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Shape isomers of light nuclei from the stability and consistency of the SU(3) symmetry

J. Cseh [a,*], G. Riczu [a], J. Darai [b]

[a] Institute for Nuclear Research, Hungarian Academy of Sciences, Debrecen, Pf. 51, 4001, Hungary
[b] Department of Experimental Physics, University of Debrecen, Debrecen, Pf. 400, 4002, Hungary

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ABSTRACT

We investigate the stability and self-consistency of the SU(3) symmetry and quadrupole deformation in light nuclei in terms of the Nilsson model and quasi-dynamical symmetry. It turns out that SU(3) is a remarkably good symmetry for commensurable major axes, similarly to the finding of the simple harmonic oscillator interaction. The method serves as an alternative to the well-known energy-minimum calculation for the determination of the shape isomers. In case of the ^{16}O, ^{20}Ne, and ^{24}Mg nuclei the results of the two different procedures are in good agreement with each other.

1. Introduction

The coexistence of different shapes is a typical phenomenon of the many body systems. For atomic nuclei it has been investigated extensively from the theoretical side, and in several cases it has been observed experimentally as well [1]. The nuclear shape isomers are usually determined from an energy-surface calculation. One applies a structure model, calculates the energy surface as a function of the deformation parameters, and its local minima correspond to the shape isomers. The absolute minimum contains the ground state region, and further local minima determine e.g. the superdeformed (SD) and the hyperdeformed (HD) states (with ratios of main axes 2:1:1, and 3:1:1, respectively). A schematic illustration is presented in the upper panel of Fig. 1.

In this Letter we discuss an alternative way for the determination of the shape isomers. It is not based on the calculation of the energy surface, rather we investigate the stability and the self-consistency of the quadrupole deformation. In fact we investigate the stability and the self-consistency of the SU(3) symmetry of the nucleus (therefore, we call it SCS-method), but this symmetry is uniquely related to the quadrupole deformation. A detailed discussion of this relation is presented in [2].

The investigation of the stability and self-consistency of the SU(3) symmetry is of great interest from two different aspects. From the theoretical side it gives an interesting contribution to the long-standing question of the validity of the SU(3) symmetry (as discussed more in detail in the concluding part). From the practical viewpoint it provides us with a new method for the determination of the shape isomers, which is an alternative to the well-known energy-minimum calculations, as mentioned above. In particular, the stability regions of the SU(3) symmetry, or quadrupole deformation, which usually satisfy to a good approximation the requirement of self-consistency, provide us with the shape isomers (see lower part of Fig. 1).

The new SCS method results in the U(3) symmetry of the shape isomers, therefore, a U(3) selection rule can be applied for the determination of its allowed and forbidden cluster configurations. Furthermore, since a cluster configuration is defined by a reaction channel, one has a direct tool to find the favoured and unfavoured reaction channels for the population or decay of the isomeric states.

The SU(3) we apply here is not the original symmetry by Elliott [3], rather its generalized version, called quasi-dynamical symmetry [4]. It is more widely applicable, but when the simple SU(3) symmetry is still well-defined, they are identical.

* Corresponding author.
E-mail addresses: cseh@atomki.mta.hu (J. Cseh), riczugabor@atomki.mta.hu (G. Riczu), darai@atomki.mta.hu (J. Darai).

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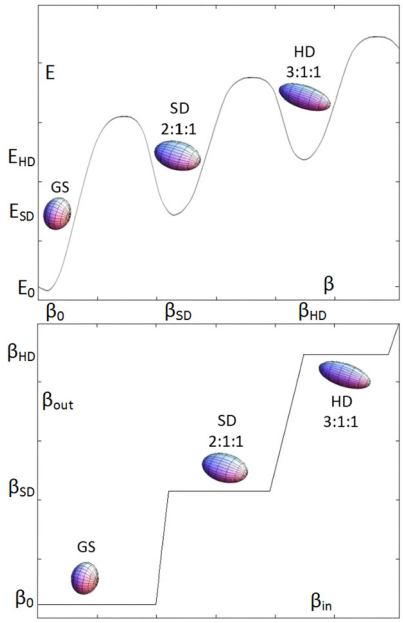

Fig. 1. Schematic illustration of the appearance of the SD and HD shapes in the energy-minimum and in the deformation stability and self-consistency calculation.

In what follows first we recall the main features of the quasi-dynamical SU(3) symmetry, then present the method of calculation and its application to the ^{16}O, ^{20}Ne, and ^{24}Mg nuclei, and finally some conclusions are drawn.

2. The quasi-dynamical SU(3) symmetry

The quasi-dynamical or effective symmetry [4,5] is one of the most general symmetry in quantum mechanics. It is a symmetry of the eigenvalue-equation, when neither the (Hamiltonian) operator is symmetric (scalar), nor its eigenvectors (transform according to some irreducible transformations) [6].

The mathematical reason for this surprising situation is provided by the embedded representation [4]. An embedded representation is obtained (for a subspace of the total Hilbert space) by calculating the matrix elements of the operators between vectors which are special linear combinations of those belonging to irreducible representations (irrep). The irreps may be, and in the interesting cases are, inequivalent. The linear combinations are special in the sense that their coefficients are the same for several vectors. When the summation and the internal product operations are exchangeable (either exactly, or approximately), then the matrices give (exactly or approximately) a representation, called embedded.

Lie-algebras of rotor model kind (semi-direct sums with Abelian ideals) have exact embedded representations [4]. Some other Lie-algebras, e.g. SU(3), contract to this kind of algebra in limiting cases, consequently they have good approximate embedded representations. E.g. let $\phi_{\delta\lambda\mu K JM}$ denote an SU(3) basis state, belonging to the $(\lambda\mu)$ irrep, with δ labelling the additional quantum numbers needed to specify the wavefunction. An arbitrary $\psi_{\alpha JM}$ shell model wavefunction can be expanded in this basis:

$$\psi_{\alpha JM} = \Sigma_{\delta\lambda\mu K} C_{\alpha\delta\lambda\mu K JM} \phi_{\delta\lambda\mu K JM}. \tag{1}$$

For a rotationally invariant Hamiltonian the expansion coefficients are independent of M. When they are also independent of J, one has an approximate quasi-dynamical SU(3) symmetry. In such a case the states with different J are said to form a soft SU(3) band.

In physical terms the embedded representation describes the adiabatic separation of variables, e.g. rotational and intrinsic degrees of freedom. The embedded representation and the related quasi-dynamical symmetry explains why some models can be successful, when they (seemingly) have no right to be so, e.g. due to the presence of symmetry-breaking interactions.

3. The method of calculation

The method of calculation is based on the observation that the asymptotic Nilsson-state (see below for more details) of the many nucleon system ($\mid \varphi_0 >$) is an intrinsic state of a soft SU(3) band [5]. It is a Q_0 highest weight state in the sense:

$$a_z^\dagger a_+ \mid \varphi_0 > = a_z^\dagger a_- \mid \varphi_0 > = 0$$
$$Q_0 \mid \varphi_0 > = (2n_z - n_+ - n_- + 3) \mid \varphi_0 > . \tag{2}$$

Here $(Q_0 - 3)$ is the $(M = 0)$ component of the $SU(3)$ quadrupole operator [5], $a_i^\dagger$, and a_j are the creation and annihilation operators of the oscillator quanta in cylindrical coordinates, and n_i is the corresponding number operator. The authors of Ref. [5] prove explicitly, based on the order of filling of the asymptotic single-particle states, that this condition is fulfilled for large $(\epsilon \geq 0.3)$ deformation.

Jarrio et al. proposed a method for the determination of the effective quantum numbers for large prolate deformation [5]. In Ref. [7] the procedure was extended to oblate shapes and to small deformation. For small deformation the states are expanded in terms of asymptotic basis. Using these results the scenario of the calculation is as follows.

i) Determine the Nilsson-orbitals as a function of the quadrupole deformation parameters.

ii) Obtain the many-particle state by filling in the Nilsson orbitals according to the energy minimum and Pauli-exclusion principle.

iii) Expand the single particle orbitals in terms of the asymptotic Nilsson-states.

iv) Determine the effective SU(3) quantum numbers from the linear combinations of iii) and from the relations of the large deformation [5].

v) The effective quantum numbers can be translated to the parameters of the quadrupole deformation. Therefore, the stability and the self-consistency can be investigated both for the effective SU(3) quantum numbers, and for the deformation.

The details of this procedure are as follows. The asymptotic Nilsson-state is defined by the eigenvalue equation of the deformed Hamiltonian with cylindrical symmetry [8]:

$$H = -\frac{\hbar^2}{2M}\Delta + \frac{M}{2}[\omega_\perp^2(x^2+y^2) + \omega_z^2 z^2] - C(\vec{l}\cdot\vec{s}) - D\vec{l}^{\,2}, \tag{3}$$

which contains in addition to the deformed harmonic oscillator potential spin-orbit, and angular momentum terms. The elongation parameter ϵ is introduced by

$$\omega_z = \omega_0(1 - \frac{2}{3}\epsilon),\ \omega_\perp = \omega_0(1 + \frac{1}{3}\epsilon),\ \epsilon = \frac{\omega_\perp - \omega_z}{\omega_0}. \tag{4}$$

Note, that the usual β parameter of deformation is related to ϵ: $\epsilon \approx 0.95\beta$.

For large deformations $(|\epsilon| > 0.3)$ the Nilsson orbitals approach straight lines [8]. These asymptotic states are characterized by the quantum numbers: $|Nn_z\Lambda\Sigma>$, where N is the total number of oscillation quanta, n_z is the number of oscillation quanta in the z direction $(n_z = N, N-1, ..., 0;\ n_\perp = N - n_z = n_x + n_y)$. Λ is

the projection of the orbital angular momentum on the z-axis ($|\Lambda| = n_\perp, n_\perp - 2, ..., 1$ or 0), and Σ is the projection of the spin [8]. Λ and Σ are coupled to Ω.

For a triaxial shape the deformed harmonic oscillator potential is:

$$V_{osc} = \frac{M}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \tag{5}$$

with $\omega_x \neq \omega_y \neq \omega_z$. The ratio of the frequencies is:

$$\omega_x = \omega_0[1 - \frac{2}{3}\epsilon\, cos(\gamma + \frac{2\pi}{3})],$$
$$\omega_y = \omega_0[1 - \frac{2}{3}\epsilon\, cos(\gamma - \frac{2\pi}{3})],$$
$$\omega_z = \omega_0[1 - \frac{2}{3}\epsilon\, cos\gamma]. \tag{6}$$

The volume conservation is expressed by: $\omega_x\omega_y\omega_z = \omega_0^3$.

We diagonalize the triaxially deformed Hamiltonian in cylindrical coordinates [9–11], thus the Nilsson orbitals $|\ \psi_\alpha >$ of a given deformation (ϵ, γ) are obtained as an expansion in terms of the asymptotic states:

$$|\ \psi_{N\alpha} > = \sum_{n_z,\Lambda,\Omega} C^{\alpha}_{n_z,\Lambda,\Omega}\ |\ N, n_z, \Lambda, \Omega > \quad . \tag{7}$$

The effective (λ, μ), quantum numbers are determined from the equations:

$$< 2\lambda + \mu > = \sum_f < f\ |\ 2n_z - n_+n_-\ |\ f >,$$
$$< \mu > (< \mu > +2) = 4\sum_{fe} |< e\ |\ a_+^\dagger a_-\ |\ f >|^2, \tag{8}$$

where the indices f and e refer to occupied (filled) and empty asymptotic Nilsson orbitals, respectively. The non-zero matrix elements of the second formula in Eq. (8) are given by [5]

$$|< Nn_z\Lambda + 2\Sigma\ |\ a_+^\dagger a_-\ |\ Nn_z\Lambda\Sigma >|^2 =$$
$$\frac{1}{4}(N - n_z - \Lambda)(N - n_z + \Lambda + 2)\ . \tag{9}$$

When evaluating (9) one has to pay attention to which orbital is filled and if the final orbital is empty or not. Λ and Σ can have negative values and each orbital is doubly occupied for the case of even-even nuclei. In the above equations only those oscillator shells enter which are open, i.e. closed oscillator shells do not contribute because the net sum of those in Eqs. (8) and (9) is zero.

The SU(3) symmetry determines the parameters of the quadrupole deformation [12]:

$$\beta^2 = \frac{16\pi}{5N_0^2}(\lambda^2 + \mu^2 + \lambda\mu),\ \gamma = arctan\left(\frac{\sqrt{3}\mu}{2\lambda + \mu}\right), \tag{10}$$

where N_0 stands for the number of oscillator quanta, including the zero point contribution:

$$N_0 = n + (A - 1)\frac{3}{2}. \tag{11}$$

Here n is the sum of the U(3) quantum numbers: $n = n_1 + n_2 + n_3$, and A is the mass number of the nucleus.

4. Application

We apply here the method described in the previous section for the determination of the shape isomers of the ^{16}O, ^{20}Ne, and ^{24}Mg nuclei. The results are presented in Fig. 2, both in 2 dimensional and in 3 dimensional plots.

The comparison with the results of the energy-minimum calculation is given in Table 1. The most systematic calculations have been carried out in Ref. [13] in terms of the Nilsson model, and in [14–16], in terms of the Bloch-Brink alpha-cluster model (for one, two, and three dimensional configurations). Therefore, we compare our results with those ones. The correspondence between the results of [13], and other works is based on the deformation parameters and the number of excitation quanta. The major-shell excitations are determined in each calculations, therefore, their number of quanta can also be compared. As for the alpha-cluster calculations, the shell model configurations of [14–16] determine also the U(3) symmetries.

When calculating the quasi-dynamical (or effective) SU(3) quantum numbers, we obtain not only integer, but also real numbers. Furthermore, some uncertainties appear occasionally (as illustrated by Fig. 2). In the Table 1 we presented the shape isomer states of the present calculation with the U(3) symmetry that corresponds to a simple shell-model configuration.

As illustrated by Table 1, the similarity of the results between the energy-minimum calculations and the new SCS method is very remarkable. In particular, for the ^{16}O nucleus there are two shape isomers (in addition to the ground state) in each calculations: a superdeformed state with a triaxial deformation and a linear alpha-chain. In ^{20}Ne there are three of them in alpha-cluster model and in the present work, and two in [13]. For ^{24}Mg our method predicts five shape isomers, as compared to the four states of the previous works. The characteristics of the shapes from different theoretical frameworks (quadrupole shape, excitation quanta, U(3) symmetry, ratios of major axes) are also in good agreement.

5. Summary and conclusion

In this paper we have presented a method for the investigation of the stability and self-consistency of the SU(3) symmetry (called SCS method). It is carried out by applying the Nilsson-model and the concept of the quasi-dynamical SU(3) symmetry. Due to the unique relations between the SU(3) quantum numbers and the quadrupole deformation parameters it can also be considered as a procedure of the study of stability and self-consistency of the quadrupole shape.

The method is applicable for the determination of the shape isomers of light nuclei. It is an alternative of the well-known energy-minimum calculation. Nevertheless, the results are in very good agreement with each other. The fact that different theoretical methods give very similar states for the shape isomers gives further credit to the theoretical prediction.

Having the SU(3) symmetry of the shape isomers a selection rule can be formulated for the determination of the allowed cluster configurations. The latter ones are directly related to the reaction channels, thus we have a simple and systematically applicable rule for finding the reactions which can populate the isomers [17].

From the theoretical viewpoint this kind of study gives an interesting contribution to the problem of the validity of the SU(3) symmetry of light nuclei. It is known to be good in the ground-state region. With the excitation, however, it breaks down. It was found for harmonic oscillator interaction that for the commensurable ratios of the major axes it recovers [18]. We obtain a similar result here for the more realistic Nilsson-Hamiltonian.

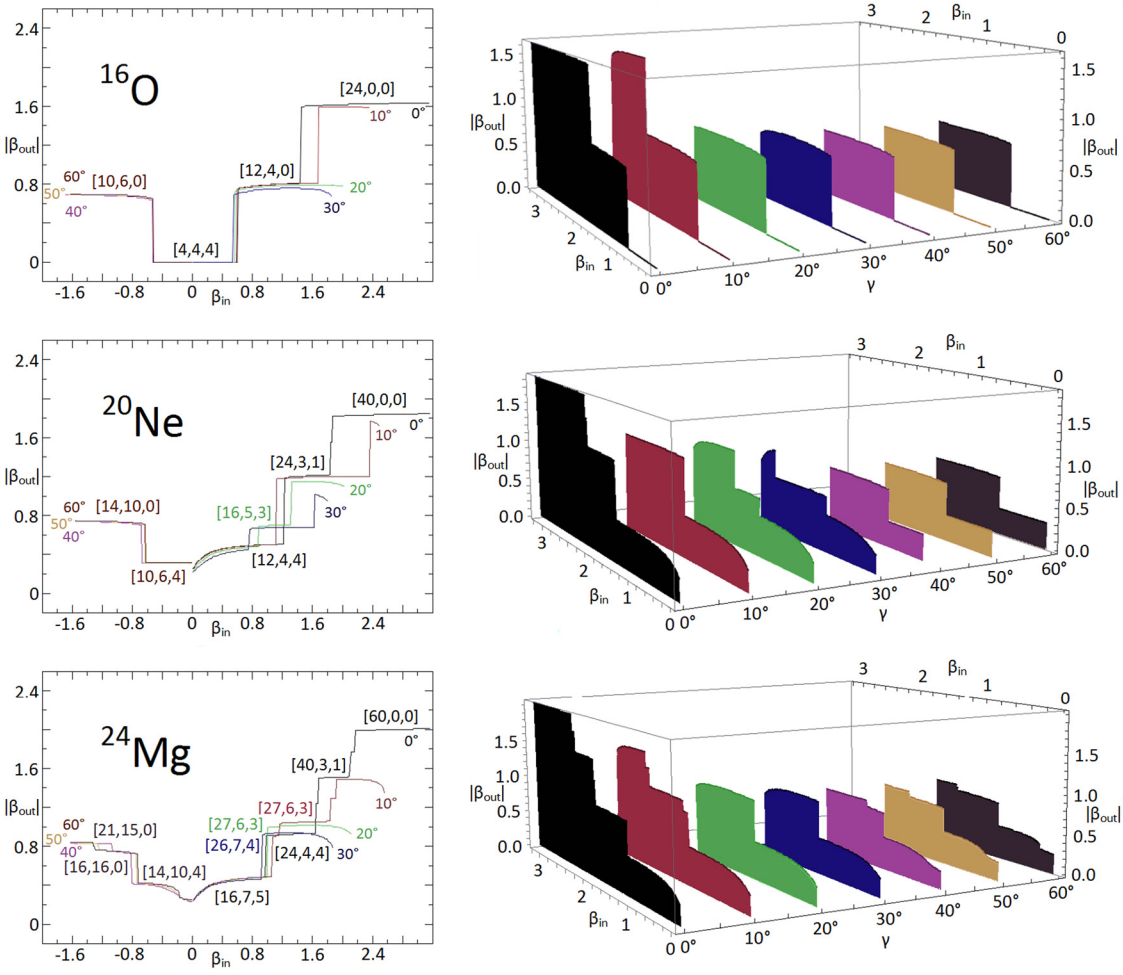

Fig. 2. Shape isomers from the symmetry stability and self-consistency calculations. In the two-dimensional plots the oblate shapes are indicated in the negative side of the β deformation.

Table 1
Shape isomers of the ^{16}O, ^{20}Ne, and ^{24}Mg nuclei from different model calculations. The results of the Nilsson-model calculations are from the work [13], the alpha-cluster model calculations (for one, two and three dimensional configurations) are presented in [14–16], while the column "shape" refers to the present work. The abbreviations are as follows: GS: ground state, SD: superdeformed, $\alpha-ch$: alpha chain, Tri: triaxial, p: prolate, o: oblate, HD: hyperdeformed. The notations of the alpha-cluster configurations are: 2d and 3d means 2 and 3 dimensional configuration, in the former case the parantheses contain the ratio of $(\omega_y:\omega_x)$, tetrahed stands for tetrahedral shape, bipyram for bipyramidal, triax for triaxial, and cylsy for cylindrically symmetric configuration. $\hbar\omega$ indicates the number of excitation quanta, (ϵ,γ) are the parameters of the quadrupole deformation (γ is given in degrees), and a:b:c stands for the ratio of the major axes of the ellipsoid.

Nucl.	Nilsson $\omega_x:\omega_y:\omega_z(\epsilon,\gamma)$	Alpha	Shape	$\hbar\omega$	U(3)	SU(3)	(ϵ,γ)	a:b:c
^{16}O	1:1:1 (0,0)	tetrahed	GS	0	[4,4,4]	(0,0)	(0,0)	1:1:1
	4:2:1 (1.04,43)	2d(2:1)	SD	4	[12,4,0]	(8,4)	(0.83,19)	2.5:1.5:1
	4:4:1 (1.2,0)	$\alpha-ch$	$\alpha-ch$	12	[24,0,0]	(24,0)	(1.56,0)	4:1:1
^{20}Ne	2:2:1 (0.40,0)	bipyram	GS	0	[12,4,4]	(8,0)	(0.50,0)	1.6:1:1
	8:3:2 (1.17,50)	2d(3:2)	Tri_{SD}	4	[16,8,0]	(8,8)	(0.80,30)	2.6:1.8:1
		2d(3:1)	HD	8	[24,4,0]	(20,4)	(1.19,9)	3.4:1.4:1
	5:5:1 (1.25,0)	$\alpha-ch$	$\alpha-ch$	20	[40,0,0]	(40,0)	(1.76,0)	5:1:1
^{24}Mg	4:3:2 (0.45,20)	3d triax	GS	0	[16,8,4]	(8,4)	(0.51,19)	1.8:1.3:1
	(1.0,0)	3d cylsy	SD(p)	4	[24,4,4]	(20,0)	(0.91,0)	2.3:1:1
	3:1:1 (1.23,60)	2d(1:1)	SD(o)	4	[16,16,0]	(0,16)	(0.72,60)	2.3:2.3:1
	5:2:1 (1.26,42)	2d(2:1)	Tri	8	[28,8,0]	(20,8)	(1.07,16)	3.3:1.7:1
			HD_{Tri}	16	[40,4,0]	(36,4)	(1.46,5)	4.3:1.3:1
	6:6:1 (1.25,0)	$\alpha-ch$	$\alpha-ch$	32	[60,0,0]	(60,0)	(1.91,0)	6:1:1

The shape isomers have good SU(3) symmetry. Furthermore, it turns out, that their connection to the cluster states are usually extremely simple. The cluster states can be expanded in the shell basis. Since the SU(3) symmetry of the shape isomers usually have only multiplicity 1, it means that a cluster configuration having this symmetry is identical with the shell configuration, sim-

ilarly to the well-known examples of the ground states of light nuclei.

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