

## CLUSTERING ALGORITHM FOR A VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

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**Abstract.** The demand for daily food purchases has increased dramatically, especially during the Covid-19 pandemic. This requires suppliers to face a huge and complex problem of delivering products that meet the needs of their customers on a daily basis. It also puts great pressure on managers on how to make day-to-day decisions quickly and efficiently to both satisfy customer requirements and satisfy capacity constraints. This study proposes a combination of the cluster-first – route-second method and  $k$ -means clustering algorithm to deal with a large Vehicle Routing Problem with Time Windows (VRPTW) in the logistics and transportation field. The purpose of this research is to assist decision-makers to make quick and efficient decisions, based on optimal costs, the number of vehicles, delivery time, and truck capacity efficiency. A distribution system of perishable goods in Vietnam is used as a case study to illustrate the effectiveness of our mathematical model. In particular, perishable goods include fresh products of fish, chicken, beef, and pork. These products are packed in different sizes and transferred by vehicles with 1000 kg capacity. Besides, they are delivered from a depot to the main 39 customers of the company with arrival times following customers' time window. All of the data are collected from a logistics company in Ho Chi Minh city (Vietnam). The result shows that the application of the clustering algorithm reduces the time for finding the optimal solutions. Especially, it only takes an average of 0.36 s to provide an optimal solution to a large Vehicle Routing Problem (VRP) with 39 nodes. In addition, the number of trucks, their operating costs, and their utilization are also shown fully. The logistics company needs 11 trucks to deliver their products to 39 customers. The utilization of each truck is more than 70%. This operation takes the total costs of 6586215.32 VND (Vietnamese Dong), of which, the transportation cost is 1086215.32 VND. This research mainly contributes an effective method for enterprises to quickly find the optimal solution to the problem of product supply.

**Keywords:** vehicle routing problem, logistics, time window, cluster, route,  $k$ -means clustering, transportation.

### Notations

*Variables and functions:*

$c$ – transportation unit cost per km [VND/km];	$\varepsilon_i$ – the earliest time that customer can receive the products;
$C$ – set of customers;	$f$ – refrigeration unit cost per km [VND/km];
$C_k$ – the be a set of $K$ centers in the $k$ -means clustering model;	$\varphi_j$ – the latest time that customer can receive the products;
$CT_k$ – maximum permissible load of truck $k$ [kg];	$G$ – graph;
$d$ – dimensional real vector;	$K_{ij}$ – maximum spending time between 2 nodes;
$d_{ij}$ – distance from node $i$ to node $j$ [km];	$L$ – labour cost per truck per day [VND];
$D_i$ – demand of customer $i$ ;	$p_i$ – processing time of customer $i$ ;
$dist(d_i, c_k)$ – the Euclidean distance between a data point $d_i$ and the cluster center $c_k$ ;	$S_j$ – the set of samples that belong to the $k$ th cluster;
$E_i$ – estimated volume of products of customer $i$ ;	

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- $S_{ik}$  – an integer variable that present the time of vehicle  $k$  starts to service a customer  $i$ ;  
 $t_{ij}$  – moving time from node  $i$  to node  $j$ ;  
 $(\varphi_i, \varepsilon_i)$  – the time window at customer  $i$  where  $\varphi_i$  the earliest time that customer  $i$  can receive the products and  $\varepsilon_i$  is the latest time that customer  $i$  can receive the products;  
 $v$  – speed [km/h];  
 $V$  – vehicle;  
 $VT_k$  – maximum volume of truck  $k$  [ $m^3$ ];  
 $x_{ijk}$  – a binary variable that equals to 1 if vehicle  $k$  drives directly from node  $i$  to node  $j$ , otherwise, it is 0;  
 $X$  – a set of observations.

#### Abbreviations:

- ASEAN – Association of South-East Asian Nations;  
 GDP – gross domestic product;  
 IMF – International Monetary Fund;  
 LIFO – last in first out;  
 MILP – mixed integer linear programming;  
 MIP – mixed integer programming;  
 VND – Vietnamese Dong;  
 VRP – vehicle routing problem;  
 VRPTW – VRP with time windows.

## Introduction

Logistics costs are costs associated with the process of distribution and circulation of goods, which have a direct impact on the efficiency of businesses, localities, and the national economy. Logistics costs are paid by businesses for supply activities in the market. Therefore, it can accelerate or inhibit product consumption and influence the competitiveness of enterprises and the whole economy. Logistics costs mainly consist of transportation costs, warehousing, packing and losses, inventory, and order processing, and administrative (Rushton *et al.* 2022).

In Vietnam, logistics costs are much higher than in many countries ASEAN region and around the world such as Thailand, Singapore, and so on. According to the data of the World Bank, logistics costs including transportation, storage, customs clearance, and so on in Vietnam are about 20.9...25.0% of GDP (VILAS 2021). In which, transportation costs account for about 60%, a high cost compared to developed countries. This cost is higher than Thailand's 6%, Malaysia's 12%, and 3 times higher than Singapore's. According to the IMF, in the US economy, logistics costs account for 9.9% of the country's GDP (\$921 billion in 2000). High logistics cost situations could cause a low level of enterprises' competitiveness. Therefore, reducing logistics costs, especially transportation costs, is a big challenge for Vietnam's logistics industry.

VRP is formed to minimize the transportation cost. The development of VRP is based on the multiple traveling salesman problems (Toth, Vigo 2014; Xu *et al.* 2018). VRP played a vital role in the field of logistics and transportation. Consequently, VRP attracts a lot of attention

from academics and managers Zhang *et al.* 2019). In the practical, VRP appears as several classes of additional constraints, such as considering customer time windows, route lengths, the limits on vehicle capacity, etc. In which, the VRPTW is an important variant of the VRP and has been paid attention from researchers (Wang *et al.* 2020). The VRPTW aims to determine the optimal set of routes for a set of restricted capacity identical vehicles and satisfy the time window constraints (El-Sherbeny 2010). Each customer is served exactly once by a vehicle within the time window defined by the earliest and latest times. Each vehicle trip starts and finishes at the given depot. The locations, demand, and time windows of customers are known as a priority. In the VRPTW, the vehicle could arrive at the customer's location before the earliest time and wait without cost or a penalty cost for example parking slots cost until service becomes allowable. However, the vehicle must not arrive at customers after the latest time. The time window constraints in the VRPTW are required strictly satisfied, however, in many practical problems, these constraints can be violated. There are 2 types of VRPs that allow violating the time windows constraints: (1) VRP with flexible time windows and (2) VRP with soft time windows.

There are different methodologies for determining the solutions of VRP and its variants such as exact algorithm, heuristic methods, and hierarchical method (Comert *et al.* 2018). In the hierarchical method, the problem is systematically split into different levels for determining solutions. In this research, one of the hierarchical methods named cluster-first – route-second was employed. This study introduces a practical problem of vehicle routing in the field of logistics and transportation. This problem mainly takes into account three constraints, including time window, vehicle capacity, and product type. The aims of this study are to determine the number of vehicles and minimize total logistics costs. In which, logistics costs include labour, fuel, and refrigeration costs. The MIP model is built to describe the product delivery network with 39 nodes. The clustering algorithm is used to solve the proposed MIP model. The results show that 11 trucks are used to serve 39 customers and the truck utilization efficiency is more than 70%. These optimal results are found in an average of 0.36 s.

This paper consists of several sections as follows. Section 1 is the literature review, which describes the overview of the research gaps of previous studies. A mathematical model of the VRP and methodology are shown in Sections 2 and 3, respectively. Section 4 presents a case study, including data collection, assumptions, and results. The finally is the conclusions.

## 1. Literature review

VRP is an important topic considered by many managers in the logistics and transportation fields. According to Cao and Yang (2017), there are two basic types of VRP, includ-

ing pure VRPs and their variations. In which, pure VRP refers to as the distribution of cargo from one destination to many other locations (Yao *et al.* 2019). The solutions of VRP bring more benefits for companies such as shortest routes and time, the number of necessary vehicles, and optimal vehicle capacity. These results contribute to reduce the total costs of logistics and increase businesses' profit (Le *et al.* 2020; Zhu *et al.* 2020).

In recent years, VRP has gained more interest from academics. There are many characteristics of VRP that are solved by different mathematical models, and their results meet the aims of businesses (Table 1). Table 1 shows that the differences in VRP characteristics between previous studies focus on objectives, problem description, and mathematical model and algorithm. First of all, the cost is the most concern of previous studies while time, distance, and the number of vehicles have less appearance in their objective function. Zhu and Hu (2019) considered the cost of opening routes in the objective equation (Ruiz *et al.* 2019). Zhang *et al.* (2017) and Spliet and Desaulniers (2015) introduced the transportation cost as a factor to be reduced in their objective function. In a more complicated way, Birim (2016) considered both transportation cost and fixed vehicle cost. Zhu and Hu (2019) minimized three types of costs that include fuel cost, labour cost, and vehicle depreciation cost. Some studies consider both costs and other factors such as distance and the number of ve-

hicles in their objective functions. Kantawong and Pravesjit (2020) implemented the optimization of distance and vehicle service cost. Londoño *et al.* (2021) also optimized distance in addition to minimizing transportation costs and penalty fees. Qi and Hu (2020) found the solution to optimize three types of cost in their cost function, including the costs of fuel, refrigeration, and cargo damage. Simultaneously, they also optimize the number of vehicles used in their problem. Tasar *et al.* (2019) determined the optimal number of vehicles to minimize the fixed and fuel costs. Similarly, Aggarwal and Kumar (2019) established the objective to drop their transportation cost and cost for the availability of vehicles and determine the optimal vehicle number. Unlike introduced studies, Afifi *et al.* (2016) and Pérez-Rodríguez, Hernández-Aguirre (2019) focused on reducing time and distance in their problem, respectively.

Secondly, problem description in almost previous studies includes network configure, constraint, and data. Networks configure from previous studies are sets of nodes ranged from 6 nodes to 200 nodes. These problems mainly deal with two constraints: time window and vehicle capacity. In which, a time window is a more popular constraint than the vehicle capacity. According to Bachu *et al.* (2021), building a model to respond to time constraints is a major challenge. Thus, almost all previous studies consider one of two constraints when determining the optimal solution.

Table 1. VRPs of previous studies (source: current study)

Study	Objective				Problem description						Mathematical model and algorithm
	cost	time	no of vehicle	dis-tance	network configuration (N – nodes)	Constraint			Data		
						time window	vehicle capacity	kind of products	experi-ence	reality	
Qi, Hu (2020)	✓		✓		13 N				✓		MIP, heuristic
Kantawong, Pravesjit (2020)	✓			✓	100 N	✓			✓		MIP, artificial bee colony
Londoño <i>et al.</i> (2021)	✓			✓	51 N				✓		MILP, local search
Aggarwal, Kumar (2019)	✓		✓		60 N	✓			✓		MIP
Pérez-Rodríguez, Hernández-Aguirre (2019)				✓	6 N	✓				✓	an estimation of distribution
Tasar <i>et al.</i> (2019)	✓		✓		100 N				✓		MIP, heuristic
Zhu, Hu (2019)	✓				200 N		✓		✓		MILP, response surface method
Ruiz <i>et al.</i> (2019)	✓				14 N		✓			✓	MIP, biased random – key genetic
Zhang <i>et al.</i> (2017)	✓				27 N	✓			✓		tabu search, the artificial bee colony
Birim (2016)	✓				10 N		✓		✓		MILP, simulated annealing
Afifi <i>et al.</i> (2016)		✓			30 N	✓			✓		MIP, simulated annealing
Spliet, Desaulniers (2015)	✓				60 N	✓	✓		✓		MIP, exact branch – price – cut algorithm
Current study	✓		✓		39 N	✓	✓	✓		✓	MIP, clustering algorithm

For example, Kantawong and Pravesjit (2020), Aggarwal and Kumar (2019), or Pérez-Rodríguez and Hernández-Aguirre (2019) solved their problem with time window. Zhu and Hu (2019) or Ruiz *et al.* (2019) presented the vehicle capacity constraint in their model. Especially, Spliet and Desaulniers (2015) determined the solution for their problem when considering both time window and vehicle capacity. However, there are some studies solving their problem without constraints such as Qi, Hu (2020) and Tasar *et al.* (2019). All VRPs are solved using data from experience or practice. Empirical data are used in more research than data from in reality. Particularly, empirical data are used to solve problems involving multiple nodes in a network configuration. Besides, it is also common in problems containing one or two constraints, even problems with no constraints.

Last, but not least, mathematical models and algorithms are used to find the suitable solution for each VRP in previous studies. Previous studies have built two mathematical models for their problems, MIP and MILP. In addition, they apply various algorithms to find the solution quickly and efficiently. Qi and Hu (2020) applied a heuristic to find solutions for MIP models with 13 nodes. Similarly, Tasar *et al.* (2019) used heuristic algorithms to deal with a MIP consisting of 100 nodes in the network configuration. Birim (2016) and Afifi *et al.* (2016) adopted simulated annealing to solve MILP and MIP, respectively. Some other studies use the algorithm of artificial bee colony (Kantawong, Pravesjit 2020), local search (Londoño *et al.* 2021), response surface method (Zhu, Hu 2019), biased random key genetic (Ruiz *et al.* 2019), or exact branch-price-and-cut algorithm (Spliet, Desaulniers 2015). Especially, Zhang *et al.* (2017) applied both tabu search and the artificial bee colony for their problems.

Similar to some previous studies, MIP is built to solve the VRP in order to determine the required number of vehicles and minimize the total logistics costs in this study. The vehicles considered in both this study and most previous studies were trucks designed to transport products (Tretjakovas, Čereška 2021). The objective function also considers three types of costs, including labour, fuel, and refrigeration costs. However, unlike previous studies, this study deals with a real VRP so the data collected from the real world will be used in our mathematical model. In addition, due to being a real problem, the mathematical model of this study has more constraints than previous studies. Time window, vehicle capacity, and kind of products are considered in the proposed mathematical model. Besides, the logistics network in this study is also larger than the network of some previous studies using real data. For example, Pérez-Rodríguez, Hernández-Aguirre (2019) and Ruiz *et al.* (2019). Furthermore, there is a difference in algorithmic application between this study and related studies. A clustering algorithm is applied in our study to solve the VRP with 39 nodes in the network structure.

## 2. Methodology

In this study, the vehicle routing time windows problem is formulated as MIP firstly. In particular, the main features of the problem are presented to provide a general description of the model being built. The objective function and constraints, as well as the key assumptions, are also formulated to specifically describe the mathematical model. In order to solve large-scale problems, we introduce the cluster-first – route-second method. This is a hierarchical approach that applied dead with a large-scale instance by using an exact algorithm in a reasonable time. In the 1st phase of the method, a clustering algorithm is applied to form the clusters. After that, the capacity of each cluster is controlled by adjusting members in clusters. In the 2nd phase, each subproblem for each cluster is solved by using the branch and bound algorithm.  $k$ -means clustering algorithm and capacitated  $k$ -means clustering algorithm are used to cluster customers. This approach is applied to a real case study given in Section 3.1.

### 2.1. Mathematical model of the VRPTW

The VRPTW is a generalization of the VRPs. The model not only aims to minimize the cost of routes but also satisfy the time window (Wang *et al.* 2020). The customer must be served within a specified time window with bound is the earliest time and the latest time. In our research, the model “one-to-all”, all vehicles need to start and return at a depot (Figure 1). In Figure 1, the warehouse icon depicts where the products are stored and where the vehicles depart. The direction of the arrows also indicates the direction of each truck in a route. Customers on different routes are presented with polygons of different colours. Ordered products will be placed on the vehicle to ship to the respective customer. The number and weight of products must meet the storage capacity of each vehicle. Based on the customer’s order requirements, two main assump-

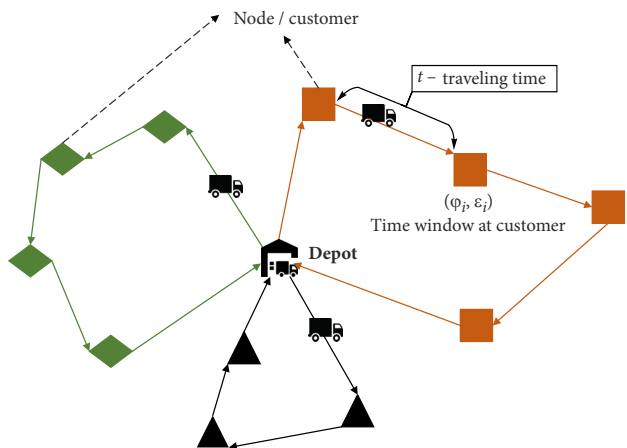


Figure 1. An example about VRPTW (Wang *et al.* 2020)

tions are considered in this mathematical model. They include: (1) the current total number of vehicles meet the total customer demand per day and (2) each customer orders the number of products for the day not to exceed the capacity of one vehicle. As a result, each vehicle is arranged on a route to serve customers and carry a variety of suitable products. This means that each vehicle can serve many customers in its trip. By contrast, one customer only receives products from only one vehicle at one time.

The VRP time windows is the VRPTW is formed by a set of  $V$  vehicles, a set of nodes that includes a set of customer  $C$  and a driving-out depot, and a returning depot. All of the nodes together generate a directed graph  $G=(V, C)$ . Customers are denoted from 1 to  $n$  with  $n$  is the number of customers, the driving – out depot and returning depot are presented by 0 and  $n + 1$ , respectively. The network is a set of  $N$  nodes that consist of  $C + 2$  nodes. An arc  $(i, j)$ , where  $i \neq j$  is an arc from node  $i$  to node  $j$ . No outgoing arc at node  $n + 1$  and no arc come to node 0. However, in some cases, node 0 and node  $n + 1$  are in the same location.

$$\min \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} (c + f) \cdot d_{ij} \cdot x_{ijk} + L \cdot \sum_{k \in V} \sum_{j \in C} x_{0jk} \quad (1)$$

subject to:

$$\sum_{k \in V} \sum_{j \in C} x_{ijk} = 1, \forall i \in C; \quad (2)$$

$$\sum_{k \in V} \sum_{j \in C} x_{0jk} \leq |V|, \forall k \in V, \forall j \in N; \quad (3)$$

$$\sum_{i \in C} D_i \cdot \sum_{j \in C} x_{ijk} \leq CT_k, \forall k \in V; \quad (4)$$

$$\sum_{i \in C} E_i \cdot \sum_{j \in C} x_{ijk} \leq VT_k, \forall k \in V; \quad (5)$$

$$\sum_{j \in C} x_{0jk} = 1, \forall k \in V; \quad (6)$$

$$\sum_{i \in N} x_{ihk} - \sum_{j \in N} x_{hjk} = 0, \forall h \in C, \forall k \in V; \quad (7)$$

$$\sum_{i \in C} x_{i,n+1,k} = 1, \forall k \in V; \quad (8)$$

$$S_{ik} + t_{ij} - K_{ij} \cdot (1 - x_{ijk}) \leq S_{jk}, \forall i, j \in C, \forall k \in V, \quad (9)$$

where:

$$t_{ij} = \frac{d_{ij}}{v} + p_i; \quad (10)$$

$$K_{ij} = \max(\varepsilon_i + t_{ij} - \varphi_j, 0); \quad (11)$$

$$a_i \leq S_{ik} \leq b_i, \forall k \in V; \quad (12)$$

$$x_{ijk} \in \{0, 1\}, \forall i, j \in N, \forall k \in V; \quad (13)$$

$$S_{ki} \in \{0, 1\}, \forall i \in N, \forall k \in V. \quad (14)$$

The model aims to determine the required numbers of vehicles to meet the needs of customers in order to minimize both labour cost and traveling cost (fuel cost and refrigeration cost) as described in Equation (1). Equation (2)

presents that customers need to be visited exactly once by a vehicle. On other words, one customer is only served by one truck. Equation (3) states that the used vehicle could not exceed the number of available vehicles. Equations (4) and (5) mean vehicle capacity need to be satisfied. This means that transporting products to customers is not exceed the maximum capacity of a truck. In which, the vehicle capacity is calculated by both permissible load with the unit of a [kg] and volume with the unit of [m<sup>3</sup>]. Equation (6) ensures that every vehicle leaves at the depot zero to a customer (Equation (7)) and then finishes the trip at the depot  $n + 1$  (Equation (8)). Equation (9) presents that every vehicle from  $i$  to  $j$  cannot arrive at  $j$  before  $S_{ik} + t_{ij}$ . Where  $t_{ij}$  is the spending time from  $i$  to  $j$  that is equal to total moving time from  $i$  to  $j$  and the moving time at customer  $j$ . The moving time is calculated based on the distance between  $i$  and  $j$  and vehicle velocity (Equation (10)). Equation (11) defines a value of  $K_{ij}$  that helps linearize a non-convex optimization. Finally, Equation (12) ensures that time windows are observed. Equations (13) and (14) show type of variables.

## 2.2. k-means clustering algorithm

$k$ -means clustering is a well-known clustering method that is used to form  $n$  observations to  $k$  clusters, where  $k$  is known as a priority. The objective of the algorithm is to minimize the objective function and to separate each compact class as far as possible.  $k$ -means clustering could be explained as below (Khan, Ahmad 2004).

Given  $X = \{x_1, x_2, \dots, x_n\}$  is a set of observations, and each observation is a  $d$  - dimensional real vector.  $C_k = \{c_1, c_2, \dots, c_n\}$  is the be a set of  $K$  centers.  $S_j = \{d | d \text{ is member of cluster } k\}$  be the set of samples that belong to the  $k$ th cluster.

$$\sum_{i=1}^n \text{dist}(d_i, c_k), \quad (15)$$

where:  $\text{dist}(d_i, c_k)$  is the Euclidean distance between a data point  $d_i$  and the cluster center  $c_k$ .

The procedure for the  $k$ -means algorithm consist of 4 steps:

» *step 1*: a set of  $c_k$  was initialized by using random sampling;

» *step 2*: decide the members of each cluster based on the minimum distance from cluster center criteria;

» *step 3*:  $c_k$  will be calculated as Equation (16).  $|S_k|$  is the number of data items in the  $k$ th cluster:

$$c_k = \frac{\sum_{d_i \in S_k}^n d_i}{|S_k|}; \quad (16)$$

» *step 4*: step 2 and step 3 could be repeated until the objective is optimal, and the algorithm reaches the maximum number of iterations.

However, the classical means clustering based on the distance among members and centers. In the VRP, the ca-

capacity of the truck needs to be taken into account. Therefore, Comert *et al.* (2018) introduced the equation for determining the number of  $k$  in  $k$ -means clustering in a capacitated VRP problem. The number of clusters is based on the capacity of trucks and demand:

$$\text{number of clusters} = \frac{\text{total demand}}{\text{truck capacity}}. \quad (17)$$

The number of clusters is round up to an integer number. For example, in this research, the total demand is 10020 kg, and a truck that has a capacity is 1000 kg. As a result, the number of clusters is calculated is 10.020 kg then the decimal rate is round up to 11. Once the number of clusters is determined, the  $k$ -means algorithm is performed in the data. The related customers are assigned to clusters has total demand could not exceed the capacity. Set number base value is specified by considering the condition: truck utilization is 70% truck. After the initial set of members in each cluster is generated from the  $k$ -means clustering algorithm based on the customer locations. The cluster demand is checked whether it is satisfied with the truck capacity and truck utilization condition (less or more than 700 to 1000 kg), the member of each group could be modified and capacity controlled repeated until set satisfied the conditions.

### 3. A case study

#### 3.1. Data collection

In this research, a transportation network from a logistics company in Ho Chi Minh city (Vietnam) is considered. The company provides warehousing and distribution services for 39 main customers in Ho Chi Minh city. As shown in Figure 2, a red star is used to represent the depot, which also is the warehouse and parking place, whereas the locations of customers are presented by blue

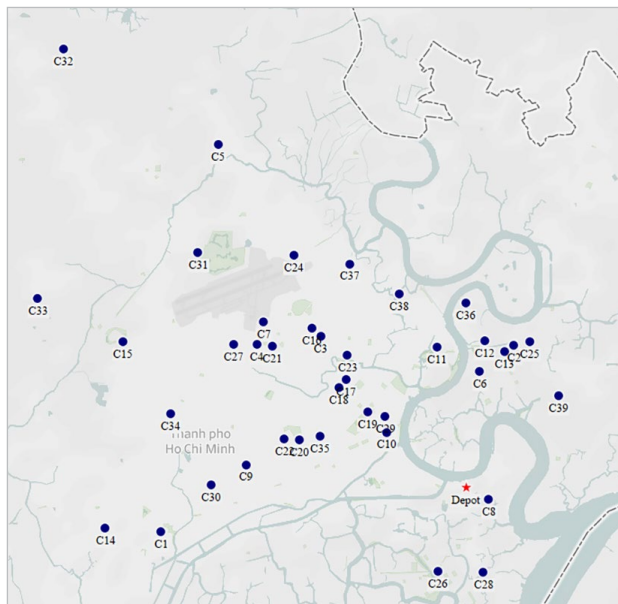


Figure 2. Customer and depot locations (source: current study)

circles. The notations for customers in the map begin with C, for example, C1 is the name of customer 1. The distance matrix among depots and customers is calculated based on latitudes and longitudes of their locations from *Google Maps*.

The logistics company has 15 1000 kg reefer trucks that can flexibly deliver goods in Ho Chi Minh city urban in rush hours and off-peak hours. Every day, the planning department in the warehouse get orders from customers and assigns customers for trucks. Next, the driver sorted, loaded, and delivered to customer. The product in a truck is arranged and grouped by customers and follows the LIFO rule. The velocity of each truck is assumed 45 km/h.

The working time is from 8:30 to 15:30 h. The earliest time and the latest time for serving by trucks and the processing time are varied by customers.

In this research, the truck cost is focused on labour cost, fuel cost, and refrigeration cost. Each truck is operated by a driver, where the salary was assumed to be 500000 VND/day. The salary is calculated based on the salary provided by the human resource department. Based on the manufacturer's technical report, each truck with full load cost 2551 VND/km for fuel. The refrigeration cost per km is assumed to equal 20% of the fuel cost.

#### 3.2. Assumptions

Assumptions are:

- » the problem is static;
- » the model uses one type of reefer truck;
- » the products of customers are packed and arranged before being shipped;
- » the number of customers, customer time window, customer demand, processing time at customer locations are deterministic and they are known as a priority;
- » the fuel cost and refrigeration cost are deterministic;
- » customer demand is changed day-by-day;
- » every truck can start at the depot at  $\phi_0 = 0$  and return the depot at  $\epsilon_0 = 420$  (7 working h);
- » each customer may be serviced by one tractor truck;
- » backhauls are not permitted;
- » 15 vehicles can meet the total demand of customers every day;
- » each customer's demand per day is less than or equal to 1000 kg.

#### 3.3. Results and analysis

The VRPTW model was implemented by IBM ILOG CPLEX Optimization Studio (version 12.10.0.0, <https://www.ibm.com/products/ilog-cplex-optimization-studio>) and the  $k$ -mean clustering algorithm was applied in MATLAB R2019a (<https://www.mathworks.com/products/matlab.html>). The results from the model were visualized by using *Tableau Desktop* 2019 version (<https://www.tableau.com/products/desktop>). All of the software was run on a computer with *Intel I7-10510U-1.80GHz* (8 CPUs), *Window 10* operation system.

3.3.1. *k*-means clustering

In order to solve a large – scale VRPTW problem, clustering methodology was applied to dispatch the original problem to subproblems that be able to determine in a practical time with a limited computational system. When applying the classical *k*-means clustering algorithm, the number of clusters is an important factor that needs to be considered. Besides, the number of clusters affects the number of members in each cluster, therefore, it affects the overall solutions of the VRPTW problem after dispatching.

A set of data for the *k*-means clustering algorithm that includes latitude and longitude of customers was used as input data, there were altogether 39 customers. It is also noted that each customer has a demand that was satisfied by a truck. The output from the classical *k*-means clustering algorithm was the cluster indices of customers. Table 2 shows an example about input and output of *k*-means clustering algorithm with the number of clusters is set to equal to 2. For instance, customer 1 and customer 3 are in cluster 1, customer 2, and customer 38 in cluster 2.

Table 3 presents a summary result from performing classical *k*-means clustering with different values of the number of clusters (value of *k*). The algorithm was executed in MATLAB. Typically, the objective function from *k*-means clustering contains local optimal solutions; therefore, the number of replications was set to 10, and the number of interactions in each replication was set to 100. When increasing the number of replications and number of interactions in each replication beyond these values, the objective was not improved.

When the number of clusters is equal to 1, all of the customers are in the sample cluster. The number of clusters is increased from *k* = 2 to *k* = 11. The increasing number of clusters (number of *k*) makes the number of members in a cluster decreased. For example, when the number of clusters is equal to 2 (*k* = 2), the maximum number of members in a is 20 while number the number of clusters is equal to 11 (*k* = 11), the maximum number of customers in a cluster is 6. This is the key point to help reduce the size of the network.

Table 2. Input-output of the classical *k*-means clustering algorithm (source: current study)

Input			Output
customer	latitude	longitude	cluster indices
1	10.74168068	106.6298856	1
2	10.80059579	106.743338	2
3	10.8034963	106.6812906	1
4	10.80097871	106.6607492	1
5	10.86402657	106.6483003	1
...	...	...	...
38	10.81670269	106.7064081	2
39	10.78474569	106.757709	2

Table 3. Classical *k*-means clustering results (source: current study)

No of cluster	Cluster	No of node	Demand	
			[kg]	[%]
<i>k</i> = 1	1	39	10020	100.00
<i>k</i> = 2	1	20	5589	55.78
	2	19	4431	44.22
<i>k</i> = 3	1	20	5051	50.41
	2	14	4459	44.50
	3	5	510	5.09
<i>k</i> = 4	1	18	3574	35.67
	2	5	2021	20.17
	3	5	510	5.09
	4	11	3915	39.07
<i>k</i> = 5	1	9	3065	30.59
	2	5	510	5.09
	3	3	1250	12.48
	4	17	3174	31.68
	5	5	2021	20.17
<i>k</i> = 6	1	3	1250	12.48
	2	9	3065	30.59
	3	3	374	3.73
	4	9	2124	21.20
	5	6	2133	21.29
	6	9	1074	10.72
<i>k</i> = 7	1	2	128	1.28
	2	9	2124	21.20
	3	9	1074	10.72
	4	3	1250	12.48
	5	9	3065	30.59
	6	5	2021	20.17
	7	2	358	3.57
<i>k</i> = 8	1	2	366	3.65
	2	9	3065	30.59
	3	8	2100	20.96
	4	2	128	1.28
	5	3	1250	12.48
	6	5	2021	20.17
	7	9	1074	10.72
	8	1	16	0.16
<i>k</i> = 9	1	3	1250	12.48
	2	2	128	1.28
	3	2	366	3.65
	4	6	1520	15.17
	5	8	2665	26.60
	6	9	1074	10.72
	7	1	16	0.16
	8	3	980	9.78
	9	5	2021	20.17

End of Table 3

No of cluster	Cluster	No of node	Demand	
			[kg]	[%]
$k = 10$	1	3	980	9.78
	2	3	1250	12.48
	3	6	1520	15.17
	4	8	2665	26.60
	5	6	524	5.23
	6	1	16	0.16
	7	2	128	1.28
	8	4	1708	17.05
	9	2	366	3.65
	10	4	863	8.61
$k = 11$	1	6	1520	15.17
	2	1	16	0.16
	3	2	128	1.28
	4	3	1250	12.48
	5	6	2455	24.50
	6	2	366	3.65
	7	2	580	5.79
	8	4	1708	17.05
	9	3	610	6.09
	10	4	863	8.61
	11	6	524	5.23

$k$ -means clustering algorithm aims to minimize the distance from members to centers. Therefore, without considering the demand of each cluster, the proportion of demand of clusters varies due to a different number of members. For example, at  $k = 2$ , the demand of cluster 1 is 55.78% and the demand of cluster 2 is 44.22%. Demands between the 2 clusters are not significantly different. However, at  $k = 3$ , the highest cluster demand is 50.41%, while the lowest cluster demand is 5.09%. Without considering demand and truck capacity, the model cannot satisfy the truck utilization condition that is at least 70%. For instance, at  $k = 9$ , sum of demand of cluster 1 is equal to 1250 kg while the requirement is 2 1000 kg trucks. So that the truck utilization is equal to 62.50% (truck capacity is 1000 kg). Similarly, the sum of demand of cluster 6 is equal to 16 kg while truck capacity is 1000 kg. Therefore, truck utilization is equal 0.16%.

Capacitated  $k$ -means clustering algorithm was applied to reduce the bias among clusters by considering demand and truck capacity (Khan, Ahmad 2004). Table 4 presents the results from capacitated  $k$ -means clustering that number of clusters equal to 11.

As shown in Table 4, the demand of each cluster was from 704 to 1000 kg, which can satisfy the truck capacity at least 70%. Although cluster 4 and cluster 9 have only one customer in a cluster, their demand already fulfilled truck capacity. Figure 3 presents customer and cluster locations from capacitated  $k$ -means clustering. There are 11 clusters or 11 groups of customers depicted in differ-

Table 4. Capacitated  $k$ -means clustering results (source: current study)

Cluster	No of node	Demand	
		[kg]	[%]
1	4	1000	9.98
2	2	995	9.93
3	3	720	7.19
4	1	1000	9.98
5	6	840	8.38
6	4	982	9.80
7	3	980	9.78
8	3	829	8.27
9	1	1000	9.98
10	4	970	9.68
11	8	704	7.03

Note: % represents the ratio between the demand of each cluster and the total demand of customers.

ent colours and shapes. Customers in the same cluster or group are presented with the same icon and colour. For example, customers 4, 7, 21 and 27 are in a cluster represented by red squares.

### 3.3.2. VRPTW

The clusters and number of members in each cluster were determined and explained in detail in Section 3.3.1. Then, the routing problem was solved by applying the branch and bound algorithm that was formulated in CPLEX. The running time is limited to 2 hours, the MIP gap is set to default setting.

The computational results from a different number of clusters from classical  $k$ -means clustering are shown in Table 5. Results from solving mathematical model include sub-objectives and overall objectives that are transportation costs, the number of used trucks, and running time for determining a solution.

As mentioned in Table 3, without considering demand and truck capacity,  $k$ -means clustering forms clusters and causes a low truck utilization level. Therefore, when the number of clusters increased, the number of trucks was used to deliver products is tended to increase. At  $k = 2$ , the model decides to use 12 trucks and at  $k = 11$ , the model uses 17 trucks. As a result, the overall cost is from  $k = 2$  to  $k = 11$  increases from 6661671.46 to 10036219.62 VND.

By dispatching a large-scale problem to subproblems, the running time is decreased from  $k = 1$ , the problem could be solved in a limited computational system, to  $k = 11$ , the problem needs [ms] to generate the results.

An interesting finding was that the solving time by applying branch and bound in CPLEX not only depend on the size of problems (number of nodes) but also the data set. The average solving time varies due to the size of problems, at  $k = 2$ , the model contains 2 subproblems that take an average solving time is 7214.43 s. The model stops solv-

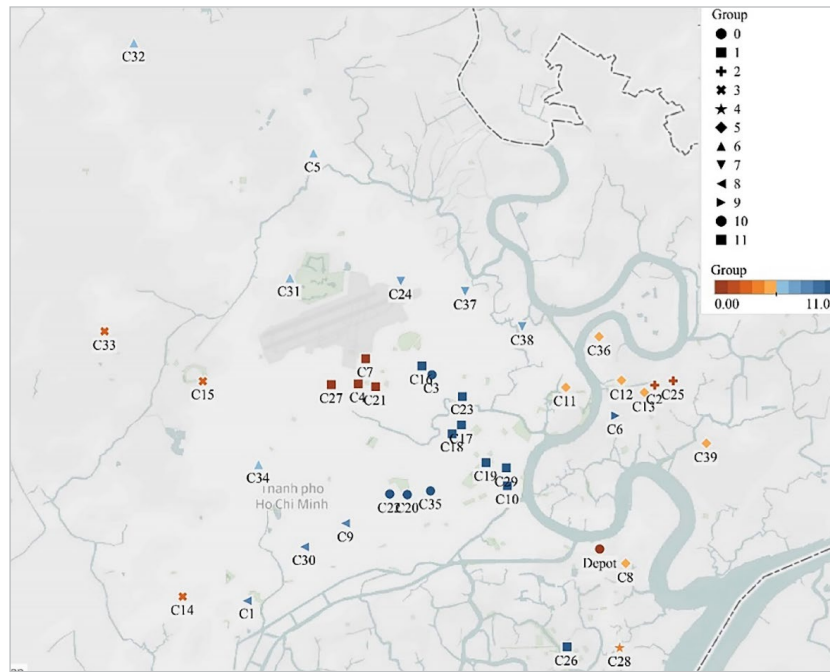


Figure 3. Customer and cluster locations from capacitated  $k$ -means clustering (source: current study)

Table 5. Computational results for different numbers of  $k$  from classical  $k$ -means clustering algorithm (source: current study)

No of cluster	No of truck	Traveling cost [VND]	Overall costs [VND]	Solving time [s]
$k = 1$	-	-	-	-
$k = 2$	12	661671.46	6661671.46	7214.43
$k = 3$	13	801425.99	7301,425.99	2548.67
$k = 4$	12	939921.40	6939921.40	1804.98
$k = 5$	14	1012314.21	8012314.21	1445.70
$k = 6$	15	1275519.08	8775519.08	57.44
$k = 7$	16	1376399.53	9376399.53	46.64
$k = 8$	17	1473159.71	9973159.71	15.65
$k = 9$	16	1417516.89	9417516.89	9.39
$k = 10$	16	1459383.50	9459383.50	0.31
$k = 11$	17	1536219.62	10036219.62	0.26

ing due to meet the time limit condition. The MIP gaps were recorded that 20.98% for the first cluster and 49.11% for the second cluster. At  $k = 11$ , there are 11 subproblems in the model, and they spend [ms] to generate a solution with a MIP gap equal to 10 – 4 (Equal to the default setting in CPLEX). Although the problems contain the same number of nodes, the solving time could be different. In some cases, if more arcs will be considered, in an optimal solution a larger number of variables will take a non-zero value that causes a larger number of solutions to need to be considered, therefore, an increase in the running time of CPLEX will be occurred (Ekşioğlu *et al.* 2009). For example, there is a problem (Table 6) that has 9 customers and 1 depots that has 1320 variables (110 integer variables and 1210 binary variables) and 1679 constraints. Though a presolve problem, a reduced MIP for problem 1

Table 6. An example demand set for problems have 9 customers (source: current study)

	Demand [kg]								
Problem 1 <sup>st</sup>	420	480	320	100	100	80	100	24	500
Problem 2 <sup>nd</sup>	915	1000	60	60	300	80	150	400	100

Table 7. Computational results according to capacitated  $k$ -means clustering algorithm (source: current study)

No of truck	Traveling cost [VND]	Overall costs [VND]	Solving time [s]
11	1086215.32	6586215.32	0.36

has 1229 rows (constraints), 1200 columns(variables), and 7630 non-zero variables while a reduced MIP for problem 2 has 979 rows, 950 columns, and 5970 non-zero variables. Therefore, the solving time for problem 1st and problem 2nd is 224.12 and 16.90 s, respectively.

Table 7 presents a summary result from VRPTW according to capacitated  $k$ -means clustering algorithm. The model uses 11 trucks to deliver products. The traveling cost and overall costs are lower than these costs from the solution at  $k = 2$  for classical  $k$ -means clustering. The subproblems take an average of 0.36 s for generating a solution. Although applying capacitated  $k$ -means clustering algorithm and branch and bound was not guarantee optimality, capacitated  $k$ -means clustering for VRPTW is suitable for the VRPTW problem due to the acceptable cost and practical running time. The data set for VRPTW could be changed every day, especially demand. Therefore, the decision-maker needs to generate solutions in a practical time with a limited configuration computer.

## Conclusions

This research presents cluster-first – route-second approach for a large – scale VRPTW. The customers were clustered by using clustering algorithms such as classical  $k$ -means clustering algorithms and capacitated  $k$ -means clustering algorithms. Classical  $k$ -means clustering algorithms cluster the customer locations and capacitated  $k$ -means clustering algorithms take into account both customer's location and truck capacity. By clustering process, the scale of VRPTW was reduced. Therefore, branch and bound algorithm could be executed to generate the solution in a practical time and limited laptop configuration.

The effectiveness of proposed method is demonstrated by the application of this method in a real case study from Vietnam. In particular, the considered logistics company has 39 customers in Ho Chi Minh city, and 1 type of truck. The truck utilization must be more than 70%. This distribution network is considered to be larger than some previous studies using real data, which are clearly presented in the literature review. The results show that without considering demand and truck capacity, classical  $k$ -means clustering forms bias clusters and causes a low truck utilization level. Besides, from results obtained from classical  $k$ -means clustering for VRPTW, an interesting finding was that the solving time by applying branch and bound in CPLEX not only depend on the size of problems (number of nodes) but also the data set. In the capacitated  $k$ -means clustering algorithm for VRPTW, the model uses 11 trucks to deliver products and can keep truck utilization more than 70%. Each subproblem takes an average of 0.36 s for generating a solution.

Rapidly finding optimal solutions plays an essential role in making the right and correct decisions related to logistics problems. Based on the results, managers can use and coordinate resources appropriately. The solution provides the necessary number of vehicles and vehicles trips. Thus, managers can plan and distribute them more efficiently. Simultaneously, the manager can understand how many vehicles need to be used to serve how many customers and who and where the customers are. The time for finding a solution is extremely short, which can support managers in planning resources to deliver products to customers quickly and exactly. This can help companies improve customer service and increase their competitive advantage in today's dynamic market. On the other hand, the total costs are also fully calculated. Managers can use them to compare with other possible alternatives or to compare current financial resources to make more suitable decisions.

The findings from this study make several contributions to the current logistics and supply chain management:

- »» VRPTW was applied by a real case study in Vietnam, therefore, the VRPTW model considers the constraints of logistics problems in reality. These results contribute in several ways to our understanding of VRPTW problems and provide a basis for cluster-first – route-second approach.

- »» although applying capacitated  $k$ -means clustering algorithm and B&B was not guarantee optimality, capacitated  $k$ -means clustering for VRPTW is suitable for the VRPTW problem due to the acceptable cost and practical running time.

The data set for VRPTW could be changed every day, especially demand. Therefore, the decision-maker needs to generate solutions in a practical time with a limited configuration computer. So that, the methodology of this study is a fundamental foundation for similar practical instances. Delivering essential food and products during Covid-19 situation is a good example. When the Covid-19 pandemic occurred, the governments of many countries ordered a lockdown to eliminate the spread of the virus. As a result, product distribution is severely affected by strict timing in each country, city, and locality. Based on our method, it can significantly reduce the vehicle scheduling time and the required products can be delivered to the customer on time. Another example is school bus routing problems. These issues face diverse needs that can change as the timetable depends on the student's grades. While the vehicle capacity is limited, re-routing is necessary to have a low operation cost while maintaining constraint satisfaction. Therefore, cluster-first – route-second is a method that can generate the solution a practical time to deal with the abundance of customers' demand sets. However, the limitations of this research have thrown up many questions in need of further investigation. In our study, there are only one type of truck was used. Hence, the method to adjust the cluster needs to be investigated. Besides, the model could be extended when the demand of a customer is more than truck capacity. Hence, the research could adjust the network. Besides, the rising environmental concern has opened an opportunity to develop a mathematical model to consider multiple objectives besides the total costs. In many practical situations such as newspaper delivery, non-emergency parcel delivery, furniture delivery, the time window constraints can be relaxed with a customer unsatisfied penalty. That could be another research direction.

## Author contributions

For research papers with several authors, a short paragraph specifying their individual contributions must be provided.

*Thi Diem Chau Le* and *Judit Oláh* conceived and designed the experiments.

*Thi Diem Chau Le* and *Duc Duy Nguyen* collected and analysed data.

*Thi Diem Chau Le* performed the experiments.

*Judit Oláh* contributed reagents/materials/analysis tools.

*Miklós Pakurár* contributes methodology.

*Thi Diem Chau Le* and *Duc Duy Nguyen* wrote original draft.

*Miklós Pakurár* and *Judit Oláh* adjusted and completed the paper.

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