

Short thesis for the degree of Doctor of Philosophy (PhD)

**Clusterization and its symmetries
in light nuclei**

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Introduction

The atomic nucleus presents two fundamental challenges. First, the exact nature of the nuclear force is still not completely understood. Second, the nucleus itself is a complex many-body quantum system. To fully describe its structure, one would need to solve the many-body Schrödinger equation, which cannot be done analytically.

Models are essential for describing nuclear structure. The fundamental structure models are built on different physical perspective, such as the shell structure in the shell model, the liquid-like behavior in the collective (liquid drop) model, or the molecular-like arrangement in the cluster model. Understanding the relationships between these models is essential for describing nuclear structure within a unified framework.

Symmetry plays a key role in connecting nuclear structure models. A common link was identified in 1958 through the dynamical $U(3)$ symmetry, when Elliott showed that collective features such as quadrupole deformation and rotation arise within the spherical shell model through $SU(3)$ symmetry. Later that same year, the equivalence between the Hamiltonians of the shell and cluster models

was demonstrated by Wildermuth and Kanellopoulos, and interpreted in $SU(3)$ terms by Bayman and Bohr. However, these connections were limited to a single-shell framework, making their extension to multi-shell systems necessary.

My supervisor and his collaborators introduced multi-configurational dynamical symmetry (MUSY), which unifies the shell model, collective model, and cluster model for the multi-major-shell problem. MUSY connects different configurations (shell and clusters) to each other. This model was applied to several nuclear systems and was able to account for the gross features of the spectrum before I started my PhD.

Within the MUSY framework, it is possible to determine the stable quadrupole shape of nuclei directly from the symmetry of the shell model. In particular, shape isomers can be identified by analyzing the stability and consistency of the underlying $U(3)$ symmetry, a method known as the Stability and Consistency of $SU(3)$ symmetry (SCS).

A cluster model known as the semi-microscopic algebraic cluster model (SACM) was developed and applied by my supervisor and collaborators. It provides a description of nuclear cluster structures while incorporating the Pauli exclusion principle.

For binary clusters, SACM has three dynamical symmetry limits: the $U(3)$, $O(4)$, and $O(3)$ limits. The $U(3)$ limit corresponds to a shell-like structure, the $O(4)$ limit represents a rigid-rotor-like (or rigid-molecule-like) structure, and the $O(3)$ limit describes the weak-coupling cluster configuration. Thus, the SACM can quantitatively address how nuclear shells evolve and whether clusterization manifests a shell-like or a rigid, molecule-like character.

Objective

Based on the discussion in the introduction, my work primarily focused on the application of the multiconfigurational dynamical symmetry (MUSY) and the semimicroscopic algebraic cluster model (SACM).

One of the main goals of my research was to use MUSY to describe the complex spectra of various nuclei in a unified and microscopic framework.

Different energy regions of a nuclear spectrum may correspond to different configurations (shell and clusters configurations). An objective of my work was to investigate the interrelation between these configurations within the MUSY framework.

In atomic nuclei both thermal and non-thermal (cold) phases (and their transitions) are present. In cold phases and their transitions, temperature plays no essential role; nevertheless, many phase-transition concepts can still be treated analogously to those in standard thermodynamics. The non-thermal phases of atomic nuclei can be studied in algebraic models of nuclear structure, where the cold phases are defined by the dynamical symmetry limits of the models. In

light of the introduction, the three dynamical symmetry limits of the SACM correspond to three phases. It is therefore of interest to investigate the nuclear spectra associated with each limit.

Thesis points

1. I used the multiconfigurational dynamical symmetry (MUSY) to provide a unified description of the ^{18}O spectrum [P1]. I used a simple dynamically symmetric Hamiltonian that contains only harmonic oscillator, quadrupole, and angular momentum interactions. It is invariant under transformations between the different configurations. As a result, the shell, $^{14}\text{C}+^4\text{He}$, and $^{12}\text{C}+2n+^4\text{He}$ cluster configurations contribute to every state in the calculated spectrum. These three configurations have been proposed previously for the interpretation of different parts of the spectrum. I considered the experimentally established band structure and, in addition, constructed candidate bands using compilation data and alpha scattering resonances to determine the Hamiltonian parameters. Then I gave a parameter-free prediction for the distribution of α -cluster states with specific spin-parities within a certain energy window and compared them with alpha scattering resonances. I found that the states of the three lowest-lying bands have identical wave functions for different configurations due to antisymmetrization. In addition, a symmetry-governed method, called Stability and self-Consistency of the SU(3) Symmetry

(SCS) was applied to determine the stable shape isomers. I predicted reaction channels in which the shape isomer states can be populated or through which they can decay. With a simple dynamically symmetric Hamiltonian, the $^{14}\text{C}+^4\text{He}$ cluster spectrum reproduces the gross features of the experimental energy spectrum of ^{18}O .

2. I have applied the multiconfigurational dynamical symmetry to provide a unified description of the spectrum of the ^{40}Ca nucleus [P2]. In the low-energy region, it shows a well-established band structure, and its superdeformed band has been confirmed experimentally. Furthermore, there are indications of hyperdeformation from high-resolution heavy-ion resonance experiments. Using the experimental compilations, I have arranged additional candidate bands. The results show that MUSY provides a unified description with good agreement.

One of the interesting findings is that the arranged bands were predicted based on the description of well-known bands. The agreement between the experimental and calculated energy levels was nearly as good as that obtained from the fitting procedure, with only about a 9% difference.

3. I systematically investigated the spectra of the phases of clustering in light nuclei [P3]. For binary clusterization, there are three phases associated with the dynamical symmetry limits: the shell-like (or soft-vibrator), rigid molecule-like (or rotator), and weakly-coupled limits. These correspond to the $U(3)$, $O(4)$, and $O(3)$ symmetries, respectively. The phase diagram of clustering is two-dimensional and can be represented by a triangle.

I applied the semimicroscopic algebraic cluster model (SACM) to demonstrate the spectra of the three phases. The spectra exhibit notable differences. The most significant differences arise from the role of the Pauli exclusion principle. By using the (strongly coupled) $U(3)$ basis, the effect of antisymmetrization can be treated exactly. This means that all Pauli forbidden states can be excluded, and all allowed states are retained. In the $O(3)$ limit it can be incorporated only partially through truncation of the relative-motion quantum numbers, whereas in the $O(4)$ limit the basis states are mixtures of Pauli allowed and Pauli forbidden components.

My analysis shows that when clustering is described within a dynamically symmetric (pure-phase) framework, the shell-like ($U(3)$) configuration emerges as the most appropriate choice, because of the crucial role of

the Pauli principle. When symmetry-breaking effects are considered, the exact location within the phase diagram can be identified by performing the calculation in the $U(3)$ basis, while employing an interaction that contains contributions from the $U(3)$, $O(4)$, and $O(3)$ dynamical symmetries. The relative strengths of these components (obtained through the best fit to experimental data) serve as the control parameters that determine the location in the phase diagram.

4. As the shell-like phase (i.e. $U(3)$ limit) provides a fully microscopic and physically meaningful description when combined with a $U(3)$ dynamically symmetric interaction, I considered the $^{12}\text{C}+\alpha$ system for illustration [P3]. I compared the $U(3)$ dynamical spectrum of $^{12}\text{C}+\alpha$ with the experimental energy spectrum of ^{16}O . The comparison showed that the band structure of ^{16}O , including the ground-state, can be reproduced to a good approximation within the $U(3)$ dynamically symmetric $^{12}\text{C}+\alpha$ model. Moreover, several experimentally observed states without definite band assignment were found to be consistent with possible $^{12}\text{C}+\alpha$ cluster configurations, although additional experimental investigations are required for a definitive identification.

Publications

In peer-reviewed Journals:

- P1. G. Riczu, **C. S. Panda**, G. Lévai, and J. Cseh, The spectrum of ^{18}O in terms of the multiconfigurational dynamical symmetry, to be submitted to Phys. Rev. C. arXiv:2606.02270.
- P2. **C. S. Panda**, J. Darai, G. Riczu, and J. Cseh, Multiconfigurational Dynamical Symmetry Description of ^{40}Ca Spectrum, Symmetry (2026), 18, 1193.
- P3. **C. S. Panda**, G. Riczu, J. Cseh, Spectra of Phases of Clusterized Nuclei, Symmetry (2026), 18, 923.

Conference proceedings:

- CP1. **C. S. Panda**, G. Riczu, J. Darai, and J. Cseh, A unified description of the spectrum of ^{18}O and ^{40}Ca , XXVIII. Spring Wind Conference (2025), 12.
- CP2. **C. S. Panda**, G. Riczu, J. Darai, and J. Cseh, Symmetry-governed approach to complex spectra of light nuclei, Proceedings of the DAE Symp. on Nucl.Phys. (2025), 69, 223.

Talks

XXVIII. Spring Wind Conference

Semmelweis University, Budapest (Hungary), 20-22 June
2025

Talk: A unified description of the spectrum of ^{18}O and ^{40}Ca .

Posters

EuNPC 2025

Caen (France), 22-26 September 2025

Poster: The spectrum of ^{18}O and ^{40}Ca nuclei in terms of the multiconfigurational dynamical symmetry.

69th DAE Symposium on Nuclear Physics, SNP-2025

NIT Jalandhar, Punjab (India), 08-12 December 2025

Poster: Symmetry-governed approach to complex spectra of light nuclei.



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PhD Publication List

Candidate: Chandra Sekhar Panda

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List of publications related to the dissertation

Foreign language scientific articles in international journals (2)

1. **Panda, C. S.**, Darai, J., Riczu, G., Cseh, J.: Multiconfigurational Dynamical Symmetry Description of ^{40}Ca Spectrum.
Symmetry. 18 (7), 1-17, 2026. ISSN: 2073-8994.
DOI: <http://dx.doi.org/10.3390/sym18071193>
IF: 2.2 (2025)
2. **Panda, C. S.**, Riczu, G., Cseh, J.: Spectra of Phases of Clusterized Nuclei.
Symmetry. 18 (6), 1-23, 2026. ISSN: 2073-8994.
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A PhD értekezés alapjául szolgáló közlemények

Idegen nyelvű tudományos közlemények külföldi folyóiratban (2)

1. **Panda, C. S.**, Darai, J., Riczu, G., Cseh, J.: Multiconfigurational Dynamical Symmetry Description of 40Ca Spectrum.
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2. **Panda, C. S.**, Riczu, G., Cseh, J.: Spectra of Phases of Clusterized Nuclei.
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A közlő folyóiratok összesített impakt faktora (az értekezés alapjául szolgáló közleményekre):
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