

ON THE STABILITY OF SUM FORM FUNCTIONAL EQUATIONS

This survey paper contains the author's results from the theory of functional equations. They are connected with the stability of certain functional equations arising in information theory.

1. INTRODUCTION

In 1940 S.M. Ulam raised the following problem in his lecture at Wisconsin University (Ulam [U64]): Let $(G, \circ)$ be a group, (H, d) be a metric group with metric d . Is the following statement true; for arbitrary $0 < \varepsilon \in \mathbb{R}$ there exists $0 < \delta \in \mathbb{R}$ such that for all functions $f: G \rightarrow H$ satisfying the inequality

$$d(f(x \circ y), f(x) \cdot f(y)) < \varepsilon, \quad x, y \in G$$

there exists a homomorphism $A: G \rightarrow H$ (that is, a solution of the so-called **Cauchy equation** $A(x \circ y) = A(x) \cdot A(y)$) satisfying the inequality

$$d(f(x), A(x)) < \delta,$$

for all $x \in G$.

Similar question can be asked in connection with other equations, too. See the following survey papers: Forti [F95], Ger [G94], Hyers-Isac-Rassias [HIR98] and Székelyhidi [S00].

We remark that Ulam did not deal with the connection between the constants ε and δ in his question. It is clear that the stability result is more informative when δ can be expressed by ε . In many cases a constant $K \in \mathbb{R}$ can be found such that $\delta = K\varepsilon$.

1.1. Notations

In the following $\mathbb{R}$ denotes the set of real numbers. Let k be a fixed positive integer. If $c \in \mathbb{R}$ then let $\underline{c} = (c, c, \dots, c) \in \mathbb{R}^k$.

If $x = (x_1, x_2, \dots, x_k) \in \mathbb{R}^k$ and $y = (y_1, y_2, \dots, y_k) \in \mathbb{R}^k$ then let

$$x \bullet y = (x_1 y_1, x_2 y_2, \dots, x_k y_k) \in \mathbb{R}^k.$$

Let $J =]0, 1[$, $J^0 =]0, 1[$, $\Lambda^k = \{(x, y) \mid x, y, x+y \in J\}$, and for all natural numbers $n \geq 2$ define the sets

$$\Gamma_n = \Gamma_n^{[k]} = \left\{ (p_1, \dots, p_n) \in J^n \mid \sum_{i=1}^n p_i = 1 \right\}$$

and

$$\Gamma_n^0 = \Gamma_n^{[d[k]]} = \left\{ (p_1, \dots, p_n) \in (J^0)^n \mid \sum_{i=1}^n p_i = 1 \right\},$$

and let $(Y, \Omega_\varepsilon) \in \{(J, \Gamma_n), (J^0, \Gamma_n^0)\}$.

A function $A: \mathbb{R}^k \rightarrow \mathbb{R}$ is **additive** if $A(x+y) = A(x) + A(y)$ for all $x, y \in \mathbb{R}^k$.

A function $M: Y \rightarrow \mathbb{R}$ is **multiplicative** on Y if $M(x \bullet y) = M(x) \cdot M(y)$ for all $x, y \in Y$. In this paper a function $M:]0, 1[\rightarrow \mathbb{R}$ is called **normal multiplicative** if M is multiplicative on $]0, 1[$ and $M(x) \neq 0$, $x \in]0, 1[$, while a function $M: [0, 1] \rightarrow \mathbb{R}$ is called **normal multiplicative** if M is multiplicative on $[0, 1]$, $M(0) = 0$, $M(1) = 1$ and $M(x) \neq 0$, $x \in]0, 1[$.

A function $L:]0, 1[\rightarrow \mathbb{R}$ is **logarithmic** on $]0, 1[$ if $L(x \cdot y) = L(x) + L(y)$ for all $x, y \in]0, 1[$, while a function $L: [0, 1] \rightarrow \mathbb{R}$ is **logarithmic** on $[0, 1]$ if $L(0) = 0$ and $L(x \cdot y) = L(x) + L(y)$ for all $x, y \in]0, 1[$.

1.2. Stability problem for sum form equations

Let $D = \{ f \mid f: Y \rightarrow \mathbb{R} \}$, $E = \{ h \mid h: Y \times Y \rightarrow \mathbb{R} \}$, $F: D \rightarrow E$ be fixed and $f \in D$. The equations investigated in this paper can be written into the form:

$$\sum_{i=1}^n \sum_{j=1}^m F(f)(p_i, q_j) = 0, \quad (p_1, \dots, p_n) \in \Omega_n, (q_1, \dots, q_m) \in \Omega_m \quad (1)$$

where $n \geq 3$ and $m \geq 3$ are fixed integers and f is unknown function. The equations having the form (1) are called **sum form functional equations**.

Following Ulam's concept the sum form functional equation (1) is said to be **stable** if for each $0 \leq \varepsilon \in \mathbb{R}$ there exists $0 \leq \delta \in \mathbb{R}$ such that for each $g \in D$ satisfying the inequality

$$\left| \sum_{i=1}^n \sum_{j=1}^m f(g)(p_i, q_j) \right| \leq \varepsilon$$

for all $(p_1, \dots, p_n) \in \Gamma_n$ and $(q_1, \dots, q_m) \in \Gamma_m$ there exists a solution f of (1) such that

$$|g(p) - f(p)| \leq \delta, \quad p \in Y.$$

We deal with the stability problem of (1) in the case $(Y, \Omega_n) = (J, \Gamma_n)$ (*closed domain case*) and $(Y, \Omega_n) = (J^\circ, \Gamma_n^\circ)$ (*open domain case*).

If $k=1$ the problem is called *one dimensional*, the (general) case $k>1$ is called *higher dimensional*.

1.3. Sum form equations arising in information theory

A sequence of functions $(I_n)_{n=2}^\infty$ where the real valued functions I_n are defined on Γ_n or Γ_n° ($n=2, 3, \dots$), is called **information measure**. (see [AD75] and [ESS98]) Define the function $\otimes: \Omega_n \times \Omega_m \rightarrow \Omega_{nm}$ by

$$P \otimes Q = (p_1 q_1, \dots, p_1 q_m, p_2 q_1, \dots, p_2 q_m, \dots, p_n q_1, \dots, p_n q_m) \in \Omega_{nm},$$

$$P = (p_1, \dots, p_n) \in \Omega_n, \quad Q = (q_1, \dots, q_m) \in \Omega_m.$$

An information measure $(I_n)_{n=2}^\infty$

is **(α, n, m) -additive** ($\alpha \in \mathbb{R}$, $n \geq 2$, $m \geq 2$ are fixed integers) if

$$I_{nm}(P \otimes Q) = I_n(P) + I_m(Q) + (2^{1-\alpha} - 1) \cdot I_n(P) \cdot I_m(Q),$$

$$P \in \Omega_n, \quad Q \in \Omega_m,$$

is **weighted (n, m) -additive of type (M_1, M_2)** ($n \geq 2$ and $m \geq 2$ are fixed integers, $M_1, M_2: Y \rightarrow \mathbb{R}$ are fixed multiplicative functions) if

$$I_{nm}(P \otimes Q) = \sum_{i=1}^n M_1(p_i) I_m(Q) + \sum_{j=1}^m M_2(q_j) I_n(P),$$

$$P \in \Omega_n, \quad Q \in \Omega_m,$$

having **sum property** if there exists a function $f: Y \rightarrow \mathbb{R}$ such that

$$I_n(p_1, \dots, p_n) = \sum_{i=1}^n f(p_i), \quad n=2, 3, \dots$$

(see [ESS98]).

Our main purpose is to prove the stability of two sum form functional equations appearing in characterization of (α, n, m) additive and weighted (n, m) -additive of type (M_1, M_2) information measures having the sum property.

The first equation we are dealing with is **BEHARA-NATH II. EQUATION**:

$$(B-N II.) \quad \sum_{i=1}^n \sum_{j=1}^m f(p_i \bullet q_j) = \sum_{i=1}^n f(p_i) \sum_{j=1}^m f(q_j)$$

where $f: Y \rightarrow R$ is an unknown function and the equation holds for all $(p_1, \dots, p_n) \in \Omega_n$ and $(q_1, \dots, q_m) \in \Omega_m$.

The second one is the **SUM FORM EQUATION OF MULTIPLICATIVE TYPE**:

$$(M) \quad \sum_{i=1}^n \sum_{j=1}^m f(p_i \bullet q_j) = \sum_{i=1}^n M_1(p_i) \sum_{j=1}^m f(q_j) + \sum_{j=1}^m M_2(q_j) \sum_{i=1}^n f(p_i)$$

where $f: Y \rightarrow R$ is unknown function, $M_1, M_2: Y \rightarrow R$ are fixed multiplicative functions and the equation holds for all $(p_1, \dots, p_n) \in \Omega_n$ and $(q_1, \dots, q_m) \in \Omega_m$.

2. STABILITY ON CLOSED DOMAIN

2.1. Stability of a simple sum form equation

In the investigation of the sum form equations the simple sum form functional equation

$$\sum_{i=1}^n \varphi(p_i) = d \quad (S)$$

where $\varphi: Y \rightarrow R$ is unknown function, $d \in R$ is fixed and (S) holds for all $(p_1, \dots, p_n) \in \Omega_n$, plays central role.

In the one dimensional case the stability of equation (S) was shown by Gy. Maksa in [M94] on closed domain.

THEOREM II.1.1. (Maksa [M94]) Let $0 \leq \varepsilon \in R$ and $d \in R$ be fixed and $\varphi: [0, 1] \rightarrow R$. If

$$\left| \sum_{i=1}^n \varphi(p_i) - d \right| \leq \varepsilon \quad (2)$$

holds for all $(p_1, \dots, p_n) \in \Gamma_n^{(1)}$ then there exists an additive function $a: R \rightarrow R$ and a function $b: [0, 1] \rightarrow R$ such that

$$\text{and} \quad \varphi(x) - \varphi(0) = a(x) + b(x), \quad x \in [0,1]$$

$$|b(x)| \leq 18 \cdot \varepsilon, \quad x \in [0,1].$$

A basic tool in the proof of Theorem II.1.1. is the stability of the Cauchy equation on Δ^1 :

THEOREM II.1.2. (Maksa [M94]) Let $0 \leq \varepsilon \in \mathbb{R}$ be fixed and $f: [0,1] \rightarrow \mathbb{R}$. If f is ε -additive on Δ^1 , that is,

$$|f(x+y) - f(x) - f(y)| \leq \varepsilon, \quad (x,y) \in \Delta^1 \quad (3)$$

then there exists an additive function $a: \mathbb{R} \rightarrow \mathbb{R}$ and a function $b: [0,1] \rightarrow \mathbb{R}$ such that

$$f(x) = a(x) + b(x), \quad x \in [0,1]$$

and

$$|b(x)| \leq 9 \cdot \varepsilon, \quad x \in [0,1].$$

To prove Theorem II.1.5. the following generalization of Theorem II.1.2. is needed.

THEOREM II.1.3. (Kocsis, [K]) Let $0 \leq \varepsilon \in \mathbb{R}$ be fixed and $f: J \rightarrow \mathbb{R}$. Suppose that (2) holds for all $(x,y) \in \Delta^k$. Then there exists an additive function $a: \mathbb{R}^k \rightarrow \mathbb{R}$ and a function $b: J \rightarrow \mathbb{R}$ such that

$$f(x) = a(x) + b(x), \quad x \in J$$

and

$$|b(x)| \leq 3^{k+2} \cdot \varepsilon, \quad x \in J.$$

Let $\emptyset \neq D \subset \mathbb{R}^k \times \mathbb{R}^k$,

$$D_x = \{x | \exists y: (x,y) \in D\},$$

$$D_y = \{y | \exists x: (x,y) \in D\},$$

$$D_{x+y} = \{z | \exists (x,y) \in D: z = x+y\},$$

$$I = D_x \cup D_y \cup D_{x+y}$$

and $f: I \rightarrow \mathbb{R}$. A function f is additive on D if the Cauchy equation holds for all $(x,y) \in D$ (see [DL67]).

Furthermore D is of uniquely additive type (Z. Daróczy, 1996), if the following statement is true: if the function f is additive on D then there exist a unique additive function A such that f is a restriction of A to I .

COLLORARY II.1.4. The set Δ^k ($k \geq 1$ is fixed integer) is of uniquely additive type, that is, if a function $f: J \rightarrow R$ is additive on Δ^k then there exists a unique additive function $A: R^k \rightarrow R^k$ such that f is the restriction of A to J . (Indeed, let $c=0$ in Theorem II.1.3.) (see also [ESS98])

The following theorem says that equation (1) is stable if $Y=J$, $\Omega_n = I_m$, and

$$F(f)(p, q) = \frac{1}{m} f(p), \quad (p, q) \in J^2,$$

that is, equation (S) is stable in the higher dimensional closed domain case, too.

THEOREM II.1.5. (Kocsis, [K]) Let $n \geq 3$ be fixed integer, $0 \leq \varepsilon \in R$ and $d \in R$ be fixed, and $\varphi: J \rightarrow R$. If (2) holds for all $(p_1, \dots, p_n) \in I_m^n$, then there exists an additive function $a: R^k \rightarrow R$ and a function $b: J \rightarrow R$ such that

$$\varphi(x) - \varphi(0) = a(x) + b(x), \quad x \in J$$

and

$$|b(x)| \leq 2 \cdot 3^{k+2} \cdot \varepsilon, \quad x \in J.$$

2.2. Stability of the sum form equation of multiplicative type in the one dimensional closed domain case

The following theorem says that equation (1) is stable if $Y=[0,1]$, $\Omega_n = I_n^{[1]}$, $\Omega_m = I_m^{[1]}$, $M_1, M_2: [0,1] \rightarrow R$ are fixed normal multiplicative functions (one of them is different from the identity function of $[0,1]$) and $F(f)(p, q) = f(p \cdot q) = M_1(p) \cdot f(q) = f(p) \cdot M_2(q)$, $(p, q) \in [0,1]^2$, that is, equation (M) is stable in the one dimensional closed domain case.

THEOREM II.2.1. (Kocsis-Maksa [KM98], Kocsis [K01]) Let $n \geq 3$ and $m \geq 3$ be fixed integers, $0 \leq \varepsilon \in R$ be fixed, $M_1, M_2: [0,1] \rightarrow R$ be fixed normal multiplicative functions, one of them is different from the identity function of $[0,1]$, and let $f: [0,1] \rightarrow R$. If

$$\left| \sum_{i=1}^n \sum_{j=1}^m f(p_i q_j) - \sum_{i=1}^n M_1(p_i) \sum_{j=1}^m f(q_j) - \sum_{j=1}^m M_2(q_j) \sum_{i=1}^n f(p_i) \right| \leq \varepsilon,$$

holds for all $(p_1, \dots, p_n) \in I_n^{[1]}$ and $(q_1, \dots, q_m) \in I_m^{[1]}$ then there exist additive functions $a_1, a_2: R \rightarrow R$, a logarithmic function $L: [0,1] \rightarrow R$, bounded functions $B_1, B_2: [0,1] \rightarrow R$ and a constant $C \in R$ such that

$$\begin{aligned} f(p) &= a_1(p) + C(M_1(p) - M_2(p)) + B_1(p), \quad p \in [0,1], \text{ if } M_1 \neq M_2, \\ f(p) &= a_2(p) + M_1(p)L(p) + B_2(p), \quad p \in [0,1], \text{ if } M_1 = M_2. \end{aligned}$$

2.3. Stability of Behara-Nath II. equation in the higher dimensional closed domain case

The following theorem says that equation (1) is stable if $Y=J$, $\Omega_n=\Gamma_n$, $\Omega_m=\Gamma_m$ and $f(f(p,q) - f(p \bullet q) - f(p) \cdot f(q))$, $(p,q) \in J^2$, that is, equation (B-N II.) is stable in the higher dimensional closed domain case.

In the one dimensional case the stability of equation (B-N II.) was proved by Gy. Maksa in [M94] on closed domain.

THEOREM II.3.1. (Maksa [M94]) Let $n \geq 3$ and $m \geq 3$ be fixed integers, $0 \leq \varepsilon \in \mathbb{R}$ be fixed and $f: [0,1] \rightarrow \mathbb{R}$. If

$$\left| \sum_{i=1}^n \sum_{j=1}^m f(p_i q_j) - \sum_{i=1}^n f(p_i) \sum_{j=1}^m f(q_j) \right| \leq \varepsilon$$

holds for all $(p_1, \dots, p_n) \in \Gamma_n^{[1]}$ and $(q_1, \dots, q_m) \in \Gamma_m^{[1]}$ then either

$$f(p) = a_1(p) + b(p), \quad p \in [0,1],$$

or

$$f(p) = a_2(p) + M(p) + f(0), \quad p \in [0,1],$$

where $a_1, a_2: \mathbb{R} \rightarrow \mathbb{R}$ are additive, $M: [0,1] \rightarrow \mathbb{R}$ is a multiplicative, $b: [0,1] \rightarrow \mathbb{R}$ is a bounded function.

The earlier results connected with the stability problem of sum form functional equations (for example Theorem II.2.1. and II.3.1.) can be applied in the closed domain case only to study our equations.

By Theorem II.1.5., we can extend the investigation to higher dimension. This theorem is a basic tool in the proof of the following theorem, too.

THEOREM II.3.2. (Kocsis, [K]) Let $n \geq 3$ and $m \geq 3$ be fixed integers, $0 \leq \varepsilon \in \mathbb{R}$ be fixed and $f: J \rightarrow \mathbb{R}$. If

$$\left| \sum_{i=1}^n \sum_{j=1}^m f(p_i \bullet q_j) - \sum_{i=1}^n f(p_i) \sum_{j=1}^m f(q_j) \right| \leq \varepsilon$$

holds for all $(p_1, \dots, p_n) \in \Gamma_n$ and $(q_1, \dots, q_m) \in \Gamma_m$ then either

$$f(p) = a_1(p) + B_1(p), \quad p \in J,$$

or

$$f(p) = a_2(p) + M(p) + B_2(p), \quad p \in J$$

where $a_1, a_2: \mathbb{R}^k \rightarrow \mathbb{R}$ are additive, $M: J \rightarrow \mathbb{R}$ is multiplicative, and $B_1, B_2: J \rightarrow \mathbb{R}$ are bounded functions.

The connection between a functional equation and the related functional inequality can be describe perfectly if we know the general solution of the

equation and the inequality, too. However we have to remark that – by our definition – the stability problems can be solved without knowing of the explicit form of the general solutions.

In the one dimensional case the general solution of equations (B-N II.), (M), and (S) is known both on closed and on open domain.

The general solution of equation (B-N II.) under the conditions supposed in paragraph II.3., that is, in the higher dimensional closed domain case, is not known if $k > 2$. The general solution in the two dimensional case is given by the author (this result is unpublished yet).

THEOREM II.3.3. (Kocsis) Let $n \geq 3$ and $m > 3$ are fixed integers and $k=2$. The general solution of equation (B-N II.) is

$$f(p) = A_1(p) + b, \quad p \in [0,1]^2,$$

or

$$f(p) = A_2(p) + M(p), \quad p \in [0,1]^2,$$

where $A_1, A_2: \mathbb{R}^2 \rightarrow \mathbb{R}$ are additive, $M: [0,1]^2 \rightarrow \mathbb{R}$ is multiplicative function, furthermore $A_2(1) = 0$ and $A_2(1) + mb = (A_1(1) + nb) \cdot (A_1(1) + mb)$.

3. STABILITY ON OPEN DOMAIN

3.1. The stability of equation (S) in the one dimensional open domain case

The following theorem says that equation (1) is stable if $Y =]0,1[$, $\Omega_n = \Gamma_n^{(1)}$, and

$$F(f)(p,q) = \frac{1}{m} f(p), \quad (p,q) \in]0,1]^2,$$

that is, equation (S) is stable in the one dimensional open domain case.

The methods used in the closed domain case are usually not applicable in the open domain case, but certain results proved on closed domain are also true on open domain. Such a result is formulated in Theorem III.1.3.

By this theorem, we can extend our investigations to the open domain case.

A basic tool in the proof of Theorem III.1.3. is the following result on the stability of Cauchy equation on an open set.

LEMMA III.1.1. (Kocsis, [K00]) Let $0 \leq \varepsilon \in \mathbb{R}$, $b, c \in \mathbb{R}$ ($b > 0$, $c > 0$) be fixed and $f:]-2b, 2c[\rightarrow \mathbb{R}$. If f is ε -additive on $] -b, c[$, that is,

$$|f(x+y) - f(x) - f(y)| \leq \varepsilon$$

holds for all $(x,y) \in] -b, c[$ then there exists an additive function $A: \mathbb{R} \rightarrow \mathbb{R}$

such that

$$|f(x) - A(x)| \leq 5\varepsilon, \quad x \in]-2b, 2c[.$$

COLLORARY III.1.2. The set $] -b, c[$ ($b, c \in \mathbb{R}$, $b > 0$, $c > 0$) is uniquely additive type, that is, if $f:]-2b, 2c[\rightarrow \mathbb{R}$ is additive on $] -b, c[$ then there exists a unique additive function $A: \mathbb{R} \rightarrow \mathbb{R}$ such that f is the restriction of A to $] -2b, 2c[$. (Indeed, let $\varepsilon = 0$ in Lemma III.1.1.) (see also [DL67], [S72])

THEOREM III.1.3. (Kocsis, [K00]) Let $n \geq 3$ be fixed integer, $0 \leq \varepsilon \in \mathbb{R}$, and $d \in \mathbb{R}$ be fixed, and $\varphi:]0, 1[\rightarrow \mathbb{R}$. If (2) holds for all $(p_1, \dots, p_n) \in \Gamma_n^{(0,1)}$ then there exists an additive function $a: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\left| \varphi(p) - a(p) + \frac{a(1) - d}{n} \right| \leq 240 \cdot \varepsilon, \quad p \in]0, 1[.$$

3.2. Stability of the equation of multiplicative type in the one dimensional open domain case

The following theorem says that equation (1) is stable if $Y =]0, 1[$, $n = m$, $\Omega_n = \Gamma_n^{(0,1)}$, $M_1, M_2:]0, 1[\rightarrow \mathbb{R}$ are fixed normal multiplicative functions, $M_1 \neq M_2$ and $F(f)(p, q) = f(p \bullet q) = M_1(p) \cdot f(q) = f(p) \cdot M_2(q)$, $(p, q) \in]0, 1[$, that is, equation (M) is stable in the one dimensional open domain case if $n = m$ and $M_1 \neq M_2$.

THEOREM III.2.1. (Kocsis, [K00]) Let $m \geq 3$ be fixed integer, $\varepsilon \geq 0$ be fixed, $M_1, M_2:]0, 1[\rightarrow \mathbb{R}$ be fixed normal multiplicative functions, $M_1 \neq M_2$ and let $f:]0, 1[\rightarrow \mathbb{R}$. If

$$\left| \sum_{i=1}^m \sum_{j=1}^m f(p_i, q_j) - \sum_{i=1}^m M_1(p_i) \sum_{j=1}^m f(q_j) - \sum_{j=1}^m M_2(q_j) \sum_{i=1}^m f(p_i) \right| \leq \varepsilon$$

holds for all $(p_1, \dots, p_m) \in \Gamma_m^{(0,1)}$ and $(q_1, \dots, q_m) \in \Gamma_m^{(0,1)}$ then there exist an additive function $a: \mathbb{R} \rightarrow \mathbb{R}$, a bounded function $B:]0, 1[\rightarrow \mathbb{R}$, and a constant $c \in \mathbb{R}$, such that

$$f(p) = a(p) + C(M_1(p) - M_2(p)) + B(p), \quad p \in]0, 1[.$$

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ÖSSZEG ALAKÚ FÜGGVÉNYEGYENLETEK STABILITÁSA

Ez az összefoglaló dolgozat a függvényegyenletek témakörében – a matematikának a XX. század második felében intenzív fejlődésnek indult területén – tartalmaz új eredményeket. Ezek az eredmények olyan függvényegyenletekhez kapcsolódnak, amelyek információ-elméleti eredetűek, úgynevezett információ-mértékek előállításához kötődnek.

Azt vizsgáljuk, hogy ezek az egyenletek rendelkeznek-e egy meghatározott – S.M. Ulam által bevezetett – stabilitási tulajdonsággal. Defináljuk az összeg alakú függvényegyenlet fogalmát, valamint az erre vonatkozó stabilitási problémát. Felírjuk a Behara-Nath II. egyenletet, valamint a multiplikatív típusú összeg alakú függvényegyenletet, és ezekre az egyenletekre vonatkozó stabilitási eredményeket mutatunk be különböző feltételek mellett.