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Applied real-valued genetic algorithm for an extended model of economic lot, purchase and delivery scheduling problem

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ABSTRACT

Supply chain management intends to integrate supply chains' activities such as material flow, information flow and financial issues. Material flow management is the most significant issue since the inventory level in the whole supply chain could be optimized by an integrated plan. In other words, when one member of the supply chain plans to reduce its inventory level solely, despite reducing inventory in this node the inventory will be stocked in other partners' warehouses. Therefore, in this paper a new mathematical model has been developed to facilitate the process of finding the optimum solution in economic production, purchase and delivery lots and their schedules in a three-echelon supply chain environment; including raw material in suppliers, manufacturer and assembly facility as a customer. The manufacturer with a flow shop system provides its requirements from supplier, assemble multiple products, and delivers products to the customer (automotive OEM alike) on an optimum multiple delivery points. The delivery cycles would be identified through the production common cycle regarding the supply chain flexibility. Finally, a modified real-valued Genetic Algorithm (MRGA), and an Optimal Enumeration Method (OEM) are developed, and some numerical experiments have been done and compared as well.

KEYWORDS

supply chain, common cycle, economic lot and scheduling problem, flow shop system, real-valued GA

1. INTRODUCTION

Recently, it has been well perceived by individual businesses to work together as a supply chain in contrast with their conventional competitions of those as solely autonomous entities. Supply chain management has been developed to integrate the related activities such as material flow, information flow and financial issues in order to obtain reliable and long-term competitive advantages. Material flow management is the most significant issue in the supply chain concept. Whereas, the inventory level in the whole supply chain can be optimized by coordinated planning. When one member of the supply chain intends to reduce its inventory level solely, consequently the supply chain's inventory management level will be weakened. To put it another way, while the inventory of one node of a supply chain is reduced, this inventory will be stocked in other suppliers or distributor warehouses. Hence, an integrated plan is critical to schedule all production and deliveries in accordance with customer's demands.

Recently, the ELSP (Economic Lot Scheduling Problem) has been extended to the supply chain as ELDSP (Economic Lot and Delivery Problem). By optimizing production and delivery schedule, the supply chain's players will be harmonized and accordingly, inventory level will be minimized. Regarding the importance of coordination in the whole supply chain

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and high effect on competitive advantage, this problem has become popular in the academic area. Consequently, many researchers have paid attention to material flow management throughout the supply chain.

In this paper, it is desired to apply the ELSP model in a three-level supply chain which the manufacturer works in multi stages-multi products manner, a case that usually occurs in the real world. The key challenge is the changes, which should be applied in traditional ELSP to handle this environment and deliver a product with lower volume and high frequency. Concerning this issue, a new mathematical model has been developed to find optimum economic production, purchase and delivery lots and schedules in a three-echelon supply chain environment. Three-echelon supply chain consists of raw material supplier, manufacturer (could be automotive part supplier) and assembly facility (OEM) as a customer. The manufacturer provides its requirements from its supplier, transforms them into multiple products, and delivers products to customers on optimum delivery points.

2. MATERIAL AND METHODS

Economic Lot and scheduling Problem was presented by Rogers in 1958 [1]. He assumed constant and continuous demand for each product, single stage production system and production capacity was considered finite. The ELSP is an important problem in scheduling which has been enormously considered by researchers during the last decades and due to its complexity, it is known as NP-hard problem demonstrated by Hsu in 1983 [2]. Santander-Mercado and Maria Jubiz-Diaz [3] surveyed ELSP and classified this problem in different aspects. They divided the problem into single facility and multi facility, based on the production system and regarding the scheduling approach, three different approaches were introduced: Common cycle approach, Basic/extended basic approach and Time-varying lot size approach. Huang, Yao [4], Ouenniche, and Boctor [5], each proposed two new heuristics approaches to solve the mathematical model for the ELSP with a flow shop system as a multi-product, multi-stage, economic lot-sizing problem. Recently, ELSP has been extended to sustainability challenges, Beck et al. [6] studied the effects of energy usage on ELSP with different approaches. Ferretti [7] considers ELSP with returns and sorting line regarding environmental legislations. Furthermore, Alaghebandha et al. [8] have explored ELSP to new branch and consider distributed flow shop in multi factories. As industrial competition increased drastically, it would be widely accepted that the competition is not just among companies and it is rival among supply chains. As ELSP entered in supply chain environment, it was named Economic Lot and Delivery Scheduling Problem. In the problem there is a simple supply chain consisting of one supplier and one customer, the supplier is in charge of producing some products and delivers them to the customer. This problem aims to optimize production and delivery lots and their schedule from supplier to the customer.

Hahm and Yano [9, 10] have conducted some studies on Two-level supply chain that established the relationship between one supplier and one customer. Moreover, they developed two heuristic algorithms under common cycle policy. In the first algorithm, production and delivery cycles are the same but in the second one there were multi deliveries during a production cycle. Torabi et al. [11, 12] developed a new mathematical model in flexible flow shop and job shop manufacturing system with an assumption of constant demand rate and finite planning horizon. They presented two effective algorithms; Optimal Enumeration Method and algorithm genetic as a meta-heuristic method. Whereas, both production and delivery cycles are assumed equivalent. Furthermore, Torabi and Jenabi [13] extended the two-echelon model under two variants of the basic period approach and considered the parallel machines in each stage in the flow shop system. Finally, two efficient hybrid genetic algorithms have been applied. Lee et al. [14] developed a new model with infinite planning horizon. They considered a manufacturer who has a single-stage production system. The manufacturer purchases the required raw material from one supplier and after transformation to final product, delivers them to the customer. They upgraded a broader supply chain and considered the process of raw material purchase. Kim et al. [15] studied a three-level supply chain including a raw material supplier, manufacturer and multi customers. The manufacturer works with single stage-multi product system under a common cycle policy and all products have a common raw material. They introduced an efficient heuristic algorithm as a solution as well.

Sarker and Diponegoro [16] developed an optimal policy in three-level supply chain environment consisting of multi-suppliers, a manufacturer with single-facility single-product system and multi-buyers. The number of production cycles was given but their lengths were not necessarily the same and the manufacturer delivers the product to the buyers in fixed intervals and lots. As a solution method, an analytical method was derived to ensure higher potential to save cost in a supply chain. Jaber and Goyal [17] developed a mathematical model in a three-level supply chain with multiple suppliers, a manufacturer and multiple buyers. In their research, the manufacturer buys the raw material and delivers the finished product to the buyer in a cyclic manner. The manufacturer's production system was a single product-single machine in infinite planning horizon in which the product requires k items from m suppliers. Eventually, a centralized decision process as an effective solution has been proposed. Osman and Demirli [18] proposed a new three-level supply chain model, in which two of three stages consist of multiple suppliers and the third stage is a single assembly facility. They employed integer multipliers policy instead of the common cycle time. The inventory profile of the finished products in the assembly facility is not considered in the model and components at each supplier node are produced on a single-stage production system. In addition, all produced components are shipped in one lot at the end of the production cycle and the production setup cost and



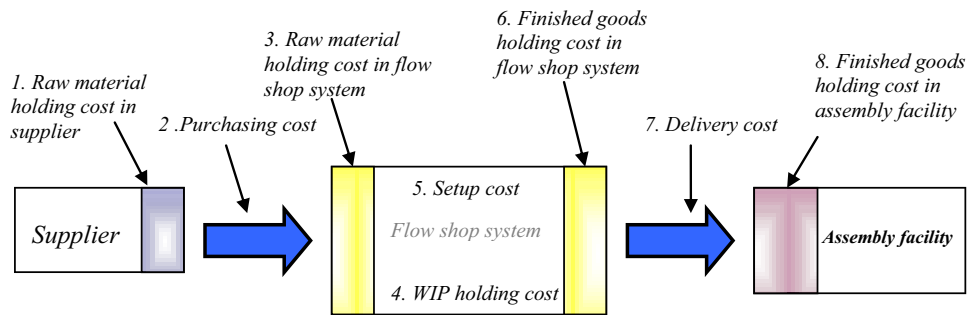


Fig. 1. Costs in considered supply chain

times at the assembly facility are neglected. In the end, a hybrid algorithm is applied to solve the developed nonlinear model. Goli and Alinaghian [19] have developed a new mathematical model where a manufacturer with an open-shop system procured raw materials from suppliers and after conversion into the final product transfer them to packaging companies. El-meehy et al. [20] developed a new mathematical model of incapacitated three-echelon supply chain and considered two raw material suppliers and two products. Finally, the model has been solved by Lingo and multiple samples by various capacities and has been experimented.

To the best of our knowledge, rare studies have been done to apply coordination in a three-level supply chain in spite of its high priority. Therefore, in this paper a new mathematical model is developed. There is a manufacturer with finite planning horizon and multi stage-multi product production system, which procures its optimum quantity of raw material from suppliers and, after assembling finished goods by flow shop system, delivers them to customers in accordance with optimum delivery cycles. The production system consists of production cost, work in process (WIP) holding cost, setup cost, purchase cost, raw material holding cost and finished goods delivery cost. As solution methodologies, an Optimal Enumeration Method (OEM) as heuristics method and a Modified Real-valued Genetic Algorithm (MRGA) as a metaheuristics method are proposed. In addition, the numerical examples are presented.

The remainder of this paper is organized as follows. In Section 3, a mathematical model is described. Section 4 proposes solution methods MRGA & OEM. Finally, in the last section numerical examples, conclusion and future study are provided.

In this paper, a new mathematical model is developed which shows a manufacturer with multi stage - multi product system and finite horizon time. The manufacturer procures its requirements from multiple suppliers and after transforming them to final product by flow shop system, delivers them to a customer. The objective function in this supply chain consists of eight costs while each of them would affect the model's output. The cost functions in the whole supply chain are:

1. Raw material holding cost in suppliers' warehouse
2. Raw material purchasing cost (consists of transportation cost except raw material price)

3. Raw material holding cost in flow shop system
4. Work in Process holding cost
5. Setup cost
6. Finished goods holding cost in flow shop system
7. Finished goods delivery cost
8. Finished goods holding cost in assembly facility warehouse

All costs are illustrated in Fig. 1.

The common cycle policy is applied in this model (i.e., the cycle times of all products are equal and there is a common cycle for all products). Therefore, an optimum common cycle time (T^*) should be calculated for the first step and, next step the production lot size of each product will be derived ($Q_i = D_i T^*$). Consequently, the purchase common cycle should be determined in a way that one lot of each product will be purchased in it. According to type of products in the automotive industry and associated logistics cost, they are usually provided more than the requirement of one production cycle. Hence, it may be assumed that the purchase common cycle is an integer multiple of the production common cycle ($B = e.T$).

It is supposed that each production common cycle (T) can consist of multiple delivery cycles (D), in order to decrease the finished goods holding cost at the end of flow shop system. The lots of products that have been finished before each delivery point are delivered to the customer. Otherwise, they should wait until the next delivery point. The relationship between production and delivery cycle is $D^* = T^*/k$ (k is an integer).

The delivered product lot to a customer can cover its requirement for period T . So during the production common cycle the products are delivered to the customer based on production sequence. In other words, the type of product would be different in each delivery and this delivery sequence duplicates in the next cycles. In Fig. 2, the delivery points of products are depicted in a case that the production sequence is (2-1-3-4) with three delivery cycles in a production cycle ($k = 3$). Products 1 and 2 are delivered at the first delivery point while products 3 and 4 are delivered at the second and third points, respectively. It is noted that product 4 is delivered to the customer at the third delivery point because its lot was not prepared at the second point and this sequence will be repeated in the next steps.

There is the same policy to identify purchase lot size (B^*). It is desirable to find a common purchase cycle in which just

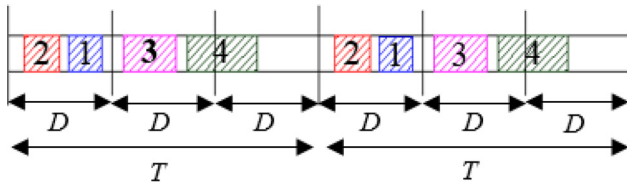


Fig. 2. Delivery cycle scheme

one lot of each product is purchased. In accordance with this type of industry, such as automotive manufacturers, the raw materials are provided more than the requirement of one production cycle. Furthermore, to simplify the problem we assume that the purchase cycle is an integer multiple of production cycle time ($B = e.T$). In the next section, the mathematical model is developed. The assumption, parameters and variables are described below:

Problem assumptions:

- Each component should be processed by one machine at each stage.
- The lot size of each component is equal in all stages.
- Machines are persistently available and each one can only process one component at a given time.
- Backlogging is not allowed.
- Setup times and costs are considered in flow shop system.
- Setup times and costs for AF are negligible.
- Delivery lead-time is assumed zero.
- The sequence of Production for each machine at each stage is unique and is optimized by the model.
- Both supplier and assembly facility have linear inventory holding costs on finished goods.
- The supplier has linear inventory holding costs on WIP components.
- Preemption is not allowed.
- Lot splitting is not allowed.
- There are unlimited buffers between successive stages and in-process inventories are allowed,
- The capacity of the production system is sufficient to meet the demands.
- Zero-switch rule is used. This means that the production of each product in each cycle begins when its inventory level reaches zero.
- The lots of products are delivered at the end of each delivery cycle.
- Raw material of each component is independent.
- Raw material purchase cycle for all components is constant and is determined by the solution method.
- The order cost for all materials is equal.
- Finished goods demands are constant and continuous.
- The production system is flow shop, semi-finished products are passing several sequential stages and there is a single machine at each stage.

Parameters:

- n : number of components
- m : number of stages
- i, u : component indices
- j : stage index

- d_i : Demand rate of component i
- R : Order Cost of the component
- p_{ij} : Production rate of component i at stage j .
- p_{im} : Production rate at last stage.
- s_{ij} : Sequence-independent setup time of component i at stage j .
- sc_i : Total setup costs of component i over all stages.
- h_{ij} : time-dependent inventory holding cost per unit of component i between stages j and $j + 1$
- h_i : Inventory holding cost (for both supplier and assembler) per unit of final component i per unit time
- h_{i0} : Inventory holding cost per unit of raw material of component i per unit time
- A : transportation cost per delivery
- M : A large real number Decision variables
- T : Common production cycle length (time interval between setups)

b_{ij} : Process beginning time of component i at stage j (after related setup operation)

Since processing each component at each stage, a value is supposed to be added for the given component, the set of h_{ij} makes an increasing series, that is

$$h_i > h_{ij}; h_{ij} > h_{i,j-1} \quad i = 1, \dots, n \quad j = 2, \dots, m-1 \quad (1)$$

Decision variables:

- σ_j : Production sequence vector at stage j
- T : Production common cycle length (time interval between setups)
- Q_i : Production lot size of component i at different stages ($Q_i = d_i \cdot T$)
- F : The number of production cycles over the planning horizon
- b_{ij} : Process beginning time of component i at stage j (after related setup operation)
- B : Raw material common purchasing cycle length that is considered greater than or equal to common production cycle length ($B = e.T$; e is an integer number)
- α_i : The delivery point number in which the component i is delivered to the customer.

The problem (Problem P) can be formulated as a mixed integer nonlinear programming. As mentioned earlier, to formulate this problem, the common cycle policy has been applied. The objective of Problem P is to minimize the average of transportation, setup, work-in-process and raw material and end component inventory holding costs per unit time for this simple supply chain. It goes without saying that the average setup cost per unit time is $\sum_{i=1}^n sc_i / T$ [12] and the average delivery cost per unit time is $A.k/T$ in the objective function. The inventory holding costs are somewhat more complicated. It is obvious that the inventory holding costs are incurred at all three levels in the supply chain, the raw material suppliers, manufacturer and the assembly facility. Figure 3 shows the inventory level of finished component i at the assembly facility. According to Fig. 3, the total cost of inventory of component i per unit time at the assembly facility is $\sum_{i=1}^n \frac{d_i \cdot T \cdot h_i}{2}$ [11].



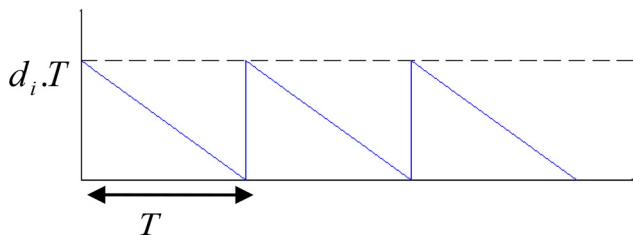


Fig. 3. Finished goods inventory level in AF

Two types of inventory are considered on the manufacturer's side: work-in-process inventory and finished goods inventory. Figures 4 and 5 show the work-in-process inventory of component *i* between two successive stages *j*-1 and *j*, and the inventory level of finished component *i*, respectively. In Fig. 4, the average WIP inventory of component *i* between two successive stages *j*-1 and *j* per unit time is as below [11]:

$$TC_{wip} = \sum_{i=1}^n \sum_{j=2}^m h_{i,j-1} \cdot d_i \cdot \left(b_{ij} + \frac{d_i \cdot T}{2p_{ij}} - b_{i,j-1} - \frac{d_i \cdot T}{2p_{i,j-1}} \right) \quad (2)$$

The finished goods inventory level in the manufacturer is depicted in Fig. 5. The producing process of semi-finished product *i* is begun at *b_{im}* in stage *m* by production rate *p_{im}*.

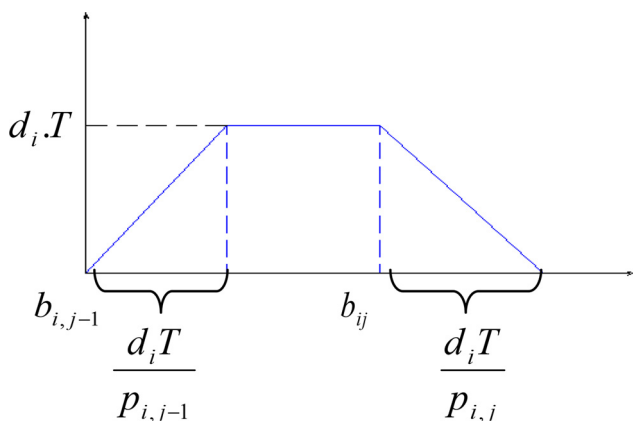


Fig. 4. Work-in-process inventory level

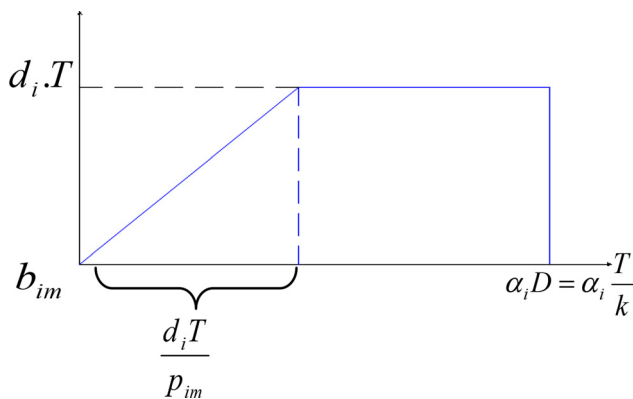


Fig. 5. Finished goods inventory level in manufacturer's site

As soon as passing *t_{im} = d_iT/p_{im}* period, the inventory level reaches *d_i.T*. Finally, inventory level vanishes as soon as production lot has been delivered to the customer at the end of the cycle.

Consequently, the total inventory holding cost of finished goods in manufacturer's site is depicted in Eq. (3) which "INT" refers to the integer part of a number in this equation.

$$TC_{FI} = \sum_{i=1}^n h_{im} \cdot d_i \cdot \left\{ \frac{d_i \cdot T}{2p_{im}} + \left(\frac{T}{k} \left(1 + INT \left(\frac{b_{im} + \frac{d_i T}{p_{im}}}{\frac{T}{k}} \right) \right) - \left(b_{im} + \frac{d_i T}{p_{im}} \right) \right) \right\} \\ = \sum_{i=1}^n h_{im} \cdot d_i \cdot T \cdot \left\{ \frac{1}{k} - \left(\frac{d_i}{2 \cdot p_{im}} \right) \right\} - \sum_{i=1}^n h_{im} \cdot d_i \cdot b_{im} \\ + \sum_{i=1}^n h_{im} \cdot d_i \cdot \frac{T}{k} \cdot INT \left(\left(\frac{k \cdot b_{im}}{T} \right) + \left(\frac{k \cdot d_i}{p_{im}} \right) \right) \quad (3)$$

By having *D*, *T* and *b_{ij}* of each item, the coefficient *α_i* as a delivery point, will be obtained through "while cycle" as below:

For *i = 1 to n*

α = 1

While *αD - [b_{im} + d_iT/p_{im}] < 0 do*

α = α + 1

End while

α_i = α

Next *i*

As it is depicted in Fig. 6, one lot of raw material of item *i* with value of *e.d_i.T* (that is equal to the requirement of *e* production cycles) is entered into the manufacturer's raw material warehouse. At *t = 0*, the inventory level of item *i* is *e.d_i.T* and at the end of each production cycle, it decreases by *d_i.T*. This inventory is reducing in every cycle with velocity of *p_{i1}* (production rate of the first stage in flow shop system) and finally, after *e* cycle, the inventory level would be zero. The average inventory of an item is calculated as below. Whereas, the first part of the formulation is the area of trapezoid and the second one is for the area of rectangle in each cycle *T*.

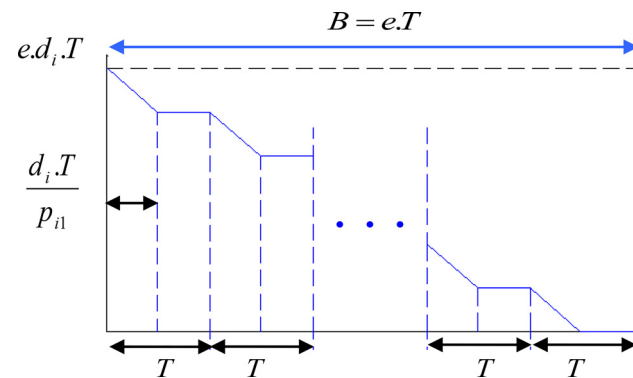


Fig. 6. Raw material inventory level in manufacturer's site



$$I_{RI} = \frac{1}{e \cdot T} \left\{ \sum_{v=1}^e \left[((e+1-v)d_i T + (e-v)d_i T) \cdot \frac{d_i T}{2p_{i1}} \right] + \sum_{v=1}^e \left[(e-v)d_i T \cdot \left(T - \frac{d_i T}{p_{i1}} \right) \right] \right\} \frac{d_i^2 T}{2p_{i1} e} \sum_{v=1}^e (2e - 2v + 1) + \frac{1}{e} \sum_{v=1}^e \left[(e-v)d_i T \left(1 - \frac{d_i}{p_{i1}} \right) \right] \tag{4}$$

Therefore, the total cost of raw material inventory holding cost on the manufacturer's side would be:

$$TC_{RI} = \sum_{i=1}^n h_{i0} \cdot \frac{d_i^2 \cdot T}{2p_{i1} e} \cdot \left(\sum_{v=1}^e (2e - 2v + 1) \right) + \frac{1}{e} \sum_{i=1}^n h_{i0} \cdot \sum_{v=1}^e \left[(e-v)d_i T \cdot \left(1 - \frac{d_i}{p_{i1}} \right) \right] \tag{5}$$

It is supposed that the order cost of all materials is given and equal. Therefore, the total cost would be calculated by the multiplication of the order cost and the number of order as follows:

$$TC_{RMI} = \frac{R}{e \cdot T} \cdot n \tag{6}$$

In raw material supplier's side, the supplier production rate is assumed equal to the demand rate of the manufacturer. That is because the model is the classic EOQ model in which the lots are delivered in each e cycle time ($e \cdot d_i \cdot T$). The inventory chart is depicted in Fig. 7 and the total cost calculation is as follows:

$$TC_{RI2} = \sum_{i=1}^n h_{i0} \cdot \frac{e \cdot d_i \cdot T}{2} \tag{7}$$

Regarding all previous calculations, the mathematical model of the proposed problem and its constraints are shown in problem 1 as follows:

Problem 1

$$(10) \text{Min } TC = \sum_{i=1}^n h_{im} \cdot d_i \cdot T \cdot \left\{ \frac{1}{k} - \left(\frac{d_i}{2 \cdot p_{im}} \right) \right\} - \sum_{i=1}^n h_{im} \cdot d_i \cdot b_{im} + \sum_{i=1}^n h_{im} \cdot d_i \cdot \frac{T}{k} \cdot INT \left(\left(\frac{k \cdot b_{im}}{T} \right) + \left(\frac{k \cdot d_i}{p_{im}} \right) \right) + \sum_{i=1}^n \sum_{j=2}^m h_{i,j-1} \cdot d_i \cdot \left(b_{ij} + \frac{d_i \cdot T}{2p_{ij}} - b_{i,j-1} - \frac{d_i \cdot T}{2p_{i,j-1}} \right) + \sum_{i=1}^n h_{i0} \cdot \frac{d_i^2 \cdot T}{2p_{i1} e} \cdot \left(\sum_{v=1}^e (2e - 2v + 1) \right) + \frac{1}{e} \sum_{i=1}^n h_{i0} \cdot \sum_{v=1}^e \left[(e-v)d_i T \cdot \left(1 - \frac{d_i}{p_{i1}} \right) \right] + \sum_{i=1}^n \frac{d_i \cdot T \cdot h_i}{2} + \frac{R}{e \cdot T} \cdot n + \frac{\sum_{i=1}^n s_{ci}}{T} + \frac{A \cdot k}{T} + \sum_{i=1}^n h_{i0} \cdot \frac{e \cdot d_i \cdot T}{2} \tag{8}$$

Subject to:

$$b_{i,j-1} + \frac{d_i \cdot T}{p_{i,j-1}} \leq b_{ij} \quad i = 1, \dots, n \quad j = 2, \dots, m \tag{9}$$

$$b_{ij} + \frac{d_i \cdot T}{p_{ij}} + s_{uj} - b_{uj} \leq M(2 - z_{i,l,j} - z_{u,l+1,j}); \quad i = 1, \dots, n \quad u \neq i, l < n \tag{10}$$

$$\sum_{l=1}^n z_{ilj} = 1; \quad i = 1, \dots, n \tag{11}$$

$$\sum_{i=1}^n z_{ilj} = 1; \quad l = 1, \dots, n \tag{12}$$

$$b_{ij} \geq s_{ij} \cdot z_{i1j}; \quad i = 1, \dots, n \tag{13}$$

$$b_{im} + \frac{d_i \cdot T}{p_{im}} \leq T; \quad i = 1, \dots, n \tag{14}$$

$$F \cdot T = H \tag{15}$$

$$e \cdot T = B \tag{16}$$

$$F, e \geq 1, \text{ integer} \tag{17}$$

$$T, B \geq 0, b_{ij} \geq 0, z_{ij} \in \{0, 1\} \tag{18}$$

Problem 1 has the following set of constraints. Constraint (9) states that no component can be processed before it is completed at the previous stage. Constraint (10) shows that at each stage, no component can be processed before the completion of its predecessor in the related production sequence. Constraints (11) and (12) are assignment constraints at stages with only one machine; state that each component has a unique position in the sequence of these stages. Constraint (13) indicates that at each stage, processing the first component in the corresponding sequence cannot be started before setting up the corresponding machine. Constraint (14) assures that the resulting schedule is cyclic in a way that the process completion time for each component at the final stage is less than or equal to cycle time. Constraint (15) implies that the common cycle is such that the planning horizon H is a multiple of T . Constraints (17) show that F and e are some integers greater than or equal to one. Finally, Constraints (18) are the non-negativity variable constraints.

To solve the developed model, two separated heuristics and meta-heuristics methods are applied. The methods are explained in the next sections.

2.1. Optimal enumeration method

Regarding literature review, the developed model is NP-hard and it is impossible to obtain a global optimum solution. This model is a mixed integer non-linear programming. Solving this problem by means of exact method on small scale, not only the global optimum is not satisfied, but also the quality of its local optimum is drastically low. In order to



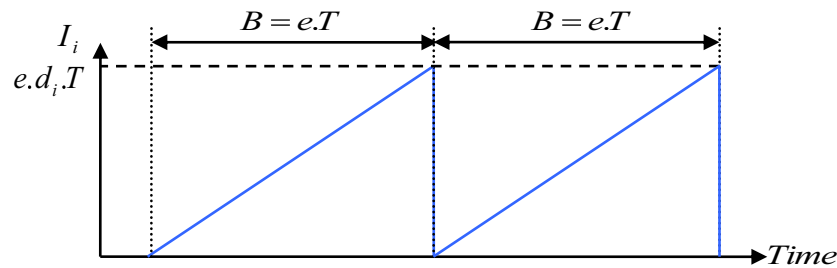


Fig. 7. Raw material inventory level in supplier's site

generate better and more reasonable outputs, an optimal enumeration method (OEM) is presented. The OEM method has been widely applied in many problems in the supply chain; see Ounniche & Boctor [21] and Torabi et al. [11, 12].

The key principle of the OEM is that the economic lot, purchase and delivery scheduling problem consists of three variables F , e , h where production, purchase and delivery cycles can be derived by them. The variable F denotes the number of production common cycles and its length (T) in finite planning horizon. Since T is a real variable, in OEM we should use the integer variable F . Regarding the high complexity of the problem, LINGO software is not able to solve this problem obviously, even on a small scale. Therefore, LINGO should be applied in OEM. In this regard, to reduce the complexity of the problem, the values of F and e are given to LINGO and converted from variables to parameters. OEM consists of two interconnected cycles as follows:

Cycle 1

$F = 1$, go to the cycle 2 and obtain the value of Z_F

set $z^* = z_1$, $e^* = e$, $T^* = T_1$, $k^* = k_1$;

Repeat until stop condition is satisfied

$F = F + 1$, and go to the cycle 2 and obtain the value of Z_F

if $Z_F \leq z^*$ then

$z^* = z_F$, $e^* = e$, $T^* = T_F$, $k^* = k_F$;

else

stop

end if

end

Cycle 2

Consider the value of e variable equal to 1 and we will have $Z_F = Z_{1,1}$

For $e = 1$ to F

Solve the resulting MILP

If $z_e \leq z_F$ then

$ZF = z_e$, $TF = T$, $kF = k$;

End

End

In cycle 1 the value of F is given at first. Then Cycle 2 finds the most convenient (least total cost) value of e with respect to F . This procedure is iterated while the value of the objective function has a decreasing trend.

It is possible that the algorithm falls into the local optimum. This is because the algorithm is not a comprehensive search and it would be terminated in the first optimum situation, even if it was a local optimum. Due to its nonlinear nature, it is highly infeasible to get global optimum. Furthermore, it is proved that the mixed-integer linear problems have an exponential complexity, known as NP-complete problems [22]. So in a large-scale problem, this algorithm has low efficiency and it is recommended to apply Meta-heuristics methodology. The next section consists of more details about Genetic Algorithm principles and its application in the developed model.

2.2. Real valued genetic algorithm

In the previous section, a new mixed-integer nonlinear programming model was proposed. It has been proved that these problems have exponential complexity. So, with larger dimensions, the solution time is increasing exponentially. The meta-heuristics methods are new and effective in solving mixed integer programming problems where all of them use an intelligent random search process to find an exact or near optimum solution. There are sufficient studies in this area and researchers have developed a GA with real-valued Genes. Right [23] developed Real-valued GA in optimization problems and compared it with the traditional binary GA method. His research shows the high efficiency of RGA as an optimization method. Muhlenbein and Voosen [24] introduced GA with continuous numbers genes and various genetic operators have been applied, as well.

2.2.1. Modified genetic algorithm structure. In this section, the details of the designed GA are explained. The principle framework of this method is Simple Genetic Algorithm (SGA) depicted in Fig. 8. However, to modify the SGA, a convergence rate is considered which decreases the probability of using mutation generation by generation based on convergence strategy.

2.2.2. Chromosome structure. In this problem, the chromosome has two divisions or sub-chromosomes. The first sub-chromosome consists of $m \times n$ genes and illustrates a sequence of production begin time of n products in all m production stages. Hence, all of them are real numbers. The other sub-chromosome has three genes. These genes demonstrate three real valued genes (e , F , k) which are purchase, production and delivery cycles respectively.



```

Procedure GA
Begin
    t = 0;
    initialize P(t);
    evaluate P(t);
    while not finished do
        begin
            t=t+1;
            select P(t) from P(t-1);
            reproduce pairs in P(t);
            evaluate P(t) ;
        end
    end.
    
```

Fig. 8. Pseudocode of SGA

Thanks to finite horizon planning in this problem, all genes are finite and have upper and lower bounds.

In the second sub-chromosome, *F* illustrates the number of production common cycles in finite horizon time. Since usually the planning horizon time in an industrial environment is one year, therefore in this problem the finite horizon time has considered 52 weeks. In accordance with our assumption, the maximum value of *F* is 52. The *e* variable shows the number of production cycles that would be covered by each purchase. The value of *e* depends on *F* value, so it varies between 1 to *F*. The third variable in this sub-chromosome is *k* that shows the number of delivery cycles in the production common cycle. In this problem, since the literature review illustrated that the length of production cycles usually is between 1 and 4 weeks, the maximum value of delivery cycles in the production cycle is assumed to 10. So this upper bound for *k* would be reasonable and changes from 1 to 10. The first sub-chromosome is a set of production beginning times of all items in all stages where genes are real numbers. These real numbers have lower and upper bounds. The former is zero and the latter is set to be *T* (*52/F*). However, the beginning times never become zero and that is due to setup time before production. Note that this constraint is applied in the developed mathematical model. Figure 9 shows the representation of chromosome with *n* products and *m* stages.

In addition, the efficiency of GA depends on the quality of the first population. That is why in this paper some simple heuristics methods are applied to find high quality vector in the first population. The first population of proposed GA consists of three groups of chromosomes. To produce the first group, the heuristics method such as LPT (Longest Process Time), SPT (Shortest Process Time) and WSPT (Weighted Shortest Process Time) have been applied. In

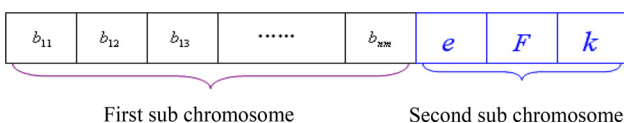


Fig. 9. Designed chromosome scheme

both LPT and SPT methods, *m* vectors are generated. The *j*th vector by SPT is obtained by sorting incremental order of *pt_{ij}* in *j*th stage of production and decreasing order in LPT. The WSPT method is similar to SPT except that the sorting criterion is based on the value of *pt_{ij}/h_j*.

Consequently, we have produced *3m* different sequential vectors. Whereas, the desired chromosome consists of begin times (*b_{ij}*). Therefore, to convert the sequential vector to begin times, a production common cycle (*T*) is required. By the given value of *T*, production lot sizes (*d_iT*) would be calculated. In addition, by means of these parameters the begin time and finish time of each item in all stages are derived and a high quality of population in GA would be generated.

By having sequence through heuristics algorithm, production common cycle and random begin time of the first item in the first stage, the begin times of all items in all stages could be extracted. The rule to get all begin times is that the begin time of one lot of item *i* at stage *j* is equal to the summation of setup time of item *i* at stage *j* (*s_{ij}*) with the maximization between finish time of one lot of item *i* at stage *j-1* and the finish time of one lot of the predecessor item (*u*) at stage *j*. This rule is as follows:

$$b_{ij} = s_{ij} + \max \left\{ b_{i,j-1} + \frac{d_i T}{p_{i,j-1}}, b_{uj} + \frac{d_u T}{p_{u,j}} \right\} \quad (19)$$

The second group of chromosomes in the first population is similar to the first group's structure. The sequence is generated randomly in the first stage and the next stages are the same. The third group might be generated to produce a more scattered population and cover more solution space. In this group, the second sub-chromosome genes are selected randomly between their ranges just like the previous groups. In addition, in the first sub-chromosome the genes are selected randomly between zero and *T*. Since the genes in the first sub-chromosome are produced randomly, as a result the probability of feasibility is low. To prevent transferring of these infeasible chromosomes to the next generations, their fitness value should be low. Regarding this issue, a penalty method has been developed as a part of the fitness function.

To survive the most qualified chromosomes in the next generations and get rid of low quality chromosomes in simulation of a natural process, a fitness function in GA is developed. This function evaluates the quality of each chromosome of the population. In the proposed problem, the solution with lower total cost is considered a better solution. Newton & Sarker [25] have implemented a genetic algorithm in ELSP. They utilized from penalty method to satisfy the constraints of the problem. The proposed penalty methods include Static penalty method, Dynamic penalty method and Adaptive penalty method. They executed the problem with all three methods and the experiments showed that the Static penalty method is the most efficient method in Economic Lot Scheduling problems.

Subsequently, the fitness function consists of two parts in this paper. The first part is objective function that is made of



eight types of cost in the supply chain area. The second part is consist of the constraints of the problem associated with the penalty. Certainly, a chromosome that satisfies all constraints is qualified as a feasible solution. To prevent the transmission of infeasible chromosome to the next generation, static penalty method has been implemented. In the static penalty method, a penalty coefficient is assigned to each constraint. If one of the constraints is not satisfied, the absolute value of its deviation multiplied by its penalty coefficient will add to the objective function in fitness value of MRGA. The coefficients are depicted in Table 1.

In this study, the GA algorithm is developed in MATLAB [26]. To find the most suitable crossover, several crossover operators have been examined. As depicted in Fig. 10, a simple two-point crossover is applied.

Finally, because of the real number nature of chromosomes, the Gaussian mutation is applied. In the next step, to identify effective crossover probability ($p_{crossover}$), the GA is evaluated by various probabilities and the GA is executed 10 times for each value. As depicted in Fig. 11, the best crossover probability is equal to 0.8.

In Gaussian mutation, a random number from Gaussian distribution is added to each chromosome to produce a new child. The Gaussian mutation in real valued chromosome is shown in Fig. 12.

Finally, a pre-defined stopping criterion identifies the algorithm stopping time. In the proposed Genetic Algorithm, the optimum iteration is determined equal to 200 by evaluating various iterations.

Table 1. Penalty coefficient for each constraint

constraint	Penalty Coefficient	constraint	Penalty Coefficient
97	9×10^9	101	3×10^6
98	9×10^7	102	3×10^6



Fig. 10. Two point crossover operator scheme

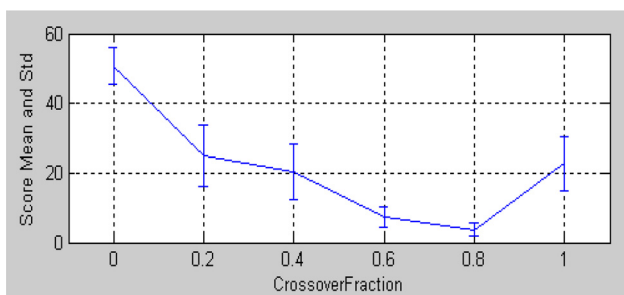


Fig. 11. Fitness function changes based on crossover probability value

$$a) \begin{bmatrix} 1.3 & 0.4 & 1.8 & 0.2 & 0.0 & 1.0 \end{bmatrix} \Rightarrow b) \begin{bmatrix} 1.2 & 0.7 & 1.6 & 0.2 & 0.1 & 1.2 \end{bmatrix}$$

Fig. 12. Gaussian mutation

3. RESULT AND DISCUSSION

To evaluate the efficiency of two proposed solution methods (optimal enumeration method (OEM) and Modified Real valued genetic algorithm), seven different examples with various dimensions have been analyzed. Every parameter value is extracted from uniform probability distribution function as depicted in Table 2.

All examples are run on a Celeron 2.4 GHz system. Each sample has been solved by OEM and Modified Real-valued GA 10 times. By increasing the problem dimensions, solving the problem by OEM becomes too much more time consuming. Consequently, the best obtained solutions for each problem after maximum 24-h runtime by OEM are in Table 3. It may be concluded that the OEM is not efficient specially for medium and large dimension problems. Because of the nonlinear nature of this problem, the obtained solutions by OEM method and LINGO software is considered local optimum and in general the greater dimensions of problems resulted from the lower quality of the solutions. As Fig. 13 shows, the efficiency of MRGA would be clarified by scaling up the problem.

Regarding the empirical result, it would be deduced that:

1. Thanks to cost reduction and mean time indexes, it is obvious that the MRGA as a metaheuristic method is more efficient than OEM as heuristic method especially on a larger scale.
2. In case of larger scale of samples, by considering mean time and max time, the scatter index of solutions grows up exponentially. It would be concluded that by increasing the scale of the problem, the calculation time of the proposed methodology is raised and the efficiency of MRGA might be weakened.

Table 2. The distributions of parameters

Parameters	Description	Distribution
d_i	Demand rate of component i	$U(100,1000)$
p_{ij}	Production rate of component i at stage j	$U(1000,15000)$
s_{ij}	Sequence-independent setup time of component i at stage j	$U(0.01,0.25)$
h_{ij}	time dependent inventory holding cost per unit of component i between stages j and $j + 1$	$U(1,20)$
A	transportation cost per delivery	$U(5000,15000)$
R	Order Cost of the component	$U(3000,6000)$

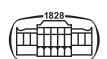


Table 3. Calculation result

No.	Problem Dimensions (stages*items)	Number of times (up to 10) that GA result is better than OEM	Mean cost reduction in GA rather than OEM	GA calculation time (per second)		
				Mean time	Max. time	Scatter index
1	3 * 3	7	911.155	26.76	27.225	0.155
2	2 * 6	8	2,409.895	29.212	31.452	0.746
3	3 * 5	8	2,913.495	29.663	32.198	0.845
4	3 * 6	9	3,496.265	31.956	35.149	1.06
5	2 * 11	10	3,968.755	34.264	38.568	1.43
6	5 * 5	10	4,381.846	36.869	43.247	2.12
7	3 * 10	10	4,966.825	39.548	46.894	2.44

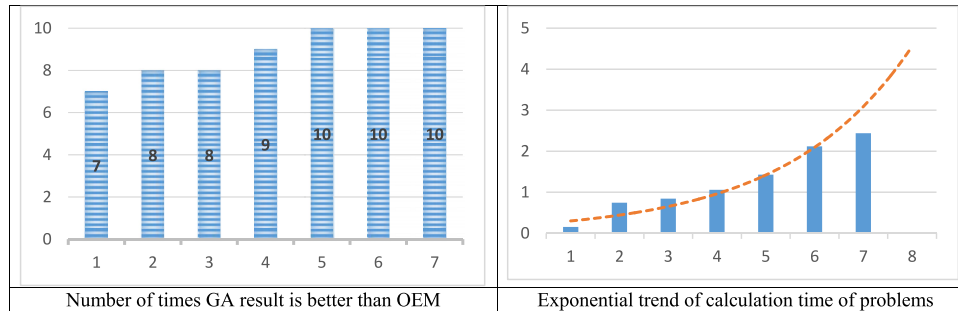


Fig. 13. The performance of MRGA in various samples

4. CONCLUSIONS

Due to the recent development in supply chain concepts and the importance of suitable decision models in order to obtain high-level coordination between supply chain partners and cost reduction in the supply chain, the Lot-sizing and Scheduling Problems are widely prioritized in this area.

The literature review shows that there are three branches of using Lot sizing and Scheduling Models. The first branch is Economic Lot and Scheduling Problem (ELSP). In this problem, a supplier with a cyclic production system produces several parts at a constant rate and delivers them to customers continuously. In this category, optimization of supplier’s production system is the main goal and the whole supply chain is neglected. The second branch is related to Economic Lot, Delivery and Scheduling Problem (ELDSP). In these problems, a supplier produces various parts in a cyclic production system and delivers the production lot to the customer at the end of each cycle. In this model, the optimization of supplier and customer is considered simultaneously and cost reduction in two-level supply chain is desirable.

The last branch that is considered by researchers recently, is Economic lot and scheduling problem in three-echelon supply chain which consists of customer (Assembly Facility), manufacturer and raw material suppliers. The manufacturer works with single-stage system in an infinite horizon and the production lot is delivered to the customer at the end of each cycle. In all the described models, the manufacturer’s production system was considered single-stage. However, in this paper this restriction is relaxed and the system is set to be multi stage (flow shop system). In the proposed Supply Chain; Assembly Facility purchases the raw

material from their suppliers in optimized lot, changes them to finished goods and finally delivers them to customers in multi delivery points during the production cycle in an optimum quantity.

In this paper, we developed a new mixed integer nonlinear programming model in accordance with the described scenario. After that, two solution methods were proposed, OEM and MRGA. Some empirical experiments were done and two proposed methods were compared together and finally, the calculation showed that MRGA is more effective than OEM in time-consumption and solution quality.

Regarding future research, there are various fields to improve decision-making in economic lot and scheduling in the supply chain. As far as the structure of the supply chain is concerned, the multi tiers of suppliers could be considered and optimization of production lots and scheduling in a comprehensive supply chain (from tier *n* suppliers to end users) would be desired to be more compatible with the real world. Another area for improvement is to develop a mathematical model and consider more limitations such as dependent setup times, backlogs, physical inventory limits, etc. It would be worthwhile to include costs related to the value of stocks in the model such as depreciation losses on inventories, insurance costs, etc. Improvement more effective optimizing methods is also considerable.

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