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A Role Of Geometry In The Frame Of Competencies Attainment

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Abstract. We discuss aspects of the Education Reform from teaching to educational system. In this context we recognize some problems in recognition of some competencies that students need to achieve and we present how we have developed the measurement method of spatial abilities and problem solving competence. Especially, we investigate how students use spatial visualization abilities in solving various problems in other mathematical course. We have tested how students use their spatial abilities previously developed in geometry courses based on conceptual approach to solve a test based on procedural concept in Mathematical Analysis course.

Key words and phrases: spatial visualization, problem solving, gender parity.

ZDM Subject Classification: D35, D65.

1. Introduction

Throughout the decades higher education has been characterized by the lecture form. On the other hand, the intensive development of information technologies has deep influence on the educational setting. The knowledge itself "has changed". It becomes available instantly; it is broader and less deep. Many problems are already solved, but the challenge is to find the existing solution. The students "have also changed". They prefer instant information; they want to see their knowledge applied. Also, in order to achieve a comprehensive, well-rounded education, integrated teamwork on several fronts is vital. We refer [10] for more details. Therefore system had to be changed through last 15-20 years - from teaching to educational system. Consequently, the Education Reform of Higher Education in the spirit of Bologna declaration is general and also involves:

- A shift from the lecture form to a student-centered approach to learning.
- A shift in the paradigm from measuring educational attainment to measuring of competencies.
- Change from focus on content in curriculum to a problem solving in new situation.

For more details, we refer to [1], [31], [25], [6], [32], [23] and [4].

Competencies might be collected into two groups:

- Generic competencies;
- Subject oriented competencies.

Among the generic competencies we point out the critical way of thinking and among subject oriented competencies one can underline the problem solving.

Most experts have considered problem solving competencies to be very important. According to them, problem solving includes the ability to structure a given problem, to relate it to the context, to identify and find the appropriate resources for getting further information and to develop strategies for decision making, even under the uncertain conditions. Problem solving also refers to the higher-order cognitive process and to the goal oriented thinking and acting in the situation where no routines are available for mastering the situation and where the solution path is not immediately obvious (see for example [23], [4]).

By [6], [32] problem solving is one of 21^{st} Century skills. Except competencies, students also need to achieve the so-called "soft skills", such as: teamwork, presentation skills, effective communication, decision making, time management, stress management, project management, interpersonal skills, motivational skills, networking skills, negotiation skills, etc.

Many authors agree that spatial skills or abilities play a very important role in development of problem solving and other competencies; we refer to [33], [3], [20], [11], [5], [26], etc. Yet the main problem is how to measure these abilities and competencies.

There are a lot of challenges for academic stuff, as well. Curricula of study programs need to be changed, as well as the approach to each course. Changing the concept of the course, learning to understand what students think about this new method, setting the goals we want to achieve in that course and finally checking if they have been met, turned out to be very challenging tasks.

The main purpose of this paper is to develop measurement methods of problem solving competence and its correlation with spatial abilities. First, we give an overview of various opinions on spatial abilities. To confirm the validity of our research, we study the interaction between the spatial abilities and learning process, with special accent on problem solving competencies development. In the next section we establish our research question: how do students use spatial visualization abilities for solving some problems in other mathematical course? To answer this question we use unconscious testing of students by analyzing their approach for solving the integration exercises in Mathematical Analysis, especially the correlation with spatial abilities shown in drawings of integration domains. Our research conclusions are presented in the last section. Also, we have accepted the hypothesis that spatial visualization abilities achieved in Geometry courses based on conceptual approach are in significant correlation with the test results from Mathematical Analysis course based on procedural concept. Additionally, we would like to address the question of gender gap. Namely, "in most countries and economies, girls underperform boys in mathematics; and among the highest-achieving students, the gender gap in favor of boys is even wider; the gender gap in mathematics performance mirrors the gender gap in students' drive, motivation and self-beliefs" (see [29]). Our research shows that, although boys achieve slightly better results, there are no statistically significant differences between genders.

2. A role of spatial ability in doing mathematics

In the world with enormous number of information, many authors have been interested in some skills which might help the students in the studies organized in the new setting as it is mentioned in Introduction. In general, spatial skills are considered to be mental skills connected with understanding, manipulating, reorganizing, or interpreting relationships visually. McGee [24] distinguished two major types of spatial skills: visualization and orientation. A different categorization was proposed by Linn and Petersen [19] who identified three spatial ability categories: spatial perception, mental rotation, and spatial visualization. They defined spatial visualization as "those spatial ability tasks which involve complicated multi-step manipulations of spatially presented information" (p. 1484). They distinguished it from the other two categories "by the possibility of multiple solution strategies" (p.1484). Let us also point out that Linn and Petersen felt that the only common characteristic of spatial visualization tasks was that the solutions required more than one step. The issue of whether spatial skills are general indicators of a particular way of mentally organizing information that might be helpful in many areas of mathematics has been discussed by many authors. One hypothesis is that mathematical reasoning and problem solving are facilitated by a 'mental blackboard' on which the activity may be organized and the interrelatedness of components 'visualized' (for more details see [3]). In addition, it is interesting to mention the results of Bishop [7] who stated that spatial training might help students to organize the situation with mental pictures during problem solving in mathematics. He proposed that the structure of the problem might be understood through a spatial format. The frequent use of tree diagrams, Venn diagrams, charts, and other figures to organize information and show relationships among components of a problem demonstrates the plausibility of this hypothesis. Smith [33] suggested an even more central role for spatial skills in problem solving. He stated that

> "the conception of spatial ability which emerges from recent research ... is so all embracing that one is led to inquire whether the process of perceiving and assimilating ... general patterns or configurations (whether spatial or non-spatial) is not in fact a process of 'abstraction' ... The process of perceiving and assimilating a gestalt ... [is] a process of abstraction (abstracting form or structure)... It is possible that any process of abstraction may involve in some degree the perception, retention in memory, recognition and perhaps reproduction of a pattern or structure" (p. 213-214).

Kozhevnikov and Thornton [15] studied how students' levels of spatial visualization ability interact with learning physics in a microcomputer-based laboratory (MBL) environment. Undergraduate students who had taken an Introductory Physics course based on MBL tools were pre and post tested at the beginning and at the end of the semester on spatial visualization ability and their conceptual understanding of mechanics. The results showed that, while spatial visualization is a reliable predictor for students' performance on physics conceptual evaluation tests before MBL instruction, the relation is not significant after the instruction. Furthermore, as a result of MBL instruction, students' level of spatial visualization increased significantly. In addition, a group of science teachers presented with different types of MBL activities also showed a significant increase in spatial visualization ability.

The above findings raise questions about the possibilities and ways to train and enhance spatial skills.

Although studies on directly trained spatial abilities have not been particularly successful, there are several studies that found significant increase in students' performance on spatial visualization tests after taking geometry or science courses (for example, see [20], [21], [22], [30], etc.). The development of spatial abilities and improvement of problem solving skills may be achieved in various ways. Therefore, let us mention the experience of L. Budai [12] using the help of plane-space analogies, combined with geometric programs GeoGebra and DGS, for the development of spatial thinking and problem solving skills in the three dimensional solid geometry. It is important to emphasize that the improvement of spatial abilities depends on the student's motivation, as well as on its strengthening with a planned, determinate development. We refer to [13] for more results, especially in the case of highly disadvantaged students.

Additionally, it has been found that integration of computer graphics with teaching mathematics, engineering or science improves students' performance on spatial tests (for example, see [5], [26], and [9]). On the other hand computer-assisted instruction (CAI) plays an important role in contemporary teaching and learning of scientific concepts. The Benchmarks for Science Literacy (American association for the Advancement of Science [2], p.18) specifically states that "computers have became invaluable in science because they speed up and extend people's ability to collect, store, compile, and analyze data, prepare research reports, and share data and ideas with investigators all over the world". Recent science education standards in the United States describe "instructional technology, which provides students and teachers with exciting tools - such as computers - to conduct inquiry and to understand science" (see [28], p.24). Science education standards in Serbia, similar to the other countries, have been established by National Council for Higher Education [27]. By these standards, graduates need to attain generic competencies such as the critical way of thinking, knowledge applicable to the real life professions etc., as well as subject specific competencies, including problem solving and usage of information and communication technologies for knowledge improvement in a corresponding field among the others. Many authors have been interested in these problems; we refer to [14] and references therein. The last three authors of this paper have studied how one can model these competencies of students in the frame of Descriptive Geometry course and the role of computers and software in this setting [11]. Electronic teaching accessories which might be used in teaching process and the role of Descriptive Geometry for strengthening one's spatial intuition important for development of problem solving and critical way of thinking competencies has been discussed in [9].

3. Research questions

The reorganization of Descriptive Geometry course in the past 15 years has been discussed in details in [9] and [11] (see also [26]). We analyzed the corresponding competencies which students achieved, especially training of spatial abilities, problem solving and critical way of thinking, the role of computers and software in the development of these competencies and new approach in education process, which is truly compatible with opinions presented in [16] and other references therein. Of course, these abilities and competencies are developed in other courses as well. Hence, it seems reasonable to study correlation between spatial abilities, based on conceptual knowledge of Analytical Geometry, and problem solving competencies, achieved in procedurally based Mathematical Analysis course through unconscious testing. More precisely, we establish the following research question:

How do students use spatial visualization abilities in solving some problems in other mathematical course?

4. Research methodology description

Survey design

To answer the research question we use an unconscious testing of students by analyzing their approach in solving of the following integration exercise which was a part of a test in Mathematical Analysis course.

EXERCISE 1. Determine bounds of integration on the right-hand side integral:

$$\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \int_{x^2+y^2}^{1} f(x,y,z)dz = \int dz \int dy \int f(x,y,z)dx$$

This exercise is a typical example of problem solving with the help of plane-spatial analogous (see [12]). In this task instruction for sketching the area of integration was not explicitly given. As a standard test exercise from Mathematical Analysis this example belongs to group of problems requiring lower level of competencies.

From the viewpoint of geometrical competencies, evaluation was performed on four different levels: 2 - full credit, 1 - partial credit, 0 - no credits, 9 - sketch missing. Figure should represent a part of circular paraboloid in space, which is also a task with low abilities requirement since the students previously have enrolled in course of Analytical Geometry. The only potential problem arises from the fact that the data needed for a full sketch of the surface has been given indirectly, through the bounds of integration.

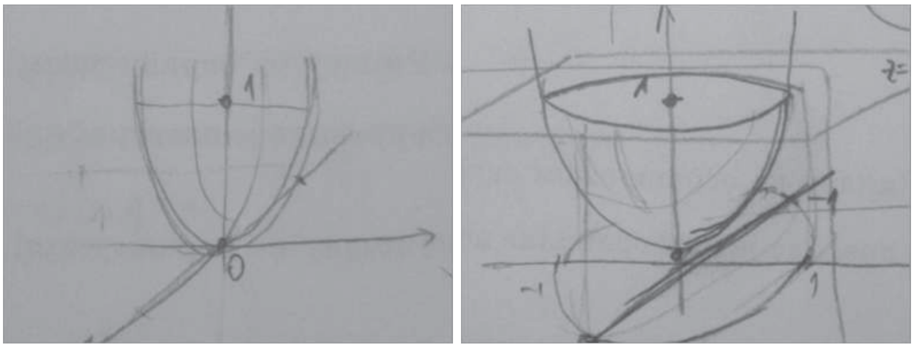

Figure 1. Example of sketch with partial (left) and full credit (right) in Exercise 1

> EXERCISE 2. Let $T = \{\,(x, y, z)|\, x \leq 1, y \geq -1, z \geq 0, y - x + z \leq 2\}$. Sketch T and calculate $\iiint\limits_T dx\,dy\,dz$.

In the second exercise, all the elements necessary for sketching the surface T are given directly. The solid T represents triangular pyramid with right-angled triangle base (whose catheti are parallel to the axis) lying in the xy-plane. One of the edges is orthogonal to xy-plane. Motivation for this type of test lies in our goal to make the students to connect knowledge from various courses and apply it on a particular task. Unlike the first exercise, here direct instruction to sketch the object was given. We used the same evaluation technology.

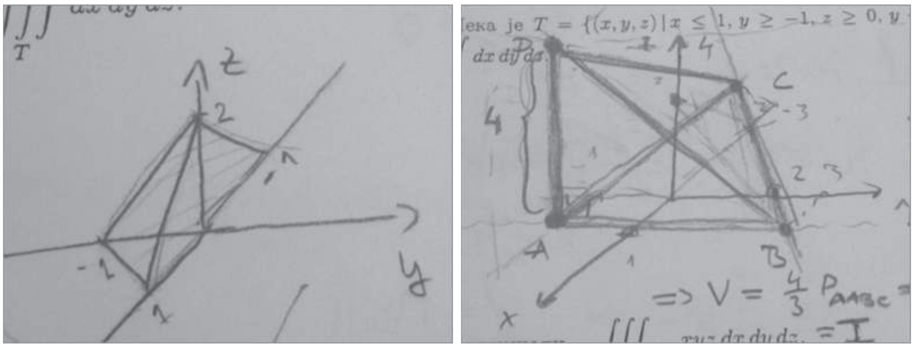

Figure 2. Example of sketch with partial (left) and full credit (right) in Exercise 2

Sample design

Our target populations were students of the second year of Faculty of Mathematics. We have randomly selected 103 students who took the course of Mathematical Analysis. Gender parity is presented in the Figure 3.

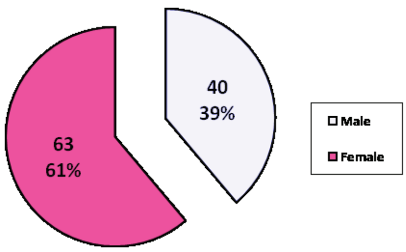

Figure 3. Gender parity

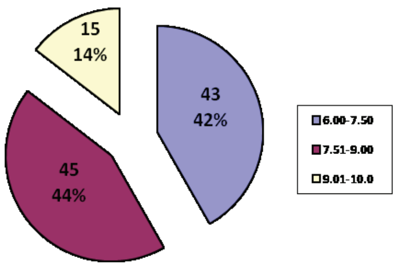

Figure 4. Distribution by GPA

5. Research conclusions

Gender differences in the drawing of 3D objects are often studied. According to [17], boys apperceive and conceptualize the shapes in a larger spatial context, while girls use an approach that focuses on small details. Also, in [18] was concluded that boys appeared to solve the tasks using a Euclidean space concept, while girls may have known the concept, but did not apply it in the drawing tasks. From our results it is obvious that, even though the male students have slightly better spatial abilities, the difference between genders has no statistical significance. For example, among the students with the full credit on their sketches in Exercise 2, the percentage of male students is almost six times higher than percentage of female students (see Table 1), yet this difference is not statistically significant, with significance level 0.1.

Table 1. Evaluation of exercises by gender

		No credits	Partial credit	Full credit	No sketch	Number of students
		Exercise 1				
Gender	Male	45.0%	7.5%	27.5%	20.0%	40
	Female	46.0%	3.2%	31.7%	19.0%	63
Total		45.6%	4.9%	30.1%	19.4%	103
		Exercise 2				
Gender	Male	47.5%	10.0%	27.5%	15.0%	40
	Female	65.1%	14.3%	4.8%	15.9%	63
Total		58.3%	12.6%	13.6%	15.5%	103

On the other hand, strong correlation between spatial abilities and achievement on the test was observed. Figure 5 represents the distribution of students by points obtained on the test. Those points serve us as a measurement of level of achievements in Mathematical Analysis. Notice that the points are slightly lower than expected - the distribution of points is rather left-skewed than Gaussian. It can be explained by the fact that the test was given in the beginning of the academic year and students are traditionally the most active in the last, not the first exam period (see results on the level of University of Belgrade - 1st year of Bologna process [34], [35], [36]).

Students with higher level of achievements in Mathematical Analysis are more likely to use sketches in solving analytical problems with geometrical background. Results of Exercise 1 are presented in Table 2.

Cells in the table are colored blue or red if the corresponding value is considerably above or below the average. We used deviations significant on the level 0.1. For example, this means that, among all students not using sketches in the

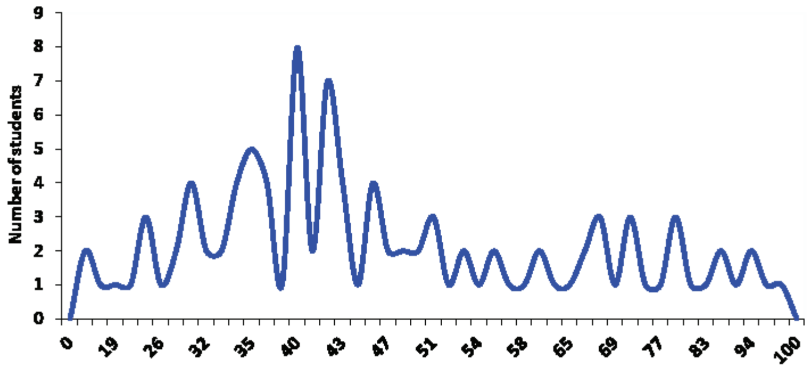

Figure 5. Distribution of students by points on the test

Table 2. Evaluation of Exercise 1 by number of points obtained on test

		No credits	Partial credit	Full credit	No sketch	Number of students
Points	0–40	46.3%	2.4%	14.6%	36.6%	41
obtained	41–60	53.8%	7.7%	25.6%	12.8%	39
on test	61–100	30.4%	4.3%	65.2%	0.0%	23
Total		45.6%	4.9%	30.1%	19.4%	103

Exercise 1, number of students with the lowest scores on the test is significantly higher than the average (with level of significance 0.1).

Students with high skill level also have high spatial ability. Correlation between achievement on the test and evaluation of sketch (where sketch is presented) is significant at level 0.01 (using 2-tailed test of significance). In Table 3, we presented results of Exercise 2 on slightly smaller target population - we considered only those students who tried to sketch required solid.

Having in mind that a result on just one test does not reflect on a students' general level of mathematical skills, we have also tested correlation between spatial abilities and overall mathematical knowledge represented by the current GPA, where 6.0 is the lowest possible score, while 10.0 is the highest one. Distribution of students by their GPA is presented in the Figure 4. Analysis similar to the

Table 3. Evaluation of Exercise 2 by number of points obtained on test

		No credits	Partial credit	Full credit	Number of students with some kind of sketch
Points	0–40	82.4%	14.7%	2.9%	34
obtained	41–60	80.0%	10.0%	10.0%	30
on test	61–100	34.8%	21.7%	43.5%	23
Total		69.0%	14.9%	16.1%	87

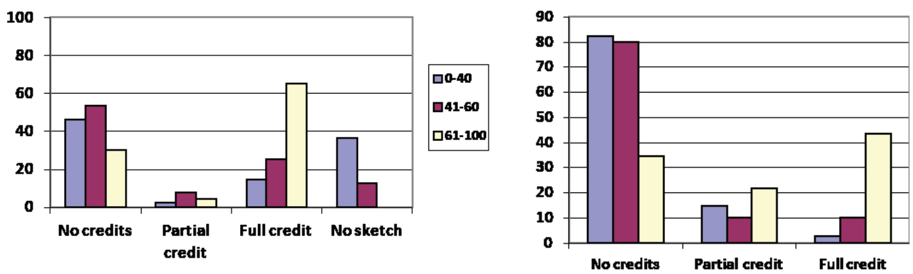

Figure 6. Evaluation of Exercise 1 (left) and Exercise 2 (right) by test achievement

previous is performed according to students' current GPA. Results are presented in Tables 4 and 5. As expected, the students with higher GPA were more inclined to use sketches as a tool to solve the presented tasks. Also, the quality of the sketches was better.

Table 4. Evaluation of Exercise 1 by GPA

		No credits	Partial credit	Full credit	No credit	Number of students
GPA	6.00–7.50	51.2%	4.7%	16.3%	27.9%	43
	7.51–9.00	44.4%	6.7%	33.3%	15.6%	45
	9.01–10.0	33.3%	0.0%	60.0%	6.7%	15
Total		45.6%	4.9%	30.1%	19.4%	103

Table 5. Evaluation of Exercise 2 by GPA

		No credits	Partial credit	Full credit	Number of students with some kind of sketch
GPA	6.00–7.50	81.8%	12.1%	6.1%	33
	7.51–9.00	69.2%	15.4%	15.4%	39
	9.01–10.0	40.0%	20.0%	40.0%	15
Total		69.0%	14.9%	16.1%	87

According to [29], boys are inclined to perform better when it comes to Mathematical concepts. Our research confirms this hypothesis, since among the students with the highest GPA there is statistically significant (at level of significance 0.05) gender difference in favor of boys (see Table 6). This may be one of the main reasons for minor advantage in spatial abilities in favor of male students.

Table 6. Gender parity according to the GPA

		Total	Male	Female	Number of students
GPA	6.00–7.50	41.7%	32.5%	47.6%	43
	7.51–9.00	43.7%	37.5%	47.6%	45
	9.01–10.0	14.6%	30.0%	4.8%	15

Having in mind the previous discussion, we can conclude that students with better spatial abilities are more predisposed to connect knowledge achieved in various courses and they perform better resulting the higher GPA and better skills performance in general.

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