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Comparative analysis of the fractional order Cahn-Allen equation

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A B S T R A C T

This current work presents a comparative study of the fractional-order Cahn-Allen (CA) equation, where the non-integer derivative is taken in the Caputo sense.The Cahn-Allen equation is an equation that assists in the comprehension of phase transitions and pattern formation in physical systems. This equation describes how different phases of matter, such as solids and liquids, change and interact throughout time. We employ two analytical methods: the Laplace Residual Power Series Method (LRPSM) and the New Iterative Method (NIM), to solve the proposed model. The LRPSM is a combination of the Laplace Transform and the Residual Power Series Method, while the New Iterative Method is a modified form of the Adomian Decomposition Method that does not require any type of polynomial or digitization. For the purpose of accuracy and reliability, we compare our findings with other methods and the exact solution used in the literature. Additionally, 2D and 3D plots are generated for various fractional order values denoted as p. These plots illustrate that as the fractional order p approaches 1, the graph of the approximate solution gradually coincides with the graph of the exact solution.

1. Introduction

In the discipline of fractional calculus (FC), one may compute various long-term dynamics and other significant information about the phenomena being studied by using the fractional order derivative of a function. Fractional calculus finds practical applications across various domains, such as the manipulation of dynamical systems, the field of electrical and optical communications, and signal processing. These applications often involve the effective description of phenomena through linear or nonlinear fractional differential equations (FDEs).[1–4] A fascinating study of fractional calculus examines the research on integrals and derivatives of fractional order. In the past 200 years, it has gained more scientific attention globally. It has remarkable applications in a variety of technical and medicinal fields. The concepts of fractional derivatives and integrals were first introduced by Riemann and Liouville.[5] The discipline of fractional calculus has seen a large number of innovative and intriguing models develop throughout time.[6–10] Caputo provided a better formula, for instance, in the field of fractional calculus. In order to demonstrate various scientific phenomena in the fields of fluid mechanics, plasma physics, solid-state physics, population dynamics, chemical kinetics, nonlinear optics, soliton theory, protein chemistry, etc., the use of nonlinear partial differential equations has grown and assumed greater significance. These nonlinear models attract an enormous amount of interest in the relevant fields, just like their scientific frameworks do. Many of the above-mentioned applied science fields[11–14] involve a variety of processes for which nonlinear models are essential. The most effective type of partial differential equations (FDEs) for modeling a variety of complicated processes in applied sciences is called fractional partial differential equations (FPDEs).[15] The El Nino-Southern oscillation mannequin and groundwater float are two significant models that are represented by FPDEs. The extended FPDE mathematical models are crucial for understanding natural processes. Researchers have attempted to numerically or analytically solve these models in order to analyze the precise dynamics of the stated events.[16–21] As a result of the ongoing research

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in this field, we take into account the fractional-order Cahn-Allen (FCA) equation. The fractional order Cahn-Allen equation is significant because it extends the classical model to better capture complex phase separation phenomena, provides improved predictive capabilities, and has applications in a variety of scientific and engineering disciplines such as materials science, nanotechnology, and others. Because of its adaptability and capacity to explain non-local interactions, it is an invaluable tool for researchers researching phase separation and related phenomena. This mathematical model holds significant importance and can be written as follows[22]:

$$D_t^{\rho}\beta(\varphi,t)-\frac{\partial^2\beta(\varphi,t)}{\partial\varphi^2}+\beta^3(\varphi,t)-\beta(\varphi,t)=0,\quad \text{where}\quad 0<\rho\le 1, \qquad (1.1)$$
$$\beta(\varphi,0)=g(\varphi).$$

The FCA equation becomes the classical Cahn-Allen equation under special conditions when $\rho = 1$. Hariharan and Kannan[23] applied the Haar wavelet technique to obtain a numerical solution for the Cahn–Allen equation. The authors of Ref. 24 applied the fractional sub-equation method to the time-fractional Cahn–Allen equation, which is a variant of the S-H equation. They obtained an approximate solution that preserves some of the properties of the original equation. The fractional-order analytical solution to the Cahn-Allen problem was constructed by Yasar and Giresunlu[25] using the homotopy analysis approach. The residual power series method (RPSM)[26–28] is an efficient way for obtaining approximate analytical solutions to fractional ordinary differential equations (FODEs). However, the procedure requires the residual function's derivative. We are all aware of how difficult it is to compute the fractional derivative of a function. As a result, the use of traditional RPSM is fairly limited. Eriqat et al. developed the Laplace residual power series method (LRPSM) to address the RPSM's drawbacks.[29] The suggested approach has been utilized for solving a number of FODEs.[30–32] Additional approaches for solving FPDEs with applications can be found in Refs. 33, 34. The Laplace transform (LT) is utilized in the LRPSM approach to reduce the concentrated problem into new algebraic equations. The RPSM is then used to compute the series solution. Finally, the inverse LT is used to achieve the desired outcome. The LRPSM requires only the most basic computations to be accomplished in less time and with improved accuracy. The main goal of this research work is to fined the solution of fractional-order Cahn-Allen equations by using LRPSM and new iterative method (NIM). Furthermore, 3D graphs are plotted for different values of fractional order p. It reveals that as the value of fractional order p approaches to 1 the graph of the approximate solution converges to the exact solution graph. In Caputo's sense, fractional-order derivatives are those whose order falls within the interval (0, 1]. The outcomes show that the techniques quickly converge and produce excellent results. The rest of the article is arranged as follows: Section 2 is devoted to a few fundamentals of fractional calculus, while Section 3 discusses the suggested approaches. Section 4 is reserved for the implementation of the proposed techniques. Section 5 contains numerical and graphical results. Conclusion remarks are described in Section 6.

2. Preliminaries

Definition 2.1. The non-integer derivative of function of order p in the sense of Caputo is defined as[35]:

$${}^{C}D_{\mu}^{m}\beta(\varphi,\mu)=\frac{1}{\Gamma(m-p)}\int_0^{\mu}(\mu-\varepsilon)^{m-p-1}\beta^{(m)}(\varphi,\varepsilon)d\varepsilon,$$
$$m-1<\alpha\le m,\quad t>0. \qquad (2.1)$$

Definition 2.2. The formula for the non-integer Riemann integral is as follows[36]:

$$I_{\mu}^{p}\beta(\varphi,\mu)=\frac{1}{\Gamma(p)}\int_0^{\mu}(\mu-\varepsilon)^{p-1}\beta(\varphi,\varepsilon)d\varepsilon. \qquad (2.2)$$

Definition 2.3. The Laplace transformation (LT) of $\beta(\varphi,\mu)$ is given as[35]

$$\beta(\varsigma,s)=\mathcal{L}_{\mu}[\beta(\varphi,\mu)]=\int_0^{\infty}e^{-s\mu}\beta(\varphi,\tau)d\mu,\quad s>p, \qquad (2.3)$$

where the Laplace transform inverse is defined as

$$\beta(\varphi,\mu)=\mathcal{L}_{\mu}^{-1}[\beta(\varphi,s)]=\int_{l-i\infty}^{l+i\infty}e^{s\mu}\beta(\varphi,s)ds,\quad l=Re(s)>l_0. \qquad (2.4)$$

Lemma 2.1. *For $n-1<\eta\le n$, $\zeta>-1$, $\mu\ge 0$ and $\delta\in R$, we have:*

1. $D_{\mu}^{\eta}\mu^{\delta}=\frac{\Gamma(\eta+1)}{\Gamma(\delta-\eta+1)}\mu^{\delta-\eta}$
2. $D_{\mu}^{\eta}\zeta=0$
3. $D_{\mu}^{\eta}I_{\mu}^{\eta}\beta(\varphi,\mu)=\beta(\varphi,\mu)$
4. $I_{\mu}^{p}D_{\mu}^{p}\beta(\varphi,\mu)=\beta(\varphi,\mu)-\sum_{i=0}^{n-1}\partial^{i}\beta(\varphi,0)\frac{\mu^{i}}{i!}$

3. Road map of the proposed methods

This section contains the general procedure of Laplace residual power series method (LRPSM) and new iterative method (NIM) for a general fractional order partial differential equation.

3.1. General procedure of LRPSM

Consider the fractional order partial differential equation

$$D_{\mu}^{p}\beta(\varphi,\mu)+N[\beta(\varphi,\mu)]+R[\beta(\varphi,\mu)]=0,\quad \text{where}\quad 0<p\le 1 \qquad (3.1)$$

subject to initial condition:

$$\beta(\varphi,0)=f_0(\varphi). \qquad (3.2)$$

Here $N[\beta(\varphi,\mu)]$ is non linear operator and $R[\beta(\varphi,\mu)]$ is linear term.

Applying Laplace transform to Eq. (3.1) and making use of Eq. (3.2) we get

$$\beta(\varphi,s)-\frac{f_0(\varphi,s)}{s}+\frac{1}{s^p}\mathcal{L}_{\mu}[N[\mathcal{L}_{\mu}^{-1}[\beta(\varphi,s)]]]+\frac{1}{s^p}R[\beta(\varphi,s)]=0. \qquad (3.3)$$

Consider Eq. (3.4) as the solution of Eq. (3.3)

$$\beta(\varphi,s)=\sum_{n=0}^{\infty}\frac{f_n(\varphi,s)}{s^{np+1}}, \qquad (3.4)$$

where the kth-truncated term series given as

$$\varphi(\varphi,s)=\frac{f_0(\varphi,s)}{s}+\sum_{n=1}^{k}\frac{f_n(\varphi,s)}{s^{np+1}},\quad n=1,2,3,4\cdots. \qquad (3.5)$$

The Laplace residual functions (LRFs)[32] are

$$\mathcal{L}_{\mu}Res(\varphi,s)=\beta(\varphi,s)-\frac{f_0(\varphi,s)}{s}+\frac{1}{s^p}\mathcal{L}_{\mu}[N[\mathcal{L}_{\mu}^{-1}[\beta(\varphi,s)]]]+\frac{1}{s^p}R[\beta(\varphi,s)], \qquad (3.6)$$

and the kth-LRFs is given as:

$$\mathcal{L}_{\mu}Res_k(\varphi,s)=\beta_k(\varphi,s)-\frac{f_0(\varphi,s)}{s}+\frac{1}{s^p}\mathcal{L}_{\mu}[N[\mathcal{L}_{\mu}^{-1}[\beta_k(\varphi,s)]]]+\frac{1}{s^p}R[\beta_k(\varphi,s)]. \qquad (3.7)$$

To illustrate a few facts, the following LRPSM features are provided:

- $\mathcal{L}_{\mu}Res(\varphi,s)=0$ and $\lim_{j\to\infty}\mathcal{L}_{\mu}Res_k(\varphi,s)=\mathcal{L}_{\mu}Res_{\varphi}(\varphi,s)$ for each $s>0$.
- $\lim_{s\to\infty}s\mathcal{L}_{\mu}Res_{\varphi}(\varphi,s)=0\Rightarrow\lim_{s\to\infty}s\mathcal{L}_{\mu}Res_{\varphi,k}(\varphi,s)=0$.
- $\lim_{s\to\infty}s^{kp+1}\mathcal{L}_{\mu}Res_{\varphi,k}(\varphi,s)=\lim_{s\to\infty}s^{kp+1}\mathcal{L}_{\mu}Res_{\varphi,k}(\varphi,s)=0,\quad 0<p\le 1,\quad k=1,2,3,\ldots$.

To calculate the coefficients using $f_n(\varphi,s)$ the following system is recursively solved:

$$\lim_{s\to\infty}s^{kp+1}\mathcal{L}_{\mu}Res_{\varphi,k}(\varphi,s)=0,\quad k=1,2,\ldots. \qquad (3.8)$$

In finally inverse Laplace transform to Eq. (3.4), to get the kth analytical result of $\beta_k(\varphi,\mu)$.

3.2. Analysis of the new iterative method

For the basic idea of the new iterative method,we consider the general functional equation :

$$\beta(\varphi) = f(\varphi) + N(\beta(\varphi)), \tag{3.9}$$

where N is non linear operator from a Banach space B to B and f is unknown function. We have been looking for a solution of (3.9) having the series form

$$\beta(\varphi) = \sum_{i=0}^{\infty} \beta_i(\varphi). \tag{3.10}$$

The nonlinear term can be decomposed as

$$N\Big(\sum_{i=0}^{\infty} \beta_i(\varphi)\Big) = N\big(\beta_0(\varphi)\big) + \sum_{i=0}^{\infty}\Big[N\Big(\sum_{j=0}^{i} \beta_j(\varphi)\Big) - N\Big(\sum_{j=0}^{i-1} \beta_j(\varphi)\Big)\Big]. \tag{3.11}$$

From (3.10) and (3.11), (3.9) is equivalent to

$$\sum_{i=0}^{\infty} \beta_i(\varphi) = f(\varphi) + N(\beta_0(\varphi)) + \sum_{i=0}^{\infty}\Big[N\Big(\sum_{j=0}^{i} \beta_j(\varphi)\Big) - N\Big(\sum_{j=0}^{i-1} \beta_j(\varphi)\Big)\Big]. \tag{3.12}$$

We define the following recurrence relation as:

$$\begin{aligned}
&\beta_0(\varphi) = f(x),\\
&\beta_1(\varphi) = N(\beta_0(\varphi)),\\
&\beta_2(\varphi) = N(\beta_0(\varphi) + \beta_1(\varphi)) - N(\beta_0(\varphi)),\\
&\beta_{n+1}(\varphi) = N(\beta_0(\varphi) + \beta_1(\varphi) + \cdots \beta_n) - N(\beta_0(\varphi) + \beta_1(\varphi) + \cdots \beta_{n-1}(\varphi)), n\\
&\quad = 1, 2, 3 \cdots .
\end{aligned} \tag{3.13}$$

Then

$$\begin{aligned}
&(\beta_0(\varphi) + \beta_1(\varphi) + \cdots \beta_n(\varphi)) = N(\beta_0(\varphi) + \beta_1(\varphi) + \cdots \beta_n(\varphi)), n = 1, 2, 3 \cdots ,\\
&\beta(\varphi) = \sum_{i=0}^{\infty} \beta_i(\varphi) = f(\varphi) + N\Big(\sum_{i=0}^{\infty} \beta_i(\varphi)\Big).
\end{aligned} \tag{3.14}$$

3.2.1. Basic road map of NIM

In this section, we discuss basic idea for solving fractional-order nonlinear PDE using the NIM. Consider the following fractional-order PDE:

$$D_\mu^p \beta(\varphi, \mu) = A(\beta, \partial\beta) + \delta(x, t), \qquad m - 1 < p \leq m, m \in N, \tag{3.15}$$

$$\frac{\partial^k}{\partial t^k} \beta(\varphi, 0) = h_k(\varphi), \quad k = 0, 1, 2, 3 \cdots m - 1. \tag{3.16}$$

Where A is non linear function of β and $\partial\beta$ (partial derivative of β with respect to φ and μ) and B is the source function. In view of the new iterative method, the initial value problem (3.15), (3.16) is equivalent to the integral equation

$$\beta(\varphi, \mu) = \sum_{k=0}^{m-1} h_k(\varphi) \frac{t^k}{k!} + I_\mu^p(A) + I_\mu^p(\delta) = f + N(\beta), \tag{3.17}$$

where

$$f = \sum_{k=0}^{m-1} h_k(\varphi) \frac{\mu^k}{k!} + I_\mu^p(\delta), \tag{3.18}$$

$$N(\beta) = I_\mu^p(A). \tag{3.19}$$

4. Numerical problems

In this section, we implement the LRPSM and NIM on some examples.

4.1. Problem 1

Consider fractional-order Cahn-Allen equations of the form:

$$D_\mu^p \beta(\varphi, \mu) - \frac{\partial^2 \beta(\varphi, \mu)}{\partial \varphi^2} + \beta^3(\varphi, \mu) - \beta(\varphi, \mu) = 0, \tag{4.1}$$

where $0 < p \leq 1$, subjected to the following IC's:

$$\beta(\varphi, 0) = \frac{1}{e^{-\frac{\varphi}{\sqrt{2}}} + 1}. \tag{4.2}$$

4.1.1. Solution by LRPSM

Applying LT to Eq. (4.1) and making use of Eq. (4.2), we get

$$\begin{aligned}
&\beta(\varphi, s) - \frac{\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}}{s} - \frac{1}{s^p}\frac{\partial^2 \beta(\varphi, s)}{\partial \varphi^2} + \frac{1}{s^p}\mathcal{L}_\mu[(\mathcal{L}_\mu^{-1}[\beta(\varphi, s)])^3]\\
&- \frac{1}{s^p}\beta(\varphi, s) = 0,
\end{aligned} \tag{4.3}$$

and so the kth-truncated term series can be given as

$$\beta(\varphi, s) = \frac{\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}}}{s} + \sum_{r=1}^{k} \frac{f_r(\varphi, s)}{s^{rp+1}}, \qquad r = 1, 2, 3, 4 \cdots \tag{4.4}$$

Laplace residual functions (LRFs)[37] are

$$\begin{aligned}
&\mathcal{L}_\mu Res(\varphi, s) = \beta(\varphi, s) - \frac{\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}}{s} - \frac{1}{s^p}\frac{\partial^2 \beta(\varphi, s)}{\partial \varphi^2} + \frac{1}{s^p}\mathcal{L}_\mu[(\mathcal{L}_\mu^{-1}[\beta(\varphi, s)])^3]\\
&- \frac{1}{s^p}\beta(\varphi, s),
\end{aligned} \tag{4.5}$$

and the kth-LRFs as:

$$\begin{aligned}
&\mathcal{L}_\mu Res_k(\varphi, s) = \beta_k(\varphi, s) - \frac{\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}}{s} - \frac{1}{s^p}\frac{\partial^2 \beta_k(\varphi, s)}{\partial \varphi^2}\\
&+ \frac{1}{s^p}\mathcal{L}_\mu[(\mathcal{L}_\mu^{-1}[\beta_k(\varphi, s)])^3] - \frac{1}{s^p}\beta_k(\varphi, s).
\end{aligned} \tag{4.6}$$

Now, to determine $f_r(\varphi, s)$, $r = 1, 2, 3, \ldots$, we substitute the rth-truncated series Eq. (4.4) into the rth-Laplace residual function Eq. (4.6), multiply the resulting equation by s^{rp+1}, and then solve recursively the relation $\lim_{s\to\infty}(s^{rp+1}\mathcal{L}_\mu Res_{\beta,r}(\varphi, s)) = 0$, $r = 1, 2, 3, \ldots$. Following are the first few terms:

$$f_1(\varphi, s) = \frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}} + 1\right)^2}, \tag{4.7}$$

$$f_2(\varphi, s) = \frac{9e^{\frac{\varphi}{\sqrt{2}}} - 9e^{\sqrt{2}\varphi}}{4\left(e^{\frac{\varphi}{\sqrt{2}}} + 1\right)^3}, \tag{4.8}$$

and so on.

Putting the values of $f_r(\varphi, s)$, $r = 1, 2, 3, \ldots$, in Eq. (4.4), we get

$$\begin{aligned}
&\beta(\varphi, s) = \frac{1}{s}\left(\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}} + 1}\right) + \frac{1}{s^{p+1}}\left(\frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}} + 1\right)^2}\right)\\
&+ \frac{1}{s^{2p+1}}\left(\frac{9e^{\frac{\varphi}{\sqrt{2}}} - 9e^{\sqrt{2}\varphi}}{4\left(e^{\frac{\varphi}{\sqrt{2}}} + 1\right)^3}\right) + \cdots .
\end{aligned} \tag{4.9}$$

Using inverse Laplace Transform,we get

$$\beta(\varphi,\mu)=\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}+\frac{\mu^{p}}{\Gamma(p+1)}\Big(\frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^{2}}\Big)+\frac{\mu^{2p}}{\Gamma(2p+1)}\Big(\frac{9e^{\frac{\varphi}{\sqrt{2}}}-9e^{\sqrt{2}\varphi}}{4\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^{3}}\Big)+\cdots. \tag{4.10}$$

4.1.2. Solution by NIM

Applying RL integral to Eq. (4.1), we get the equivalent form

$$\beta(\varphi,\mu)=\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}-I_{t}^{\sigma}\Big[\frac{\partial^{2}\beta(\varphi,\mu)}{\partial\varphi^{2}}-\beta^{3}(\varphi,\mu)+\beta(\varphi,\mu)\Big]. \tag{4.11}$$

According to NIM procedure, we get the following few terms

$$\beta_{0}(\varphi,\mu)=\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1},$$
$$\beta_{1}(\varphi,\mu)=\frac{3\mu^{p}\operatorname{sech}^{2}\big(\frac{\varphi}{2\sqrt{2}}\big)}{8\Gamma(p+1)},$$
$$\beta_{2}(\varphi,\mu)=\frac{9e^{\frac{\varphi}{\sqrt{2}}}\mu^{2p}}{32\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)^{6}}\Big(-\frac{8\big(e^{\frac{\varphi}{\sqrt{2}}}-1\big)\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)^{3}}{\Gamma(2p+1)}-\frac{9e^{\sqrt{2}\varphi}\Gamma(3p)\mu^{2p}}{\Gamma(4p)\Gamma(p+1)^{3}}-\frac{3(2)^{2p+3}e^{\sqrt{2}\varphi}\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)\Gamma\big(p+\frac{1}{2}\big)\mu^{p}}{\sqrt{\pi}\Gamma(p+1)\Gamma(3p+1)}\Big). \tag{4.12}$$

By NIM algorithm final solution is under

$$\beta(\varphi,\mu)=\beta_{0}(\varphi,\mu)+\beta_{1}(\varphi,\mu)+\beta_{2}(\varphi,\mu)+\cdots \tag{4.13}$$

$$\beta(\varphi,\mu)=\frac{1}{e^{-\frac{\varphi}{\sqrt{2}}}+1}+\frac{3\mu^{p}\operatorname{sech}^{2}\left(\frac{\varphi}{2\sqrt{2}}\right)}{8\Gamma(p+1)}+\frac{9e^{\frac{\varphi}{\sqrt{2}}}\mu^{2p}}{32\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)^{6}}\Big(-\frac{8\big(e^{\frac{\varphi}{\sqrt{2}}}-1\big)\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)^{3}}{\Gamma(2p+1)}-\frac{9e^{\sqrt{2}\varphi}\Gamma(3p)\mu^{2p}}{\Gamma(4p)\Gamma(p+1)^{3}}-\frac{3(2)^{2p+3}e^{\sqrt{2}\varphi}\big(e^{\frac{\varphi}{\sqrt{2}}}+1\big)\Gamma\big(p+\frac{1}{2}\big)\mu^{p}}{\sqrt{\pi}\Gamma(p+1)\Gamma(3p+1)}\Big)+\cdots. \tag{4.14}$$

4.2. Problem 2

Consider fractional-order Cahn-Allen equations of the form:

$$D_{\mu}^{p}\beta(\varphi,\mu)-\frac{\partial^{2}\beta(\varphi,\mu)}{\partial\varphi^{2}}+\beta^{3}(\varphi,\mu)-\beta(\varphi,\mu)=0,\quad \text{where}\quad 0<p\leq 1. \tag{4.15}$$

Subjected to the following IC's:

$$\beta(\varphi,0)=\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}. \tag{4.16}$$

4.2.1. Solution by LRPSM

Applying LT to Eq. (4.15) and making use of Eq. (4.16), we get

$$\beta(\varphi,s)-\frac{\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}}{s}-\frac{1}{s^{p}}\frac{\partial^{2}\beta(\varphi,s)}{\partial\varphi^{2}}+\frac{1}{s^{p}}\mathcal{L}_{\mu}[(\mathcal{L}_{\mu}^{-1}[\beta(\varphi,s)])^{3}]-\frac{1}{s^{p}}\beta(\varphi,s)=0, \tag{4.17}$$

and so the kth-truncated term series are

$$\beta(\varphi,s)=\frac{\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}}}{s}+\sum_{r=1}^{k}\frac{f_{r}(\varphi,s)}{s^{rp+1}},\quad r=1,2,3,4\cdots \tag{4.18}$$

Laplace residual functions (LRFs)[32] are

$$\mathcal{L}_{\mu}Res(\varphi,s)=\beta(\varphi,s)-\frac{\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}}{s}-\frac{1}{s^{p}}\frac{\partial^{2}\beta(\varphi,s)}{\partial\varphi^{2}}+\frac{1}{s^{p}}\mathcal{L}_{\mu}[(\mathcal{L}_{\mu}^{-1}[\beta(\varphi,s)])^{3}]-\frac{1}{s^{p}}\beta(\varphi,s), \tag{4.19}$$

and the kth-LRFs as:

$$\mathcal{L}_{\mu}Res_{k}(\varphi,s)=\beta_{k}(\varphi,s)-\frac{\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}}{s}-\frac{1}{s^{p}}\frac{\partial^{2}\beta_{k}(\varphi,s)}{\partial\varphi^{2}}+\frac{1}{s^{p}}\mathcal{L}_{\mu}[(\mathcal{L}_{\mu}^{-1}[\beta_{k}(\varphi,s)])^{3}]-\frac{1}{s^{p}}\beta_{k}(\varphi,s). \tag{4.20}$$

Now, to determine $f_r(\varphi,s)$, $r=1,2,3,\ldots$, we substitute the rth-truncated series Eq. (4.18) into the rth-Laplace residual function Eq. (4.20), multiply the resulting equation by s^{rp+1}, and then solve recursively the relation $\lim_{s\to\infty}(s^{rp+1}\mathcal{L}_{\mu}Res_{\beta,r}(\varphi,s))=0$, $r=1,2,3,\ldots$. Following are the first few terms:

$$f_{1}(\varphi,s)=\frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^{2}}, \tag{4.21}$$

$$f_{2}(\varphi,s)=\frac{9}{2}\sinh^{4}\left(\frac{\varphi}{2\sqrt{2}}\right)\operatorname{csch}^{3}\left(\frac{\varphi}{\sqrt{2}}\right), \tag{4.22}$$

and so on.

Putting the values of $f_r(\varphi,s)$, $r=1,2,3,\ldots$, in Eq. (4.18), we get

$$\beta(\varphi,s)=\frac{1}{s}\Big(\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}\Big)+\frac{1}{s^{p+1}}\Big(\frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^{2}}\Big)+\frac{1}{s^{2p+1}}\Big(\frac{9}{2}\sinh^{4}\left(\frac{\varphi}{2\sqrt{2}}\right)\operatorname{csch}^{3}\left(\frac{\varphi}{\sqrt{2}}\right)\Big)+\cdots. \tag{4.23}$$

Using inverse Laplace Transform,we get

$$\beta(\varphi,\mu)=\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}+\frac{\mu^{p}}{\Gamma(p+1)}\Big(\frac{3e^{\frac{\varphi}{\sqrt{2}}}}{2\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^{2}}\Big)+\frac{\mu^{2p}}{\Gamma(2p+1)}\Big(\frac{9}{2}\sinh^{4}\left(\frac{\varphi}{2\sqrt{2}}\right)\operatorname{csch}^{3}\left(\frac{\varphi}{\sqrt{2}}\right)\Big)+\cdots. \tag{4.24}$$

4.2.2. Solution by NIM

Applying RL integral to Eq. (4.1), we get the equivalent form

$$\beta(\varphi,\mu)=\frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1}-I_{\mu}^{p}\Big[\frac{\partial^{2}\beta(\varphi,\mu)}{\partial\varphi^{2}}-\beta^{3}(\varphi,\mu)+\beta(\varphi,\mu)\Big]. \tag{4.25}$$

According to NIM procedure, we get the following few terms

$$\beta_0(\varphi,\mu) = \frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1},$$

$$\beta_1(\varphi,\mu) = \frac{3\mu^p \operatorname{sech}^2\left(\frac{\varphi}{2\sqrt{2}}\right)}{8\Gamma(p+1)},$$

$$\beta_2(\varphi,\mu) = \frac{9e^{\frac{\varphi}{\sqrt{2}}}\mu^{2p}}{32\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^6}\Bigg(8\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)\Bigg(\frac{\left(e^{\frac{\varphi}{\sqrt{2}}}-1c\right)\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^2}{\Gamma(2p+1)} - \frac{3\,4^p e^{\frac{\varphi}{\sqrt{2}}}\Gamma\left(p+\frac{1}{2}\right)\mu^p}{\sqrt{\pi}\,\Gamma(p+1)\Gamma(3p+1)}\Bigg) - \frac{9e^{\sqrt{2}\varphi}\Gamma(3p)\mu^{2p}}{\Gamma(4p)\Gamma(p+1)^3}\Bigg). \tag{4.26}$$

By NIM algorithm final solution is under

$$\beta(\varphi,\mu) = \beta_0(\varphi,\mu) + \beta_1(\varphi,\mu) + \beta_2(\varphi,\mu) + \cdots, \tag{4.27}$$

$$\beta(\varphi,\mu) = \frac{1}{e^{\frac{\varphi}{\sqrt{2}}}+1} + \frac{3\mu^p \operatorname{sech}^2\left(\frac{\varphi}{2\sqrt{2}}\right)}{8\Gamma(p+1)} + \frac{9e^{\frac{\varphi}{\sqrt{2}}}\mu^{2p}}{32\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^6}\Bigg(8\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)\Bigg(\frac{\left(e^{\frac{\varphi}{\sqrt{2}}}-1c\right)\left(e^{\frac{\varphi}{\sqrt{2}}}+1\right)^2}{\Gamma(2p+1)} - \frac{3\,4^p e^{\frac{\varphi}{\sqrt{2}}}\Gamma\left(p+\frac{1}{2}\right)\mu^p}{\sqrt{\pi}\,\Gamma(p+1)\Gamma(3p+1)}\Bigg) - \frac{9e^{\sqrt{2}\varphi}\Gamma(3p)\mu^{2p}}{\Gamma(4p)\Gamma(p+1)^3}\Bigg) + \cdots. \tag{4.28}$$

5. Results and discussion

Two analytical methods LRPSM and NIM are used in the present work to examine the fractional-order Cahn-Allen equations. To show that the suggested approaches are capable of handling complicated nonlinear problems we investigated two nonlinear problems. The outcomes are extremely favorable and show the effectiveness the approaches under discussion are. The fractional operator offers additional degrees of freedom and includes the nonlocal influence into the projected models in the current framework. These methods' simplicity allows us to quickly reach a solution and identify a considerable area of convergence, which is that makes them innovative. The presented numerical simulations assure the veracity of the results. When compared to the exact solution, tables provide significantly superior results. Finally, we argue that the approaches we suggest are highly reliable and can be applied to large study classifications involving fractional-order nonlinear scientific methods, which improves our understanding of nonlinear compound phenomena in interconnected fields of innovation and science. For the sake of accuracy, we could investigate the plots for the fractional/classical order, which show the strong connection of the LRPSM and NIM solutions with the exact solution.

Figs. 1 and 2 show 2D graphs of approximate solutions for problem 1 using LRPSM and NIM with dissimilar values of fractional order $p = 0.4$, $p = 0.6$, $p = 0.8$ and $p = 1.0$. Figs. 3 and 4 show 2D plots $\beta(\varphi,\mu)$ for problem 2 by using LRPSM and NIM with dissimilar values of fractional order $p = 0.4$, $p = 0.6$, $p = 0.8$ and $p = 1.0$.

Figs. 5 and 6 show 3D behavior of approximate solution by using LRPSM and NIM at various values of fractional order $p = 0.4$, $p = 0.6$, $p = 0.8$ and $p = 1.0$ for problem 1. Figs. 7 and 8 show 3D plots $\beta(\varphi,\mu)$ by using LRPSM and NIM for problem 2.

Tables 1 and 3 show the comparison of LRPSM and NIM solutions and their absolute (Abs) error for problems 1 and 2 respectively at fractional order $p = 1$. Tables 2 and 4 presents the numerical values of $\beta(\varphi,\mu)$ using NIM and LRPSM solution at various values of fractional order p for problems 1 and 2 respectively.

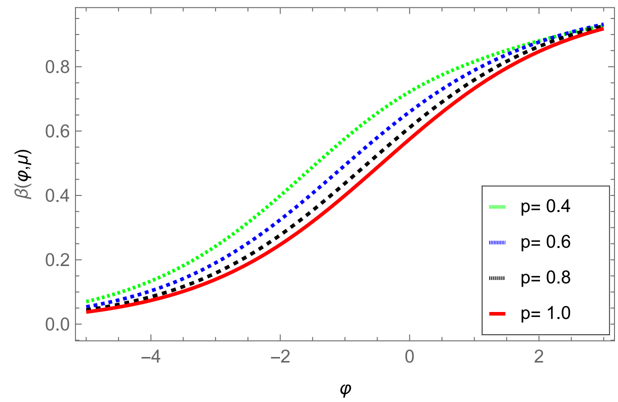

Fig. 1. Impact of fractional order p on $\beta_{LRPSM}(\varphi,\mu)$ solution.

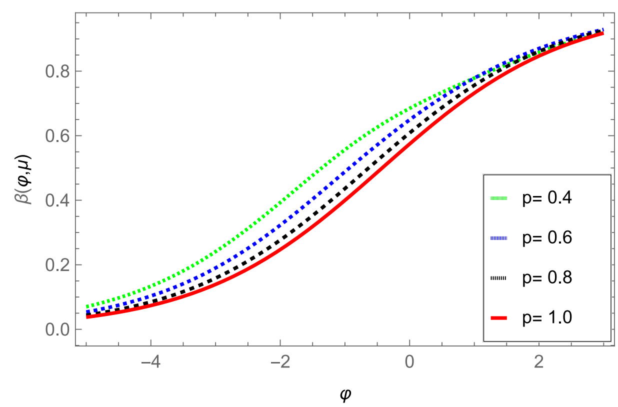

Fig. 2. Impact of fractional order p on $\beta_{NIM}(\varphi,\mu)$ solution.

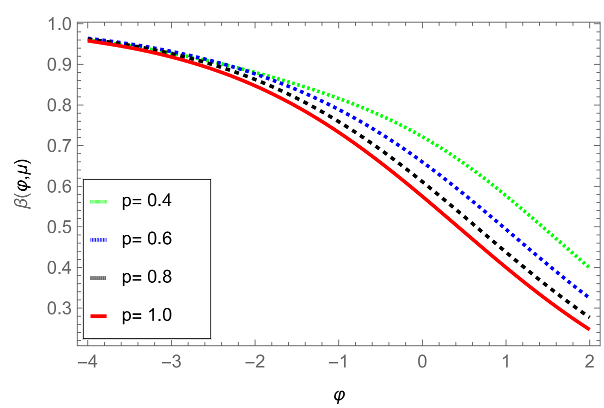

Fig. 3. Impact of fractional order p on $\beta_{LRPSM}(\varphi,\mu)$ solution.

Table 5 shows the comparison of absolute error utilizing the projected techniques LRPSM and NIM with the Elzaki iterative transform method. It reveals that the LRPSM is more accurate than the Elzaki iterative transform method[22] for solving fractional order Cahn-Allen equations.

6. Conclusion

In this study, we implement LRPSM and NIM to accurately solve the fractional-order Cahn-Allen equation. The Cahn-Allen equation is a mathematical model which facilitates us understand phase transitions and pattern development in physical systems. This equation illustrates how distinct phases of matter, such as solids and liquids, change and

Table 1
Numerical values of $\beta(\varphi,\mu)$ by using NIM and LRPSM and their absolute error at $t = 0.0002$ and $p = 1$.

φ	$\beta(\varphi,\mu)_{NIM}$	$\beta(\varphi,\mu)_{LRPSM}$	$\beta(\varphi,\mu)_{Exact}$	$Abs.error(NIM)$	$Abs.error(LRPSM)$
0.	0.500007	0.500008	0.500007	4.996×10^{-16}	5.65234×10^{-17}
0.1	0.517678	0.517678	0.517678	5.55112×10^{-16}	2.81554×10^{-18}
0.2	0.535304	0.535304	0.535304	5.55112×10^{-16}	9.92985×10^{-17}
0.3	0.552842	0.552842	0.552842	5.55112×10^{-16}	5.51817×10^{-17}
0.4	0.57025	0.57025	0.57025	4.44089×10^{-16}	7.16598×10^{-17}
0.5	0.587486	0.587486	0.587486	5.55112×10^{-16}	9.77417×10^{-17}
0.6	0.60451	0.60451	0.60451	4.44089×10^{-16}	1.31359×10^{-16}
0.7	0.621285	0.621285	0.621285	4.44089×10^{-16}	1.32785×10^{-16}
0.8	0.637774	0.637774	0.637774	4.44089×10^{-16}	1.64504×10^{-16}
0.9	0.653945	0.653945	0.653945	3.33067×10^{-16}	3.03211×10^{-16}
1.	0.669768	0.669768	0.669768	3.33067×10^{-16}	2.5297×10^{-16}

Table 2
Numerical values of $\beta(\varphi,\mu)$ by using NIM and LRPSM with dissimilar values of fractional order $p = 0.4$, $p = 0.6$ and $p = 0.8$.

φ	$NIM_{p=0.4}$	$LRPSM_{p=0.4}$	$NIM_{p=0.6}$	$LRPSM_{p=0.6}$	$NIM_{p=0.8}$	$LRPSM_{p=0.8}$
0.1	0.58044	0.581291	0.54238	0.542418	0.526938	0.526939
0.2	0.597319	0.59819	0.55985	0.559889	0.544522	0.544523
0.3	0.613967	0.614855	0.577173	0.577213	0.561996	0.561997
0.4	0.630351	0.63125	0.594308	0.594348	0.579317	0.579319
0.5	0.646439	0.647343	0.611217	0.611257	0.596445	0.596447
0.6	0.662202	0.663107	0.627862	0.627902	0.613342	0.613343
0.7	0.677614	0.678513	0.64421	0.64425	0.62997	0.629971
0.8	0.69265	0.693539	0.660228	0.660268	0.646295	0.646296
0.9	0.70729	0.708163	0.675888	0.675927	0.662286	0.662287
1.	0.721516	0.722369	0.691164	0.691203	0.677914	0.677916

Table 3
Numerical values of $\beta(\varphi,\mu)$ by using NIM and LRPSM and their absolute error at $t = 0.0002$ and $p = 1$.

φ	$\beta(\varphi,\mu)_{NIM}$	$\beta(\varphi,\mu)_{LRPSM}$	$\beta(\varphi,\mu)_{Exact}$	$Abs.error(NIM)$	$Abs.error(LRPSM)$
0.1	0.482337	0.482337	0.482337	5.55112×10^{-16}	1.44072×10^{-17}
0.2	0.464711	0.464711	0.464711	5.55112×10^{-16}	7.38198×10^{-17}
0.3	0.447172	0.447172	0.447172	4.9960×10^{-16}	2.16637×10^{-17}
0.4	0.429764	0.429764	0.429764	5.55112×10^{-16}	9.22936×10^{-17}
0.5	0.412528	0.412528	0.412528	5.55112×10^{-16}	1.15908×10^{-16}
0.6	0.395504	0.395504	0.395504	4.44089×10^{-16}	2.4724×10^{-17}
0.7	0.378729	0.378729	0.378729	4.9960×10^{-16}	1.05893×10^{-16}
0.8	0.36224	0.362240	0.362240	3.88578×10^{-16}	8.36149×10^{-17}
0.9	0.346068	0.346068	0.346068	3.88578×10^{-16}	1.00612×10^{-16}
1.	0.330245	0.330245	0.330245	3.88578×10^{-16}	6.71968×10^{-17}

Table 4
Numerical values of $\beta(\varphi,\mu)$ by using NIM and LRPSM with dissimilar values of fractional order $p = 0.4$, $p = 0.6$ and $p = 0.8$.

φ	$NIM_{p=0.4}$	$LRPSM_{p=0.4}$	$NIM_{p=0.6}$	$LRPSM_{p=0.6}$	$NIM_{p=0.8}$	$LRPSM_{p=0.8}$
0.1	0.54672	0.547538	0.507515	0.507552	0.491785	0.491786
0.2	0.529383	0.530165	0.489861	0.489896	0.474131	0.474132
0.3	0.511968	0.512712	0.472231	0.472264	0.456542	0.456543
0.4	0.494516	0.495221	0.45467	0.454702	0.43906	0.439061
0.5	0.477071	0.477733	0.437221	0.43725	0.421728	0.421729
0.6	0.459673	0.460291	0.419925	0.419953	0.404587	0.404588
0.7	0.442364	0.442938	0.402823	0.402849	0.387675	0.387676
0.8	0.425185	0.425716	0.385954	0.385977	0.37103	0.371031
0.9	0.408177	0.408663	0.369353	0.369375	0.354685	0.354686
1.	0.391377	0.391821	0.353056	0.353076	0.338673	0.338674

Table 5
Error analysis of the proposed methods with the Elzaki iterative transform method (EITM)[22].

φ	Error (EITM)	Error (NIM)	Error (LRPSM)
0	7.5×10^{-6}	8.78908×10^{-12}	8.12989×10^{-16}
0.1	7.49063×10^{-6}	8.745×10^{-12}	3.3281×10^{-13}
0.2	7.46262×10^{-6}	8.61389×10^{-12}	7.02822×10^{-13}
0.3	7.41625×10^{-6}	8.39917×10^{-12}	1.10362×10^{-12}
0.4	7.35197×10^{-6}	8.10463×10^{-12}	1.52819×10^{-12}
0.5	7.27042×10^{-6}	7.73659×10^{-12}	1.96837×10^{-12}
0.6	7.17237×10^{-6}	7.30238×10^{-12}	2.41631×10^{-12}
0.7	7.05875×10^{-6}	6.81077×10^{-12}	2.86373×10^{-12}
0.8	6.9306×10^{-6}	6.27054×10^{-12}	3.30329×10^{-12}
0.9	6.78908×10^{-6}	5.692×10^{-12}	3.7277×10^{-12}
1	6.63542×10^{-6}	5.08504×10^{-12}	4.13074×10^{-12}

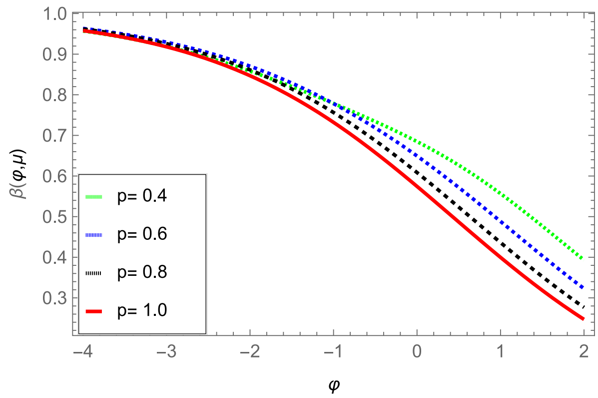

Fig. 4. Impact of fractional order p on $\beta_{NIM}(\varphi,\mu)$ solution.

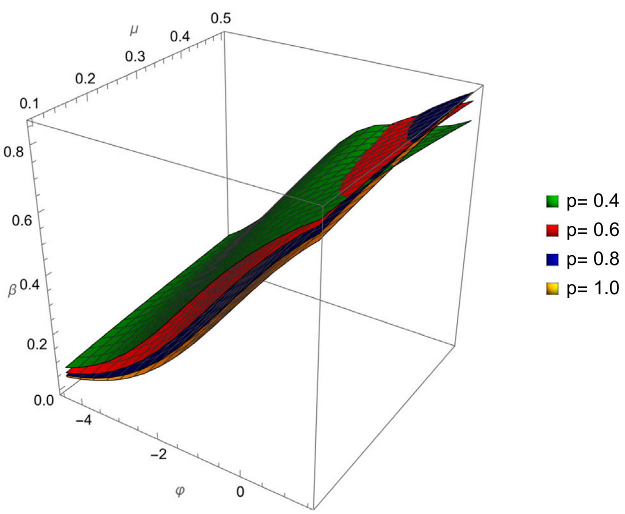

Fig. 5. Impact of fractional order p on $\beta_{LRPSM}(\varphi,\mu)$ solution.

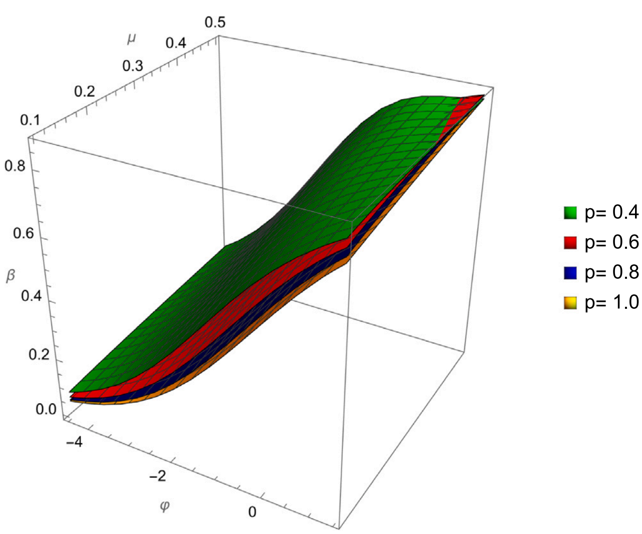

Fig. 6. Impact of fractional order p on $\beta_{NIM}(\varphi,\mu)$ solution.

interact with one another throughout time. It has several applications in materials science, condensed matter physics, and other scientific fields. The accuracy and convergence of the projected techniques was analyzed form Tables 1 and 3. The convergence rate for the NIM technique ranged between 10^{-16} and 10^{-17}, while the convergence rate for LRPSM ranged from 10^{-16} to 10^{-18}. Furthermore, 2D and 3D graphs was analyzed using NIM and LRPSM solutions. It reveals that there exists an excellent mutual understanding between LRPSM and NIM solutions of the problems. On the basis of sufficient degree of accuracy the current techniques are preferred choice for addressing complex non-linear fractional-order partial differential equations.

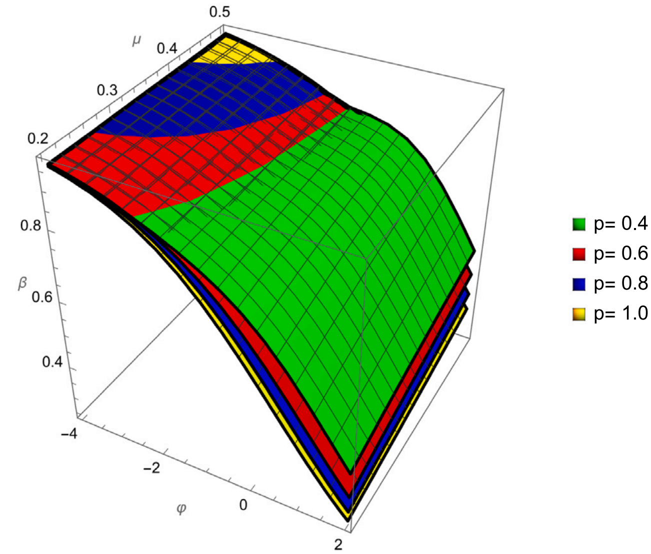

Fig. 7. Impact of fractional order p on $\beta_{LRPSM}(\varphi,\mu)$ solution.

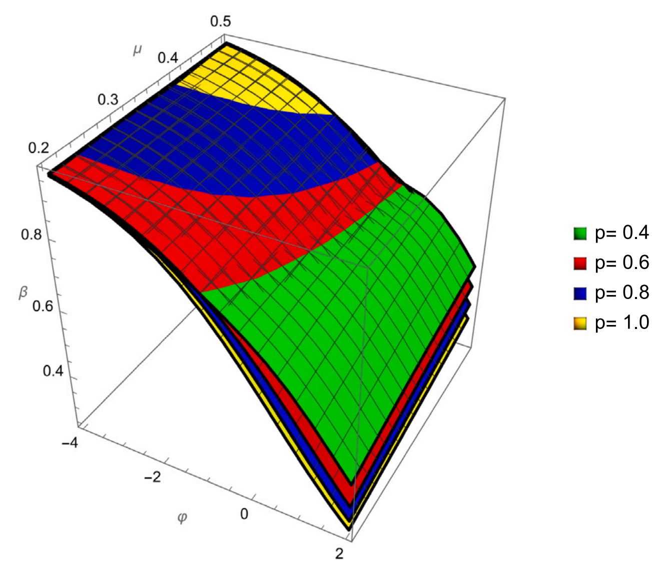

Fig. 8. Impact of fractional order p on $\beta_{NIM}(\mu,\mu)$ solution.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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