

SUMMARY

In the thesis the development and the establishment of the concept of length and perimeter are put forward.

For young learners essential mathematical concepts including geometrical concepts are introduced and their basis is laid down. The use of the concept of length and perimeter both in everyday life and in various fields of science is of great importance. In order to acquire these concepts definite concepts of quantities are needed so that learners could sort out the various units of measurements and their transformation into other units. When establishing the concept of length we also relied on the children's knowledge gained earlier and their level of knowledge has been continually tested. Prior to the introduction of concepts the level of constancy of quantity was examined.

The reliable concept of length and the proper skill of estimation are integral part of the development of space perception.

The concepts of length and perimeter have been established by means of a large amount of activities and measurements.

In terms of the subject matter, the linguistic and the communication aspects should also be taken into consideration. In the worksheets children were asked to come up with reasons for their solutions and the various use of terms expressing space has been presented.

As well as examining the establishment of concepts related to the subject matter, the methods of teaching were also dealt with.

Our research can be called a natural experiment, since it was carried out in natural situations and in the everyday activities of children such as games, learning and working. In natural situations selected appropriately under carefully controlled conditions we created the pedagogical situation and applied the method of teaching which enhanced the level of knowledge of the learners in the experiment to a defined degree, and there was a continuous follow-up of the effect on the learners' achievement.

The hypotheses and the research question were formulated according to the above.

The hypothesis is that the abstract concepts of mathematics, such as length and perimeter can be established for young learners only through concrete, practical experience which is interesting for them and involves them as long as they grasp the concepts.

In our opinion the evolvement of the concept of length, the comprehension of the concept of measurement, the acquisition of the relationship between the units of measurement and index number as well as the measurement of length in other fields of science and in everyday life can be assisted by the following:

- estimations in the learners' intermediate and wider surrounding, measurements by comparison, by means of natural and standard units, and in various activities.
- developing learners' linguistic awareness in terms of length.

The evolvement of the concept of perimeter is developed by

- determining the length of open and closed broken and curved lines.
- creating triangles, rectangles and squares through concrete activities.
- determining the length of the sides of polygons and their sums, focusing on the fact that the perimeter is an amount of length.
- taking notes of counting the perimeter in various ways.

The research question

Can we properly motivate children to study the subject matter through estimations and measurements, drawing open and closed broken lines and determining their length, and can we properly lay the basis of the concept perimeter through a required amount of activities?

In **chapter 3** the historical background of the research topic is presented.

The history of the metre, the changes of its definitions, the history of Hungarian units of measurements of length are presented. Methodological references to units of measurements and measurements and transformations of units in old mathematical course books are also described.

In Hungarian folk tales and legends old Hungarian units of measurements and terms referring to space are highly important. Thus making children aware of them was an essential part of the experiment.

In **chapter 4** a brief analysis of the works of Zoltán Kovács, György Hajós and Hans Freudenthal is presented.

The metric foundation of length is an essential part of the thesis. It was the works of Zoltán Kovács that we mainly relied on.

The objective of the thesis is to determine the length of broken and curved lines, the perimeter of polygons and plane figures, and the theoretical background is based on the definitions and the theorems by György Hajós.

The operations with length display the relationship between the mathematical and everyday knowledge of the topic.

The concepts and theorems presented in the theory have contributed to the improvement of the topics of development and their mathematical foundation.

In **chapter 5** the methodological background and the pedagogical and psychological implications can be found.

When designing the research an overview of the trends in reform pedagogy was relevant and both the Hungarian and international trends were taken into consideration.

The main aspects of realistic mathematics teaching, the gradual mathematical abstraction and the importance of models were also taken account in designing the experimental teaching. We consider the learner as the most important part of the learning process and we attempted to put the theory and practice of learning on a new basis. According to the theory of constructivism the construction of new knowledge is patterned by the existing knowledge, attitudes and cognitive models, and the new knowledge is to be constructed by everyone on their own. Personal involvement is required for the creation of knowledge and in our experimental teaching a wide range of reform pedagogy was used, such as group work, project work, case studies and discussions both in lessons and in extracurricular situations.

Great emphasis was laid on the achievement of Hungarian reform pedagogues and their theories and methodologies of teaching mathematics, dating back to György Maróthi who was the first to write a textbook which contained examples and explanations.

We have also examined the development of mathematics curricula and these achievements have also been used in the planning of the teaching experiment.

The characteristics of children's cognition were analysed primarily in terms of Piaget's theory, whereas in the algorithm of concept formation we mainly relied on Skemp's theory. In the evolution of the concept of length, the perception constancies and the levels of retaining of operations it was the theory of Piaget and Freudenthal that we considered most relevant.

When teaching the measurement of length difficulties may arise from the fact that at school entry there can be huge differences in children's language use. Children who come from families which language-wise are more competent are in a favourable position when they start „science”. Thus, the study of the linguistic level required for the terms of space related to length, estimations, and giving reasons for their solutions and the relevant theoretical background are integral part of the thesis.

In **chapter 6** the underlying principles of the curriculum and subject are described. In designing the experiment the development tasks highlighted in the National Core Curriculum were taken into consideration. We have also examined in what ways the developmental areas of the National Core Curriculum are related to the formation of the concepts of length and perimeter and the activities applied in teaching.

After analysing and comparing the frame curricula we came to the conclusion that although the teaching material has been structured and elaborated in various ways, teaching measurements can be found in several topics.

The cross-curricular aspects of the topic are also presented and it was also shown at what level and to what extent the concept of length and perimeter is linked with other fields of education.

In **chapter 7** the educational environment, the school itself is briefly introduced and a short description of the learners participating in the experiment is provided.

In **chapter 8** a detailed description of the research is given. The research consists of three main parts: measurement before planning the research, the research plan and the experimental teaching.

The evaluation of the test paper prior to the research, which focused on the old Hungarian measurement units and the standard units and the relationship between them. The test papers were filled in by the learners of class 4 in two schools, whom their teachers considered to be pupils of average ability. The selection of school and the planning of the experimental teaching were based on the results of the test papers. It was in 2003 autumn that the test papers were filled in and the experimental teaching started in 2004 spring.

The experimental teaching was divided into six stages and in the first stage the experimental teaching happened during the classes, but from class 2 they were held during the afternoon sessions.

Teaching in the course of six stages took place in the same class from class 1 to class 4 where the formation of the concept of length and perimeter, estimation skills and the use of terms expressing space were followed up.

The requirements in the six stages were as follows:

Stage 1

- introduction to old Hungarian units of length based on Hungarian folk tales (fathom, arm, span, foot;
- standard units (meter, decimeter, centimeter);
- measurements, estimations.

Stage 2

- the introduction of inch and mile, which perhaps requires a higher level of imagination;
- estimations, measurements in the wider environment of children;
- determining the length of curved lines;
- making rectangles and squares from strings in the class and in the schoolyard.

Stage 3

- estimations, measurements;
- drawing broken lines according to verbal instructions and determining their length;
- creating triangles and squares and rectangles from straws;
- determining the length of the sides of polygons and also their sum.

Stage 4

- estimations and measurements;
- drawing broken lines according to verbal instructions;
- determining the perimeter of rectangles in various ways;
- creating rectangles by cutting various polygons;
- interpretation of words describing space, which can be used to express length.

Stage 5

- the use of words and terms describing space in our immediate and wider environment;
- determining the perimeter of squares and rectangles in various ways and taking notes of them by means of symbols;
- creating rectangles and squares of equal perimeter on grid;
- estimation of larger and taller objects.

Stage 6

- the use of words expressing spatiality and the units of measurement of length in other fields of science;
- estimation and measurement of distance, height and length in real life, maps and pictures;
- determining the perimeter of rectangle and square by formula;
- making learners aware of the connection between the units of measurement.

At the end of each stage learners' level of knowledge was evaluated by means of testing and interviews or group discussions.

From stage 2 during the first lesson a diagnostic test was applied in order to measure the constancy of knowledge and the level of knowledge required for the areas to be developed.

In **chapter 9** the outcomes and the hypothesis of the developmental experiment were evaluated and the research question was answered.

Finally we came to the conclusion that after the measurements

- in groups and pairs the learners' estimation skills have greatly improved during the four years of the teaching experiment;
- despite the large amount of measurement and estimation, the discussion and the analysis of the results of the measurements the majority of learners still were not able to estimate realistically long and high objects;
- children's awareness of units of length seemed to be good, which was also shown by the evaluations of the test papers;
- development can be observed in the use of words expressing spatiality both in mathematics and everyday life and also in their use of estimations;
- by the end of stage 3 hardly any learners had problems with directions when they were drawing broken lines;
- when determining the length of broken lines more and more learners used ruler even if the shapes were given on grid;
- the use of ruler as a means measurement has improved a lot in pair work, only one or two children forgot that measuring should start not from the edge of the ruler but from the unit of measurement;
- when creating polygons from straws children were able to discover several characteristics of squares, rectangles and triangles;
- according to the results of test papers the level of determining the perimeter of polygons was quite good and this is a sound basis to build on.

According to the above we believe that our hypothesis was realistic and a definite answer can be given to the research question, as the answer can be found in the outcomes.

In **chapter 10** the following conclusions of the experimental teaching can be formulated:

- Learners have a large amount of knowledge on the size of objects and the relationship between them even before admission to school, and this knowledge can be efficiently used in education.
- On the radio, in television, at home, in the streets, at school, in various tales children can come across several words which are related to the shape, position, characteristics and the interdependence of objects. In this way they can acquire several geometrical technical terms, even if sometimes they are not really aware of their meaning or can have misconceptions about them. At school it is up to the teacher to provide the proper content of the existing geometrical concepts.
- When designing course material for the development of learners' measurement skills we have to focus on the formation of the concept of geometric quantity

and this knowledge can be used when children learn the concept of number and the operations with numbers. And all these should be closely related with the teaching of shapes.

- Children should use various means of measurement and acquire skills in their use.
- When teaching measurement efforts should be made to apply this knowledge in other fields of education.
- Developing the estimation skills of children should be a continuous task, which is of great importance in everyday life and in improving space perception.
- The majority of children even in class 2 have difficulties with transformations into various measurements. E.g. children know that $1\text{m} = 10\text{dm}$, however some of them had serious problems when they had to transform five metres into decimetres. They were able to carry out transformations in which they had gained experience earlier. In class 1 only few children are able to transform, and even in class 2 children can have difficulties in understanding the inverse proportion of the unit of measurement and the index number.
- Probably more verbal explanation and discussions are needed in the course of teaching so that the inadvertent mistakes and inaccuracies could be revealed in time.