Short thesis for the degree of doctor of Philosophy (PhD)

FIBER BUNDLE MODELLING OF FAILURE PHENOMENA

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Introduction

In the framework of my PhD research I investigated failure phenomena emerging in complex systems which are composed of a large number of interacting elements. From the mechanically induced fracture of solid bodies[1], through the breakdown of traffic in urban areas[2,3] and avalanche activity of neural networks[4], to the blackout of high voltage power grids [5], a large diversity of failure phenomena can be mentioned which share interesting common features. First of all, the local failure of an element of the system can trigger further failure events due to the redistribution of load over the remaining intact elements. Together with the constraint of load conservation this load sharing mechanism can induce cascades of failures which may spread over a large fraction of the system eventually leading to global failure. The interplay of the cascading failure dynamics and of the network topology has recently been studied using discrete models on various types of complex networks. In these approaches either the nodes[6,7] or the links[8] of the network are assumed to undergo a degradation process accompanied by a mechanism of load rearrangement on the intact elements which can give rise to cascades of failure events. However, it has not been clarified how the failure dynamics driven by localized load redistribution and load conservation is affected by the topology of the underlying network of load transmitting connections, when structural disorder has to compete with the randomness of local strength.

The process of spreading and the overall stability of the system is substantially affected by the degree of disorder of the load bearing

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[1] M. J. Alava, et al, Adv. Phys. **55**, 349 (2006).

[2] S. Boccaletti et al, Physics Reports **424**, 175 (2006)

[3] Y. Moreno et al, Europhysics Letters **58**, 630 (2002).

[4] N. Jung et al, Chaos **10**, 063118 (2020).

[5] I. Dobson et al, Chaos **17**, 026103 (2007).

[6] S. Siddique et al, Phys. Rev. E. **89**, (2014).,

[7] J. Awrejcewicz et al, Chaos **31**, 7 (2021).

[8] D. V. Stäger et al, Phys. Rev. E. **90**, (2014).

capacity of the individual elements[1,9]. Disorder usually reduces the ultimate strength of the system, however, at the micro-level, it can help to arrest propagating failure avalanches, hence, increasing the damage tolerance[1]. Recent investigations have revealed that such failure phenomena are characterized by scaling laws both on the macro- and micro-scales and their characteristics exhibit a high degree of robustness against specific details of the failing system[1,10]. These results led to the emergence of the idea that the theory of phase transitions and critical phenomena provide the adequate framework for the description of such failure phenomena. Most notably, for the fracture processes of heterogeneous materials the phase transition description proved to provide novel insights[1,11,12]. However, studies on the phase transition analogy of fracture phenomena have been limited to uniaxial loading of heterogeneous materials[1].

During my PhD work, I performed research in this prospering field using the approaches and tools of statistical physics, phase transitions and critical phenomena, the physics of complex systems and network science.

Objectives

I carried out research in two main directions: First, I tried to obtain a deeper understanding of the phase transition analogy of fracture processes. For this purpose I studied how a solid body perforates due to dynamic impact loading as the energy of impact is varied at different amounts of materials' disorder. Based on a simple model I wanted to clarify how the system approaches the critical point of perforation, i.e. the point where the bar breaks through. My goal was to identify

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[9] M. J. Alava, et al, J. Phys. D: Appl. Phys. **42**, 214012 (2009).

[10] A. Petri, et al, Phys. Rev. Lett. **73**, 3423 (1994).

[11] O. Ramos et al, Phys. Rev. Lett. **110**, 165506 (2013).,

[12] J. P. Sethna et al, Nature **410**, 242 (2001)

the order parameter of the damage to perforation transition and to determine the value of the critical exponents characterizing the system in the vicinity of the critical point. It was also a crucial question how robust the behaviour of the system is against the degree of disorder.

As the next step, my research focused on the cascading failure mechanism of complex systems initiated by external load increments. Due to the similarities of the emergence of crackling avalanches in heterogeneous solids under an external load and of the spreading of failure cascades e.g. in technological networks, I investigated the local failure mechanism of heterogeneous materials on complex networks. My primary goal was to understand how the interplay of the topology of the network of load transmitting connections and the disordered local strength affects the statistical and dynamical features of failure cascades. Most notably, I showed that as the network topology changes a transition emerges from the localized to the mean field behaviour of fiber bundle model. It was my goal to understand the mechanism of this transition and to clarify how the degree of strength disorder of fibers affects the transition.

Failure avalanches gradually spread on the interaction network through consecutive steps of local failure and load redistribution. Analyzing the temporal evolution of failure avalanches I wanted to give a detailed description of the relation of the temporal profile of avalanches and the structure of the underlying network.

Methodology

Due to the decisive role of disorder that is present in the microstructure of the system and in the local physical properties of its elements, analytical approaches have limited capabilities in the investigation of failure phenomena. Hence, most of the studies in the field are largely based on computer simulations of discrete models[1]. In my research I used the fiber bundle model (FBM) which was originally introduced to describe the fracture of heterogeneous materials[13]. FBM well captures the key features of failure processes and allows to control the degree of disorder on the meso- or micro-scale in the system. FBM has proven very successful in studying a large diversity of failure phenomena since it grasps the essential mechanisms of the intermittent failure spreading yet being simple enough to offer analytic solutions in certain limiting cases[3,13,14].

In case of impact fracture, the bar shape specimen was composed of two rigid blocks which were glued together by an elastic interface. I constructed an FBM to discretize the interface where cracking was induced by impact. The rigidity of the two blocks ensured global (but not equal) load sharing over the intact fibers of the interface in a strain gradient, which made it possible to perform analytical calculations up to some extent. I analyzed the transition from partial failure to perforation by means of computer simulations varying the impact energy in a broad range.

For the network study, to generate complex networks with controllable structural features I used the Watts-Strogatz model (WS)[15]. Starting from a square lattice of fibers the network of load transmitting connections was gradually randomized by rewiring the links. I applied localized load sharing to redistribute the load over the near-

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[13] S. Pradhan et al, Phys. Mod. Phys. **82**, 499 (2010).

[14] R. C. Hidalgo et al, Phys. Rev. E **80**, 051108 (2009).

[15] D. J. Watts and S. H. Strogatz, Nature **393**, 440 (1998).

est neighbors of failed fibers, and used computer simulations to reveal how failure cascades change when the structure of the network is tuned from completely regular to completely random.

I developed effective simulation algorithms which made it possible to study systems composed of up to $N = 4 \times 10^6$ fibers, where the periodic boundary conditions used in both directions resulted in $L = 8 \times 10^6$ load transferring connections. The random strength of fibers was sampled from a Weibull distribution, which allowed for controlling the amount of disorder in a broad range. I examined 30 different values of the rewiring probability in the WS model spanning the entire range of allowed values. For each parameter set I calculated averages over 2000 samples.

My research is of purely theoretical nature, based on computer simulations and analytical calculations. Hence, I profited a lot from the supercomputer of the University of Debrecen.

New scientific results

1. I investigated the impact induced failure of a bar shaped specimen of heterogeneous materials using a three point bending model. Varying the energy of impact and the amount of disorder of the material I analyzed in details how breaking of the bar occurs [P1].

   (a) I showed that depending on the imparted energy the impact process has two different outcomes: there exists a critical impact energy below which the specimen gets damaged in the sense that it suffers partial failure and keeps its integrity, while above the critical point perforation occurs and the specimen breaks into two pieces. Computer simulations revealed that this damage to perforation transition has analogies to continuous phase transitions characterized by critical power laws [P1].

   (b) I performed a careful numerical analyzes to determine the critical exponents of the phase transition. In particular, I identified the fraction of intact interface and the deflection rate as the order parameter and susceptibility of the damage to perforation transition, respectively, and determined their critical exponents. I showed that at finite disorder different exponents are obtained farther and closer to the critical point and explained this crossover in terms of the geometrical structural of damage [P1].

   (c) I deduced scaling laws of the damage threshold and critical deflection of the system in terms of the amount of disorder and number of fibers used for discretization in the model of three-point bending. I demonstrated that if the number of fibers is sufficiently high the results are independent of the details of discretization [P1].

2. I carried out a theoretical study of the failure dynamics of the fiber bundle model on complex networks with the goal to understand how the interplay of the topology of the network of load transmitting connections and of the amount of disorder of the strength of fibers determines the behaviour of the system both on the micro- and macro-scales. My study was limited to one type of networks, i.e. the Watts-Strogatz rewiring technique was applied to interpolate between a completely regular square lattice and a completely random network. This choice had the advantage that detailed investigations could be performed varying the degree of structural randomness and strength disorder at the same time [P2, T1, PT1].

   (a) I demonstrated that a transition occurs from the localized to the mean field behaviour of FBMs as the network of load transmitting connections is gradually randomized by rewiring. The transition is limited to a range of the rewiring probability so that below the lower bound of the transition regime structural randomization has no significant effect on the failure dynamics, while above its upper bound the transition is completed. In the transition regime the avalanche size distribution displays a crossover between two power laws of different exponents. Increasing the rewiring probability, the critical stress and strain where eventual failure occurs, increase and tend to limits that are close to their mean field values [P2].

(b) I pointed out that two competing mechanisms govern the network's response: rewiring of the underlying network establishes long range random links in the load transmitting network that reduce the load localization and allows the system to bear larger avalanches. However, with increasing structural randomness the degree distribution of the network widens increasing the fraction of low degree nodes. Hence, a counter effect emerges since the strong load localization around low degree nodes can trigger catastrophic failure making the system more vulnerable to early avalanches especially on highly randomized networks with a narrow distribution of node strength. I proposed an analytical description which captures the main ingredients of the above mechanism and provides a reasonable description of the numerical findings [P2].

(c) My computer simulations revealed that the threshold disorder of fibers has a substantial effect on the LLS to ELS transition: the transition regime becomes narrower and shifts to higher rewiring probabilities by decreasing the strength disorder of nodes. Most importantly, the transition is restricted to a well-defined range of strength disorder: there exists a threshold amount of strength disorder of nodes below which network randomization does not provide any improvement neither of the load bearing capacity nor of the avalanche tolerance of the system. I identified an optimal network structure with the highest robustness against cascading failure [P2].

3. I performed a comprehensive study of the temporal evolution of the spreading process of failure avalanches driven by the gradual redistribution of load in the fiber bundle model focusing on the role of the underlying structure of the load transmitting network and the degree of strength disorder of fibers [P3, T1, PT1].

   (a) My calculations revealed that not only the size but also the duration of failure avalanches has a scale free statistics: at low structural disorder when load concentration is strong, large avalanches of long duration occur very rarely in the system. Decreasing the strength disorder of fibers, both the duration and size exponents increase and tend towards a common limit value. Increasing the structural randomness of the load transmission network, the exponents decrease and approach their mean field values, which do not depend on the amount of strength disorder of nodes of the network. Simulations showed that longer avalanches have a larger size expressed by a power law relation. I demonstrated that at any rewiring probability the exponents of avalanche size, duration, and their correlation obey a scaling relation [P3].

   (b) I showed that the average temporal profile of avalanches on all network topologies is a distorted parabola having a right handed asymmetry. The result implies that avalanches start slowly then speed up and finally stop suddenly. My simulations showed that as the network becomes more and more random, avalanches of the same duration evolve to larger sizes and the degree of their profile's asymmetry decreases. I demonstrated that at a given network topology, profiles of different duration can be collapsed on the top of each other by means of a scaling transformation. The result proves that the degree of asymmetry of avalanche profiles is an attribute of the network structure [P3].

Publications

Publications related to the dissertation

Papers in refereed journals

P1 A. Batool, G. Pal and F. Kun, *Impact-induced transition from damage to perforation*, Phys. Rev. E **102**, 9 (2020). **IF: 2.529**

P2 A. Batool, G. Pal, Z. Danku, and F. Kun, *Transition from localized to mean field behaviour of cascading failures in the fiber bundle model on complex networks*, Chaos Solit. Fractals **159**, 112190 (2022). **IF: 9.992**

P3 A. Batool, G. Pal, Z. Danku, and F. Kun. *Temporal evolution of failure avalanches of the fiber bundle model on complex networks.* Chaos: An Interdisciplinary Journal of Nonlinear Science **32**, 063121 (2022). *Selected as Editor's Pick by Chaos.* **IF: 3.741**

Talks

T1 A. Batool, *Transition from localized to mean field behaviour of cascading failures in the fiber bundle model on complex networks*, Avalanche 2022, Debrecen, Hungary, 29.8-2.09.2022.

Posters

PT1 A. Batool, G. Pal and F. Kun, *Failure avalanches on complex networks*, 46th Conference of the Middle European Cooperation in Statistical Physics (MECO46), Riga Latvia, 11-13.05.2021.

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List of publications related to the dissertation

Foreign language scientific articles in international journals (3)

1. **Batool, A.**, Danku, Z., Pál, G., Kun, F.: Temporal evolution of failure avalanches of the fiber bundle model on complex networks.
   Chaos. 32 (6), 1-10, 2022. ISSN: 1054-1500.
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   IF: 3.741 (2021)

2. **Batool, A.**, Pál, G., Danku, Z., Kun, F.: Transition from localized to mean field behaviour of cascading failures in the fiber bundle model on complex networks.
   Chaos Solitons Fractals. 159, 1-11, 2022. ISSN: 0960-0779.
   DOI: http://dx.doi.org/10.1016/j.chaos.2022.112190
   IF: 9.922 (2021)

3. **Batool, A.**, Pál, G., Kun, F.: Impact-induced transition from damage to perforation.
   Phys. Rev. E. 102 (4), 1-11, 2020. ISSN: 2470-0045.
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