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THEORETICAL INVESTIGATION OF ROPE STRAND SUBJECTED TO AXIAL TENSILE LOAD

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Wire ropes or rope strands are one of the most important load carrying components of transportation systems such as bridge, elevator, crane and mine hoisting. Wire ropes must have high tensile strength in order to carry high tensile load. Wire ropes are mainly subjected to axial tensile load in service. This axial tensile load is exerted upon wires of rope or strand. Due to this reason tensile load causes elongation, strain and stress on the center wire (core wire) and outer wires. In this study, theoretical calculations proposed by Feyrer are adopted in order to determine wire loads, stresses, elongations and strains for axially loaded rope strands. An illustrative example is given.

Keywords: *strand tensile stress, axial loaded strand, wire strain, rope strand*

1. Introduction

It is inevitable to increase the interest in living with more comfort, growing in parallel with the development of technology, and to increase interest in transportation systems in the near future. One of the most stressed elements of material handling systems is steel wire ropes which are frequently used in these systems. Ropes comprised of strands which are helically wrapped around a fiber or a steel core. The helical direction of the outer strands is called as winding direction of the rope [1, 2]. The strand is obtained by winding the thin steel wires around the core in one or more layers according to a certain rule [3]. Tensile load is exerted upon strand wires where outer helical wires are laid by certain array around a wire or fiber core in order to form a strand. Numerous studies have been carried out in the literature on the axially loaded rope strands. Feyrer [4] presented formulas for the calculation of wire loads, stresses and elongations for axially loaded rope strands, spiral rope, rotation resistant rope and stranded rope in his book. Love [5] discussed the general theory of bending and torsion of thin rods. Costello [6] exposed a theory for estimating stresses that occur on a multilayer rope exposed to bending, axial and torsional loads. Phillips and Costello [7] determined stresses occurring on an axially loaded 6×25 Filler rope which bent on the sheave. Costello [8] expanded the thin rod theory derived by Love to analytically formulate the static behavior of a wire, strand and a whole rope, respectively, in such a way that effects of curvature and torsional change in a thin wire are also considered. Kumar and Cochran [9] presented an analytical approach on the static behavior of a rope with fiber core which axially loaded and is subjected to torsional moment. Jiang [10] presented general formula for linear and non-linear analyses of wire ropes. Gnanavel et al. [11] performed a numerical study taking into consideration core–wire and wire–wire contacts together and compared the results with other theories. Onur [12] investigated the response of the prestressing strands to axial tensile load theoretically and experimentally. Onur et al. [13, 14] put effort to determine fatigue lifetime of steel wire ropes subjected to bending over sheave fatigue. The objective of this study is to determine stress and strain values on a specific rope strand subjected to axial tensile load. In addition, load carrying capacity of each wires of strand

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and elongations are determined. In this study, Feyrer theory has been considered in order to determine load carrying capacity of each wire of strand, elongations and strains of center wire and outer helical wires and tensile stresses occurring on each wire of strand where rope strand is subjected to axial tensile load. An illustrative example is given.

2. Forces and stresses on a rope strand

Wire ropes or rope strands are frequently subjected to axial loads in the application areas. Those axial loads cause tensile stress on the cross section of rope strand perpendicular to the carried axial load. It is inevitable to calculate loads and stresses carried by each wire of strand in order to evaluate safety factor of steel strand. Certain assumptions are made to derive analytical solution of strand axially loaded. If an axially loaded strand is free to rotate, the torsional moment occurs because of the helical form of the wires. In the installation, the strand rotation should be prevented so that the structure of the strand is not distorted and uneven stress distributions in strand wires do not occur. Strand rotation can be prevented by restraining strand's each end against rotation. In this study, the equations are presented and used for the state that strand is not allowed to rotate. Solid model of a rope strand is shown in Fig. 1. In Fig. 1, strand has a center wire and six outer wires wrapped around this center wire with a certain winding angle. The first layer is entitled as layer 0 where the core wire is located and outer layer where six outer wires are helically wrapped around core wire is entitled as layer 1.

There are two external forces exerted upon a single wire at the i. layer of the strand. These are strand tensile force, S_i and circumference force, U_i. Both outer forces S_i and U_i on a wire must be in balance with the inner forces that are the wire tensile force, F_i and the wire shear force, Q_i. Wire shear force and bending and torsion moments on a wire are small and can be neglected. Loads exerted upon wire are shown in Fig. 2 [4].

F_i in a wire in the layer i shown in Fig. 2 can be found by using Eq. (1):

$$F_i = S_i / \cos \alpha_i, \tag{1}$$

where α_i is lay angle.

U_i is found by using Eq. (2)

$$U_i = F_i \sin \alpha_i. \tag{2}$$

Radial force at unit length exists between center wire and wire helix under tensile load. Radial force at unit length is found by using equation (3) [4]:

$$q_i = F_i / \rho_i = F_i \sin^2 \alpha_i / r_i, \tag{3}$$

where r_i is wire winding radius in layer i. For example, winding radius of outer wires (r_1) is calculated by $r_1 = R_0 + R_1$. Here, R_0 is center wire radius and R_1 is outer wire radius.

S_i is found by using Eq. (4):

$$S_i = F_i \cos \alpha_i. \tag{4}$$

Tensile force of the strand (S) is the sum of all wire tensile force components and can be found by using Eq. (5):

$$S = \sum_{i=0}^{n} z_i S_i = \sum_{i=0}^{n} z_i F_i \cos \alpha_i, \tag{5}$$

where n is number of wire layers. It is counted beginning from inside with $n = 0$ which is the layer of centre wire and z_i is the number of wires in the wire layer i.

It is supposed that strand cross section rests as plane if the strand with the length l_s is elongated with Δl_s by a tensile force. The tensile force of a wire in a wire layer i is found by using Eq. (6):

$$F_i = (\Delta l_i / l_i) E_i A_i, \tag{6}$$

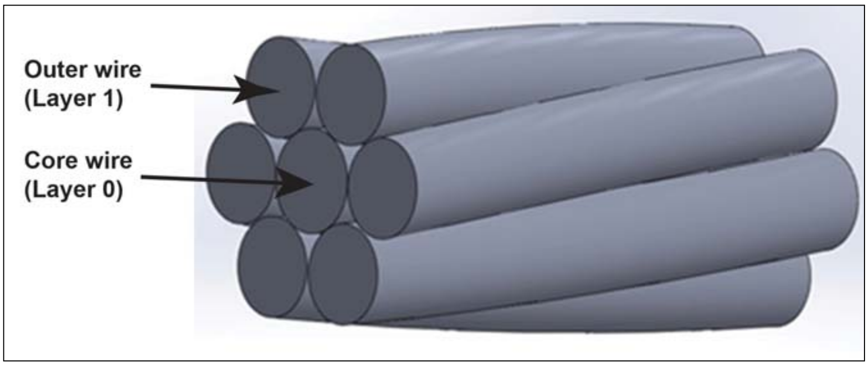

Fig. 1. Solid model of rope strand

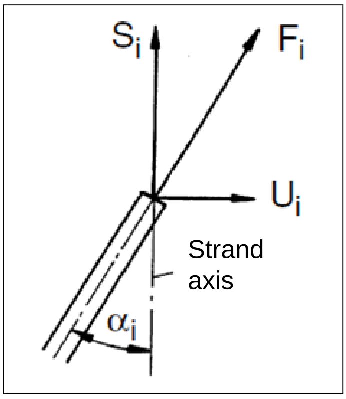

Fig. 2. Forces acting on a wire of strand (shear forces are neglected)

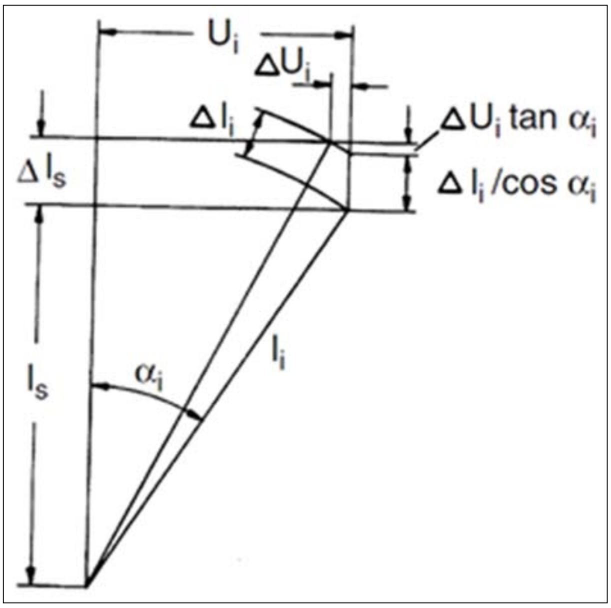

Fig. 3. Elongation of strand wire

where l_i is the wire length, Δl_i is the wire elongation, E_i is the modulus of elasticity and A_i is the cross-sectional area of a wire in the layer i. The wire strain (ε_i) is found by using Eq. (7) [4]:

$$\varepsilon_i = \Delta l_i / l_i . \tag{7}$$

The wire length (l_i) is obtained by using Eq. (8):

$$l_i = l_s / \cos\alpha_i . \tag{8}$$

Figure 3 shows the unwound wire about the strand axis before and after the strand elongation.

Δl_i is found by using Eq. (9) by considering Fig. 3.

$$\Delta l_i = (\Delta l_s - \Delta u_i \tan\alpha_i)\cos\alpha_i . \tag{9}$$

Poisson's ratio of the wire helix is given by Eq. (10):

$$\upsilon_i = (\Delta u_i / u_i)/(\Delta l_i / l_i) , \tag{10}$$

where u_i is the winding circumference and Δu_i is its contraction. Equation (11) can be found when Eq. (10) and Eq. (7) are used together:

$$\Delta u_i = \varepsilon_i v_i u_i . \tag{11}$$

Because of winding circumference is $u_i = l_i \sin\alpha_i$, Eq. (11) becomes $\Delta u_i = \varepsilon_i v_i l_i \sin\alpha_i$. Wire elongation (Eq. (12)) can be found when this equation and Eq. (9) are used together.

$$\Delta l_i = \Delta l_s \cos\alpha_i - \varepsilon_i v_i l_i \sin^2\alpha_i . \tag{12}$$

A wire elongation in the wire layer i is found by using Eq. (13):

$$\Delta l_i = \frac{\Delta l_s \cos\alpha_i}{1 + v_i \sin^2\alpha_i} . \tag{13}$$

Eqs (13), (6) and (8) are used together to calculate tensile force of a wire (F_i) in the wire layer i related to elongation of strand (Δl_s) that is given by Eq. (14):

$$F_i = \frac{\Delta l_s \cos^2\alpha_i}{l_s (1 + v_i \sin^2\alpha_i)} E_i A_i . \tag{14}$$

Component of tensile force of a wire in the strand axis is S_i and is found by using Eq. (15):

$$S_i = \frac{\Delta l_s \cos^3\alpha_i}{l_s (1 + v_i \sin^2\alpha_i)} E_i A_i . \tag{15}$$

Total tensile force of whole strand, S is obtained by using Eq. (16) [4]:

$$S = \frac{\Delta l_s}{l_s} \sum_{i=0}^{n} \left(\frac{z_i \cos^3\alpha_i}{1 + v_i \sin^2\alpha_i} E_i A_i \right) . \tag{16}$$

The tensile force in a wire of a specific wire layer k obtained by using together Eqs (14) and (16) can be found by using Eq. (17):

$$F_k = \frac{\dfrac{\cos^2\alpha_k}{1 + v_k \sin^2\alpha_k} E_k A_k}{\sum\limits_{i=0}^{n} \left(\dfrac{z_i \cos^3\alpha_i}{1 + v_i \sin^2\alpha_i} E_i A_i \right)} S . \tag{17}$$

Wire tensile stress of a specific wire layer k is obtained by utilizing Eq. (18):

$$\sigma_{tk} = \frac{F_k}{A_k} = \frac{\dfrac{\cos^2\alpha_k}{1 + v_k \sin^2\alpha_k} E_k}{\sum\limits_{i=0}^{n} \left(\dfrac{z_i \cos^3\alpha_i}{1 + v_i \sin^2\alpha_i} E_i A_i \right)} S . \tag{18}$$

If a strand is subjected to axial tensile load, strand torque occurs because of the helical structure of the outer wires. Strand torque is calculated regarding rope tensile load and geometric data of the strand. Torque for a strand can be found by using Eq. (19):

$$M = c_{1s} d_s S , \tag{19}$$

where d_s is the strand diameter. The torque constant c_{1s} depends on geometry of rope. c_{1s} can be found by using Eq. (20) [4]:

$$c_{1s} = \frac{\sum\limits_{i=1}^{n} z_i r_i A_i \cos^2\alpha_i \sin\alpha_i}{d_s \sum\limits_{i=0}^{n} z_i A_i \cos^3\alpha_i} . \tag{20}$$

Table 1. Properties of investigated rope strand

Parameter	Value	Parameter	Value
Center wire radius (R_0) (mm)	2.7	Minimum breaking load ($F_{\min}$) (kN)	279
Wire diameter at outer layer (R_1) (mm)	2.61	Wire modulus of elasticity (E) (MPa)	196500
Pitch of outer layers (p) (mm)	240	Number of outer wires	6
Poisson's ratio (v)	0.3	Helix angle (α_1)	7.915°
Strand length (l_s) (mm)	500	Strand diameter (d_s) (mm)	15.7

3. A case study

In this study, a strand that has 15.70 mm in diameter has been used. Technical properties of investigated rope strand are presented in Table 1.

A case study is performed by making certain assumptions as such that strand length (l_s) is selected to be 500 mm, strain of the center wire or strand is assumed to be $\varepsilon_s = \varepsilon_{\text{strand}} = \varepsilon_0 = 0.0003$. Strain of the center wire of the strand is equal to the strain of the strand. Load carrying capacity of each of the wires of strand, elongations and strains of center wire and outer helical wires and tensile stresses emerged on strand wires where strand is exposed to tensile load are determined and presented by considering Feyrer's theory. The following steps in the calculations made can be summarized as follows: strand elongation Δl_s is calculated by using formula $\varepsilon_{\text{strand}} = \Delta l_s / l_s$ or conversely, the strand strain is calculated by using the measured elongation amount of strand to which an axial load is applied. Strand elongation Δl_s is found to be 0.15 mm. Outer wire length is found to be $l_i = l_1 = 504.8$ mm by using Eq. (8). A wire elongation Δl_i that is located on the outer layer of strand is found by using Eq. (13). Strain of wires, ε_1 that are located on the outer layer of strand is found by using Eq. (7). Tensile force $F_i = F_1$ in the direction of axis of an outer wire is found by using Eq. (14) or Eq. (17). Tensile force $S_0 = F_0$ in the direction of axis of center wire is found by using Eq. (14), (15) or (17). Total carried tensile load by outer layer (S_{1t}) of strand is six times greater than S_1 value that is found by using Eq. (15) since outer layer of strand has six wires. Total carried axial tensile load by whole strand, S is found by using Eq. (16). Axial tensile stresses emerged on a wire that is located at outer layer and center wire are found by using Eq. (18). Radial force at unit length, q_1 between center wire and helical outer wire under tensile load is found by using Eq. (3) and torque, M at each layer under tension is found by using Eq. (19). Load carrying capacity of each of the wires of strand and strain of outer helical wires and strand wire's tensile stresses where strand is under tension have been calculated and presented in Table 2. Subscript 0 expresses center wire and subscript 1 expresses wire of outer layer in Table 2.

It is readily seen from Table 2 that carried tensile force, S_0 by center wire in the strand axis direction is 1350 N.

Total carried axial tensile load of six outer wires S_{1t} is 7314 N. Total carried load by whole strand is determined to be $S = S_0 + S_{1t} = 8664$ N. It can be concluded that center wire carries 15.582% of total load and helical wires at outer layer carry 84.418%. Strain of an outer helical wire, ε_1 is found to be 0.000293186. Since the center wire is straight it is not twisted under the tensile load and therefore the torsional moment at the center is not generated. Total torsional moment exerted upon outer wires, M is found to be 5400.184 Nmm. Radial force at unit length, q_1 between center wire and outer helical wire is found to be 4.387 N/mm. Tensile stress on center wire σ_{t0} is found to be 58.949 MPa and tensile stress occurred on a helical outer wire σ_{t1} is found to be 57.507 MPa where strand is exposed to tensile load. It can be concluded that tensile stress on center wire is little greater than outer wire.

4. Conclusions

In this study, theoretical calculations proposed by Feyrer are adopted in order to determine wire loads,

Table 2. Loads, stresses and strains occurred on investigated strand

Load/Moment/Stress/Strain	Value	Load/Moment/Stress/Strain	Value
ε_1	0.000293186	F_1 (N)	1231
S_{1t} (N)	7314	$F_0 = S_0$ (N)	1350
S (N)	8664	σ_{t0} (MPa)	58.949
q_1 (N/mm)	4.387	σ_{t1} (MPa)	57.507
M (Nmm)	5400.184		

stresses, elongations and strains for axially loaded rope strands. An illustrative example is given. In case strand is subjected to 8664 N in the axial direction it is found that the strain of strand and the central wire becomes 0.0003 and strain of outer wire becomes 0.000293186. Center wire carries 1350 N and six outer wires carry 7314 N of total carried load in the strand axis direction. It can be concluded that center wire carries 15.582% of total load and helical wires at outer layer carry 84.418%. Total torsional moment exerted upon outer wires is found to be 5400.184 Nmm. In addition, tensile stress on center wire is found to be 58.949 MPa and tensile stress on outer wire is found to be 57.507 MPa. It can be concluded that tensile stress on center wire is little greater than outer wire.

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