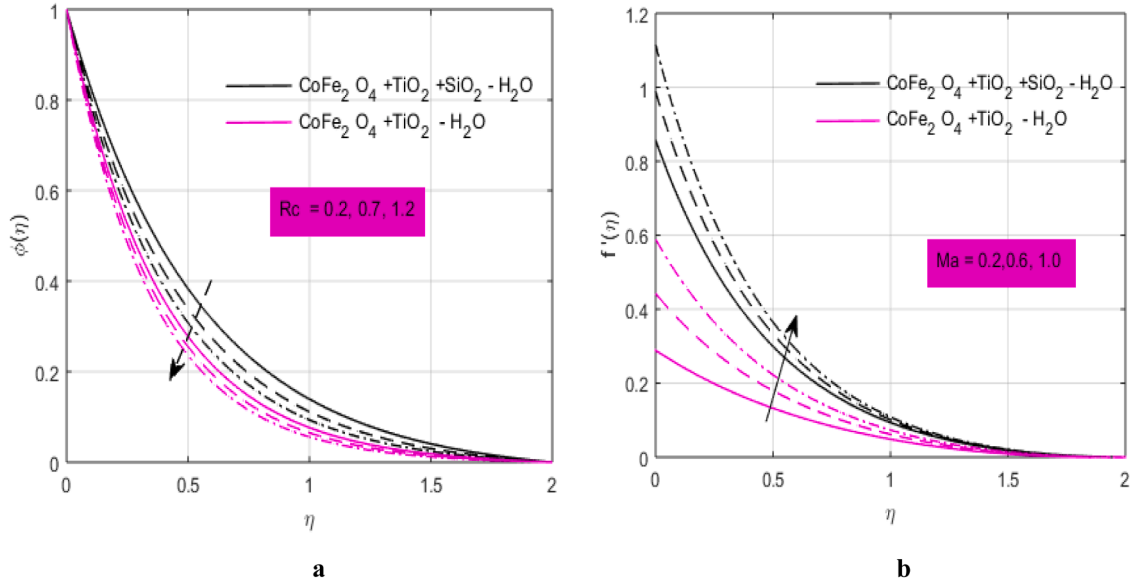
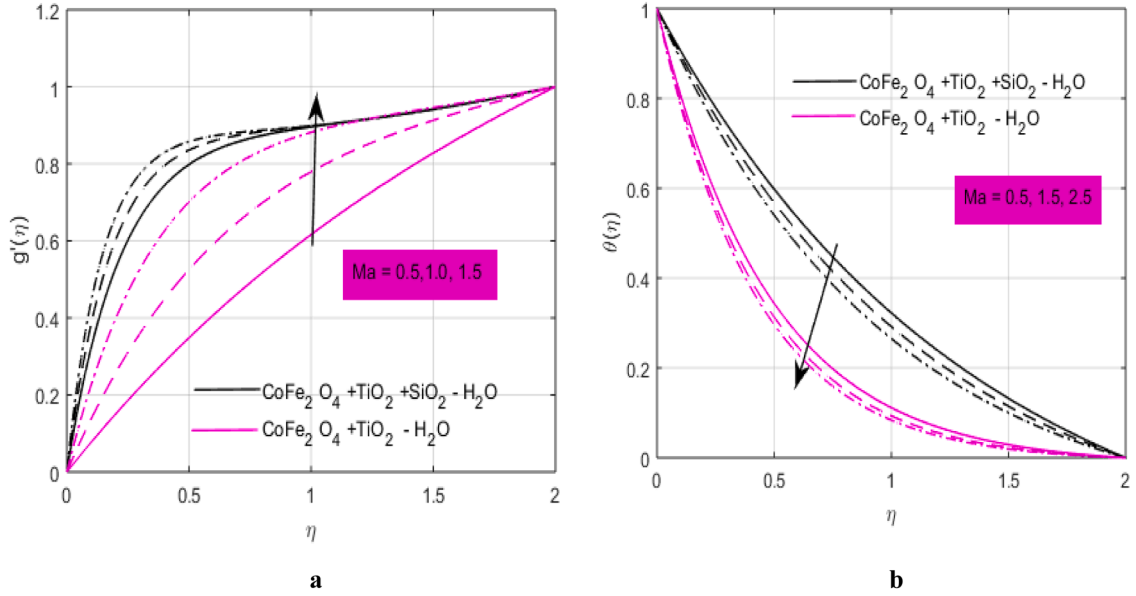
The boundary conditions are [37–41]:

$$\mu_{thnf} \frac{\partial u}{\partial r} = \frac{\partial \sigma_1}{\partial x} = -\sigma_0 \left(\gamma_r \frac{\partial T}{\partial x} + \gamma_c \frac{\partial C}{\partial x}\right), \quad v = 0, \quad \frac{\partial H_1}{\partial r} = H_2 = 0, \quad (7)$$

$$T = T_w, \quad C = C_w \text{ at } y = 0, \quad (8)$$

$$u \rightarrow 0, \quad H_1 = H_e = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ at } y \rightarrow \infty. \quad (9)$$

Surface tension

Fig. 5. (a and b) Outcome of Rc on ϕ and Ma on f .Fig. 6. (a and b) Result of Ma on g' and θ .

$$\sigma = \sigma_0[1 - \gamma_T(T - T_\infty) - \gamma_C(C - C_\infty)] \quad \gamma_C = -\frac{\partial \sigma}{\partial C}\bigg|_T, \quad \gamma_T = -\frac{\partial \sigma}{\partial T}\bigg|_C.$$

Equations use the transformation variables [38] shown below (1)–(6):

$$\psi = a(v_f U_w x)^{1/2} f(\eta), \quad \eta(x, r) = \frac{r^2 - a^2}{2a} \left(\frac{U_w}{v_f x} \right)^{1/2}, \quad H_1 = r H_0 g(\eta), \quad T_w = T_\infty + T_0 \left(\frac{x}{l} \right), \quad (10)$$

$$T(x, r) = T_\infty + T_0 \left(\frac{x}{l} \right) \theta(\eta), \quad C(x, r) = C_\infty + C_0 \left(\frac{x}{l} \right) \phi(\eta). \quad (11)$$

The following are the similarity variables for momentum equations:

$$u = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x}. \quad (12)$$

We reach at the subsequent, using Eq. (13):

$$P_1((1+2\gamma)f'' + 2\gamma f''') + P_2(f''f - (f')^2) + \beta((g')^2 - gg'') = 0, \quad (13)$$

$$\lambda((1+2\gamma)g'' + 2\gamma g''') + (fg'' - f'g) + \beta((g')^2 - gg'') = 0, \quad (14)$$

$$P_4((1+2\gamma)\theta' + 2\gamma\theta'') + P_5 \text{Pr}(f\theta' - f'\theta) + \frac{P_2 P_4}{P_1}(A^*f' + B^*\theta) = 0, \quad (15)$$

$$((1+2\gamma)\phi'' + 2\gamma\phi') + \text{Sc}(f\phi' - f'\phi) - \text{Sc}Rc(\phi)^n = 0. \quad (16)$$

The transform conditions are:

$$f''(0) = -\frac{2Mn(1+Ma)}{P_1}, \quad f(0) = 0, \quad g'(0) = 0, \quad \theta(0) = 1, \quad \phi(\infty) \rightarrow 0, \quad \phi(0) = 1, \quad (17)$$

$$f(\infty) \rightarrow 0, \quad g(0) = 0, \quad g'(\infty) \rightarrow 1, \quad \theta(\infty) \rightarrow 0. \quad (18)$$

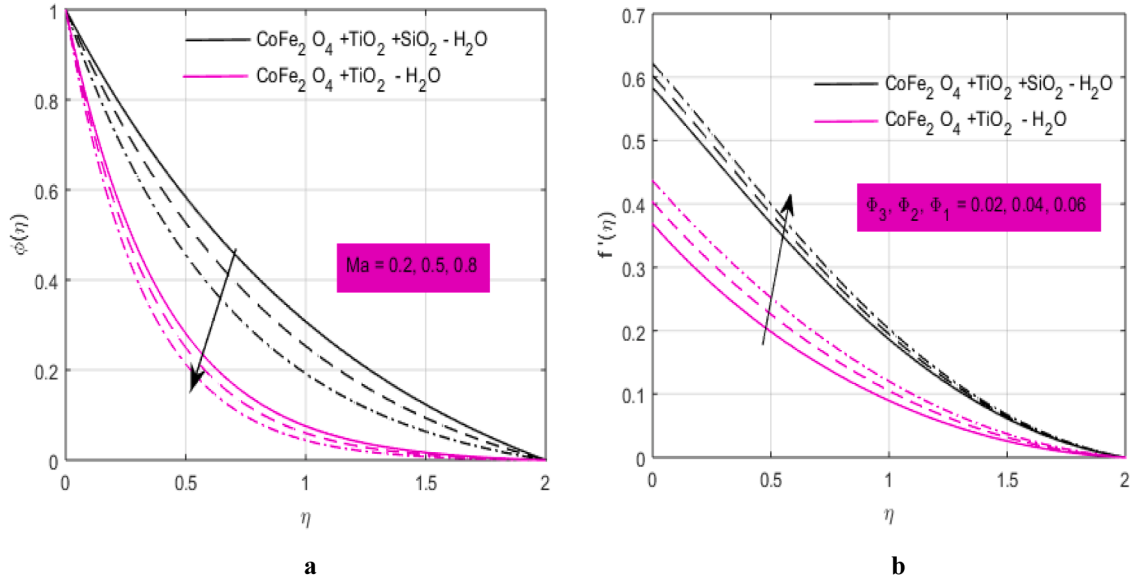
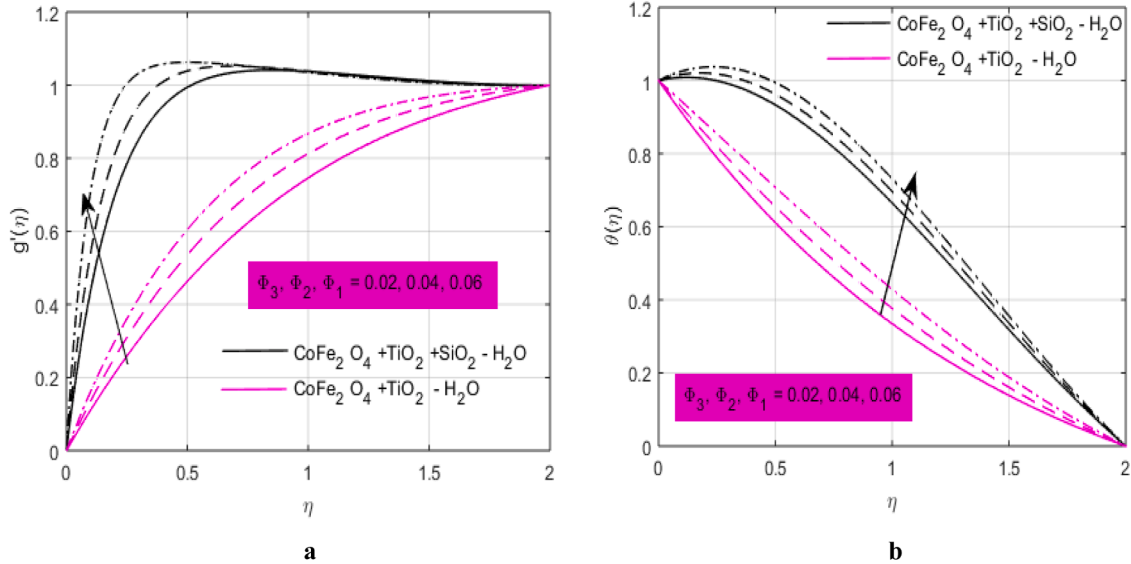
Fig. 7. (a and b) Result of Ma on ϕ, Φ_2 and Φ_3 on g' .Fig. 8. (a and b) Impact of Φ_1, Φ_2 and Φ_3 on g' and θ .

Table 1

The thermophysical assets of base fluid and NPs [21,24].

Properties constituents	c_p	k	σ	ρ
TiO ₂	686.2	8.9538	10^{-10}	4250
CoFe ₂ O ₄	700	3.7	5.51×10^9	4907
SiO ₂	730	1.5	1.0×10^{-18}	2650
Water	4179	0.613	0.05	997.1

Here, β - magnetic parameters, γ - curvature term, Pr – Prandtl number, Sc – Schmidt number, Ma – Marangoni convection parameter, Mn – Marangoni number.

$$\beta = \frac{\mu_e}{4\pi\rho_f} \left(\frac{IH_0}{U_0} \right)^2, \quad \gamma = \left(\frac{lv_f}{a^2 U_0} \right)^{1/2}, \quad \lambda = \frac{\eta_0}{\nu_f}, \quad Pr = \frac{(\rho c_p)\mu_f}{k_f \nu_f}, \quad Sc = \frac{\nu_f}{D_f}, \quad Rc = \frac{l^n k_c}{(U_0)^n},$$

$$Ma = \frac{C_0 \gamma_c}{T_0 \gamma_T}, \quad Mn = \frac{\sigma_0 T_0 \gamma_T L}{\mu_f \nu_f}.$$

The definitions of the Nusselt number, skin friction coefficient and Sherwood number are as follows [38]:

$$C_{fx} = \frac{\tau_w}{u_w^2 \rho_f}, \quad Nu_x = \frac{x q_w}{(T_w - T_\infty) k_f}, \quad Sh_x = \frac{x q_m}{D_B (C_w - C_\infty)}, \quad (19)$$

$$\tau_w = \mu_{thnf} \left[\frac{\partial u}{\partial r} \right]_{r=0}, \quad q_w = - (k_{thnf}) \frac{\partial T}{\partial r} \bigg|_{r=0}, \quad q_m = - D_B \frac{\partial C}{\partial y} \bigg|_{y=0}. \quad (20)$$

Where Heat flux is represented by q_w , surface shear stress is denoted by τ_w and mass flux is represented by q_m .

$$C_{fx}(Re_x)^{-0.5} = P_1 f''(0), \quad Nu_x(Re_x)^{-0.5} = -(P_4) \theta'(0), \quad Sh_x(Re_x)^{-0.5} = -P_6 \phi'(0). \quad (21)$$

Table 2

Thermal characteristics of the THNF [21,24] ($\Phi_3 = \Phi_{\text{SiO}_2}$, $\Phi_2 = \Phi_{\text{TiO}_2}$, $\Phi_1 = \Phi_{\text{CoFe}_2\text{O}_4}$).

$P_1 = \frac{\mu_{\text{thnf}}}{\mu_f} \mu_{\text{thnf}} = \frac{\mu_f}{(1 - \Phi_1)^{2.5} (1 - \Phi_2)^{2.5} (1 - \Phi_3)^{2.5}}$
$P_2 = \frac{\rho_{\text{thnf}}}{\rho_f} \rho_{\text{thnf}} = (1 - \Phi_2) [(1 - \Phi_3) \rho_f + \rho_3 \Phi_3] + \rho_2 \Phi_2 + \rho_1 \Phi_1$
$P_3 = \frac{\sigma_{\text{thnf}}}{\sigma_f}$, $\frac{\sigma_{\text{thnf}}}{\sigma_{\text{nf}}} = \frac{\sigma_1 + (S - 1) \sigma_{\text{thnf}} - (S - 1) \Phi_1 (\sigma_{\text{thnf}} - \sigma_1)}{\sigma_1 + (S - 1) \sigma_{\text{thnf}} + \Phi_1 (\sigma_{\text{thnf}} - \sigma_1)}$, $\frac{\sigma_{\text{thnf}}}{\sigma_{\text{nf}}} = \frac{\sigma_2 + (S - 1) \sigma_{\text{nf}} - (S - 1) \Phi_2 (\sigma_{\text{nf}} - \sigma_2)}{\sigma_2 + (S - 1) \sigma_{\text{nf}} + \Phi_2 (\sigma_{\text{nf}} - \sigma_2)}$, $\frac{\sigma_{\text{thnf}}}{\sigma_f} = \frac{\sigma_3 + (S - 1) \sigma_f - (S - 1) \Phi_3 (\sigma_f - \sigma_3)}{\sigma_3 + (S - 1) \sigma_f + \Phi_3 (\sigma_f - \sigma_3)}$
$k_{\text{thnf}} = \frac{k_{\text{thnf}}}{k_{\text{nf}}} = \left(\frac{k_3 + (S - 1) k_{\text{thnf}} - (S - 1) \Phi_3 (k_{\text{thnf}} - k_3)}{\Phi_3 (k_{\text{thnf}} - k_3) + k_3 + (S - 1) k_{\text{thnf}}} \right)$, $P_5 = \frac{k_{\text{thnf}}}{k_f}$, $\frac{k_{\text{thnf}}}{k_{\text{nf}}} = \left(\frac{k_2 + (S - 1) k_{\text{nf}} - (S - 1) \Phi_2 (k_{\text{nf}} - k_2)}{\Phi_2 (k_{\text{nf}} - k_2) + k_2 + (S - 1) k_{\text{nf}}} \right)$, $\frac{k_{\text{nf}}}{k_f} = \left(\frac{k_1 + (S - 1) k_f - (S - 1) \Phi_1 (k_f - k_1)}{\Phi_1 (k_f - k_1) + k_1 + (S - 1) k_f} \right)$
$(\rho C_p)_{\text{thnf}} = \frac{(\rho C_p)_{\text{thnf}}}{(\rho C_p)_f}$, $(\rho C_p)_{\text{thnf}} = (1 - \Phi_1) \{ (1 - \Phi_2) [(1 - \Phi_3) (\rho C_p)_f + (\rho C_p)_{x_2} \Phi_3] + (\rho C_p)_{x_2} \Phi_2 + (\rho C_p)_{x_1} \Phi_1$

Table 3

Effect of various constraints on Nusselt number.

Ma	A^*	B^*	M	Mn	Nu_x
0.2	0.3	0.4	0.1	0.6	2.489140
0.3					2.489448
0.4					2.489753
	0.3				2.493535
0.4	0.4	0.3	0.1	0.6	2.493538
	0.5				2.498020
		0.2			2.483435
0.1	2.0	0.3	0.1	0.6	2.483538
		0.4			2.488920
			0.2		2.493541
0.2	2.0	0.3	0.3	0.6	2.493041
			0.4		2.488020
				0.2	2.487722
0.2	2.0	0.3	0.1	0.3	2.493527
				0.4	2.490478

Table 4

Effect of different factors on Sherwood number.

Ma	Rc	Mn	A^*	B^*	M	Sh_x
0.2	0.3	0.4	0.1	0.6	0.7	1.506115
0.3						1.506430
0.4						1.506743
	0.3					1.510647
0.4	0.4	0.3	0.1	0.6	0.7	1.510639
	0.5					1.504995
		0.2				1.504666
0.1	2.0	0.3	0.1	0.6	0.7	1.510630
		0.4				1.527558
			0.2			1.510639
0.2	2.0	0.3	0.3	0.6	0.7	1.510642
			0.4			1.515001
				0.2		1.511639
0.2	2.0	0.3	0.1	0.3	0.7	1.511642
				0.4		1.515401
					0.2	1.510645
0.2	2.0	0.3	0.1	0.6	0.3	1.503173
					0.4	1.487163

Table 5

Contrasting the current results with earlier published studies by setting all additional parameters to zero.

γ	Nu_x	
	Present results	Alharbi et al. [38]
0.1	0.0574242	0.0574437
0.3	0.0454153	0.0454040
0.5	0.0464643	0.0464745
0.7	0.0391112	0.0391005

3. Numerical method

It seems that obtaining an adequate response will be challenging due to the existing model's extreme non-linear behavior. The RKF-45th technique uses a high-level language to solve these equations mathematically in a collaborative environment. Substitutions are used to convert the higher order ODEs into a system of first-order differential equations. (Fig. 1c)

$$u_2 = f', \quad u_1 = f, \quad u_3 = f'', \quad u_4 = g, \quad u_5 = g', \quad u_3 = f'', \quad u_6 = g'', \quad u_6 = g'', \quad (22)$$

$$u_7 = \theta, \quad u_8 = \theta', \quad u_8 = \theta', \quad u_9 = \phi, \quad u_{10} = \phi', \quad u_{10} = \phi'. \quad (23)$$

4. Results and discussion

The suggested model provides useful insights for optimizing heat and mass transfer in industrial systems with cylindrical geometries, such as heat exchangers, reactors, and cooling systems in nuclear and chemical engineering. This model is useful for advanced thermal management in nanofluid-based technologies such as biomedical devices, energy systems, and microfluidic cooling of electronic components, as it takes into account higher-order chemical reactions, heat generation, and magnetic induction effects. The outcome of λ on the magnetic gradient function and velocity profile is seen in Figs. 2(a&b). The flow field but magnetic induction field increases with λ . The stream magnetic gradient curve is reinforced when the value of λ is altered. In the observed situations, the flow allocation in the far movement field grows to a unity incurable. The relationship between the rate of sticky diffusion and the frequency of magnetic dispersion is known as the magnetic Prandtl number. λ stands for the joint ratio, which is the viscous dissemination rate divided by the magnetic diffusion rate. The velocity, concentration, and energy profiles decrease as the curvature constant increases, as seen in Figs. 3(a&b). Figs. 4(a&b) show how A^* and B^* affect the θ field of the THNF. It is exposed that thermal fields are improved as the A^* and B^* values are increased. The A^* and B^* appear to be transmitting heat energy into the liquid flow, as shown by their positive values, which enhances the thermal distribution's uniformity. Fig. 5(a) illustrate how Rc affect mass transmission. In this case, an increase in Rc declines the concentration gradient. Ma , Φ_1 , Φ_2 and Φ_3 effects on f, g, θ and ϕ profiles are shown in Figs. 5(b), 6(a&b), 7(a&b) and 8(a&b). The graph shows how improved velocity and magnetic induction profiles follow from enhanced Ma . Surface differences are the cause of this phenomenon. A higher influence will typically produce a larger gradient in velocity because the Marangoni impact serves as a pouring force for liquid streams. These figures show that when the value of Ma increases, the solutal and thermal fields drastically decline. Surface tension, a physical feature of liquid surfaces, is caused by the liquid's attraction to particles on the surface layer. Consequently, when ST rises, temperatures decrease, indicating that particles have a greater attraction to one another. Additionally, an increase in the volume friction of NPs outcomes in a deterioration in the concentration profile, a thickening of the associated BL (boundary layer), and an enhancement in the thermal profile. When the volume fraction parameters for THNF and hybrid nanofluids increase, the temperature, velocity profiles, magnetic induction, and width of the

corresponding BL all increase, but the concentration boundary layer decreases Table 1 presentations the features of the NPs and base fluid. Table 2 displays the thermophysical characteristics of the THNF. Tables 3 and 4 show how different factors affect Sherwood and Nusselt numbers. The heat transmission rates from the current study and those from previous research are displayed in Table 5. There is a major connection between the two results. This demonstrates that the current study is valid.

5. Conclusions

The Marangoni convection ternary hybrid nanofluid flow made of metallic nanoparticles over a stretching cylinder that is electro-conductive and transmits both heat and mass is described in this work. The primary conclusions made from the previous observation are listed below:

- v The inverse magnetic Prandtl number, curvature constant and magnetic term all have an impact on the velocity profile, which increases when the volume percentage of nanoparticles is taken into consideration.
- v A decrease in the curvature constant and magnetic term results in a drop in the induced MF, which upsurges with variations in the inverse magnetic Prandtl number, volume fraction of NPs, and Marangoni convection parameter.
- v The thermal profile upsurges when the heat source and nanoparticle volume fraction both vary positively.
- v The mass transfer profile is negatively impacted by the curvature constant, Marangoni convection parameter, and chemical reaction
- v The basic fluid's thermos-physical characteristics are greatly enhanced by the addition of ternary hybrid nanoparticles.

CRedit authorship contribution statement

Munawar Abbas: Supervision, Methodology, Conceptualization. **Mostafa Mohamed Okasha:** Methodology, Conceptualization. **Dilsora Abduvalieva:** Methodology, Formal analysis. **Ali Akgül:** Methodology, Formal analysis. **Murad Khan Hassani:** Methodology, Formal analysis. **Ali Hasan Ali:** Methodology, Formal analysis. **Zuhair Jastanayah:** Writing – review & editing. **Hakim AL Garalleh:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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