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Research Article

On Fuzzy $\mathfrak{F}^*$-Simply Connected Spaces in Fuzzy $\mathfrak{F}^*$-Homotopy

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In this paper, the notions of fuzzy $\mathscr{F}^*$-simply connected spaces and fuzzy $\mathfrak{F}^*$-structure homeomorphisms are introduced, and further fuzzy $\mathfrak{F}^*$-structure homeomorphism between fuzzy $\mathfrak{F}^*$-path-connected spaces are studied. Also, it is shown that every fuzzy $\mathscr{F}^*$-structure subspace of fuzzy $\mathscr{F}^*$-simply connected space is fuzzy $\mathscr{F}^*$-simply connected subspace. Further, the concepts of fuzzy $\mathscr{F}^*$-contractible spaces and fuzzy $\mathscr{F}^*$-retracts are introduced, and it is proved that every fuzzy $\mathscr{F}^*$-contractible space is fuzzy $\mathscr{F}^*$-simply connected.

1. Introduction

Homotopy theory is the main part of algebraic topology which studies topological objects up to homotopy equivalence. Homotopy equivalence is a weaker relation than topological equivalence. The homotopy theory is one among the foremost branches of algebraic topology. The thoughts and techniques of homotopy theory have pervaded many components of topology. Several topologists like Massey [1], Munkres [2], and Hatcher [3], introduced and studied concepts of homotopy theory and fundamental groups. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [4]. The fuzzy homotopy theory was introduced by Culvacioglu and Citil [5]. Salleh and Tap [6, 7] introduced the concept of the fundamental group to fuzzy topological spaces based on the definition of fuzzy topology introduced by Chang [8]. Based on Chang's [8] description of fuzzy topology, Salleh and Tap [6, 7, 9] introduced the concept of basic fundamental group of fuzzy topological spaces. Several recent papers contain extensive investigations and applications about fuzzy topological space and fuzzy homotopy theory; see [10–12].

Later on, Guner [13] intensively investigated the concept of fuzzy contractible spaces. Rodabaugh [14, 15] introduced the concept of L-fuzzy retract in 1981. Further fuzzy homotopy theory was intensively developed by Aras [16], Citil and Cuvalcioglu [17], Palmeira and Bedregal [18], etc. and gave many interesting results on fuzzy homotopy theory. In this connection, Madhuri and Amudhambigai [19, 20] introduced the concepts of fuzzy $\mathfrak{F}^*$-homotopy and $\mathfrak{F}^*$-fundamental group. Also the concept of fuzzy $\mathfrak{F}^*$-structure isomorphisms between $\mathfrak{F}^*$-fundamental groups are studied in [19, 20]. In this recent work, the concept of fuzzy $\mathfrak{F}^*$-structure homeomorphism between fuzzy $\mathfrak{F}^*$-path-connected spaces are studied. The notion of fuzzy $\mathfrak{F}^*$-simply connected space is introduced, and it is shown that every fuzzy $\mathfrak{F}^*$-structure subspace of fuzzy $\mathfrak{F}^*$-simply connected space is fuzzy $\mathfrak{F}^*$-simply connected. Further, the concepts of fuzzy $\mathfrak{F}^*$-contractible spaces and fuzzy $\mathfrak{F}^*$-retracts are introduced and it is proved that every fuzzy $\mathfrak{F}^*$-contractible space is fuzzy $\mathfrak{F}^*$-simply connected. Also in fuzzy $\mathfrak{F}^*$-contractible space, each fuzzy $\mathfrak{F}^*$-loop based at any fuzzy point is equivalent to the constant fuzzy $\mathfrak{F}^*$-loop.

2. Preliminaries

In this section, some basic concepts of fuzzy homotopy have been recalled from the previous literature. Also related results and propositions are studied from various research articles. Some definitions and preliminary results are presented in this section.

Definition 1. Let (X,τ) be a fuzzy topological space. A fuzzy set $\mu \in I^X$ is called fuzzy irreducible if $\mu \neq 0_X$, and for all fuzzy closed sets $\gamma,\delta \in I^X$ with $\mu \leq (\gamma \vee \delta)$, it follows that either $\mu \leq \gamma$ or $\mu \leq \delta$.

Remark 2. Let (X,τ) be a fuzzy topological space. Any $\lambda \in I^X$ is said to be fuzzy irreducible closed if it is both fuzzy irreducible and fuzzy closed.

Definition 3. Suppose that (X,τ) be a fuzzy topological space and assume that $\alpha \in I^X$ be a fuzzy open set in (X,τ). Then the collection $\mathfrak{F} = \{\sigma \in I^X : \alpha \mathfrak{q} \sigma$ and $1-\sigma$ is a fuzzy irreducible closed set in $(X,\tau)\}$. Then the collection $\mathfrak{F}$ which is finer than the fuzzy topology τ on X is said to be a $\mathfrak{F}$-structure on X. A nonempty set X with a $\mathfrak{F}$-structure denoted by $(X,\mathfrak{F})$ is said to be fuzzy $\mathfrak{F}$-structure space. Each member of $\mathfrak{F}$ is said to be fuzzy $\mathfrak{F}$-structure open set, and the complement of each fuzzy $\mathfrak{F}$-structure open set is said to be fuzzy $\mathfrak{F}$-structure closed. A $\mathfrak{F}$-structure on a nonempty set X together with 0_X is said to be fuzzy $\mathfrak{F}^*$-structure. Then $(X,\mathfrak{F}^*)$ is called a fuzzy $\mathfrak{F}^*$-structure space generated by τ.

Definition 4. Suppose that (X,τ) be any topological space and assume that $(X,\mathfrak{F}^*)$ be fuzzy $\mathfrak{F}^*$-structure space. Assume also that $U \subset X$ and χ_U denotes the so-called fuzzy characteristic function of the subset U. Then the fuzzy $\tilde{\xi}$-structure given by τ is $\mathfrak{F}^*(\tau) = \{\chi_U : U \in \tau\}$, and the pair $(X,\mathfrak{F}^*(\tau))$ is called a fuzzy $\tilde{\xi}$-structure space given by (X,τ).

Note 5. Let I be the unit interval. Let ζ be an Euclidean topology on I, and then $(I,\mathfrak{F}^*(\zeta))$ is a fuzzy $\mathfrak{F}^*$-structure space introduced by the (usual) topological space (I,ζ).

Definition 6. Let $\pi_1((X,\mathfrak{F}^*),x_\lambda)$ and $\pi_1((X,\mathfrak{F}^*),x'_\mu)$ be any two fuzzy $\mathfrak{F}^*$-fundamental groups of $(X,\mathfrak{F}^*)$ at x_λ and x'_μ, respectively. A function $f:\pi_1((X_1,\mathfrak{F}_1{}^*),x_\lambda) \longrightarrow \pi_2((Y,\mathfrak{F}_2{}^*),x'_\mu)$ is called a fuzzy $\mathfrak{F}^*$-structure homomorphism if $f([\theta]\circ[\eta]) = f([\theta])\circ f([\eta])$ for every $[\theta],[\eta] \in \pi_1((X_1,\mathfrak{F}_1{}^*),x_\lambda)$. Moreover the fuzzy $\mathfrak{F}^*$-structure homomorphism is called a fuzzy $\mathfrak{F}^*$-structure isomorphism if it is bijective.

Proposition 7. *Let $(X,\mathfrak{F}^*)$ be any fuzzy $\mathfrak{F}^*$-structure space and let $x_\lambda \in \mathscr{FP}(X)$. Let $\eta_0,\eta_1,\theta_0,\theta_1 \in Y((X,\mathfrak{F}^*),x_\lambda)$ be any fuzzy $\mathfrak{F}^*$-loops in $(X,\mathfrak{F}^*)$. If $\eta_0 \cong_{\mathfrak{P}} \eta_1$ and $\theta_0 \cong_{\mathfrak{P}} \theta_1$, then $\eta_1 * \theta_1 \cong_{\mathfrak{P}} \eta_0 * \theta_0$.*

Proposition 8. *Let $(X,\mathfrak{F}^*)$ be any fuzzy $\mathfrak{F}^*$-structure space. Let $[\alpha],[\beta],[\gamma] \in \pi_1((X,\mathfrak{F}^*),x_\lambda)$, where x_λ is a fuzzy point in X. Then $([\alpha]\circ[\beta])\circ[\gamma] = [\alpha]\circ([\beta]\circ[\gamma])$.*

Proposition 9. *Let $(X,\mathfrak{F}^*)$ be any fuzzy $\mathfrak{F}^*$-structure space and let $(I,\mathfrak{F}^*(\zeta))$ be any fuzzy $\mathfrak{F}^*$-structure space introduced by (I,ζ). Also, let $e:(I,\mathfrak{F}^*(\zeta)) \longrightarrow (X,\mathfrak{F}^*)$ be the fuzzy $\mathfrak{F}^*$-path defined by $e(t_\zeta) = x_\lambda$ for each t_ζ in $(I,\mathfrak{F}^*(\zeta))$, and x_λ is fuzzy point in X. Then $[\alpha]\circ[e] = [e]\circ[\alpha] = [\alpha]$, for each $[\alpha] \in \pi_1((X,\mathfrak{F}^*),x_\lambda)$.*

Proposition 10. *Let $(X,\mathfrak{F}^*)$ be any fuzzy $\mathfrak{F}^*$-structure space and x_λ be fuzzy point in X. Let $[\alpha] \in \pi_1((X,\mathfrak{F}^*),x_\lambda)$. Then there exists a $[\bar{\alpha}] \in \pi_1((X,\mathfrak{F}^*),x_\lambda)$ such that $[\alpha]\circ[\bar{\alpha}] = [\bar{\alpha}]\circ[\alpha] = [e]$.*

Remark 11. From Proposition 8, Proposition 9, and Proposition 10, it is seen that $\pi_1((X,\mathfrak{F}^*),x_\lambda)$ forms a group under an operation (namely, multiplication). It is called $\mathfrak{F}^*$-fundamental group of $(X,\mathfrak{F}^*)$ based at x_λ.

Definition 12. (see [21]). Let $f,g : X \longrightarrow Y$ be fuzzy continuous maps and $f \cong g$. If g is a constant, then f is called a fuzzy nulhomotopic map.

Definition 13. Let $A \subset X$. A is an L-fuzzy retract of X in (X,T) (abbreviated F-retract) if there is an F-continuous $r:(X,T) \longrightarrow (A,T(A))$ such that $r(x) = x$ for each $x \in A$.

Definition 14. Let $1_X : (X,\tau) \longrightarrow (X,\tau)$ be an identity mapping. If 1_X is fuzzy homotopic to a constant, then (X,τ) is called a fuzzy contractible space.

Proposition 15. *Let $(X_1,\mathfrak{I}_1)$, $(X_2,\mathfrak{I}_2)$, and $(X_3,\mathfrak{I}_3)$ be any three fuzzy $\mathfrak{F}^*$-structure spaces. If $\varphi:(X_1,\mathfrak{I}_1) \longrightarrow (X_2,\mathfrak{I}_2)$ and $\phi:(X_2,\mathfrak{I}_2) \longrightarrow (X_3,\mathfrak{I}_3)$ are fuzzy $\mathfrak{F}^*$-structure continuous functions, then $\phi\circ\varphi:(X_1,\mathfrak{I}_1) \longrightarrow (X_3,\mathfrak{I}_3)$ is a fuzzy $\mathfrak{F}^*$-structure continuous function.*

Remark 16. Let $(X,\mathfrak{I})$ be any fuzzy $\mathscr{F}^*$-structure space and let $x_\lambda, y_\mu \in \mathscr{FP}(X)$. Let β be any fuzzy $\mathscr{F}^*$-path joining x_λ with y_μ. Also, let $l_{x_\lambda}, l_{y_\mu} : (I,\mathfrak{I}(\zeta)) \longrightarrow (X,\mathfrak{I})$ be any two fuzzy $\mathscr{F}^*$-loops defined such that

$$\begin{gathered} l_{x_\lambda}(t_\rho) = x_\lambda \text{ for all } t_\rho \in \mathscr{FP}(I) \text{ in } (I,\mathfrak{I}(\zeta)), \\ l_{y_\mu}(t_\rho) = y_\mu \text{ for all } t_\rho \in \mathscr{FP}(I) \text{ in } (I,\mathfrak{I}(\zeta)). \end{gathered} \tag{1}$$

Then

(i) $[l_{x_\lambda}]\circ[\beta] = [l_{x_\lambda} * \beta] = [\beta]$, on replacing $[e]$ by $[l_{x_\lambda}]$ in the proof of Proposition 9

(ii) $[\beta]\circ[l_{y_\mu}] = [\beta * l_{y_\mu}] = [\beta]$, on replacing $[e]$ by $[l_{y_\mu}]$ in the proof of Proposition 9

(iii) $[\beta]\circ[\bar{\beta}] = [\beta * \bar{\beta}] = [l_{x_\lambda}]$. where $\bar{\beta}(t) = \beta(1-t)$, on replacing $[e]$ by $[l_{x_\lambda}]$ in the proof of Proposition 10

(iv) $[\bar{\beta}] \circ [\beta] = [\bar{\beta} * \beta] = [l_{y_\mu}]$, on replacing $[e]$ by $[l_{y_\mu}]$ in the proof of Proposition 10

(v) If $l_1 \cong_{\mathfrak{L}} l_2$, then $(\bar{\beta} * l_1) * \beta \cong_{\mathfrak{L}} \bar{\beta} * (l_2 * \beta)$.

Therefore, $[l_{x_\lambda}]$ serves as the left identity and $[l_{y_\mu}]$ serves as the right identity for any [l].

Proposition 17. *Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathcal{F}^*$-structure spaces. Let $\psi, \varphi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathcal{F}^*$-structure continuous functions, where $\mathcal{Q}$ is fuzzy $\mathcal{F}^*$-structure homotopy between ψ and φ. Let $(I, \mathfrak{I}(\zeta))$ be any fuzzy $\mathcal{F}^*$-structure space introduced by (I, ζ). Also if $\beta : (I, \mathfrak{I}(\zeta)) \longrightarrow (X_2, \mathfrak{I}_2)$ is a fuzzy $\mathcal{F}^*$-path joining $\psi(x_\lambda)$ with $\varphi(x_\lambda)$ defined by $\beta(t) = \mathcal{Q}(x_\lambda, t)$, where $t \in I$ and $x_\lambda \in \mathcal{FP}(X_1)$, then the following Figure 1 of induced fuzzy $\mathcal{F}^*$-structure homomorphisms is commutative.*

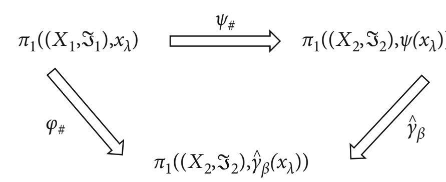

Figure 1

3. Properties of Fuzzy $\mathfrak{F}^*$-Simply Connected Spaces

Throughout this paper, the collection of all fuzzy points x_t, where $0 < t \leq 1$ over X, is denoted by $\mathcal{FP}(X)$, and the set of all fuzzy points over I is denoted by $\mathcal{FP}(I)$. In this section, the concepts of fuzzy $\mathfrak{F}^*$-simply connected spaces, fuzzy $\mathfrak{F}^*$-null-homotopic functions, fuzzy $\mathfrak{F}^*$-contractible spaces, and fuzzy $\mathfrak{F}^*$-retractions are introduced, and some interesting properties are studied.

Definition 18. Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces. Let $\phi, \varphi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure continuous functions and $\phi \cong \varphi$. If φ is a constant function, then ϕ is said to be a fuzzy $\mathfrak{F}^*$-null-homotopic function.

Definition 19. Any fuzzy $\mathfrak{F}^*$-structure space $(X, \mathfrak{I})$ is said to be a fuzzy $\mathfrak{F}^*$-simply connected space if it is fuzzy $\mathfrak{F}^*$-path-connected and every fuzzy $\mathfrak{F}^*$-loop in $(X, \mathfrak{I})$ is a fuzzy $\mathfrak{F}^*$-null-homotopic function.

A fuzzy $\mathfrak{F}^*$-structure space which is not fuzzy $\mathfrak{F}^*$-simply connected is said to be fuzzy $\mathfrak{F}^*$-multiply connected.

Definition 20. Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces. If the bijective function $\varphi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ and its inverse function are fuzzy $\mathfrak{F}^*$-structure continuous functions, then the function φ is said to be a fuzzy $\mathfrak{F}^*$-structure homeomorphism.

Proposition 21. *Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces and let $\varphi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be a fuzzy $\mathfrak{F}^*$-structure homeomorphism. If $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-path-connected, then $(X_2, \mathfrak{I}_2)$ is also a fuzzy $\mathfrak{F}^*$-path-connected space.*

Proof. Let $(X_1, \mathfrak{I}_1)$ be fuzzy $\mathfrak{F}^*$-path-connected and $\varphi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be a fuzzy $\mathfrak{F}^*$-structure homeomorphism. Let $(I, \mathfrak{I}(\zeta))$ be a fuzzy $\mathfrak{F}^*$-structure space introduced by (I, ζ). For $t_1, t_2 \in I$, let $x_{t_1}, y_{t_2} \in \mathcal{FP}(X)$ be any two fuzzy points. Since $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-path connected, there exists a fuzzy $\mathfrak{F}^*$-path $\alpha : (I, \tau(\zeta)) \longrightarrow (X_1, \mathfrak{I}_1)$ such that $\alpha(0) = \varphi^{-1}(x_{t_1})$ and $\alpha(1) = \varphi^{-1}(y_{t_2})$. Thus, $\varphi \circ \alpha : (I, \tau(\zeta)) \longrightarrow (X_2, \mathfrak{I}_2)$ is such that

$$\varphi \circ \alpha(0) = \varphi(\alpha(0)) = \varphi\left(\varphi^{-1}\left(x_{t_1}\right)\right) = x_{t_1}, \text{ as } \varphi \text{ is onto.} \quad (2)$$

Also, $\varphi \circ \alpha(1) = \varphi(\alpha(1)) = \varphi(\varphi^{-1}(y_{t_2})) = y_{t_2}$, as φ is onto.

Therefore, $\varphi \circ \alpha$ is a fuzzy $\mathfrak{F}^*$-path in $(X_2, \mathfrak{I}_2)$ joining from x_{t_1} to y_{t_2}. Hence, $(X_2, \mathfrak{I}_2)$ is a fuzzy $\mathfrak{F}^*$-path-connected space. □

Proposition 22. *Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces and $(I, \mathfrak{I}(\zeta))$ be a fuzzy $\mathfrak{F}^*$-structure space introduced by (I, ζ). Let $\varphi, \psi : (I, \mathfrak{I}(\zeta)) \longrightarrow (X_1, \mathfrak{I}_1)$ be any two fuzzy $\mathfrak{F}^*$-paths joining from x_{t_1} to y_{t_2} such that $\varphi \cong_{\mathfrak{P}} \psi$, where $x_{t_1}, y_{t_2} \in \mathcal{FP}(X_1)$. If $\phi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ is a fuzzy $\mathfrak{F}^*$-structure continuous function, then $\phi \circ \varphi, \phi \circ \psi : (I, \mathfrak{I}(\zeta)) \longrightarrow (X_2, \mathfrak{I}_2)$ are fuzzy $\mathfrak{F}^*$-structure continuous functions and $\phi \circ \varphi \cong_{\mathfrak{P}} \phi \circ \psi$.*

Proof. Let $(I, \mathfrak{I}(\zeta))$ be any fuzzy $\mathfrak{F}^*$-structure space introduced by (I, ζ). Since ϕ, φ, ψ are fuzzy $\mathfrak{F}^*$-structure continuous functions and by Proposition 15, $\phi \circ \varphi$, $\phi \circ \psi$ are also fuzzy $\mathfrak{F}^*$-structure continuous functions. Furthermore, $\varphi \cong_{\mathfrak{P}} \psi$ implies that there exists a fuzzy $\mathfrak{F}^*$-structure continuous function $\mathcal{G} : (I, \mathfrak{I}(\omega)) \times (I, \mathfrak{I}(\zeta)) \longrightarrow (X_1, \mathfrak{I}_1)$ such that

$$\begin{aligned} &\mathcal{G}(0, s_t) = x_{t_1} \text{ and } \mathcal{G}(1, s_t) = y_{t_2}, \text{ for each } s_t \in \mathcal{FP}(I) \text{ in } (I, \mathfrak{I}(\zeta)), \\ &\mathcal{G}(p_t, 0) = \varphi(p_t) \text{ and } \mathcal{G}(p_t, 1) = \psi(p_t), \text{ for each } p_t \in \mathcal{FP}(I) \text{ in } (I, \mathfrak{I}(\omega)), \end{aligned} \quad (3)$$

where $x_{t_1}, y_{t_2} \in \mathcal{FP}(X_1)$. Now, $\mathcal{H} : (I, \mathfrak{I}(\omega)) \times (I, \mathfrak{I}(\zeta)) \longrightarrow (X_2, \mathfrak{I}_2)$ is such that $\mathcal{H}(p_t, s_t) = \phi(\mathcal{G}(p_t, s_t))$, for each fuzzy point $p_t \in \mathcal{FP}(I)$ in $(I, \mathfrak{I}(\omega))$ and $s_t \in \mathcal{FP}(I)$ in $(I, \mathfrak{I}(\zeta))$. Since ϕ and $\mathcal{G}$ are fuzzy $\mathfrak{F}^*$-structure continuous functions, by Proposition 15, $\phi \circ \mathcal{G} = \mathcal{H}$ is also a fuzzy $\mathfrak{F}^*$-structure continuous function. Moreover, $\mathcal{H}$ satisfies the following conditions.

$$\begin{aligned} &\mathcal{H}(0, s_t) = \phi(\mathcal{G}(0, s_t)) = \phi\left(x_{t_1}\right), \\ &\mathcal{H}(1, s_t) = \phi(\mathcal{G}(1, s_t)) = \phi\left(y_{t_2}\right), \\ &\mathcal{H}(p_t, 0) = \phi(\mathcal{G}(p_t, 0)) = \phi(\varphi(p_t)) = (\phi \circ \varphi)(p_t), \\ &\mathcal{H}(p_t, 1) = \phi(\mathcal{G}(p_t, 1)) = \phi(\psi(p_t)) = (\phi \circ \psi)(p_t), \end{aligned} \quad (4)$$

for each fuzzy point $p_t \in \mathcal{FP}(I)$ in $(I, \mathfrak{I}(\omega))$ and $s_t \in \mathcal{FP}(I)$ in $(I, \mathfrak{I}(\zeta))$. Hence, $\phi \circ \varphi \cong_{\mathfrak{P}} \phi \circ \psi$. □

Proposition 23. *Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces. Let $\phi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be any fuzzy $\mathfrak{F}^*$-structure homeomorphism. Then $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-simply connected if and only if $(X_2, \mathfrak{I}_2)$ is fuzzy $\mathfrak{F}^*$-simply connected.*

Proof. Let $(I, \mathfrak{I}(\zeta))$ be any fuzzy $\mathfrak{F}^*$-structure space introduced by (I, ζ). Since $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-simply connected, by Definition 19, $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-path-connected. Let $\phi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ be a fuzzy $\mathfrak{F}^*$-structure homeomorphism. Then by Proposition 21, $(X_2, \mathfrak{I}_2)$ is fuzzy $\mathfrak{F}^*$-path-connected. Thus, it is enough to prove that every fuzzy $\mathfrak{F}^*$-loop in $(X_2, \mathfrak{I}_2)$ is fuzzy $\mathfrak{F}^*$-null-homotopic. Let $\mathrm{l} : (I, \tau(\zeta)) \longrightarrow (X_2, \mathfrak{I}_2)$ be a fuzzy $\mathfrak{F}^*$-loop in $(X_2, \mathfrak{I}_2)$. Since $(X_1, \mathfrak{I}_1)$ is fuzzy $\mathfrak{F}^*$-simply connected, $\bar{\phi} \circ \mathrm{l} : (I, \tau(\zeta)) \longrightarrow (X_1, \mathfrak{I}_1)$ is a fuzzy $\mathfrak{F}^*$-loop in $(X_1, \mathfrak{I}_1)$ which is fuzzy $\mathfrak{F}^*$-path-homotopic to some constant fuzzy $\mathfrak{F}^*$-loop $\beta : (I, \tau(\zeta)) \longrightarrow (X_1, \mathfrak{I}_1)$, i.e., $\bar{\phi} \circ \mathrm{l} \cong_{\mathfrak{P}} \beta$.

Since ϕ is a fuzzy $\mathfrak{F}^*$-structure continuous function and by Proposition 22

$$\phi \circ \left(\bar{\phi} \circ \mathrm{l}\right) \cong_{\mathfrak{P}} \phi \circ \beta, \tag{5}$$

which implies that $(\phi \circ \bar{\phi}) \circ \mathrm{l} \cong_{\mathfrak{P}} \phi \circ \beta$, by using associative property, and so $\mathcal{I}_{X_2} \circ l \cong_{\mathfrak{P}} \phi \circ \beta$, as $\mathcal{I}_{X_2}$ is an identity function.

Thus, $l \cong_{\mathfrak{P}} \phi \circ \beta$.

Also, l is fuzzy $\mathfrak{F}^*$-path-homotopic to some constant fuzzy $\mathfrak{F}^*$-loop $\phi \circ \beta$. Thus, every fuzzy $\mathfrak{F}^*$-loop l in $(X_2, \mathfrak{I}_2)$ is fuzzy $\mathfrak{F}^*$-null-homotopic. Thus, $(X_2, \mathfrak{I}_2)$ is fuzzy $\mathfrak{F}^*$-simply connected.

The converse part is also proved in the reverse direction of ϕ and $\bar{\phi}$. □

Proposition 24. *Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-structure space. Then, any fuzzy $\mathfrak{F}^*$-path-connected space $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected if and only if any two fuzzy $\mathfrak{F}^*$-paths in $(X, \mathfrak{I})$ having same endpoints are fuzzy $\mathfrak{F}^*$-path-homotopic.*

Proof. Let $(I, \mathfrak{I}(\zeta))$ be any fuzzy $\mathfrak{F}^*$-structure space introduced by (I, ζ). Let $\alpha, \beta : (I, \mathfrak{I}(\zeta)) \longrightarrow (X, \mathfrak{I})$ be any two fuzzy $\mathfrak{F}^*$-paths in $(X, \mathfrak{I})$ such that

$$\alpha(0) = \beta(0) = x_{t_1} \text{ and } \alpha(1) = \beta(1) = y_{t_2}, \tag{6}$$

where $x_{t_1}, y_{t_2} \in \mathcal{FP}(X)$. Thus, $\alpha * \bar{\beta}$ is a fuzzy $\mathfrak{F}^*$-loop based at x_{t_1}. Since $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected, every fuzzy $\mathfrak{F}^*$-loop based at x_{t_1} in $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-null-homotopic, that is

$$\alpha * \bar{\beta} \cong_{\mathfrak{P}} l_{x_{t_1}}, \tag{7}$$

which implies that $[\alpha * \bar{\beta}] = [l_{x_{t_1}}]$, then $[(\alpha * \bar{\beta}) * \beta] = [l_{x_{t_1}} * \beta]$.

Thus, $[\alpha * (\bar{\beta} * \beta)] = [\beta]$, as in (i) of Remark 16, $[\alpha * l_{yt_2}] = [\beta]$, as in (iv) of Remark 16, $[\alpha] = [\beta]$,as in (ii) of Remark 3.

Hence, $\alpha \cong_{\mathfrak{P}} \beta$.

Therefore, any two fuzzy $\mathfrak{F}^*$-paths in $(X, \mathfrak{I})$ having same endpoints are fuzzy $\mathfrak{F}^*$-path-homotopic.

Conversely, let $\alpha, \beta : (I, \mathfrak{I}(\zeta)) \longrightarrow (X, \mathfrak{I})$ be any two fuzzy $\mathfrak{F}^*$-paths in $(X, \mathfrak{I})$ such that $\alpha \cong_{\mathfrak{P}} \beta$. Thus, $\alpha \cong_{\mathfrak{P}} \beta$, $[\alpha] = [\beta]$, and $[\alpha * l_{yt_2}] = [\beta]$, as in (ii) of Remark 16.

Thus,$[(\alpha * \bar{\beta}) * \beta] = [\beta]$, as in (iv) of Remark 16, and then$[(\alpha * \bar{\beta}) * \beta] = [\mathrm{l}_{x_{t_1}} * \beta]$, as in (i) of Remark 16, which implies that$[\alpha * \bar{\beta}] = [\mathrm{l}_{x_{t_1}}]$.

Thus, $\alpha * \bar{\beta} \cong_{\mathfrak{P}} l_{x_{t_1}}$.

Therefore, for every fuzzy $\mathfrak{F}^*$-loop based at x_{t_1} in $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-null-homotopic. Thus, $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected. □

Definition 25. Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-structure space and $Y \subseteq X$. Let $(Y, \mathfrak{I}_Y)$ be a fuzzy $\mathfrak{F}^*$-structure subspace and $\mathcal{R} : (\mathrm{X}, \mathfrak{I}) \longrightarrow (\mathrm{Y}, \mathfrak{I}_{\mathrm{Y}})$ be any fuzzy $\mathfrak{F}^*$-structure continuous function. Then $(Y, \mathfrak{I}_Y)$ is said to be fuzzy $\mathfrak{F}^*$-retract of $(\mathrm{X}, \mathfrak{I})$ if there exists a fuzzy $\mathfrak{F}^*$-structure continuous function $\mathcal{R}$ such that $\mathcal{R}(x_t) = x_t$, for all $x_t \in \mathcal{FP}(Y)$. Also, the function $\mathcal{R}$ is called as fuzzy $\mathfrak{F}^*$-retraction.

Proposition 26. *Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-structure space and $Y \subseteq X$. If $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected and $(Y, \mathfrak{I}_Y)$ is the fuzzy $\mathfrak{F}^*$-retract of $(X, \mathfrak{I})$, then $(Y, \mathfrak{I}_Y)$ is also a fuzzy $\mathfrak{F}^*$-simply connected space.*

Proof. Let $(X, \mathfrak{I})$ be fuzzy $\mathfrak{F}^*$-simply connected and $(Y, \mathfrak{I}_Y)$ be the fuzzy $\mathfrak{F}^*$-retract of $(X, \mathfrak{I})$. Then $\mathcal{R} : (X, \mathfrak{I}) \longrightarrow (Y, \mathfrak{I}_Y)$ is a fuzzy $\mathfrak{F}^*$-retraction function. If $x_{t_1}, y_{t_2} \in \mathcal{FP}(Y)$ are any two fuzzy points, then there is a fuzzy $\mathfrak{F}^*$-path $\alpha : (I, \mathfrak{I}(\zeta)) \longrightarrow (X, \mathfrak{I})$ such that $\alpha(0) = x_{t_1}$ and $\alpha(1) = y_{t_2}$. Then $\mathcal{R} \circ \alpha : (I, \tau(\zeta)) \longrightarrow (Y, \mathfrak{I}_Y)$ is a fuzzy $\mathfrak{F}^*$-path in $(Y, \mathfrak{I}_Y)$ such that

$$\begin{aligned}(\mathcal{R} \circ \alpha)(0) &= \mathcal{R}(\alpha(0)) = \mathcal{R}\left(x_{t_1}\right) = x_{t_1},\\ (\mathcal{R} \circ \alpha)(1) &= \mathcal{R}(\alpha(1)) = \mathcal{R}\left(y_{t_2}\right) = y_{t_2},\end{aligned} \tag{8}$$

where $x_{t_1}, y_{t_2} \in \mathcal{FP}(Y)$. Thus, $\mathcal{R} \circ \alpha$ is a fuzzy $\mathfrak{F}^*$-path in $(Y, \mathfrak{I}_Y)$. Therefore, $(Y, \mathfrak{I}_Y)$ is fuzzy $\mathfrak{F}^*$-path-connected.

Let $\mathcal{J} : (Y, \mathfrak{I}_Y) \longrightarrow (X, \mathfrak{I})$ be the inclusion function. Thus, $\mathcal{R} \circ \mathcal{J} : (Y, \mathfrak{I}_Y) \longrightarrow (Y, \mathfrak{I}_Y)$, that is, $\mathcal{R} \circ \mathcal{J} = \mathcal{I}_Y$ where $\mathcal{I}_Y$ is an identity function on $(Y, \mathfrak{I}_Y)$. Also, let $\mathrm{l} : (I, \tau(\zeta)) \longrightarrow (Y, \mathfrak{I}_Y)$ is a fuzzy $\mathfrak{F}^*$-loop in $(Y, \mathfrak{I}_Y)$ at $x_t \in \mathcal{FP}(Y)$. Thus, $\mathcal{J} \circ \mathrm{l} : (I, \tau(\zeta)) \longrightarrow (X, \mathfrak{I})$ is a fuzzy $\mathfrak{F}^*$-loop in $(X, \mathfrak{I})$ at x_t. Since $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected, $\mathcal{J} \circ \mathrm{l}$ is fuzzy $\mathfrak{F}^*$-path homotopic in $(X, \mathfrak{I})$ to the constant fuzzy $\mathfrak{F}^*$-loop $\mathscr{C} : (I, \tau(\zeta)) \longrightarrow (X, \mathfrak{I})$ such that $\mathscr{C}(p_{t_1}) =$

s_{t_2}, for all $p_{t_1} \in \mathcal{FP}(I)$ and $s_{t_2} \in \mathcal{FP}(X)$ and hence $\mathcal{J} \circ \mathrm{l} \cong_{\mathfrak{P}} \mathcal{C}$. Also, by Proposition 22

$$\mathcal{R} \circ (\mathcal{J} \circ \mathrm{l}) \cong_{\mathfrak{P}} \mathcal{R} \circ \mathcal{C}, \tag{9}$$

implies that $(R \circ J) \circ l \cong \_P R \circ C$, and so $\mathcal{I}_Y \circ \mathrm{l} \cong_{\mathfrak{P}} \mathcal{R} \circ \mathcal{C}$.

Thus, $l \cong_{\mathfrak{P}} \mathcal{R} \circ \mathcal{C}$ and $\mathcal{R} \circ \mathcal{C}$ is a constant fuzzy $\mathfrak{F}^*$-loop (i.e.,) $\mathcal{R} \circ \mathcal{C}(p_{t_1}) = \mathcal{R}(\mathcal{C}(p_{t_1})) = \mathcal{R}(s_{t_2}) = s_{t_2}$, for all $p_{t_1} \in \mathcal{FP}(I)$. Therefore l is fuzzy $\mathfrak{F}^*$-null-homotopic in $(Y, \mathfrak{I}_Y)$. Hence, $(Y, \mathfrak{I}_Y)$ is a fuzzy $\mathfrak{F}^*$-simply connected space. □

Definition 27. Let $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ be any two fuzzy $\mathfrak{F}^*$-structure spaces and $\phi : (\mathrm{X}_1, \mathfrak{I}_1) \longrightarrow (\mathrm{X}_2, \mathfrak{I}_2)$. Any fuzzy $\mathfrak{F}^*$-structure continuous function ϕ is said to be a fuzzy $\mathfrak{F}^*$-homotopy equivalence if there exists a fuzzy $\mathfrak{F}^*$-structure continuous function $\varphi : (X_2, \mathfrak{I}_2) \longrightarrow (X_1, \mathfrak{I}_1)$ such that $\varphi \circ \phi$ is fuzzy $\mathfrak{F}^*$-homotopic to $\mathcal{I}_{X_1}$ where $\mathcal{I}_{X_1}$ is an identity function on $(X_1, \mathfrak{I}_1)$ and $\phi \circ \varphi$ is fuzzy $\mathfrak{F}^*$-homotopic to $\mathcal{I}_{X_2}$ where $\mathcal{I}_{X_2}$ is an identity function on $(X_2, \mathfrak{I}_2)$.

Also, any two fuzzy $\mathfrak{F}^*$-structure spaces $(X_1, \mathfrak{I}_1)$ and $(X_2, \mathfrak{I}_2)$ are said to be of the same fuzzy $\mathfrak{F}^*$-homotopy type if there exists a fuzzy $\mathfrak{F}^*$-structure continuous function $\phi : (X_1, \mathfrak{I}_1) \longrightarrow (X_2, \mathfrak{I}_2)$ which is fuzzy $\mathfrak{F}^*$-homotopy equivalence.

Definition 28. Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-structure space and $\mathcal{I}_\mathrm{X} : (\mathrm{X}, \mathfrak{I}) \longrightarrow (\mathrm{X}, \mathfrak{I})$ be an identity function. If $\mathcal{I}_\mathrm{X}$ is a fuzzy $\mathfrak{F}^*$-null-homotopic function, then $(\mathrm{X}, \mathfrak{I})$ is said to be a fuzzy $\mathfrak{F}^*$-contractible space.

Proposition 29. *Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-structure space. If $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-contractible, then each fuzzy $\mathfrak{F}^*$-loop α based at any fuzzy point $x_t \in \mathcal{FP}(X)$ is equivalent to the constant fuzzy $\mathfrak{F}^*$-loop l_{x_t} at x_t.*

Proof. Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-contractible space. Then $\mathcal{I}_X : (X, \mathfrak{I}) \longrightarrow (X, \mathfrak{I})$ is an identity function and it is fuzzy $\mathfrak{F}^*$-homotopic to the constant fuzzy $\mathfrak{F}^*$-loop l_{x_t} by a fuzzy $\mathfrak{F}^*$-homotopy $\mathcal{H}$. Thus there is a fuzzy $\mathfrak{F}^*$-loop δ at x_t which is defined as $\delta(s) = \mathcal{H}(x_t, s)$, where $s \in I$. Let $(\mathcal{I}_X)_\# : \pi_1((X, \mathfrak{I}), x_t) \longrightarrow \pi_1((X, \mathfrak{I}), x_t)$ and $(\mathrm{l}_{x_t})_\# : \pi_1((X, \mathfrak{I}), x_t) \longrightarrow \pi_1((X, \mathfrak{I}), x_t)$ be any two induced fuzzy $\mathfrak{F}^*$-structure homomorphisms. Also, let $\widehat{\gamma}_{\underline{\delta}} : \pi_1((X, \mathfrak{I}), x_t) \longrightarrow \pi_1((X, \mathfrak{I}), x_t)$ be defined as $\widehat{\gamma}_\delta[\alpha] = [\bar{\delta} * \alpha * \delta]$, where $[\alpha] \in \pi_1((X, \mathfrak{I}), x_t)$. Then by Proposition 17, the following Figure 2 shows that $\widehat{\gamma}_\delta \circ (\mathcal{I}_X)_\# = (\mathrm{l}_{x_t})_\#$.

For each $[\alpha] \in \pi_1((X, \mathfrak{I}), x_t)$,$\widehat{\gamma}_\delta \circ (\mathcal{I}_X)_\#[\alpha] = (\mathrm{l}_{x_t})_\#[\alpha]$,$\widehat{\gamma}_\delta[\mathcal{I}_X \circ \alpha] = [\mathrm{l}_{x_t} \circ \alpha]$,and$[\bar{\delta} * (\mathcal{I}_X \circ \alpha) * \delta] = [\mathrm{l}_{x_t}]$, as in (i) of Remark16;$[\bar{\delta} * \alpha * \delta] = [\mathrm{l}_{x_t}]$,$[\bar{\delta}] \circ [\alpha] \circ [\delta] = [\mathrm{l}_{x_t}]$,$[\alpha] \circ [\bar{\delta}] \circ [\delta] = [\mathrm{l}_{x_t}]$,and$[\alpha] \circ [\mathrm{l}_{y_t}] = [\mathrm{l}_{x_t}]$, as in (iv) of Remark16; and$[\alpha] = [\mathrm{l}_{x_t}]$, as in (ii) of Remark16,$\alpha \cong \mathrm{l}_{x_t}$.

Therefore, each fuzzy $\mathfrak{F}^*$-loop α is equivalent to the constant fuzzy $\mathfrak{F}^*$-loop l_{x_t}. □

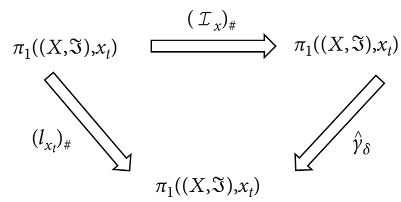

Figure 2

Proposition 30. *Every fuzzy $\mathfrak{F}^*$-contractible space is fuzzy $\mathfrak{F}^*$-simply connected.*

Proof. Let $(X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-contractible space and let l_{x_t} be the constant fuzzy $\mathfrak{F}^*$-loop at $x_t \in \mathcal{FP}(X)$. Since $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-contractible, $\mathcal{I}_X : (X, \mathfrak{I}) \longrightarrow (X, \mathfrak{I})$ is a fuzzy $\mathfrak{F}^*$-null-homotopic function by a fuzzy $\mathfrak{F}^*$-homotopy $\mathcal{H}$, so $\mathcal{H} : \mathcal{I}_X \cong \mathrm{l}_{x_t}$. Thus, the fuzzy $\mathfrak{F}^*$-homotopy $\mathcal{H}$ is defined as:

$$\mathcal{H}(y_t, 0) = y_t, \mathcal{H}(y_t, 1) = x_t, \mathcal{H}(x_t, s) = x_t, \tag{10}$$

where $x_t, y_t \in \mathcal{FP}(X)$ and $s \in I$. Let $\alpha : (I, \mathfrak{I}(\zeta)) \longrightarrow (X, \mathfrak{I})$ be a fuzzy $\mathfrak{F}^*$-path such that $\alpha(s) = \mathcal{H}(y_t, s)$. Thus, $\alpha(0) = \mathcal{H}(y_t, 0) = y_t$ and $\alpha(1) = \mathcal{H}(y_t, 1) = x_t$. Thus, α is a fuzzy $\mathfrak{F}^*$-path from y_t to x_t. Similarly, β is also fuzzy $\mathfrak{F}^*$-path from x_t to z_t where $z_t \in \mathcal{FP}(X)$. Therefore, $\alpha * \beta$ is also a fuzzy $\mathfrak{F}^*$-path from y_t to z_t. Thus, $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-path connected.

Let $[\alpha] \in \pi_1((X, \mathfrak{I}), x_t)$ and also let us define a fuzzy $\mathfrak{F}^*$-homotopy $\mathcal{G}(s, t) = \mathcal{H}(\alpha(s), t)$. Thus, $\mathcal{G}(s, 0) = \mathcal{H}(\alpha(s), 0) = \alpha(s)$ and $\mathcal{G}(s, 1) = \mathcal{H}(\alpha(s), 1) = l_{x_t} = x_t$, where l_{x_t} is a constant fuzzy $\mathfrak{F}^*$-loop at x_t. Thus, $\mathcal{G} : \alpha \cong l_{x_t}$. Thus, $[\alpha] = [l_{x_t}]$. Hence, every fuzzy $\mathfrak{F}^*$-loop in $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-null-homotopic. Therefore, $(X, \mathfrak{I})$ is fuzzy $\mathfrak{F}^*$-simply connected. Hence, every fuzzy $\mathfrak{F}^*$-contractible space is fuzzy $\mathfrak{F}^*$-simply connected. □

4. Conclusion

In this paper, the notions of fuzzy $\mathfrak{F}^*$-simply connected spaces and fuzzy $\mathfrak{F}^*$-contractible spaces are introduced, and some important characterizations related to fuzzy $\mathfrak{F}^*$-homotopy are discussed. Also it is proved that in fuzzy $\mathfrak{F}^*$-contractible space, each fuzzy $\mathfrak{F}^*$-loop based at any fuzzy point is equivalent to the constant fuzzy $\mathfrak{F}^*$-loop. This is just a beginning of studying fuzzy $\mathfrak{F}^*$-simply connected spaces. There is a huge scope of further study in extending the results of fuzzy $\mathfrak{F}^*$-homotopy to fuzzy $\mathfrak{F}^*$-covering spaces. Further using fuzzy compact and fuzzy Lindelof spaces, types of fuzzy $\mathfrak{F}^*$-homotopy can be developed.

Data Availability

No data were used to support this study.

Conflicts of Interest

All authors have declared they do not have any competing interests.

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