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BENDING DYNAMIC MODEL ADAPTABLE TO CUTTING

Abstract: Cutting tools represent one of very important parts in the chain of production. Due to its importance, knowledge of its behavior gives an opportunity to increase efficiency of cutting tools design. In this paper the bending, torsion dynamic models of driving chains and the computational simulation program based on the models with its adaptation on cutting tools are elaborated.

Keywords: dynamic model, excitation effects, natural bending frequencies, simulation program

1. INTRODUCTION

The steps and problem of dynamic model construction, the derivation of the frequency equations and the application of the simulation program in several papers [1, 2, 3] are outlined. The models elaborated for rotating, flexibly supported or suspended shafts are applicable to cutting tools in simply cases. The simulation program is able to model the bending vibration of some cutting tools with the appropriate input data. The model and the simulation program are capable to determine: the natural bending frequencies of the tools and chucking devices; and the method of the detuning, as well. The appropriate surface quality and dimensional accuracy can be assured by the detuning of the natural frequency from the operation frequency away.

In this paper the derivation and the use of the bending dynamic model will be represented.

2. THE CLARIFICATION OF THE EXCITATION EFFECTS

The excitation effects acting on the cutting tools (turning tools, shaper-and-planer tools), arising from the cutting process, can't be eliminated. These excitation forces can be determined with suitable measurement system during the cutting process [4]. The knowledge of the alteration of the excitation forces in the time is absolutely necessary in case of dynamic analysis to determine the frequency of the excitation. It can be determined with the load model and an experimental verification can be carried out [5].

3. MODEL OF BENDING VIBRATIONS

The natural frequencies of bending vibrations can be determined with the continuum model (Fig. 1, Fig. 2).

The equation system in the case of a constant cross-section rod and flexible mountings, disregarding the damping:

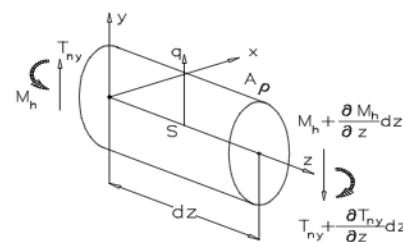


Fig. 1. Type of continuum model

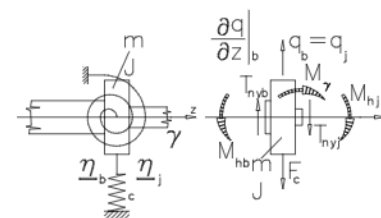


Fig. 2. Limitation and load of continuum model

$$\rho \frac{\partial^2 q}{\partial t^2} + IE \frac{\partial^4 q}{\partial z^4} = 0 \quad (1)$$

where: $q(z,t)$: general coordinate of the rod cross-section

E : flexibility modulus

I : secondary torque of the rod section

ρ : density

The solution of the differential equation can be sought in the following form:

$$q = v_z \cos(\alpha t + \varepsilon) \quad (2)$$

where: $q(z)$: general coordinate of optional cross-section

α : angular velocity of shaft

After substitution the following differential equation can be obtained ($k^4 = \alpha^2 \frac{A\rho}{IE}$):

$$v^{IV} - k^4 v = 0 \quad (3)$$

The solution can be sought in the following form:

$$v_{(z)} = D_1 S_{(kz)} + D_2 T_{(kz)} + D_3 U_{(kz)} + D_4 V_{(kz)} \quad (4)$$

where D_1, D_2, D_3, D_4 , can be established from the boundary conditions.

The mechanical characteristics of an optional cross-section are contained in the state-vector:

$$\underline{n}_i = \begin{bmatrix} v_i \\ \varphi_i \\ M_i \\ \bar{T}_i \\ 1 \end{bmatrix} \quad (5)$$

where: $v_{(z)}$ [mm] the cross-section displacement
 $\varphi_{(z)} = v'_{(z)}$ [rad] the cross-section rotation
 $M_{(z)} = -IE v''_{(z)}$ [Nm] bending moment acting on the cross-section
 $\bar{T}_{(z)} = IE v'''_{(z)}$ [N] shearing force acting on the cross-section

Mechanical characteristics of an optional cross-section with the help of a transform matrix:

$$\underline{n}_1 = \underline{F} \underline{n}_0 \quad (6)$$

which:

$$\underline{F} = \begin{bmatrix} S_{(kz)} & \frac{1}{k} T_{(kz)} & -\frac{1}{k^2 IE} U_{(kz)} & \frac{1}{k^3 IE} V_{(kz)} & 0 \\ kV_{(kz)} & S_{(kz)} & -\frac{1}{KIE} T_{(kz)} & \frac{1}{k^2 IE} U_{(kz)} & 0 \\ -k^2 IE U_{(kz)} & -kIE V_{(kz)} & S_{(kz)} & -\frac{1}{k} T_{(kz)} & 0 \\ k^3 IE T_{(kz)} & k^2 IE U_{(kz)} & -kV_{(kz)} & S_{(kz)} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

transpose matrix

$\underline{P}$ connecting matrix for: flexible support excitation force and torque:

$$\underline{P}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & J\alpha^2 - \frac{1}{\gamma} & 1 & 0 & 0 \\ m\alpha^2 - \frac{1}{c} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$\underline{P}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & H \\ 0 & 0 & 0 & 1 & R \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

where: H, R: Supplementary loading due e.g. to unbalancing effect or eccentricity.

The frequency equation can be derived from the matrix equation after applying the boundary conditions:

Unknowns of the equation system: v_0, ϕ_0 in total 2

The number of equations: $M_1 = 0; \bar{T}_1 = 0$; in total 2

Thus the equation system can be solved.

4. APPLICATION OF MODEL FOR A SINGLE ROD ELEMENT

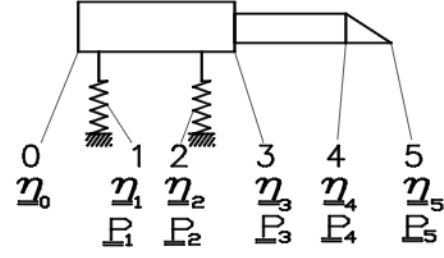


Fig. 3. Designation of section boundaries of the turning tools

The mechanical characteristics of the last cross-section:

$$\underline{n}_5 = \underline{F}_5 \underline{P}_4 \underline{F}_4 \underline{P}_3 \underline{F}_3 \underline{P}_2 \underline{F}_2 \underline{P}_1 \underline{F}_1 \underline{n}_0 \quad (10)$$

Every $\underline{F}$ and $\underline{P}$ of the matrix equation can be written

Boundary conditions: $M_0 = T_0 = 0$

The following equation system is deduced:

$$av_0 + b\phi_0 = K_1 \quad (11)$$

$$cv_0 + d\phi_0 = K_2$$

The coefficients appearing in the equation system are denoted with the letters $a \dots m$ for lucidity.

The number of equations and unknowns is 2.

The natural frequencies are given from the zero point of the determinant derived from coefficients of the equation system's homogeneous section.

The unknowns can be calculated with the following Cramer's method, e.g.: the ϕ_0 .

$$\phi_0 = \frac{\begin{vmatrix} a & K_1 \\ c & K_2 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \quad (12)$$

After determining the unknowns, the mechanical characteristics of the cross-section can be determined at any section boundary. The displacement, rotation, bending moment and concrete value of the shearing force can be calculated at the given section boundary, if the operating frequency is substituted into the transform matrix.

5. CONCLUSION

In this paper the derivation and the usage of the bending dynamic model with its application for calculation of displacement, rotation, bending moment and concrete value of the shearing force and also detuning in the natural frequencies are elaborated. The parameters of the cutting tool are input data of the developed computerised simulation programme. With the help of the programme the effect of the modifications on the natural frequencies can be easily followed.

6. REFERENCES

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