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ORIGINAL RESEARCH PAPER

An application of filtered back projection method for computed tomography images

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ABSTRACT

Mathematical methods take an important part in reconstruction technologies of radiographic image. Back projection, iterative, and analytical (Two-dimensional Fourier, Filtered Back Projection) methods are the most important procedures for image reconstruction. Whenever there exists numerous projections, analytical methods have a great performance in speed and accuracy and due to these advantages they are comprehensively used for X-ray imaging. One of the widespread used methods in tomographic image reconstruction is Filtered Back Projection (FBP) algorithm. This paper presents an application of this reconstruction algorithm for a generated image of the object. Shepp-Logan filter is used to form the filtered back projection image and performance improvement is investigated. The obtained images indicate that FBP algorithm can be substantial for various applications in the field of medicine and industry.

KEYWORDS

reconstruction, computed tomography (CT) imaging, filtered back projection algorithm (FBP), image enhancement

1. INTRODUCTION

Computed tomography (CT) is a noninvasive medical procedure and it produces cross-sectional images of the body by using specialized X-ray equipment. Each cross-sectional image representing a “slice” of the imaged person is used for various therapeutic and diagnostic purposes. EMI Ltd. presented the first computerized commercial tomography of X-rays in medical applications, and a great impact was achieved on radiological diagnosis by means of it [1, 2]. Thereafter, another mathematical technique for image reconstruction by means of projections was presented and tomography was developed [3]. Based on the experiments performed, a coefficient matrix related to the slice sections was proposed by measuring X-rays transmission widthwise an object at various angles and acquiring images of cross sections [4]. Thus, CT is accepted as an efficient tool to examine the interior features of the object and it is important to obtain high quality images [5]. Image reconstruction has major effects on image quality and therefore on radiation dose. For a given radiation dose, it is preferable to reconstruct minimal noisy images without disturbing image accuracy and spatial resolution.

Reconstructions that enhance image quality can reduce radiation dose, since lower dose can be used to reconstruct the same quality images. In addition, if a CT scan is indicated clinically, the techniques that reduce dose are necessary. There have been various dose reducing techniques such as tube current modulation [6], organ-specific care [7], beam-shaping filters [8] and CT parameters optimizations [9]. The combination of some CT parameters such as thickness of slice, reconstruction filters, rotation numbers, etc. affects the quality of the image even if the delivered radiation dose to the patients is the same. Therefore, algebraic reconstruction technique (ART) which is an iterative method was preferred for the reconstruction of CT images [10].

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However, all the mentioned methods for image reconstruction present in the literature have some inadequacies, for example iteration methods suffer from poor computation speed and convergence accuracy in presence of noise [11]. Fourier techniques are direct but require two-dimensional inverse transform, interpolations and coordinate transformations [12]. Because of these deficiencies, an alternative method with computational advantages for image reconstruction has been sought and back projection and filtered back projection (FBP) algorithms have been proposed [13].

It is important to investigate the reconstruction of CT images, because CT imaging is widely used in various areas for treatment and diagnostics due to its ability to image the anatomical details. Also, advantages such as high spatial resolution and slice selection in any direction make CT imaging preferable. In addition, as it can be concluded from the present researches, high diagnostic quality CT images can be achieved by using FBP in most circumstances.

Although there have been some researches which claim that the quality of the CT scan images reconstructed by using FBP technique deteriorates with the increasing obesity rates [14], it has still major benefit with the shorter reconstruction duration. In other respects, although some other studies present that radiation dose can be reduced further with some other approches compared to FBP [15], they still fail to detect low contrast lesions [16]. Therefore, in this paper, reconstruction implementation is performed on a generated image by using FBP algorithm and the enhancement for the image quality is investigated by using a Shepp-Logan filter to filter the projections. There have been various studies in the literature and a comparison of these works is summarized in Table 1.

For instance, Qi and Chen have proposed a FBP algorithm to reconstruct the DPC-CT images. They conducted various simulations and verified the accuracy of the proposed reconstruction algorithm [17]. Al Hussani and Al Hayani applied FBP on CT scanner and tried to reconstruct head phantom image. They saw that FBP algorithm is an efficient and useful method for reconstructing CT images. They also decided that the back projection calculations can be reduced by using different filter types [18]. Willeming et al. evaluated the reconstructed image quality of CT images for FBP and Adaptive Statistical Iterative Reconstruction algorithms and used Shepp-Logan filter for FBP algorithm [19]. Sauter et al. compared the pulmonary embolism detectability of FBP and Iterative Model Reconstruction algorithms and used Shepp-Logan filter for FBP algorithm [20]. Ramadhan et al. proposed a comparative study between FBP algorithm and Algebraic Reconstruction Technique to detect the cancers or tumors from artificial tissue models. They showed that FBP algorithm provides better results than Algebraic Reconstruction Technique due to the noise calculation in Algebraic Reconstruction Technique. In recent studies, artificial intelligence has gained interest in the reconstruction of medical images. Willemink and Noël proposed a review of CT image reconstruction for FBP algorithm and artificial intelligence methods. They claim although huge data sets are necessary for artificial intelligence methods, they will play a major role in medical image reconstruction processes in the near future.

The difference of this study from related studies in the literature is that synthetic or real data is not used in this study. In addition, the original input image is produced in MATLAB 2018b program and the projections of the object are used to reconstruct it by using the FBP algorithm. The reason of using Shepp-Logan filter in the filtering process is that it demonstrates superior performance for both synthetic and real images in the research presented by Prabhat et al. in 2012 [23]. In addition, Hann filtering, which is one of the other preferred filter types, is also used to compare the performance of Shepp-Logan filter.

In Section 2, the detailed information about FBP algorithm and Shepp-Logan filter is reviewed. Section 3 is devoted to the framework of the reconstruction model and simulation results. The generated test image, reconstruction

Table 1. Relevant studies dealing with FBP for the types of different images

Authors	Purpose of the study	Used method	Image type
Qi and Chen (2008) [17]	To investigate the reconstruction performance for differential phase-contrast computed tomography	FBP algorithm	Differential phase-contrast computed tomography (DPC-CT)
Al Hussani and Al Hayani (2014) [18]	To evaluate the performance of Ram Lak filter and Hann window for the reconstruction of a head phantom	FBP algorithm	CT head phantom
Willeming et al. (2014) [19]	To compare the image quality of reconstructed CT images	FBP algorithm, Adaptive Statistical Iterative Reconstruction	CT images
Sauter et al. (2016) [20]	To compare the performances on detecting pulmonary embolism in CT pulmonary angiography	FBP algorithm, Iterative Model Reconstruction	CT pulmonary angiography (CTPA)
Ramadhan et al. (2018) [21]	To compare the performance on detecting cancers or tumors	FBP algorithm, Algebraic Reconstruction Technique (ART)	Artificial tissue models
Willemink and Noël (2019) [22]	To evaluate the CT image reconstruction	FBP algorithm, Artificial intelligence methods	CT images

process, spatial impulse response and transfer functions of the used Shepp-Logan and Hann filters and the back projections and filtered back projections of the object are presented. Finally, in Section 4, the concluding remark is included.

2. BACKGROUND

2.1. FBP algorithm

If $f(x,y)$ and $p(t,\theta)$ are considered as the image slice and projections set, respectively, for the image reconstruction $f(x,y)$ is estimated from $p(t,\theta)$. In CT, in order to obtain image reconstruction, several algorithms have been used. FBP algorithm is most widely used for the reconstruction of an object from its X-ray projections [24]. Since each projection is considered only once, FBP provides fast computing.

If a single projection is handled, the two-dimensional Fourier transform of this object is obtained by means of Central Slice Theorem in this projection. In accordance with this theorem, the one-dimensional Fourier transform of the projection of a two-dimensional function $f(x,y)$ is equal to a line through the origin of the two-dimensional Fourier transform of the same function [25]. Thus, a simple reconstruction can be performed if the proper places for Fourier transform values of this projection are selected in this process, the other projections are assumed to be zero.

By using two dimensional inverse Fourier transform, the object function is defined in Eq. (1) [26],

$$f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u, v)e^{j2\pi(ux+vy)}dudv, \tag{1}$$

where u and v variables represent rectangular coordinate system. If rectangular coordinate system in Eq. (1) is replaced with polar coordinate system, new coordinate system and differentials can be defined as in Eq. (2) [26],

$$u = wcos\theta, \quad v = wsin\theta, \quad dudv = wdwd\theta. \tag{2}$$

By using Eqs (1) and (2), the inverse Fourier transform can be defined as,

$$f(x, y) = \int_0^{2\pi}\int_0^{\infty} F(w, \theta)e^{j2\pi w(xcos\theta+ysin\theta)}wdwd\theta + \int_0^{\pi}\int_0^{\infty} F(w, \theta+180^{\circ})e^{j2\pi w[xcos(\theta+180^{\circ})+ysin(\theta+180^{\circ})]}wdwd\theta \tag{3}$$

and with symmetry property $F(w, \theta+180°) = F(-w, \theta)$, Eq. (3) can be simplified as given in Eq. (4),

$$f(x, y) = \int_0^{\pi}\left[\int_{-\infty}^{\infty} F(w, \theta)|w|e^{j2\pi wt}dw\right]d\theta, \tag{4}$$

where $t = x\cos\theta + y\sin\theta$. In order to provide convenience, the Fourier transform of the projection obtained at θ angle can be given by,

$$f(x, y) = \int_0^{\pi}\left[\int_{-\infty}^{\infty} S_\theta|w|e^{j2\pi wt}dw\right]d\theta \tag{5}$$

Equation (5) includes a filtering process where $|w|$ term represents the frequency response of the filter. Therefore, this process is called as filtered projection. Thus, Eq. (5) can be considered in the form as,

$$f(x, y) = \int_0^{\pi} Q_\theta(xcos\theta + ysin\theta)d\theta, \tag{6}$$

where

$$Q_\theta(t) = \int_{-\infty}^{\infty} S_\theta(w)|w|e^{j2\pi wt}dw. \tag{7}$$

By regarding Eq. (6), in order to estimate f(x,y) for the different angles projections in the discrete form,

$$f(x, y) = \frac{\pi}{N}\sum_{i=1}^{n} Q_{\theta_i}(xcos\theta_i + ysin\theta_i) \tag{8}$$

can be used where N is the number of θ_i angles for the projections. By taking into consideration the equations given in (1)–(8), The FBP algorithm can be summarized as given in the following:

- Acquire projections from object.
- Take one dimensional Fourier transform and apply $|w|$ filter.
- Find inverse one dimensional Fourier transform.
- Back Project result.

By examining Eqs (1)–(8) and the summary of the FBP algorithm, it can be seen that only one dimensional Fourier transform is used and therefore less computations are needed.

There are several frequency domain filters which are commonly used for filtering process in the FBP algorithm [27–29]. Even though there are a number of filters, it is an important issue to select the optimal filter for a robust image reconstruction. Some of the filters which are used are;

- **Ramanathan-Lakshminarayan (Ram-Lak)**: It is a ramp filter that is sensitive to noise.
- **Shepp-Logan**: It is obtained by the multiplication of the Ram-Lak filter and a "sinc" function.
- **Cosine:** It is obtained by the multiplication of the Ram-Lak filter and a cosine function.
- **Hamming:** It is obtained by the multiplication of the Ram-Lak filter and a Hamming window.
- **Hann:** It is obtained by the multiplication of the Ram-Lak filter and a Hann window.

In this paper, Shepp-Logan filter is proposed to realize the filtering process given in Eq. (7). In addition, Hann filter, which is another filtering frequently preferred in the literature due to its superiority, is used to provide a performance comparison [30].

2.2. Shepp-Logan filter

Back projection of $f(x,y)$ at θ angles is the same as generating a line over the origin of the two dimensional Fourier space. Back projection causes higher density in the regions close to the origin, lower density farther than the origin, correspondingly higher spatial frequencies are suppressed by means of $1/|w|$. Thus, a low-pass filtered version of the image is formed. However, a $|w|$ filter can improve it. For this purpose, Shepp-Logan filter, which is obtained by the multiplication of the Ram-Lak filter and sinc function, can be used [31].

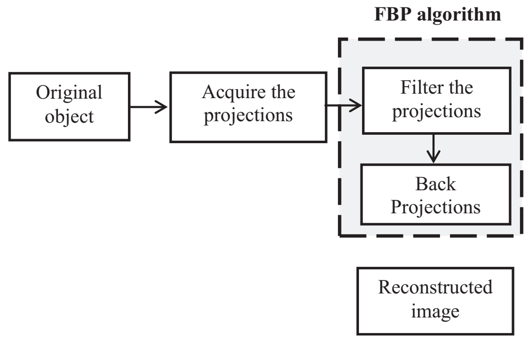


Fig. 2. The reconstruction process

3. MODEL AND SIMULATION

In this paper, the projections of the object are used to reconstruct it by using the FBP algorithm. A blurred reconstructed object is formed by using simple back projection. To eliminate this blur, a Shepp-Logan filter is used to filter the projections. The MATLAB 2018b simulation program is preferred for the programming of the proposed process. Test image is generated in MATLAB 2018b as shown in Fig. 1. Thereafter original input image is generated, FBP algorithm is applied with the steps given in Section 2.1. Figure 2 demonstrates the reconstruction process block diagram.

However, it can be considered that there are some assumptions such as size of the image, projections number, theta angle increment that effect the robustness of the algorithm, consequently the image quality. These parameters are determined by trials on the simulation. Increments of the theta angle is decided to be 5° degrees in the interval [0° 180°]. Shepp-Logan and Hann filters are used for performing filtering processes. Each one of these projections will be weighted using these filters which prevent blurry images for back projection method [31]. The ramp filter will be multiplied by sinc function in Shepp-Logan filter. Figure 3a and b show the spatial impulse responses and the transfer functions of the used Shepp-Logan and Hanning filters, respectively.

By using Eq. (8), the reconstructed object can be obtained by means of filtered projections. The acquired data of the back projection and FBP algorithm for 37 projections of an input test image are shown in Fig. 4. Figure 4a, b and c show the obtained results of the back projection and filtered back projections of object for Shepp-Logan and Hann filters, respectively.

In the plot of Fig. 4, number of projections are chosen to provide the best quality for the image. Also, the step size and image size are determined with trials by taking into account the relationship of the performance with these parameters [32]. It can be seen in Fig. 4 that while the object is not very clear in back projection of the object, it is more distinct and less blurred in the filtered back projection one by using Shepp-Logan filter. In addition, it can be clearly seen from Fig. 4c that when back projection of object is filtered by using Hann filter, the object is no longer noticeable and more blurred and bright back projection is obtained with 37 projections.

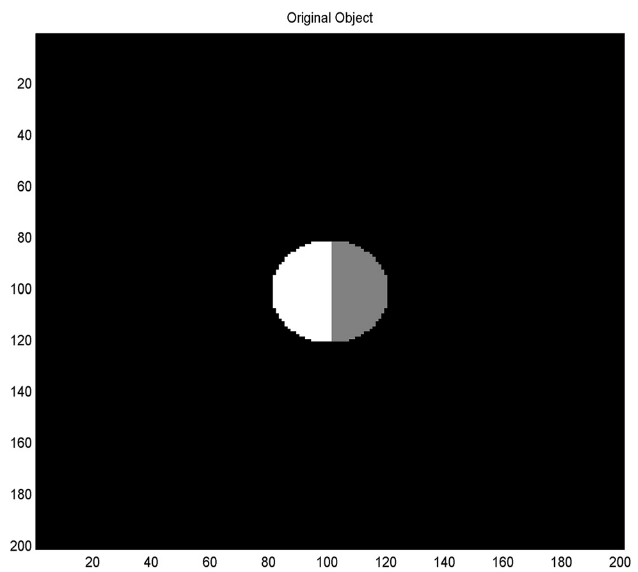
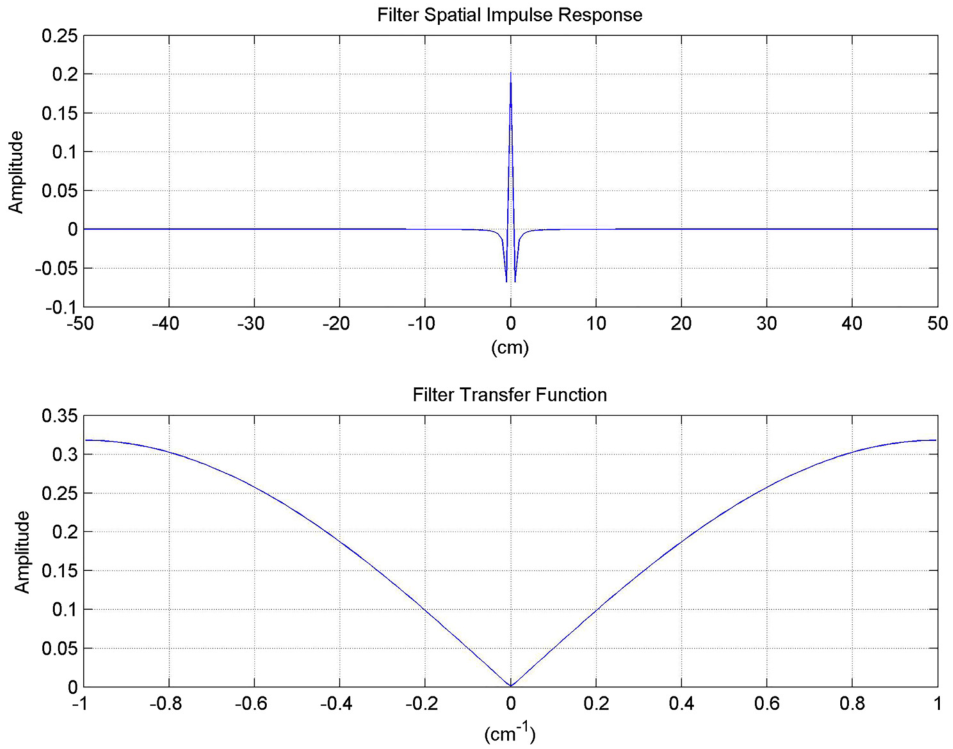


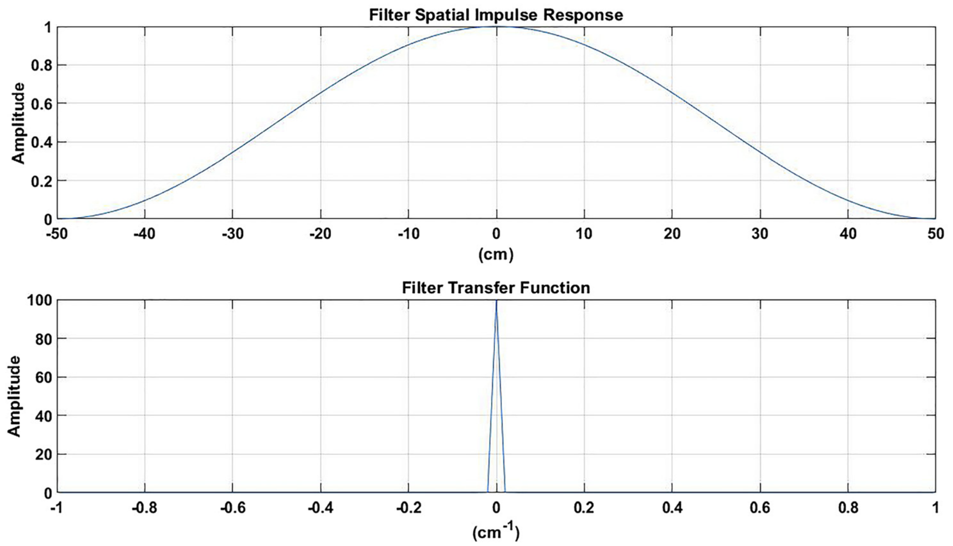
Fig. 1. Original input image

4. CONCLUSION

FBP algorithm can be considered as a useful and effective reconstruction method for CT images. In this paper, a test image is formed by using MATLAB simulation program and reconstruction performance is investigated, however, it will be the most important part of the real time scanning process specially in medical applications because the radiation time the patient is exposed should be as short as possible. For this purpose in this paper, the filtering performances of Shepp-Logan and Hann filters are compared. According to the obtained results, the usage of Shepp-Logan filter both reduces the back projection calculations and enhances the quality of the reconstructed image. In other respects, it provides less blurred image. Furthermore, the effect of the noise in the filtered back projection image given in Fig. 4b, can be decreased by using other types of windows before filtering the projections. It can be considered as a future issue by using different types of filters and windows together



a)



b)

Fig. 3. Spatial impulse response and transfer function of the used filters a) Shepp-Logan filter b) Hann filter

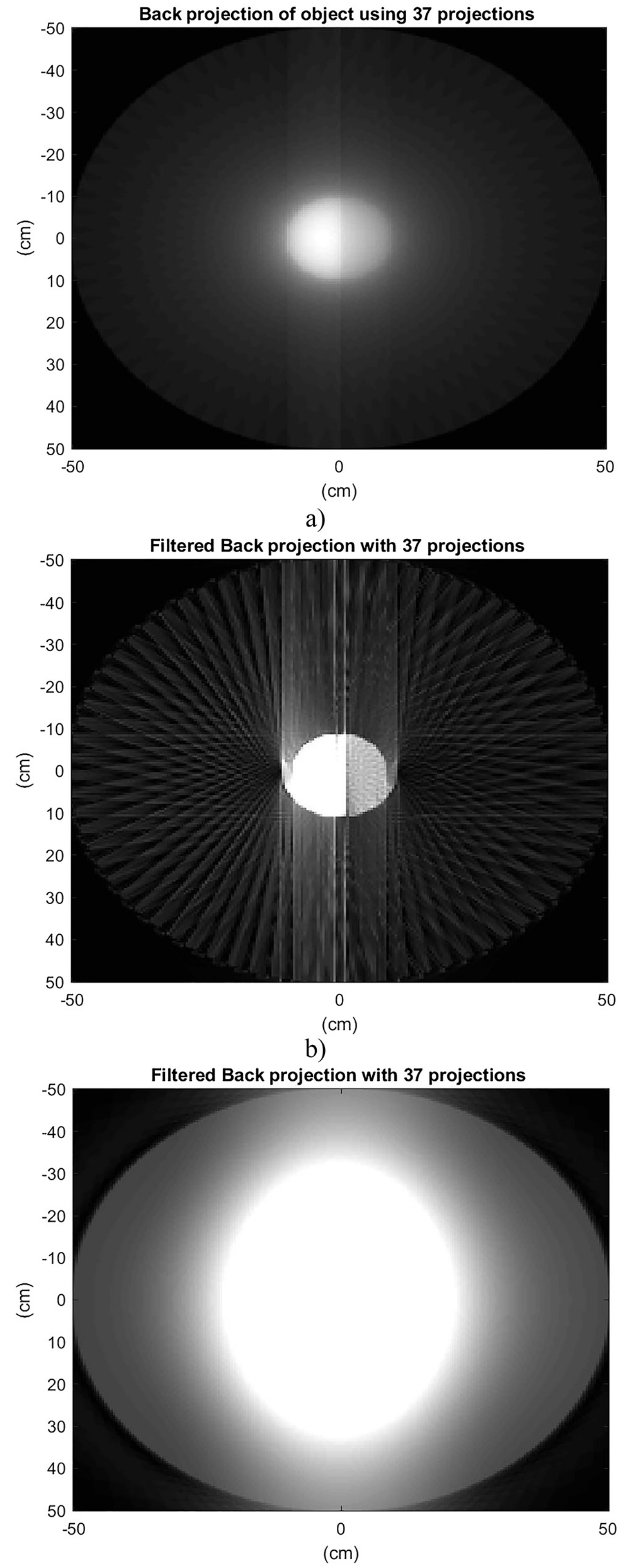


Fig. 4. Back projection and filtered back projections of object for 37 projections. a) Back projection b) Shepp-Logan filtered back projections c)Hann filtered back projections

in order to investigate the enhancement in the quality of the image.

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