Afrika Matematika (2025) 36:54
https://doi.org/10.1007/s13370-025-01253-5

On metric dimension of symmetrical planer pyramid related graphs

Saqib Nazeer[1] · Muhammad Hussain[1] · Atiq ur Rehman[1] · Ali Hasan Ali[2,3,4]

Received: 14 May 2023 / Accepted: 7 January 2025

Abstract
Robotics and networking have transformed the world in numerous ways. Nowadays, computer networks play a vital role not only in business, education, and medical treatments, but also in entertainment. Graph theory has wide-ranging applications in robotics, tricky games, computer networking, map formations, image processing, Loran or Sonar models, pattern recognition, artificial intelligence, medical networks (such as oxygen or other treatment wiring), navigation problems, electrical networks, and more. Metric dimension has left its mark on artificial intelligence, map navigation, image recognition, pattern formation, facility location problems, and resource management (cost, time, labor, and resources). Robots are used in almost every field of life, including saving lives in the medical field, and metric dimension plays a crucial role in ensuring their accuracy. In this recent work, we enhance the concept of reflection of graphs and introduce new graphs, which we have called horizontal reflection graphs. We also compute the metric dimension of $h - Vrl(mid(Tower_p))$ and h-Vrl(Middle tower Path Graph) and found it to be constant.

Keywords Resolving set · Reflection graph · Path graph · Metric dimension · Connectivity

Mathematics Subject Classification Primary 05C12 · 05C40 · 05C80

✉ Ali Hasan Ali
ali.hasan@science.unideb.hu

Saqib Nazeer
saqibnazeer455@gmail.com

Muhammad Hussain
mhmaths@gmail.com

Atiq ur Rehman
dr.maths81@gmail.com

1 Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Lahore, Punjab 53710, Pakistan

2 Institute of Mathematics, University of Debrecen, Pf. 400, H-4002 Debrecen, Hungary

3 Department of Business Management, Al-imam University College, Balad, Iraq

4 Technical Engineering College, Al-Ayen University, 64001 Dhi Qar, Iraq

Published online: 12 February 2025

1 Introduction

Metric dimension is a term firstly explaind by Slater [21] named mathematician. He revealed the working of Sonar, Loran model stations by the very concept. Another Hararry named mathematician along with Melter independently revealed the under discussion concept [4].

Let U is finite, undirected,simple graph with $E = \{q_1, q_2, \ldots, q_c\}$ any ordered subset of vertex set of U so that $r(q|E)$ is a representation of any vertex q with respect to E is $c - tuple$ and is given by $(d(g, q_1), d(g, q_2), d(g, q_3), \ldots, d(g, q_c))$.

If every element of vertex set has representation that is unique with E then E is termed as resolving set of U. If E is having smallest cardinality among resolving sets then cardinality of E is metric dimension of U and is symbolized by $dim(U)$.

No graph other than path graph has 1 metric dimension [23]. Metric dimension possessed by cycle graph equals to 2. Rich applications of resolvability are found in artificial intelligence example: tricky games, recognition of images, networks of robot navigation, pattern recognitions,processing of maps [26]. Many details about usefulness of metric dimension are in [13]. M. Perc et al. worked on evolution dynamics of group interaction on saturated populations [18]. K. AZhar et el.computed fault tolarant partitions of Mesh related networks [14]. A. Szolonki et al. calculated metric dimension of join of two graphs [1]. I. G. Yero et al.computed metric dimension of corona product of graphs [10] and R Naeem et el. computed resolvabity of graphs [24]. G. Chartrand et al. computed metric dimension of important graphs [5]. M. Imran et al calculated metric dimension of circulant graphs [16]. M. Imran et al. found the metric dimension of generalized Peterson multigraphs [17]. R. A. Melter et al. computed metric basis in digital geometry [23]. M.perc et el. worked on evolutionary and coevolutionary games [22] and made us ready to have good understanding of the under discussion concepts..

A worth while work is done by some authors in computation metric dimention of path related graphs. Ali et al. got success in 2012 in computation of metric dimension of middle nd power graphs of path graph. [15]. Alholi et al. put their contribution in 2017 to finding out the bounded, constant metric dimension [20]. Shahida et al. able to compute metric dimension of join of any two graphs [2]. Ali et al. successfully calculated in 2012 constant metric dimension of total path graph and proved that it is constant [15]. Pan et al. In 2019 found unbounded metric dimension of cyle and path related splitting graphs [9]. Again Peng et al. got success in finding metric dimension of Kenser related graphs in 2016 [8].
In the year 2021 Nawaz et al. were able to prove that path power three and four possess unbounded metric dimension, they also proved results on the edges of power of total path [29]. In the year 2021 Saqib et al. found bounded and constant metric dimension of middle tower and reflection of middle tower graphs [27].

Kamran et el. worked on above concept and determined Fault-tolerance partition dimension tower related graphs [14]. This research is enhancement of the very research work and we determined the constant metric dimension of graphs such as $h - Vrl(mid(Tower_p))$ and $h - Vrl(SSP_p)$. We introduced h copies of middle tower graph vertically with one common vertex and named it as $h - Vrl(mid(Tower_p))$ similarly we introduced $h - Vrl(SSP_p)$.

Definition 1 A simple, planer, undirected graph having vertex set $V = \{V_q^c | 1 \leqslant g \leqslant p, 1 \leqslant q \leqslant c\}$ where $t = 1, 2, 3, \ldots, p$ [27].

Each vertex of Middle $Tower_p$ is adjacent with other vertex if:

1- $v_q^c \sim v_g^{c+1}$: $q = 1, 2, 3, \ldots, c$ and $g = 1, 2, 3, \ldots, p - 1$.

2-$v_q^c \sim v_{g+1}^{c+1}$ when $q = 1, 2, 3, \ldots, c$ and $g = 1, 2, 3, \ldots, p - 1$.

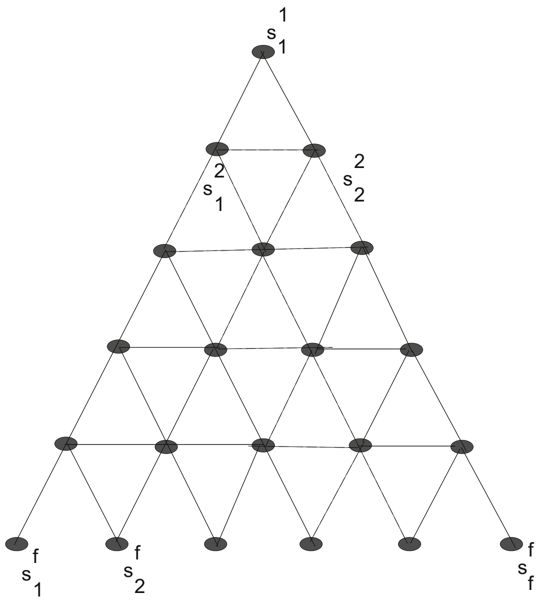

Fig. 1 Middle $Tower_p$ graph

3-$v_q^c \sim v_{g+1}^c$ when $q = 1, 2, 3, \ldots, c$ and $g = 1, 2, 3, \ldots, p-1$.

Figure 1 shows tower graph with p vertices in its base:

Definition 2 SPP_p having p vertices present in its base with $\bigcup_{q=1}^{p}\{V_p^q\}$ as vertex set where $q = 1, 2, 3, \ldots, p$ and $c \leqslant q$ [27].

Every v_c^q of SPP_p is adjacent with other vertex if.
1- $v_c^q \sim v_c^{q+1}$:$c = 1, 2, 3, \ldots, p-1$ and $q = 1, 2, 3, \ldots, p-1$: $c \leqslant q$
2-$v_c^q \sim v_{c+1}^{q+1}$: $c = 1, 2, 3, \ldots, p-1$ and $q = 1, 2, 3, \ldots, p-1$.
3-$v_c^q \sim v_{c+1}^q$: $c = 1, 2, 3, \ldots, p$ and $q = 1, 2, 3, \ldots, p-l$.

Figure 2 shows symmetrical planer pyramid graph with p vertices in its base:

Metric Dimension of $h - Vrl(mid(Tower_f))$

The concept of reflection $mid(Tower_p)$ was introduced by Saqib et al. we enhanced this for vertical mirror image for h reflections and computed the metric dimension of said graphs. We firstly calculated metric dimension $2 - Vrl(mid(Tower_f))$ and then for $3 - Vrl(mid(Tower_f))$ and in the end generalized results for any h.

Theorem 3 *Metric Dimension of 2-vertical reflections of middle tower path graph with f vertices in base is 2 i.e., $dim 2 - Vrl(mid(Tower_f))$ is 2.*

Proof As $2 - Vrl(mid(Tower_f))$ is other than path graph so $dim\, 2 - Vrl(mid(Tower_f))$ is not 1. Now we consider the $V(2 - Vrl(Tower_f) = \{u_f^l\} \bigcup \{v_f^l\}$ where $f = 1, 2, 3, 4, \ldots, f$,

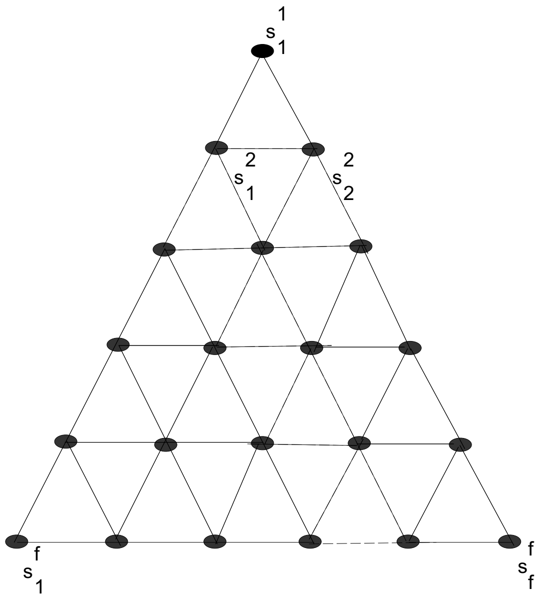

Fig. 2 Symmetrical planer pyramid graph

and $l = 1, 2, 3, \ldots, l$. Let $Q = \{u_1^f, v_1^f\}$ be the resolving set of $2 - Vrl(mid(Tower_f))$ such that:

$$r(u_i^1|Q) = (f - 1, 2f - i).i = 1, 2, 3, \ldots, f$$
$$r(u_i^p|Q) = (f - p, 2f - i - 1).p = 1, 2, 3, \ldots, f$$
$$r(v_i^1|Q) = (f + i - 1, f - 1).i = 1, 2, 3, \ldots, f$$
$$r(v_i^p|Q) = (f + i + p - 3, f - p).i = 1, 2, 3, \ldots, f$$

Now we shall discuss uniqueness of each representation.

When $q \neq c$ and $r(u_q^1|Q), r(u_c^1|Q)$ i.e.,

$$r(u_q^1|Q) = r(u_c^1|Q).$$
$$\Longrightarrow (f - 1, 2f - q) = (f - 1, 2f - c).$$
$$\Longrightarrow q = c \ (contradiction).$$

When $q \neq c$ and $r(u_q^p|Q), r(u_c^p|Q)$ i.e.,

$$r(u_q^p|Q) = r(u_c^p|Q).$$
$$\Longrightarrow (f - p, 2f - q - 1) = (f - p, 2f - c - 1).$$
$$\Longrightarrow q = c \ (contradiction).$$

When $q \neq c$ and $r(u_i^q|Q), r(u_i^c|Q))$ i.e.,

$$r(u_i^q|Q) = r(u_i^c|Q).$$
$$\Longrightarrow (f - q, 2f - i - 1) = (f - c, 2s - i - 1).$$
$$\Longrightarrow f = c \ (contradiction).$$

When $f \neq c$ and $r(v_q^1|Q), r(v_c^1|Q)$ i.e.,

$$r((v_q^1|Q) = r(v_c^1|Q).$$
$$\implies (f+q-1, f-1) = (f+c-1, f-1).$$
$$\implies q = c \ (contradiction).$$

When $q \neq c$ and $r(v_q^p|Q), r(v_c^p|)$ i.e.,

$$r(v_q^p|Q) = r(v_c^p|Q).$$
$$\implies (f+q+p-3, f-p) = (f+c+p-3, f-p).$$
$$\implies f = c \ (contradiction).$$

When $q \neq c$ and $r(v_i^q|Q), r(v_i^c|Q)$ i.e.,

$$r(v_i^q|Q) = r(v_i^c|Q).$$
$$\implies (f+i+q-3, f-q) = (f+i+c-3, s-c).$$
$$\implies f = c \ (contradiction).$$

So W is resolving set with minimum number of elements hence metric dimension of $2 - Vrl(mid(Tower_f))$ is 2.

Figure 3 shows 2- Vrl(mid(Tower) with f vertices in its base:

□

Theorem 4 *Metric Dimension of* 3*-vertical reflections of middle tower path graph with* f *vertices in base is* 2 *i.e.dim* $3 - Vrl(mid(Tower_f))$ *is* 3.

Proof As $3 - Vrl(mid(Tower_f))$ is other than path graph so its metric dimension is greater than 1. Now we consider the $V(3 - Vrl(Tower_f) = \{u_f^q\} \bigcup \{v_f^q\} \bigcup \{w_f^q\}$ where $f =$

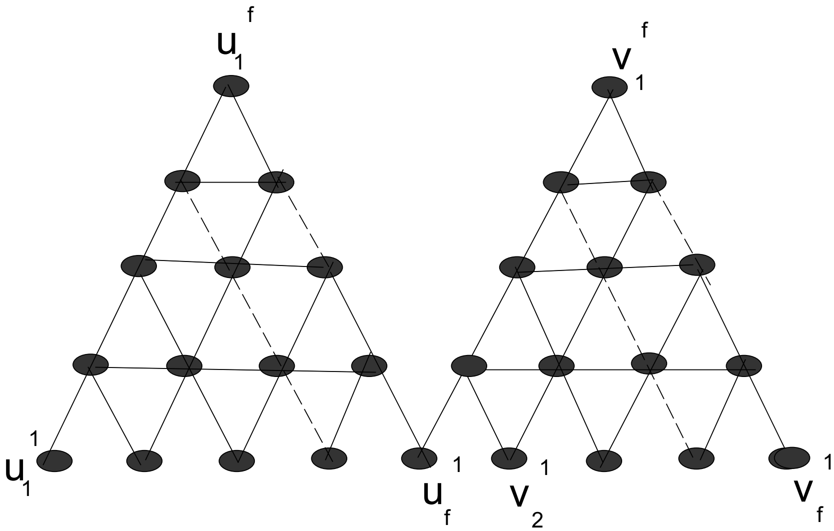

Fig. 3 $2 - Vrl(mid(Tower_f))$

$1, 2, 3, 4, \ldots, f$,and $q = 1, 2, 3, \ldots, f$. Let $Q = \{u_1^f, v_1^f, w_1^f\}$ be the resolving set of $3 - Vrl(mid(Tower_f))$ such that:

$$r(u_s^1|Q) = (f-1, 2f-s, 3f-s).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2, 3, \ldots, f.$$
$$r(u_s^p|Q) = (f-p, 2f-s-1, 3f-s-1).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2, 3...., f.$$
$$r(v_s^1|Q) = (f+s-1, f-1, 2f+s-1).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2, 3, \ldots, f.$$
$$r(v_s^p|Q) = (f+s+p-3, f-p, 2f-s-1).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2.3, \ldots, f.$$
$$r(w_s^1|Q) = (2f+s-1, f+s-1, f-1).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2, 3, ...f.$$
$$r(ws^p|Q) = (2f+s+p-3, f+s-3, f-p).s = 1, 2, 3, \ldots, f-p+1, p = 1, 2, 3, \ldots, f.$$

Now we shall discuss uniqueness of each representation.

When $q \neq c$ and $r(u_q^1|Q), r(u_c^1|Q)$ i.e.,

$$r(u_q^1|Q) = r(u_c^1|Q).$$
$$\Longrightarrow (f-1, 2f-q, 3f-l) = (f-1, 2f-c, 3f-l).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(u_s^q|Q), r(u_s^c|Q)$ i.e.,

$$r(u_s^q|Q) = r(u_s^c|Q).$$
$$\Longrightarrow (f-q, 2f-s-1, 3f-s-1) = (f-c, 2f-s-1, 3f-s-1).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(u_q^p|Q), r(u_c^p|Q)$ i.e.,

$$r(u_q^p|Q) = r(u_c^p|Q).$$
$$\Longrightarrow (f-p, 2f-q-1, 3f-q-1) = (f-p, 2f-c-1, 3f-c-1).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(v_q^1|Q), r(v_c^1|Q)$ i.e.,

$$r(v_q^1|Q) = r(v_c^1|Q).$$
$$\Longrightarrow (f+q-1, f-1, 2f+q-1) = (f+q-1, f-1, 2f+q-1).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(v_s^q|Q), r(v_s^c|Q)$ i.e.,

$$r(v_s^q|Q) = r(v_s^c|Q).$$
$$\Longrightarrow (f+s+q-3, f-q, 2f-s-1) = (f+s+c-3, f-c, 2f-s-1).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(w_q^1|W), r(w_c^1|W)$ i.e.,

$$r(w_q^1|Q) = r(w_c^1|Q).$$
$$\Longrightarrow (2f+q-1, f+q-1, f-1) = (2f+c-1, f+c-1, f-1).$$
$$\Longrightarrow q = c\ (contradiction).$$

When $q \neq c$ and $r(w_s^q|Q), r(w_s^c|Q)$ i.e.,

$$r(w_s^l|Q) = r(w_s^l|Q).$$

$\Longrightarrow (2f+s+q-3,\ f+s-3,\ f-q) = (2f+s+c-3,\ f+s-3,\ f-c).$
$\Longrightarrow q = c\ (contradiction).$

When $q \neq c$ and $r(w_q^p|Q), r(w_c^p|Q)$ i.e.,

$r(w_q^p|Q) = r(w_c^p|Q).$
$\Longrightarrow (2f+q+p-3,\ f+q-3,\ f-p) = (2f+c+p-3,\ f+c-3,\ f-p).$
$\Longrightarrow f = c\ (contradiction).$

Let on contrary $H = \{u_1^f, v_1^f\}$ be metric bases possessing cardinality 2 then

$$r(u_3^2|Q) = r(v_2^3|Q)(contradiction).$$

Let on contrary $H = \{u_1^f, w_1^f\}$ be metric bases possessing cardinality 2 then

$$r(u_2^2|H) = r(w_1^3|H)(contradiction).$$

Let on contrary $H = \{v_1^f, v_1^f\}$ be metric bases possessing cardinality 2 then

$$r(v_2^2|H) = r(w_2^3|H)(contradiction).$$

Hence H is the resolving set containing minimum number of elements so $3 - Vrl(mid(Tower_f))$ has metric dimension 3.

Figure 4 shows 3- Vrl(mid(Tower) with f vertices in its base:

□

Theorem 5 *Metric Dimension of h-vertical reflections of middle tower path graph with k vertices in base is h i.e.dim* $h - Vrl(mid(Tower_k))$ *is h.*

Proof As $h - Vrl(mid(Tower_k))$ is other than path graph so its metric dimension cannot be 1. Now we consider the $V(h - Vrl(mid(Tower_k))) = \{u_{k(h)}^l\} \bigcup \{u_{1(1)}^1\}$ where $p = 2, 3, 4, \ldots, k, h = 1, 2, 3, \ldots, h$ and $l = 1, 2, 3, \ldots, l$.

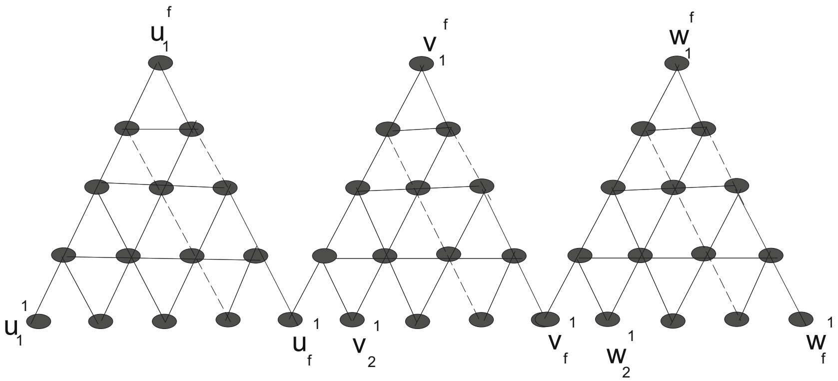

Fig. 4 $3 - vrl(mid(Tower_f))$

Let $Q = \{u^k_{1(1)}, u^k_{1(2)}, u^k_{1(3)}, \ldots, u^k_{1(h)}\}$ be the resolving set of $h - Vrl(mid(Tower_k))$ such that:t

$$\begin{aligned} r(u^1_{s(h)}|Q) &= ((h-1)k+s-1, (h-2)k+s-1, (h-3)k+s-1, (h-4)k \\ &\quad +s-1, \ldots, (h-(h-1))k-1, 2k-s-1, \ldots, (f-h+1)k-s-1). \\ r(u^1_{k(h)}|Q) &= (hk-1, (h-1)k-1, (h-2)k-1, (h-3)k \\ &\quad -1, \ldots, (k-1)_{h^{th}place}, k-1, 2k-1, \ldots, (k-h)k-1). \\ r(u^q_{s(h)}|Q) &= ((h-1)k+q+s-3, (h-2)k+q+s-3, (h-3)k+q+s \\ &\quad -3, (h-4)k+q+s-3, \ldots, (k-l)_{h^{th}place}, 2k-s-1, \ldots, \\ &\quad (p-h+1)k-s-1). \end{aligned}$$

When $q \neq c$ $u^1_{q(h)}$ and $u^1_{c(h)}$ s i.e.,

$$\begin{aligned} & r(u^1_{q(h)}|Q) = r(u^1_{c(h)}|Q). \\ & \Longrightarrow ((h-1)k+q-1, (h-3)k+q-1, (h-4)k+q-1, \ldots, \\ & \quad (h-(h-1))k-1, 2k-q-1, \ldots, (p-h+1)k-q-1)) \\ & \quad = ((h-1)k+c-1, (h-2)k+c-1, (h-3)k+c-1, (h-4)k+c-1, \ldots, \\ & \quad (h-(h-1))k-1, 2k-c-1, \ldots, (p-h+1)k-c-1). \\ & \Longrightarrow (h-1)k+q-1 = (h-1)k+c-1. \\ & \Longrightarrow q = c \ (contradiction). \end{aligned}$$

when $q \neq c$

$$\begin{aligned} & r(u^1_{s(q)}|Q) = r(u^1_{s(c)}|Q). \\ & \Longrightarrow ((q-1)k+s+-1, (q-3)k+s-1, (q-4)k+s-1, \ldots, \\ & \quad (q-(q-1))k-s, 2k-s-1, \ldots, (p-s+1)k-s-1)) \\ & = ((c-1)k+s-1, (c-2)k+s-1, \\ & \quad (c-3)k+s-1, (c-4)k+s-1, \ldots, \\ & \quad (c-(c-1))k-1, 2k-s-1, \ldots, (p-c+1)k-s-1). \\ & \Longrightarrow (h-1)k+q+-1 = (h-1)k+c+-1. \\ & \Longrightarrow q = c \ (contradiction). \end{aligned}$$

Similarly when $q \neq c$

$$\begin{aligned} & r(u^1_{s(t)}|Q) = r(u^1_{g(h)}|Q). \\ & \Longrightarrow (qk-1, (q-1)k-1, (q-2)k-1, (q-3)k-1, \ldots, \\ & \quad (k-1)_{q^{th}place}, k-1, 2k-1, \ldots, (p-q)k-1) \\ & = (ck-1, (c-1)k-1, (c-2)k-1, (c-3)k-1, \ldots, (k-1)_{c^{th}place}, \\ & \quad k-1, 2k-1, \ldots, (p-c)k-1). \\ & \Longrightarrow (q-1)k+q-1 = (c-1)k+c-1. \\ & \Longrightarrow q = c \ (contradiction). \end{aligned}$$

When $t \neq g$

$$\begin{aligned} & r(u^l_{i(q)}|Q) = r(u^l_{i(c)}|Q). \\ & \Longrightarrow ((q-1)k+l+s-3, (q-2)k+l+s-3, (q-3)k+l \end{aligned}$$

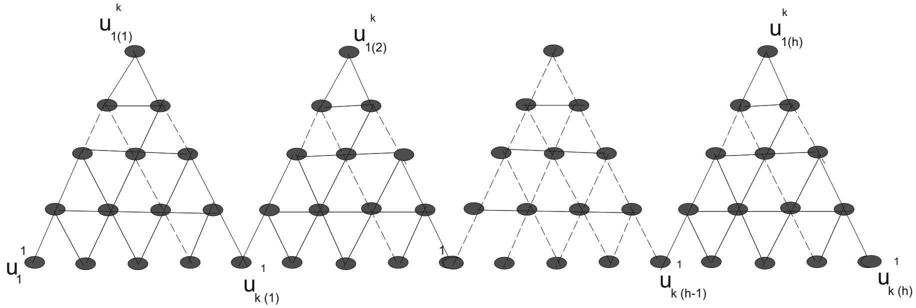

Fig. 5 $h - Vrl(mid(Tower_k))$

$$
\begin{aligned}
&+s-3,(q-4)k+l+s-3,\\
&\ldots,(k-l)_{t^{th}place},2k-s-1,\ldots,(p-q+1)k-s-1)\\
&=((c-1)k+l+s-3,(c-2)k+l+s-3,(c-3)k+l+s-3,\\
&(c-4)k+l+s-3,\ldots,(k-l)_{c^{th}place},2k-s-1,\ldots,(p-g+1)k-s-1)\\
&\Longrightarrow (q-1)k+l+s-3=(c-1)k+l+s-3.\\
&\Longrightarrow q=c\ (contradiction).
\end{aligned}
$$

When $q \neq c$

$$
\begin{aligned}
&r(u^q_{s(h)}|Q)=r(u^c_{s(h)}|Q).\\
&\Longrightarrow ((h-1)k+q+s-3,(h-2)k+q+s-3,(h-3)k+q+s-3,(h-4)k\\
&\quad +q+s-3,\ldots,(k-q)_{h^{th}place},2k-s-1,\ldots,(p-h+1)k-s-1)\\
&=((h-1)k+c+s-3,(h-2)k+c+s-3,(h-3)k+c+s-3,(h-4)k\\
&\quad +c+s-3,\ldots,(k-l)_{h^{th}place},2k-s-1,\ldots,(p-h+1)k-s-1)\\
&\Longrightarrow (h-1)k+q+s-3=(h-1)k+c+s-3.\\
&\Longrightarrow q=c\ (contradiction).
\end{aligned}
$$

So W is a resolving set of the under discussion $h-Vrl(mid(Tower_k))$. We shall establish that Q is resolving set with least number of elements. On contrary let W^1 is the resolving set with $h-1$ elements so metric dimension of $h-Vrl(mid(Tower_k))$ is $h-1$. For particular value $h=3$ the metric dimension of $3-Vrl(mid(Tower_k))$ is 3 contradiction to previous theorem. Hence proved.

Figure 5 shows h- Vrl(mid(Tower) with k vertices in its base:

□

Corollary 6 *Metric Dimension of h-vertical reflections of middle tower path graph with p vertices in base is independent of number of vertices in basei.e.* $dim\ (s-Vrl(mid(Towere_p)))$ *is independent of* p.

Proof $dim\ (s-Vrl(mid(Tower_p)))$ is not independent of p i.e.,
When $dim\ (s-Vrl(mid(Tower_l))) \neq dim\ (s-Vrl(mid(Tower_q)))$ where $l \neq q$.
but from above theorem contradiction.

□

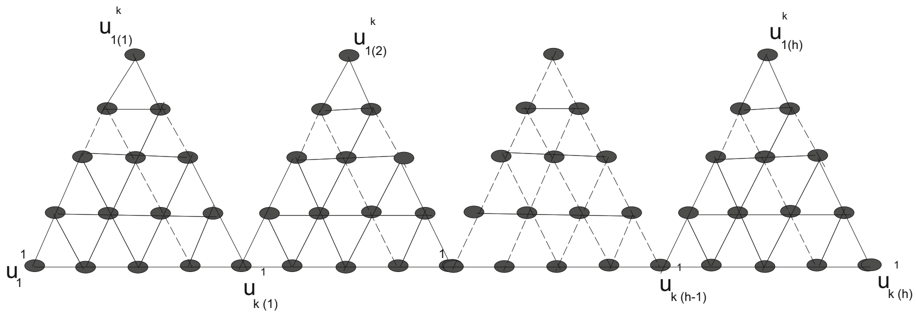

Fig. 6 $s - Vrl(mid(SSP_p))$

The concept of reflection SPP_s was introduced by Saqib et al. we enhanced this for vertical mirror image for h reflections and computed the metric dimension of these graphs. We firstly calculated metric.

Theorem 7 *Metric Dimension of p-vertical reflections of symmetrical planer pyramid path graph with s vertices in base is* 2 *i.e.,* $dim\ p - Vrl(mid(SPP_s))$ *is* 2*.*

Proof As $p - Vrl(mid(SPP_s))$ is other than path graph so its metric dimension is greater than 1. Now we consider the $V(p - Vrl(SPP_s) = \bigcup\{u^l_{p(h)}\}$ where $p = 1, 2, 3, 4, \ldots, p$, $h = 1, 2, 3, \ldots, h$ and $l = 1, 2, 3, \ldots, l$. Let $H = \{u^s_{1(1)}, u^s_{p(s}\}$ be the resolving set of $p - Vrl(SPP_s)$ such that:

$r(u^l_{g(i)}|H) = ((g-1)(s-1) + i + l - 2, (p-g+1)(s-1) - i + 1)$.

When $v \neq t$ and $u^l_{g(v)}, u^l_{g(t)}$ i.e.,

$$r(u^l_{g(v)}|H)) = r(u^l_{g(t)}|H).$$
$$\Longrightarrow ((g-1)(s-1) + v + l - 2, (p-g+1)(s-1) - v + 1)$$
$$= (((g-1)(s-1) + t + l - 2, (p-g+1)(s-1) - t + 1).$$
$$\Longrightarrow v = t\ (contradiction).$$

When $v \neq t$ and $u^v_{g(i)}|H), u^t_{g(i)}|H)$ i.e.,

$$r(u^v_{g(i)}|H)) = r(u^l_{g(t)}|H).$$
$$\Longrightarrow ((g-1)(s-1) + i + v - 2, (p-g+1)(s-1) - i + 1)$$
$$= (((g-1)(s-1) + i + t - 2, (p-g+1)(s-1) - i + 1).$$
$$\Longrightarrow v = t\ (contradiction).$$

When $v \neq t$ and $u^l_{v(i)}|H), u^l_{t(i)}|H)$ i.e.,

$$r(u^l_{v(i)}|H)) = r(u^l_{t(i)}|H).$$
$$\Longrightarrow ((v-1)(s-1) + i + l - 2, (p-v+1)(s-1) - i + 1)$$
$$= ((t-1)(s-1) + i + l - 2, (p-t+1)(s-1) - i + 1).$$
$$\Longrightarrow v = t\ (contradiction).$$

So H is a resolving set of $p - Vrl((SPP_p))$ and its metric dimension is 2.
Figure 6 shows $p - Vrl((SPP_p))$ with p vertices in its base:

□

Corollary 8 *Metric Dimension of S-vertical reflections of symmetrical planer pyramid path graph with p vertices in base is independent of number of vertices in base i.e., $dim\ s - Vrl((SPP_p))$ is independent of p.*

Proof $dim\ (s - Vrl((SPP_p)))$ is not independent of p i.e.,
When $dim\ (s - Vrl((SPP_l))) \neq dim\ (s - Vrl((SPP_q)))$ where $l \neq q$.
but from above theorem contradiction.

□

2 Conclusions

In this dissertation, concept of some planer graphs, as s vertical reflections of symmetrical planer pyramid of path $(s - Vrl(mid(SSP_p)))$ and h vertical reflections of f- middle tower graph of path $(h - Vrl(mid(Tower_f)))$ is generalised and their metric dimension is computed. Metric dimension of all discussed graphs is constant. Further this concept can be extended to the cycle related graphs such as h vertical reflections of total and middle of cycle graph with p vertices. Also it would be interesting to investigate half reflections of said graphs.

Funding Open access funding provided by University of Debrecen.

Data availability This study is not supported by any other data.

Declarations

Conflict of interest There are no Conflict of interest among the authors.

References

1. Szolnoki, A., Perc, M.: Correlation of positive and negative reciprocity fails to confer an evolutionary advantage: phase transitions to elementary strategies. Phys. Rev. X **3**(4), 041021 (2013)
2. Shahida, A.T., Sunitha, M.S.: On the metric dimension of joins of two graphs. Global J. Pure Appl. Math. **5**(9), 33–38 (2014)
3. Caceres, C., Hernando, C., Mora, M., Pelayo, I., Puertas, M.L.: On the metric dimension of infinite graphs. Electron. Notes Discret. Math. **35**, 15–20 (2009)
4. Harary, F., Melter, R.A.: On the metric dimension of a graph. Ars Combin. **2**, 191–195 (1976)
5. Chartrand, G., Eroh, L., Johnson, M.A., Oellermann, O.R.: Resolvability in graphs and the metric dimension of a graph. Discret. Appl. Math. **105**, 99–113 (2000)
6. Sudhakara, G., Hemanth Kumar, A.R.: Graphs with metric dimension two-A characterization. Proc. World Acad. Sci., Eng. Technol., Int. J. Math. Comput. Sci. **3**(12), 1128–1133 (2009)
7. Heinig, G., Rost, K.: Algebraic Methods for Toeplitz-Like Matrices Operators. Birkhäuser, Boston, MA, USA (1984)
8. Peng, H., Wang, X., Wang, J., Chen, J.: On the metric dimension of some Kneser graphs. J. Comput. Theor. Nanosci. **13**(5), 3013–3018 (2016)

9. pan, H., Ali, M., Ali, G., Ali, M.T., Yang, X.: On the families of graphs with unbounded metric dimension. IEEE Access **7**, 165060–165064 (2019)
10. Yero, I.G., Kuziak, D., Rodríguez-Velázquez, J.A.: On the metric dimension of corona product graphs. Comput. Math. Appl. **61**(9), 2793–2798 (2011)
11. Yero, I.G., Jakovac, M., Kuziak, D., Taranenko, A.: The partition dimension of strong product graphs and Cartesian product graphs. Disbcret. Math. **331**, 43–52 (2014)
12. Tomescu, I., Imran, M.: Metric dimension and R-sets of connected graphs. Gr. Comb. **27**(4), 585–591 (2010)
13. Kratica, J., Kovacevic-Vujcic, V., Cangalovic, M., Stojanovic, M.: Minbimal doubly resolving sets and the strong metric dimension of some convex polytopes. Appl. Math. Comput. **218**(19), 9790–9801 (2012)
14. Azhar, K., Zafar, S., Kashif, A., Aljaedi, A.: Fault-tolerant partition mesolvabilty in mesh related networks and applications. IEEE Access **10**, 71521–71529 (2022)
15. Ali, M., Rahim, M.T., Ali, G.: On path related graphs with constant metric dimension. Utilitas. Math. **88**(1), 203–209 (2012)
16. Imran, M., Baig, A.Q., Bokhary, S.A.U.H., Javaid, I.: On the metric dimension of circulant graphs. Appl. Math. Lett. **25**, 320–325 (2012)
17. Imran, M., Siddiqui, M.K., Naeem, R.: On the metric dimension of generalized Petersen multigraphs. IEEE Access **6**, 74328–74338 (2018)
18. Perc, M., Gomez-Gardeñes, J., Szolnoki, A., Floría, L.M., Moreno, Y.: Evolutionary dynamics of group interactions on structured populations: a review. J. Roy. Soc. Interface **10**(80), 20120997 (2013)
19. Perc, M., Szolnoki, A.: Coevolutionary games-A mini review. Biosystems **99**(2), 109–125 (2010)
20. Alholi, M.M., Abughneim, O.A., Ezeh, H.A.: Metric dimension of some path related graphs. Global J. Pure Appl. Math **3**(2), 149–157 (2017)
21. Slater, P.J.: Leaves of trees. Congr. Numer. **14**, 549–559 (1975)
22. Slater, P.J.: Dominating and reference sets in a graph. J. Math. Phys. Sci. **22**(4), 445–455 (1988)
23. Melter, R.A., Tomescu, I.: Metric bases in digital geometry. Comput. Vis. Graph. Image Process. **25**, 113–121 (1984)
24. Naeem, R., Imran, M.: On resolvability and exchange property in antiweb-wheels. Utilitas Math. **104**, 187–200 (2017)
25. van Dal, R., Tijssen, G., Tuza, Z., van der Veen, J.A.A., Zamfirescu, C., Zamfirescu, T.: Hamiltonian properties of Toeplitz graphs. Discret. Math. **159**(1–3), 69–81 (1996)
26. Khuller, S., Raghavachari, B., Rosenfeld, A.: Localization in graphs, Univ. Maryland, College Park, College Park, MD, USA, Tech. Rep. UMIACS-TR-94-92, 1994
27. Nazeer, S., Hussain, M., Alrawajeh, F.A., Almotairi, S.: Metric dimension on path-related graphs. Math. Probl. Eng. **2021**, 2085778 (2021)
28. Saputro, S.W., Simanjuntak, R., Uttunggadewa, S., Assiyatun, H., Baskoro, E.T., Salman, A.N.M., Baa, M.: The metric dimension of the lexicographic product of graphs. Discret. Math. **313**(9), 1045–1051 (2013)
29. Nawaz, S., Ali, M., Ali, G., Khan, S.: Computing metric dimension of power od total graph. IEEE Access **9**, 74550–745561 (2021)
30. Shang, Y.: Percolation in a hierarchical lattice. Zeitschrift Naturbforschung **67**(5), 225–229 (2012)