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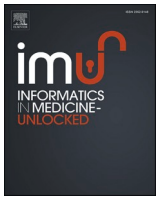

Optimization and control in rubella transmission dynamics: A boundedness-preserving numerical model with vaccination

Samiullah Salim [a], Fazal Dayan [a], Muhammad Aziz ur Rehman [a], Husam A. Neamah [b,c,d,*]

[a] Department of Mathematics, School of Science, University of Management and Technology, Lahore, 54000, Pakistan
[b] Department of Electrical Engineering and Mechatronics, Faculty of Engineering, University of Debrecen, Ótemetö Utca 2-4, 4028, Debrecen, Hungary
[c] Technical Engineering College, Al-Ayen University, Thi-Qar 64001, Iraq
[d] Department of Business Management, Al-imam University College, Balad, Iraq

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ABSTRACT

Rubella is an infectious disease that can spread globally. It spreads in the subtropics as well as the tropics. Although it is commonly thought to be a non-fatal condition, there are several circumstances in which it can be fatal. Pregnant women infected with the Rubella virus face a high risk of fetal development. The main goal of this study is to look into a model for the spread of Rubella while considering a vaccination campaign as a control measure. The positivity and boundedness of the re-infection rubella transmission and vaccination model are investigated. The local and global stability of the equilibrium points is examined. The sensitivity of parameters is also investigated. Three different techniques forward Euler, RK-4, and non-standard finite difference (NSFD) method are developed for the numerical solution, and their simulation results are examined. Among these three, the NSFD method is superior due to its convergence and positive behavior for all step size values. For some values of the step size, the other two techniques failed to produce positivity and convergent solutions. Analytically, the proposed model's convergence, positivity, boundedness, and consistency are investigated. Finally, the impact of the Rubella vaccine on infected populations has been examined, revealing that vaccination is one of the most effective ways to prevent rubella transmission.

1. Introduction

German measles, also known as rubella, is an infectious viral infection caused by the rubella virus. When an infected person sneezes or coughs, the virus is typically spread through airborne droplets. The disease can also be passed from a pregnant woman to her unborn child, which can result in serious birth defects and complications. A mild fever, rash, and swollen lymph nodes are common Rubella symptoms. The symptoms are often minor and resolve within a week or two. The virus poses a considerable risk to pregnant women because it can cause congenital rubella syndrome (CRS) in their unborn children. CRS can cause various issues, including hearing and vision difficulties, heart defects, and developmental delays. Immunization is critical in preventing rubella because it protects not just the individual but also fosters herd immunity, which protects vulnerable people such as infants and those with impaired immune systems. Rubella symptoms include a low fever, sore throat, headache, and a characteristic rash that spreads from the face to the body, usually lasting a few days. Additional symptoms include swollen lymph nodes, joint aches, and red eyes. Rubella normally cures spontaneously and without consequences, but it has a significant impact on pregnant women because the virus can cross the placenta and impair fetal development, leading to CRS. Rubella has no specific therapy and usually goes away on its own within a short period. Treatment is generally aimed at reducing symptoms like temperature and discomfort. Rubella control is primarily based on prevention. Rubella is highly preventable with the MMR vaccine, which all children should receive as part of their usual vaccination plan. Adults who have not been vaccinated or have never had the disease should consider getting vaccinated, especially if they intend to visit locations where Rubella is common [1–4]. The virus that causes the sickness flourishes in both tropical and subtropical climates. Rubella has the potential to be severely damaging to a developing fetus when pregnant. The primary modes of transmission for postnatal rubella are direct contact with contaminated nasopharyngeal secretions and inhalation of virus-carrying airborne droplets. In susceptible pregnant women, the virus can penetrate the placenta and propagate throughout the fetus's

* Corresponding author.
E-mail address: husam@eng.unideb.hu (H.A. Neamah).

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vascular system. Nonimmune pregnant women who have measles risk harming their fetuses and neonates. Prenatal immunization can help prevent the sickness. Vaccination is not advised during pregnancy due to the presence of live, weakened viruses, which could potentially pose risks to the fetus [5].

Gao et al. [6] examined the influence of rubella vaccination on the disease's epidemiology in Australia. Thompson conducted a systematic review of the literature, identifying quantitative models of measles and rubella transmission dynamics and highlighting crucial findings pertinent to future modeling endeavors [7]. Thompson and Badizadegan developed a dynamic disease model for the propagation of measles and rubella. They subsequently applied this model to 180 WHO member states and three additional regions, collectively representing over 99.5 % of the global population in 2013 [8]. The model took into account seasonality, age-specific mixing patterns, and the potential for preferential mixing among under-vaccinated subgroups, all factors that could contribute to disparities in vaccination coverage affecting disease transmission. Saito et al. applied this model to rubella notification data from 2012 to 2013, presenting a discrete-time and spatial epidemic model [9]. In a separate study, Alleman et al. conducted a rubella antibody seroprevalence assessment using serum samples collected in 2008–2009 from 1605 pregnant women aged 15–46 attending seven prenatal care facilities across three regions in the Democratic Republic of the Congo [10].

The rubella virus was widespread across all three regions, with most cases of viral exposure and seroconversion occurring before the age of 15. However, between 10 % and 20 % of the mothers were at risk of contracting the rubella virus and having a child with CRS. Wu et al. used an age-structured transmission model to compare the effectiveness of seven different vaccine approaches in lowering the CRS illness burden in East Java [11]. The model predictions were tested to see how resistant they were to changes in vaccine coverage and other important epidemiological parameters. Chen et al. [12] performed a phylogenetic study on rubella virus isolates and constructed a phylogenetic tree based on the World Health Organization standard sequencing window. Gerberry and Milner extend the SIR model to include individuals who have the disease but are unable to transmit it to others to analyze the dynamics of the latent class. For the model, both a homoclinic bifurcation and the existence of Hopf bifurcations are demonstrated [13]. Baleanu et al. used the Caputo-Fabrizio fractional derivative to investigate the rubella disease model and developed a three-step Adams-Bashforth scheme for the mathematical solution of the liver model. The existence and uniqueness of the solution were argued using fixed point theory [14]. Buonomo has explored an SEIR epidemic model with vertical transmission where the optimal control theory has been applied to assess the vaccination strategy on the model dynamics. The optimality system was derived and solved numerically [15]. Ahmad et al. came up with a deterministic SEITR model of Rubella by applying the Non-Standard Finite Difference (NSFD) method to achieve the numerical solution of the disease [16]. Xu et al. studied a Rubella model with a fractal-fractional exponential decay kernel [17]. Al Qurashi has researched the Rubella virus by fractal-fractional differential and integral operators [18]. Tilahun proposed both a deterministic and stochastic model to discuss Rubella dynamics by considering vertical transmission and environmental factors [19]. Rasit and Tuncer applied a measles model that tends to forecast measles in Turkey from 1970 to 2021 [20]. Bhavithra and Devi analyzed the dynamics of a measles pandemic with the help of a fuzzy SIR model [21]. Khan et al. studied the discrete-time phytoplankton-zooplankton model with some impacts of external toxicity in the phytoplankton population [22]. Abbas et al. have proposed a model representing the dynamics of methamphetamine and performed local and global stability and sensitivity analysis [23]. Alqahtani et al. presented deterministic and stochastic models for the study of the dynamics of MERS-CoV transmission and its epidemic potential [24]. Sheergojri et al. offered a fuzzy Gompertz growth model to reduce uncertainty in modeling tumor population dynamics [25]. Li et al. investigated the Sasa-Satsuma equation using ideas from planar dynamical theory and the beta differential operator. The bifurcation, equilibrium, and sensitivity of the system are also analyzed [26]. Xu et al. studied the plankton population dynamical system accompanying delay [27]. Xu et al. investigated fractional neural networks involving delays [28,29]. Baber et al. investigated the stochastic Chen–Lee–Liu equation numerically and analytically [30]. Xu et al. studied the dynamics and chaotic behavior of a fish farm model containing populations of nutrients and mussels with different kernels insight of fractal-fractional operator [31]. Li and Wu examined SCIR models for COVID-19 dynamics with and without immigration to derive conditions for stability and properties at the Hopf bifurcation [32]. Tong et al. discussed deterministic and stochastic models for rumor propagation that also incorporate media coverage and age-dependent education [33]. The dynamics for a reaction-diffusion SEIR epidemic model driven by a mass action infection mechanism in a heterogeneous environment are analyzed [34]. Researchers also investigated the local and global stabilities of the disease dynamical models [35–41], just to mention a few.

The NSFD theory was first proposed by Mickens [42]. It plays a crucial role in the mathematical and numerical modeling of diseases. It has proven to be one of the most valuable strategies in recent years [43]. One of the most important aspects is the complete absence of the fundamental numerical instabilities that plague traditional finite difference schemes. Numerous differential equations have been solved using the EFD and NSFD techniques [44–46] for example. Verma and Kayenat investigated the generalized Burgers-Huxley (GBH) equation using the NSFD approach. The maximum error of NSFD solutions is compared to the maximum error of several other methods to demonstrate the superiority of the proposed method. The CPU time for all computations is also compared, with the results demonstrating that the NSFD scheme produces accurate results in a matter of seconds, saving time [47]. Raza et al. explored a stochastic HIV/AIDS model within a bisexual community, employing NSFD theory, which incorporates counseling and antiviral medication. The findings suggest that the stochastic model proves to be more practical compared to the deterministic model [48]. The NSFD stochastic scheme retains all of the disease dynamical model's important attributes. Iqbal et al. investigated NSFD theory-based fractional order HIV/AIDS transmission [49]. Ahmed et al. [50] developed a new HIV CD4+T cell reaction-diffusion model. Allehiany et al. [51] applied the NSFD scheme to study a Covid-19 model incorporating fuzziness for its numerical resolution. Similarly, Alhebshi et al. [52] explored a computer virus model utilizing fuzzy criteria. The investigated models undergo equilibrium and reproduction analysis. Both the forward Euler method and NSFD schemes were utilized for numerical solutions. It was noted that while the NSFD scheme maintained stability, convergence, and positivity, the forward Euler method did not uphold these fundamental characteristics of epidemic models.

The primary objective of this study is to investigate a dependable implicit numerical integration method for modeling rubella disease. We contend that the tool for the proposed implicit numerical integration technique is trustworthy, efficient, and encompasses all the dynamic properties necessary for modeling long-term illness behavior. The current work is interesting since it is the first to study the dynamics of the rubella virus disease using NSFD conditions. This work additionally investigates the constructed scheme's convergence, positivity, boundedness, and consistency. Vaccination plays a crucial role in preventing rubella. Rubella vaccination, typically given as part of the MMR vaccine, is highly effective in preventing rubella infection. The vaccine contains a weakened live virus that stimulates the body's immune system to produce antibodies to fight the virus. By providing immunity to the virus, the rubella vaccine can prevent the spread of the disease, particularly to vulnerable populations, such as pregnant women, who can experience serious complications if they contract rubella during pregnancy. The effects of vaccination on infected populations are also studied in this work. The novelty of the paper consists of constructing, implementing,

Table 1
The description of the parameter.

Parameter	Description
α	Birth rate
μ	Rate of natural death
p	percentage of those at risk who have had vaccinations
q	The effectiveness level of vaccination
β	Rate of contact
δ	Rate of transformation from latent to infectious
γ	Recovery rate

and analyzing a first-order numerical scheme under the novel conditions of the NSFD senses. This approach is carefully framed to a specific model of the transmission dynamics of Rubella. To the best of our knowledge, this model has never been examined using NSFD conditions; hence, it constitutes a novel contribution to the literature.

2. Rubella virus transmission and vaccination model

We considered the rubella model [53].

$$\frac{dS}{dt} = \alpha N - pqS - (1-pq)\beta SI - \mu S, \tag{1}$$

$$\frac{dE}{dt} = (1-pq)\beta SI + \beta RI - \delta E - \mu E, \tag{2}$$

$$\frac{dI}{dt} = \delta E - \gamma I - \mu I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I - \beta RI - \mu R, \tag{4}$$

$$\frac{dV}{dt} = pqS - \mu V. \tag{5}$$

Here, the parameters S, E, I, R, V and N are used to denote susceptible, exposed, infectious, recovered, vaccinated and total populations respectively. The variables used above are described in Table 1 and Fig. 1 shows the flowchart of the above model.

2.1. Positivity and boundedness of the model

For the positivity, consider system (1–5) as follows.

$$\frac{dS}{dt}|_{s=0} = \alpha N \geq 0$$

$$\frac{dE}{dt}|_{E=0} = (1-pq)\beta SI + \beta RI \geq 0$$

$$\frac{dI}{dt}|_{I=0} = \delta E \geq 0$$

$$\frac{dR}{dt}|_{R=0} = \gamma I \geq 0$$

$$\frac{dV}{dt}|_{V=0} = pqS \geq 0$$

as desired.

For boundedness, consider the population function as follows at any time t.

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} + \frac{dV}{dt}$$

$$\frac{dN}{dt} = \alpha N - \mu N$$

$$\frac{dN}{dt} = N(\alpha - \mu)$$

$$\frac{dN}{dt} \leq N(\alpha - \mu)$$

Since, $S(t) + E(t) + I(t) + R(t), V(t) \leq N$.

$N(t) \leq N(0)e^{(\alpha-\mu)t}, t \geq 0$

as desired.

2.2. Stability analysis

The model has a DFE point $p_0 = \left(\frac{\alpha}{pq+\mu}, 0, 0, 0, \frac{pq\alpha}{(pq+\mu)\mu}\right)$ and an EE point $p(s^*, e^*, i^*, r^*, v^*)$ as an inherent balance. A second equilibrium has no simple form, so it is only stated by $(s^*, e^*, i^*, r^*, v^*)$. As a result, the analysis was limited to an equilibrium that was devoid of sickness. Jacobian of system (1–5) is

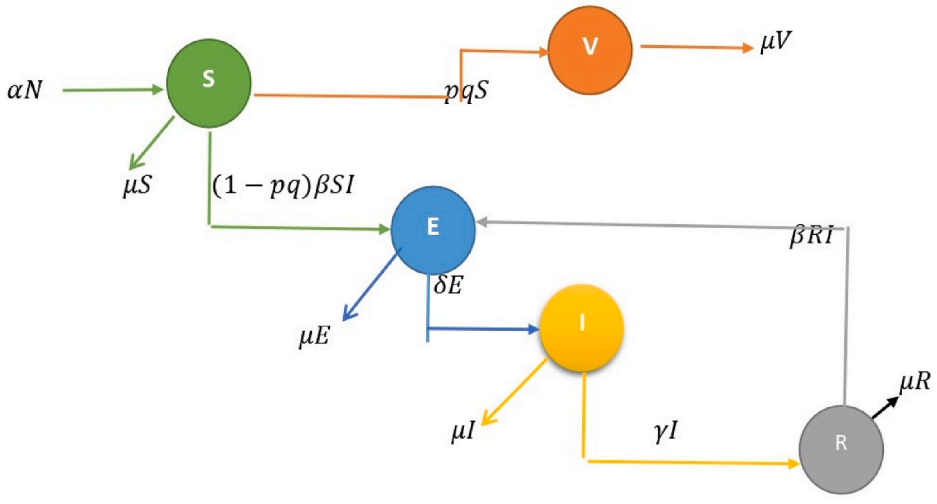

Fig. 1. Flowchart of the model.

$$J=\begin{bmatrix} -pq-(1-pq)\beta I-\mu & 0 & -(1-pq)\beta S & 0 & 0 \\ (1-pq)\beta I & -\delta-\mu & (1-pq)\beta S+\beta R & \beta I & 0 \\ 0 & \delta & -\gamma-\mu & 0 & 0 \\ 0 & 0 & \gamma-\beta R & -\mu & 0 \\ pq & 0 & 0 & 0 & -\mu \end{bmatrix}$$

and

$$J(p_0)=\begin{bmatrix} -pq-\mu & 0 & \frac{-\alpha\beta(1-pq)}{pq+\mu} & 0 & 0 \\ 0 & -\delta-\mu & \frac{\alpha\beta(1-pq)}{pq+\mu} & 0 & 0 \\ 0 & \delta & -\gamma-\mu & 0 & 0 \\ 0 & 0 & \gamma & -\mu & 0 \\ pq & 0 & 0 & 0 & -\mu \end{bmatrix}$$

The eigenvalues of the above matrix are $\lambda_1=\lambda_2=-\mu$, $\lambda_3=-\mu-pq$, $\lambda_{4,5}=-\frac{1}{2(pq+\mu)}\left(A\pm\sqrt{B}\right)$, where $A=(pq+\mu)(\gamma+\delta+2\mu)$ and $B=(pq+\mu)^2(\delta-\gamma)^2+(pq+\mu)(1-pq)4\alpha\beta\delta$ [53]. Since all eigenvalues are less than one, it indicates that system (1–5) is locally stable at the DEF point p_0.

Theorem 1. *The system at DFE* $\left(S^1,E^1,I^1,R^1,V^1\right)=\left(\frac{\alpha}{pq+\mu},0,0,0,\frac{pq\alpha}{(pq+\mu)\mu}\right)$ *is globally asymptotically stable if* $R_0<1$.

Proof. *Consider the Lyapunov function,* $U:\Omega\longrightarrow R$ *as follows*:

$$U=\left(S-S^1-S^1\ln\frac{S}{S^1}\right)+\left(E-E^1-E^1\ln\frac{E}{E^1}\right)+\left(I-I^1-I^1\ln\frac{I}{I^1}\right)+\left(R-R^1-R^1\ln\frac{R}{R^1}\right)+\left(V-V^1-V^1\ln\frac{V}{V^1}\right)$$

$$\frac{dU}{dt}=\left(1-\frac{S^1}{S}\right)\frac{dS}{dt}+\frac{dE}{dt}+\frac{dI}{dt}+\frac{dR}{dt}+\left(1-\frac{V^1}{V}\right)\frac{dV}{dt}$$

$$\frac{dU}{dt}=\left(1-\frac{S^1}{S}\right)(\alpha-pqS-(1-pq)\beta SI-\mu S)+(1-pq)\beta SI+\beta RI-\delta E-\mu E+\delta E-\gamma I-\mu I+\gamma I-\beta RI-\mu R+\left(1-\frac{V^1}{V}\right)(pqS-\mu V)$$

$$\frac{dU}{dt}=-\frac{\alpha\left(S-S^1\right)^2}{SS^1}-\frac{pqS\left(V-V^1\right)^2}{VV^1}$$

$$\frac{dU}{dt}=-\left(\frac{\alpha\left(S-S^1\right)^2}{SS^1}+\frac{pqS\left(V-V^1\right)^2}{VV^1}\right)<1$$

From the above calculation, it can be observed that $\frac{dU}{dt}<0$, if $R_0<1$ and $\frac{dU}{dt}=0$ if $S=S^1,E=E^1,I=I^1,R=R^1,V=V^1$. As a result, the system is globally asymptotically stable at C_1.

2.3. Sensitivity analysis

The basic reproduction number of the studied model [53] is

$$R_0=\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}$$

The sensitivity of a parameter χ can be calculated as [54]

$$\xi(\chi)=\frac{\chi}{R_0}\frac{\partial R_0}{\partial\chi}$$

To find the sensitivity of the parameter β we have,

$$\xi(\beta)=\frac{\beta}{R_0}\frac{\partial R_0}{\partial\beta}=\frac{\beta}{\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}\frac{\partial\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}{\partial\beta}=1$$

$$\xi(\alpha)=\frac{\alpha}{\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}\frac{\partial\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}{\partial\alpha}=1$$

$$\xi(\delta)=\frac{\delta}{\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}\frac{\partial\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}{\partial\delta}=1$$

$$\xi(p)=\frac{p}{\frac{\alpha\beta\delta-pq\alpha\beta\delta}{pq\mu^2+pq\mu\gamma+\mu^3+\mu^2\gamma+pq\mu\delta+pq\gamma\delta+\mu^2\delta+\mu\delta\gamma}}\frac{\partial\left(\frac{\alpha\beta\delta-pq\alpha\beta\delta}{pq\mu^2+pq\mu\gamma+\mu^3+\mu^2\gamma+pq\mu\delta+pq\gamma\delta+\mu^2\delta+\mu\delta\gamma}\right)}{\partial p}$$

$$=\frac{p(-\alpha\beta\delta q)((pq+\mu)(\mu+\delta)(\mu+\gamma)-((q\mu^2+q\mu\gamma+q\mu\delta+q\gamma\delta)(\alpha\beta\delta-\alpha\beta\delta pq))}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}$$

$$\xi(q)=\frac{q}{\frac{\alpha\beta\delta-pq\alpha\beta\delta}{pq\mu^2+pq\mu\gamma+\mu^3+\mu^2\gamma+pq\mu\delta+pq\gamma\delta+\mu^2\delta+\mu\delta\gamma}}.\frac{\partial\left(\frac{\alpha\beta\delta-pq\alpha\beta\delta}{pq\mu^2+pq\mu\gamma+\mu^3+\mu^2\gamma+pq\mu\delta+pq\gamma\delta+\mu^2\delta+\mu\delta\gamma}\right)}{\partial q}$$

$$=\frac{p(-\alpha\beta\delta p)((pq+\mu)(\mu+\delta)(\mu+\gamma)-((p\mu^2+p\mu\gamma+p\mu\delta+p\gamma\delta)(\alpha\beta\delta-\alpha\beta\delta pq))}{(pq+\mu)(\mu+\delta)(\mu+\gamma)(1-pq)}$$

$$\xi(\mu)=\frac{\mu}{\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}\frac{\partial\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}{\partial\mu}=\frac{-\mu((2pq\mu+pq\gamma+3\mu^2+2\mu\gamma+pq\delta+2\mu\delta+\gamma\delta)(\alpha\beta\delta-\alpha\beta\delta pq))}{(pq+\mu)(\mu+\delta)(\mu+\gamma)(1-pq)(\alpha\beta\delta)}$$

$$\xi(\gamma)=\frac{\gamma}{\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}\frac{\partial\frac{(1-pq)\alpha\beta\delta}{(pq+\mu)(\mu+\delta)(\mu+\gamma)}}{\partial\gamma}=\frac{-\gamma((pq\mu+\mu^2+pq\delta+\mu\delta)(\alpha\beta\delta-\alpha\beta\delta pq))}{(pq+\mu)(\mu+\delta)(\mu+\gamma)(1-pq)(\alpha\beta\delta)}$$

The calculated findings indicate that β, α and δ are sensitive parameters. Increasing sensitive parameters increases R_0, and vice versa. On the other side, the parameters p, μ and γ are insensitive and any increase or decrease in these variables do not affect the value of R_0.

3. Numerical modelling

In this section, we will develop three different numerical schemes for the solution of the studied model.

3.1. Forward Euler method

Considering the system of equations (1)–(5), we develop a forward Euler scheme. Constructing a numerical scheme for the forward Euler involves making the following supposition.

$S(t) \approx S_n, E(t) \approx E_n, I(t) \approx I_n, V(t) \approx V_n$ and $R(t) \approx I_n$.

$$S^{n+1} = S^n + h(\alpha - pqS^n - (1-pq)\beta S^n I^n - \mu S^n) \tag{6}$$

$$E^{n+1} = E^n + h((1-pq)\beta S^n I^n + \beta R^n I^n - \delta E^n - \mu E^n) \tag{7}$$

$$I^{n+1} = I^n + h(\delta E^n - \gamma I^n - \mu I^n) \tag{8}$$

$$R^{n+1} = R^n + h(\gamma I^n - \beta R^n I^n - \mu R^n) \tag{9}$$

$$V^{n+1} = V^n + h(pqS^n - \mu V^n) \tag{10}$$

3.2. Runge- Kutta scheme of Fourth order (R-K4 method)

3.2.1. Step 1

$$k_1 = h(\alpha - pqS^n - (1-pq)\beta S^n I^n - \mu S^n)$$

$$l_1 = h((1-pq)\beta S^n I^n + \beta R^n I^n - \delta E^n - \mu E^n)$$

$$m_1 = h(\delta E^n - \gamma I^n - \mu I^n)$$

$$n_1 = h(\gamma I^n - \beta R^n I^n - \mu R^n)$$

$$o_1 = h(pqS^n - \mu V^n)$$

3.2.2. Step 2

$$k_2 = h\left(\alpha - pq\left(S^n + \frac{k_1}{2}\right) - (1-pq)\beta\left(S^n + \frac{k_1}{2}\right)\left(I^n + \frac{m_1}{2}\right) - \mu\left(S^n + \frac{k_1}{2}\right)\right.$$

$$l_2 = h\left((1-pq)\beta\left(S^n + \frac{k_1}{2}\right)\left(I^n + \frac{m_1}{2}\right) + \beta\left(R^n + \frac{n_1}{2}\right)\left(I^n + \frac{m_1}{2}\right) - \delta\left(E^n + \frac{l_1}{2}\right) - \mu\left(E^n + \frac{l_1}{2}\right)\right)$$

$$m_2 = h\left(\delta\left(E^n + \frac{l_1}{2}\right) - \gamma\left(I^n + \frac{m_1}{2}\right) - \mu\left(I^n + \frac{m_1}{2}\right)\right)$$

$$n_2 = h\left(\gamma\left(I^n + \frac{m_1}{2}\right) - \beta\left(R^n + \frac{n_1}{2}\right)\left(I^n + \frac{m_1}{2}\right) - \mu\left(R^n + \frac{n_1}{2}\right)\right)$$

$$o_2 = h\left(pq\left(S^n + \frac{k_1}{2}\right) - \mu\left(V^n + \frac{o_1}{2}\right)\right)$$

3.2.3. Step 3

$$k_3 = h\left(\alpha - pq\left(S^n + \frac{k_2}{2}\right) - (1-pq)\beta\left(S^n + \frac{k_2}{2}\right)\left(I^n + \frac{m_2}{2}\right) - \mu\left(S^n + \frac{k_2}{2}\right)\right.$$

$$l_3 = h\left((1-pq)\beta\left(S^n + \frac{k_2}{2}\right)\left(I^n + \frac{m_2}{2}\right) + \beta\left(R^n + \frac{n_2}{2}\right)\left(I^n + \frac{m_2}{2}\right) - \delta\left(E^n + \frac{l_2}{2}\right) - \mu\left(E^n + \frac{l_2}{2}\right)\right)$$

$$m_3 = h\left(\delta\left(E^n + \frac{l_2}{2}\right) - \gamma\left(I^n + \frac{m_2}{2}\right) - \mu\left(I^n + \frac{m_2}{2}\right)\right)$$

$$n_3 = h\left(\gamma\left(I^n + \frac{m_2}{2}\right) - \beta\left(R^n + \frac{n_2}{2}\right)\left(I^n + \frac{m_2}{2}\right) - \mu\left(R^n + \frac{n_2}{2}\right)\right)$$

$$o_3 = h\left(pq\left(S^n + \frac{k_2}{2}\right) - \mu\left(V^n + \frac{o_2}{2}\right)\right)$$

3.2.4. Step 4

$$k_4 = h(\alpha - pq(S^n + k_3) - (1-pq)\beta(S^n + k_3)(I^n + m_3) - \mu(S^n + k_3)$$

$$l_4 = h((1-pq)\beta(S^n + k_3)(I^n + m_3) + \beta(R^n + n_3)(I^n + m_3) - \delta(E^n + l_3) - \mu(E^n + l_3))$$

$$m_4 = h(\delta(E^n + m_3) - \gamma(I^n + m_3) - \mu(I^n + m_3))$$

$$n_4 = h(\gamma(I^n + m_3) - \beta(R^n + n_3)(I^n + m_3) - \mu(R^n + n_3))$$

$$o_4 = h(pq(S^n + k_3) - \mu(V^n + o_3))$$

3.2.5. Final step

$$S^{n+1} = S^n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{11}$$

$$E^{n+1} = E^n + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \tag{12}$$

$$I^{n+1} = I^n + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4) \tag{13}$$

$$R^{n+1} = R^n + \frac{1}{6}(n_1 + 2n_2 + 2n_3 + n_4) \tag{14}$$

$$V^{n+1} = V^n + \frac{1}{6}(o_1 + 2o_2 + 2o_3 + o_4) \tag{15}$$

3.3. NSFD scheme

$$S^{n+1} = \frac{S^n + h\alpha}{1 + h(pq + (1-pq)\beta I^n + \mu)} \tag{16}$$

$$E^{n+1} = \frac{S^n + h((1-pq)\beta S^n I^n + \beta R^n I^n)}{1 + h(\delta + \mu)} \tag{17}$$

$$I^{n+1} = \frac{I^n + h\delta E^n}{1 + h(\gamma + \mu)} \tag{18}$$

$$R^{n+1} = \frac{R^n + h\gamma I^n}{1 + h(\beta I^n + \mu)} \tag{19}$$

$$V^{n+1} = \frac{V^n + hhpqS^n}{(1 + h\mu)} \tag{20}$$

3.4. Convergence analysis

In this section, we will conduct the convergence analysis of the NSFD scheme applied to the SEIRV model, focusing on the DFE point. Jacobian of the NSFD scheme is

$$J=\begin{bmatrix} \frac{1}{1+h(pq+(1-pq)\beta I+\mu)} & 0 & -\frac{h(1-pq)(S+h\alpha)}{(1+h(pq+(1-pq)\beta I+\mu))^2} & 0 & 0 \\ \frac{h(1-pq)\beta I}{1+h(\delta+\mu)} & \frac{1}{1+h(\delta+\mu)} & \frac{h((1-pq)\beta S+\beta R)}{1+h(\delta+\mu)} & \frac{h\beta I}{1+h(\delta+\mu)} & 0 \\ 0 & \frac{h\delta}{1+h(\gamma+\mu)} & \frac{1}{1+h(\gamma+\mu)} & 0 & 0 \\ 0 & 0 & \frac{h\gamma(1+h(\beta I+\mu)-h\beta(R+h\gamma I)}{(1+h(\beta I+\mu))^2} & \frac{1}{1+h(\beta I+\mu)} & 0 \\ \frac{hpq}{1+h\mu} & 0 & 0 & 0 & \frac{1}{1+h\mu} \end{bmatrix}$$

$$J(p_0)=\begin{bmatrix} \frac{1}{1+h(pq+\mu)} & 0 & -\frac{h(1-pq)\left(\frac{\alpha}{pq+\mu}+h\alpha\right)}{(1+h(pq+\mu))^2} & 0 & 0 \\ 0 & \frac{1}{1+h(\delta+\mu)} & \frac{h\left((1-pq)\beta\left(\frac{\alpha}{pq+\mu}\right)\right)}{1+h(\delta+\mu)} & 0 & 0 \\ 0 & \frac{h\delta}{1-h(\gamma+\mu)} & \frac{1}{1-h(\gamma+\mu)} & 0 & 0 \\ 0 & 0 & \frac{h\gamma(1+h\mu)}{(1+h\mu)^2} & \frac{1}{1+h\mu)} & 0 \\ \frac{hpq}{1+h\mu} & 0 & 0 & 0 & \frac{1}{1+h\mu} \end{bmatrix}$$

Three eigenvalues of the above matrix are $\lambda_1=\lambda_2=\frac{1}{1+\mu}$, $\lambda_3=\frac{1}{1+h(pq+\mu)}$. The following matrix can be used to get the remaining eigenvalues.

$$A=\begin{bmatrix} \frac{1}{1+h(\delta+\mu)} & \frac{h\left((1-pq)\beta\left(\frac{\alpha}{pq+\mu}\right)\right)}{1+h(\delta+\mu)} \\ \frac{h\delta}{1-h(\gamma+\mu)} & \frac{1}{1-h(\gamma+\mu)} \end{bmatrix}$$

Now, by using the Bauer F lemma [55], we have

$$\lambda^2-c\lambda+d=0$$

where c is the trace and d is the determinant of the above matrix. Here, $d<0$, $1+c+d>0$ and $1-c+d>0$ as desired.

3.5. Positivity analysis

Theorem. *Assume that S,E,I,R and V, the state variables of the NSFD scheme are all positive at $t=0$; furthermore, if all of the parameters are positive, then $S^{n+1}\geq 0, E^{n+1}\geq 0, I^{n+1}\geq 0, R^{n+1}\geq 0, V^{n+1}\geq 0$.*

Proof. *Taking into account the state variables S,E,I,R and V of the NSFD scheme* (16–20) *as*

$$S^{n+1}=\frac{S^n+h\alpha}{1+h(pq+(1-pq)\beta I^n+\mu)}$$

$$E^{n+1}=\frac{S^n+h((1-pq)\beta S^n I^n+\beta R^n I^n)}{1+h(\delta+\mu)}$$

$$I^{n+1}=\frac{I^n+h\delta E^n}{1+h(\gamma+\mu)}$$

$$R^{n+1}=\frac{R^n+h\gamma I^n}{1+h(\beta I^n+\mu)}$$

$$V^{n+1}=\frac{V^n+hhpqS^n}{(1+h\mu)}$$

By combining all the equations in the aforementioned system with $n=0$, we arrive at the following expression.

$$S^1=\frac{S^0+h\alpha}{1+h\left(pq+(1-pq)\beta I^0+\mu\right)}\geq 0$$

$$E^1=\frac{S^0+h\left((1-pq)\beta S^0 I^0+\beta R^0 I^0\right)}{1+h(\delta+\mu)}\geq 0$$

$$I^1=\frac{I^0+h\delta E^0}{1+h(\gamma+\mu)}\geq 0$$

$$R^1=\frac{R^0+h\gamma I^0}{1+h\left(\beta I^0+\mu\right)}\geq 0$$

$$V^1=\frac{V^0+hhpqS^0}{(1+h\mu)}\geq 0$$

Now, by substituting $n=1$ in, we can proceed to the next step.

$$S^2=\frac{S^1+h\alpha}{1+h\left(pq+(1-pq)\beta I^1+\mu\right)}\geq 0$$

$$E^2=\frac{S^1+h\left((1-pq)\beta S^1 I^1+\beta R^1 I^1\right)}{1+h(\delta+\mu)}\geq 0$$

$$I^2=\frac{I^1+h\delta E^1}{1+h(\gamma+\mu)}\geq 0$$

$$R^2=\frac{R^1+h\gamma I^1}{1+h\left(\beta I^1+\mu\right)}\geq 0$$

$$V^2=\frac{V^1+hhpqS^1}{(1+h\mu)}\geq 0$$

Assume that the given equations maintain positivity for $n=2,3,4,\ldots,n-1$, i.e. $S^{n+1}\geq 0, E^{n+1}\geq 0, I^{n+1}\geq 0, R^{n+1}\geq 0, V^{n+1}\geq 0$; for $n=2,3,4,\ldots,n-1$.

We will now analyze the positivity for a random positive integer n in

Z. We find that

$$S^{n+1}=\frac{S^n+h\alpha}{1+h(pq+(1-pq)\beta I^n+\mu)}\geq 0$$

$$E^{n+1}=\frac{S^n+h((1-pq)\beta S^n I^n+\beta R^n I^n)}{1+h(\delta+\mu)}\geq 0$$

$$I^{n+1}=\frac{I^n+h\delta E^n}{1+h(\gamma+\mu)}\geq 0$$

$$R^{n+1}=\frac{R^n+h\gamma I^n}{1+h(\beta I^n+\mu)}\geq 0$$

$$V^{n+1}=\frac{V^n+hhpqS^n}{(1+h\mu)}\geq 0$$

Thus, for any positive integer values of n, the suggested approach ensures positivity for the state variables S, E, I, R and V.

3.6. Boundedness of the NSFD scheme

Theorem. *Let S^0, E^0, I^0, R^0 and V^0 are finite, such that $S^0+E^0+I^0+R^0+V^0\leq 1$. Moreover, in the model μ,β,V,α are all positive. Then, S^{n+1}, E^{n+1}, I^{n+1}, and R^{n+1} are bounded by recrossing defined real constant b_{n+1} such that $S^{n+1}, E^{n+1}, I^{n+1}$ and $R^{n+1}<b_{n+1}\ \forall n\in Z^+$ where $b_{n+1}=5b_n+h\mu+h\beta {b_n}^2+hVb_n+h\alpha b_n$ and $b_1=5+h\mu+h\beta S^0I^0+hVE^0+h\alpha I^0$.*

Proof. *Taking into account the state variables S, E, I, R and V of the NSFD scheme* (16–20) *as,*

$$S^{n+1}(1+h(pq+(1-pq)\beta I^n+\mu))=S^n+h\alpha$$

$$E^{n+1}(1+h(\delta+\mu))=E^n+h((1-pq)\beta S^nI^n+\beta R^nI^n)$$

$$I^{n+1}(1+h(\gamma+\mu))=I^n+h\delta E^n$$

$$R^{n+1}(1+h(\beta I^n+\mu))=R^n+h\gamma I^n$$

$$V^{n+1}(1+h\mu)=V^n+hpqS^n$$

Adding all of the equations in the preceding system yields the following expression.

$$\begin{aligned}&S^{n+1}+E^{n+1}+I^{n+1}+R^{n+1}+V^{n+1}+S^{n+1}(h(pq+(1-pq)\beta I^n+\mu))\\&+E^{n+1}(h(\delta+\mu))+I^{n+1}(h(\gamma+\mu))+R^{n+1}(h(\beta I^n+\mu))+V^{n+1}(h\mu)=S^n\\&+h\alpha+E^n+h((1-pq)\beta S^nI^n+\beta R^nI^n)+I^n+h\delta E^n+R^n+h\gamma I^n+V^n+hpqS^n\end{aligned}$$

$$\begin{aligned}&S^{n+1}+E^{n+1}+I^{n+1}+R^{n+1}+V^{n+1}+S^{n+1}(h(pq+(1-pq)\beta I^n+\mu))\\&+E^{n+1}(h(\delta+\mu))+I^{n+1}(h(\gamma+\mu))+R^{n+1}(h(\beta I^n+\mu))+V^{n+1}(h\mu)=S^n\\&+E^n+I^n+R^n+V^n+h\alpha+h((1-pq)\beta S^nI^n+\beta R^nI^n)+h\delta E^n+h\gamma I^n+hpqS^n\end{aligned}\tag{21}$$

Using mathematical induction theory and restrictions placed on the parameters and state variables, we show that the nonstandard numerical scheme is bounded. We get the following when we substitute $n=0$ in equation (21).

$$\begin{aligned}&(S^1+E^1+I^1+R^1+V^1)+S^1(1+h(pq+(1-pq)\beta i^0+\mu))\\&+E^1(1+h(\delta+\mu))+I^1(1+h(\gamma+\mu))+R^1(1+h(\beta I^0+\mu))\\&+V^1(1+h\mu)=S^0+E^0+I^0+R^0+V^0+h\alpha+h((1-pq)\beta S^0I^0+\beta R^0I^0)\\&+h\delta E^0+h\gamma I^0+hpqS^0\end{aligned}$$

$$\begin{aligned}&(S^1+E^1+I^1+R^1+V^1)+S^1(1+h(pq+(1-pq)\beta I^0+\mu))\\&+E^1(1+h(\delta+\mu))+I^1(1+h(\gamma+\mu))+R^1(1+h(\beta I^0+\mu))\\&+V^1(1+h\mu)<b_1\end{aligned}$$

$$\begin{aligned}&\Rightarrow(S^1+E^1+I^1+R^1+V^1)+S^1(1+h(pq+(1-pq)\beta I^0+\mu))\\&+E^1(1+h(\delta+\mu))+I^1(1+h(\gamma+\mu))+R^1(1+h(\beta I^0+\mu))\\&+V^1(1+h\mu)<5+h\mu+h\beta S^0I^0+hVE^0+h\alpha I^0\end{aligned}$$

$$\Rightarrow S^1<b_1,$$

$$E^1<b_1,$$

$$I^1<b_1,$$

$$R^1<b_1,$$

$$V^1<b_1,$$

Now, by substituting $n=1$ in equation (21) to reach the following step.

$$\begin{aligned}&(S^2+E^2+I^2+R^2+V^2)+S^2(1+h(pq+(1-pq)\beta i^1+\mu))\\&+E^2(1+h(\delta+\mu))+I^2(1+h(\gamma+\mu))+R^2(1+h(\beta i^1+\mu))\\&+V^2(1+h\mu)<b_2,\end{aligned}$$

$$\begin{aligned}&\Rightarrow(S^2+E^2+I^2+R^2+V^2)+S^2(1+h(pq+(1-pq)\beta i^1+\mu))\\&+E^2(1+h(\delta+\mu))+I^2(1+h(\gamma+\mu))+R^2(1+h(\beta i^2+\mu))\\&+V^2(1+h\mu)<b_{n+1}=5b_n+h\mu+h\beta {b_n}^2+hVb_n+h\alpha b_n,\end{aligned}$$

$$\Rightarrow S^2<b_2,$$

$$E^2<b_2,$$

$$I^2<b_2,$$

$$R^2<b_2,$$

$$V^2<b_2.$$

Now, let us assume that, for positive integers 2, 3, 4, …, $n-1$, the expression (21) is bounded i.e. $S^n<b_n, E^n<b_n, I^n<b_n, R^n<b_n, V^n<b_n$, $\forall n\epsilon\{2,3,4,\ldots,n-1\}$. Now, we look into the boundedness of an integer n.

$$\begin{aligned}&(S^{n+1}+E^{n+1}+I^{n+1}+R^{n+1}+V^{n+1})+S^{n+1}(1+h(pq+(1-pq)\beta I^n+\mu))\\&+E^1(1+h(\delta+\mu))+I^{n+1}(1+h(\gamma+\mu))+R^{n+1}(1+h(\beta I^n+\mu))\\&+V^{n+1}(1+h\mu)=S^n+E^n+I^n+R^n+V^n+h\alpha+h((1-pq)\beta S^nI^n\\&+\beta r^nI^n)+h\delta E^n+h\gamma I^n+hpqS^n<b_n\end{aligned}$$

$$S^{n+1}<b_{n+1},$$

$$E^{n+1}<b_{n+1},$$

$$I^{n+1}<b_{n+1},$$

$$R^{n+1}<b_{n+1},$$

$$V^{n+1}<b_{n+1},$$

Therefore, for all positive values of , S^{n+1}, E^{n+1}, I^{n+1}, R^{n+1}, and V^{n+1} are constrained within a real number denoted by b_{n+1}.

3.7. Consistency analysis

This section employs Taylor's series expansion to carry out the consistency analysis of the NSFD scheme. First, we use the Taylor's

Table 2
Values of variable.

Variable	Value
S	0.3
E	0.05
I	0.12
R	0.33
V	0.2

Table 3
Values of parameter.

Parameter	Value
α	0.015
μ	0.015
γ	0.01
δ	0.85
p	0.3
q	0.85

series expansion (TSE) of S^{n+1} to the first equation of the implicit numerical integration technique system (16–20).

$$S^{n+1} = S^n + h\frac{dS}{dt} + \frac{h^2}{2!}\frac{d^2S}{dt^2} + \frac{h^3}{3!}\frac{d^3S}{dt^3} + \ldots,$$

in the following expression:

$$\left(S^n + h\frac{dS}{dt} + \frac{h^2}{2!}\frac{d^2S}{dt^2} + \frac{h^3}{3!}\frac{d^3S}{dt^3} + \ldots\right)(1 + h(pq + (1-pq)\beta I^n + \mu)) = S^n + h\alpha,$$

$$S^n + S^n h(pq + (1-pq)\beta I^n + \mu) + h\frac{dS}{dt}(1 + h(pq + (1-pq)\beta I^n + \mu)) + \frac{h^2}{2!}\frac{d^2S}{dt^2}(1 + h(pq + (1-pq)\beta I^n + \mu)) + \frac{h^3}{3!}\frac{d^3S}{dt^3}(1 + h(pq + (1-pq)\beta I^n + \mu)) + \ldots = S^n + h\alpha,$$

We get the following by applying $h \to 0$,

$$S^n(pq + (1-pq)\beta I^n + \mu) + \frac{dS}{dt} = \alpha,$$

$$\frac{dS}{dt} = \alpha - S^n(pq + (1-pq)\beta I^n + \mu),$$

$$\frac{ds}{dt} = \alpha - (pq + (1-pq)\beta IS + \mu).$$

This result shows that our discretized equation is in agreement with the initial equation of system (1–5). Similarly, we take the following equation and apply the TSE of E^{n+1}:

$$E^{n+1} = E^n + h\frac{dE}{dt} + \frac{h^2}{2!}\frac{d^2E}{dt^2} + \frac{h^3}{3!}\frac{d^3E}{dt^3} + \ldots,$$

In the following expression:

$$E^{n+1}(1 + h(\delta + \mu)) = E^n + h((1-pq)\beta S^n I^n + \beta R^n I^n),$$

$$\left(E^n + h\frac{dE}{dt} + \frac{h^2}{2!}\frac{d^2E}{dt^2} + \frac{h^3}{3!}\frac{d^3E}{dt^3} + \ldots\right)(1 + h(\delta + \mu)) = E^n + h((1-pq)\beta S^n I^n + \beta R^n I^n),$$

We get the following by applying $h \to 0$,

$$E^n + \frac{dE}{dt} = (1-pq)\beta S^n I^n + \beta R^n I^n,$$

$$\frac{dE}{dt} = (1-pq)\beta S^n I^n + \beta R^n I^n - E^n.$$

$$\frac{dE}{dt} = (1-pq)\beta SI + \beta RI - E.$$

Similarly, applying TSE on the third equation of the developed NSFD scheme, we obtain as follows

$$I^{n+1} = I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \ldots,$$

In the following expression:

$$I^{n+1}(1 + h(\gamma + \mu)) = I^n + h\delta E^n,$$

$$\left(I^n + h\frac{dI}{dt} + \frac{h^2}{2!}\frac{d^2I}{dt^2} + \frac{h^3}{3!}\frac{d^3I}{dt^3} + \ldots\right)((1 + h(\gamma + \mu))) = I^n + h\delta E^n,$$

We get the following by applying $h \to 0$,

$$I^n + \frac{dI}{dt} = \delta E^n,$$

$$\Rightarrow \frac{dI}{dt} = \delta E - I.$$

Similarly, TSE of the next variable is

$$R^{n+1} = R^n + h\frac{dR}{dt} + \frac{h^2}{2!}\frac{d^2R}{dt^2} + \frac{h^3}{3!}\frac{d^3R}{dt^3} + \ldots,$$

Equation four of the NSFD scheme gives us

$$R^{n+1}(1 + h(\beta I^n + \mu)) = R^n + h\gamma I^n,$$

$$\left(R^n + h\frac{dR}{dt} + \frac{h^2}{2!}\frac{d^2R}{dt^2} + \frac{h^3}{3!}\frac{d^3R}{dt^3} + \ldots\right)(1 + h(\beta I^n + \mu)) = R^n + h\gamma I^n,$$

We get the following by applying $h \to 0$,

$$R^n + \frac{dR}{dt} = \delta E^n,$$

$$\Rightarrow \frac{dR}{dt} = \delta E - R.$$

The TSE for the last equation of the studied model is

$$V^{n+1} = V^n + h\frac{dV}{dt} + \frac{h^2}{2!}\frac{d^2V}{dt^2} + \frac{h^3}{3!}\frac{d^3V}{dt^3} + \ldots,$$

From equation (20) of the NSFD scheme, we have

$$V^{n+1}(1 + h\mu) = V^n + hpqS^n,$$

$$\left(V^n + h\frac{dV}{dt} + \frac{h^2}{2!}\frac{d^2V}{dt^2} + \frac{h^3}{3!}\frac{d^3V}{dt^3} + \ldots\right)(1 + h\mu) = V^n + hpqS^n,$$

Applying $h \longrightarrow 0$ results in the following,

$$V^n\mu + \frac{dV}{dt} = pqS^n,$$

$$\Rightarrow \frac{dV}{dt} = pqS - \nu\mu.$$

As a result, the ODES above system and our discretized implicit numerical integration technique are consistent.

4. Numerical simulations

Tables 2 and 3 show the values of the variables and parameters

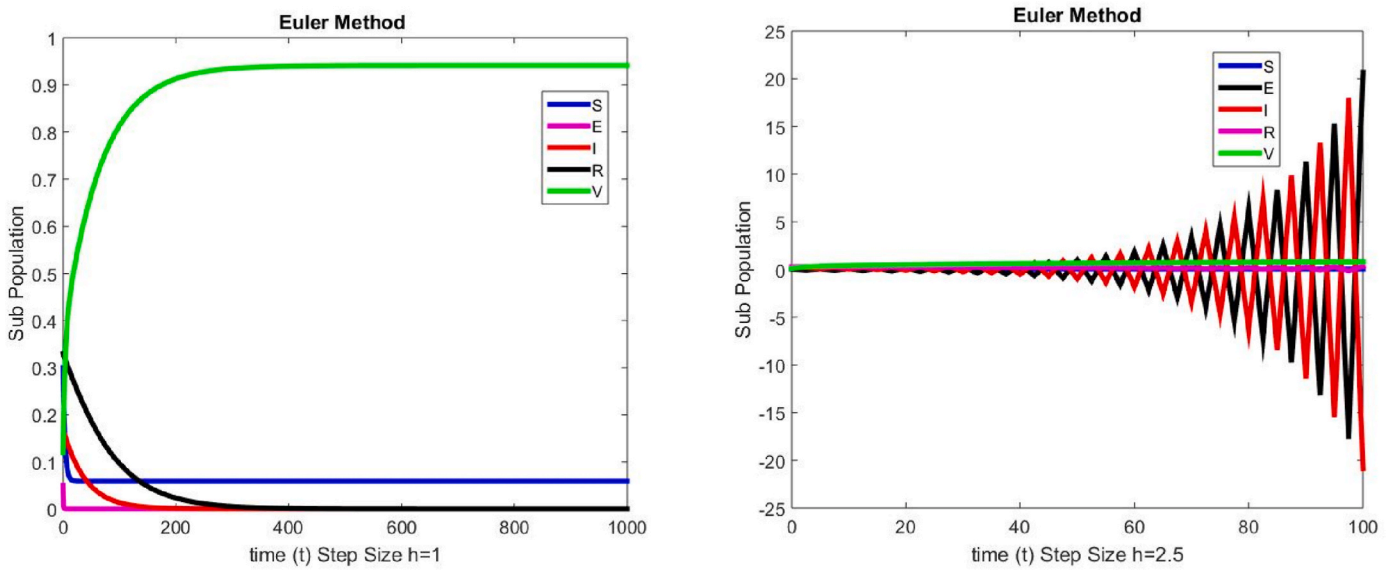

Fig. 2. Subpopulations using Euler scheme.

utilized in the numerical simulations.

Fig. 2 depicts the graphical behaviors of the subpopulations using the Euler technique. It can be observed that the Euler scheme converges at small step sizes and begins nonphysical oscillations, producing negative values when the step size increases somewhat. Negative values in these models are meaningless as the population cannot be negative. This behavior of the graphs shows that the approach is not suitable for representing rubella illness dynamics.

Fig. 3 represents the population dynamics using the RK-4 method. The method is stable, and gives positive and convergent solutions at small values of the time step sizes. However, higher step size yield negative outcomes, which are not feasible for population dynamics and this behavior shows that the RK-4 method is also not suitable for representing rubella illness dynamics.

The subpopulations using the NSFD method are shown in Fig. 4. The NSFD scheme remains positive and convergent despite changing step size values. We can conclude that the NSFD scheme is a reliable tool for reflecting rubella virus disease dynamics. These results demonstrate that the NSFD method is a reliable tool while analyzing the dynamics of rubella transmission.

Fig. 5 depicts the effects of vaccination. It can be seen that vaccination is inversely proportional to infection, with an increase in vaccination decreasing infection and vice versa. It can be concluded that vaccination is the most effective method of preventing rubella transmission. The MMR vaccine is extremely effective against rubella infection. The vaccine contains live, weakened viruses that stimulate the

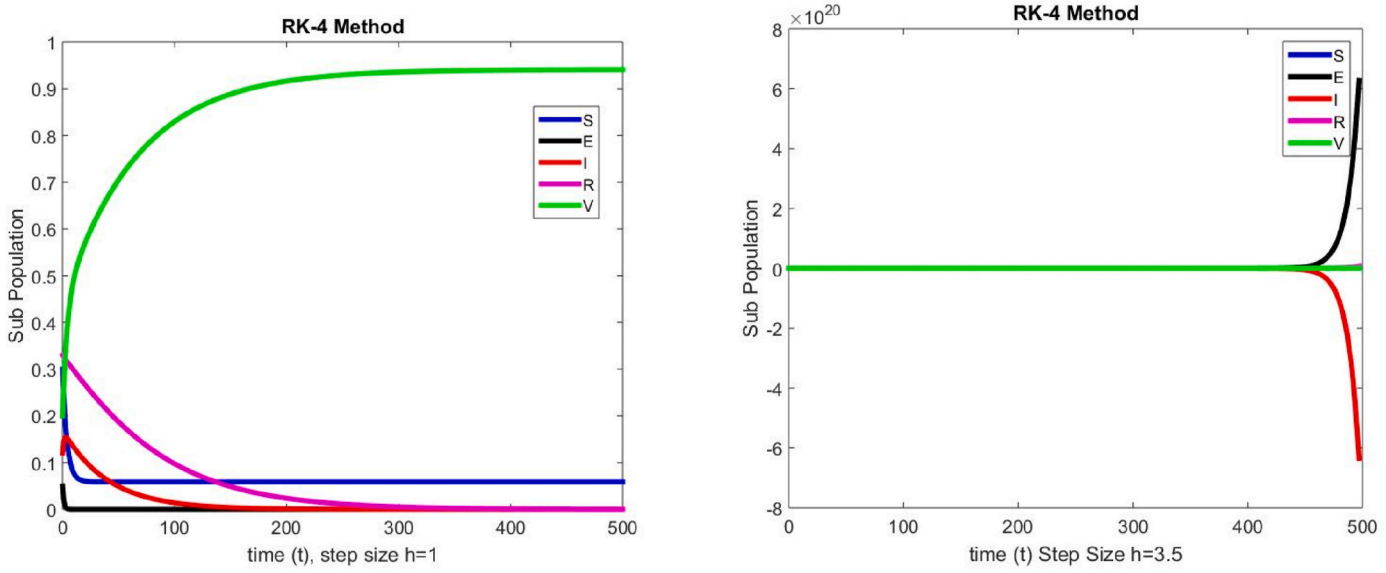

Fig. 3. Subpopulations using the RK-4 scheme.

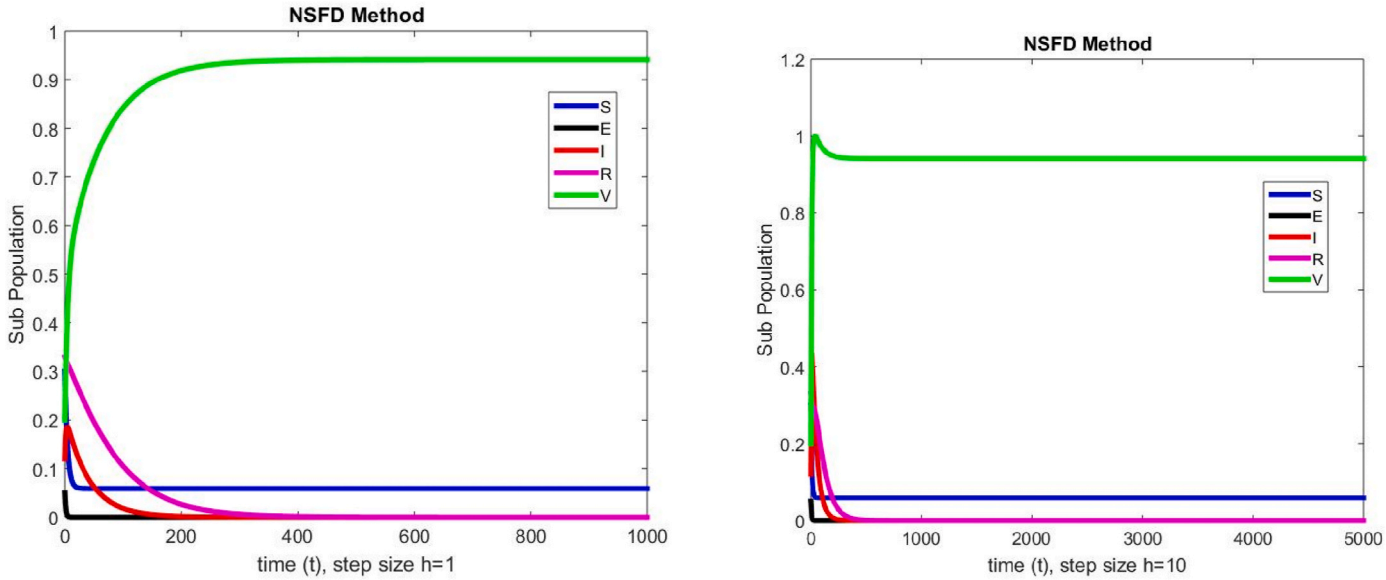

Fig. 4. Subpopulations using the NSFD scheme.

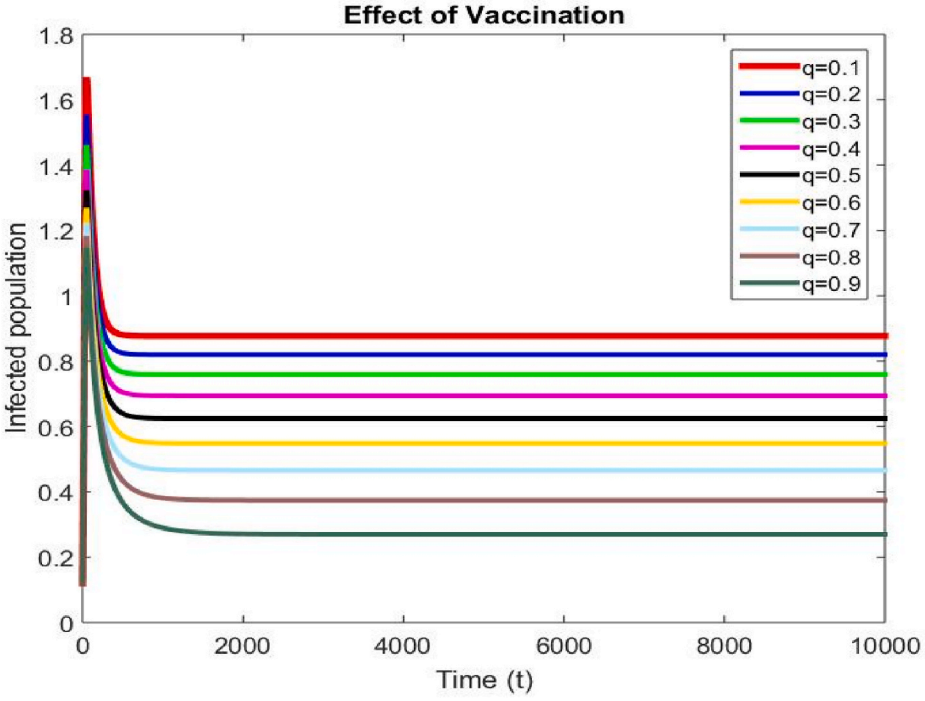

Fig. 5. Effects of vaccination on the infected population.

immune system to produce anti-rubella antibodies. When a large proportion of the population is immunized against rubella, herd immunity develops, which aids in the prevention of the virus's spread.

5. Conclusion

Rubella is typically a mild infectious disease presenting with a rash and swollen lymph nodes for about three days. The illness spreads in the air and through direct contact. It is found among children and adults, though it can be quite severe and risky for pregnant women and their fetuses when developing in the uterus. Vaccination is an important tool in preventing the spread of Rubella and protecting vulnerable populations. This study presents an accurate and trustworthy numerical solution for a Rubella epidemic model using an NSFD approach. Two equilibrium points that can be used to symbolize DFE and endemic, respectively, have been established by the model from a mathematical perspective. Local and global stability for the DFE point has been determined. The sensitivity of parameters is investigated. The model is solved numerically with three different techniques. The graphical results of the developed three methods are compared. According to the simulation, neither presented the conventional, well-known methods Euler and RK-4 are capable of producing reliable results even at very small step sizes. On the other hand, the suggested method gives accurate and convergent solutions for all choices of the values of the step sizes. Furthermore, the NSFD scheme's positivity, convergence, boundedness, and consistency, are investigated. The scheme preserves positivity

which is the main feature of these types of models as the compartments in the model consist of the population which cannot be negative. So negative values are meaningless in these models. The Euler and RK-4 methods give negative values at some choices of the values of the step size which makes these methods not suitable for the study of disease dynamics of the studied model. The effects of the Rubella vaccination are also being investigated. Rubella vaccination is typically administered as part of routine childhood immunizations. Furthermore, vaccination is critical in preventing rubella transmission. It not only protects those who receive the vaccine, but it also contributes to herd immunity, lowering the likelihood of outbreaks and protecting vulnerable populations who are unable to receive the vaccine. The benefits of rubella vaccination extend beyond the prevention of transmission and congenital rubella syndrome. Vaccination also provides economic benefits, such as lowering the cost of disease treatment and complications, as well as reducing lost productivity due to illness. Vaccination is substantially less expensive than treating rubella and its consequences, such as hospitalization for pregnant women with rubella or infants born with CRS. Maintaining high vaccination coverage rates is important for achieving and maintaining herd immunity and protecting vulnerable populations. The current work mainly focuses on the construction of an NSFD scheme for the solution of the Rubella virus disease dynamics model. The work can be extended to fractional, fuzzy, delayed and stochastic senses and many more directions.

CRediT authorship contribution statement

Samiullah Salim: Writing – original draft, Validation, Software, Methodology, Data curation. **Fazal Dayan:** Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Methodology, Formal analysis, Conceptualization. **Muhammad Aziz ur Rehman:** Writing – original draft, Visualization, Methodology, Conceptualization. **Husam A. Neamah:** Writing – review & editing, Supervision, Resources, Investigation, Formal analysis.

Availability of data and materials

The datasets used and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Ethical statement

The study does not involve human participants, animals, or any sensitive data. All authors affirm that the manuscript is original and has not been published elsewhere nor is it under consideration for publication elsewhere.

Furthermore, all authors have read and approved the manuscript for submission to *Informatics in Medicine Unlocked*. The authors declare that there are no conflicts of interest regarding this study.

No external funding was received specifically for this study. Data supporting the findings of this study are available from the corresponding author upon reasonable request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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