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ARTICLE

Dynamic Optimisation of Public Pension Systems

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Abstract
This paper presents a dynamic optimisation algorithm for public pension systems, developed by using Bellman's principle of optimality. The algorithm minimises a weighted objective function over a limited time frame (2024–2036), where decisions are optimised annually for seven variables: retirement age (men and women), contribution rate, national average wage, early retirement rate, active pensioner share, and employment rate. The system is controlled by deterministic demographic forecasts and policy-dependent limitations. The paper presents the algorithm underlying the model, which produces an ideal decision path that minimises deviations from a policy baseline for 2024. The structure of the model can be adapted to pay-as-you-go pension systems under demographic pressure in other countries.

Keywords Dynamic optimisation · Bellman equation · Pension system modelling · Objective function

1 Introduction

The long-term sustainability of public pension systems is a complex, high-dimensional optimisation problem influenced by demographic change, economic volatility, and policy constraints. Static economic models are insufficient to capture the interdependencies among the multiple variables shaping pension dynamics over time. Short-term and single-variable sensitivity analyses do not support robust policy design under dynamic conditions [1, 10]. Consequently, there is a growing demand for formalised modelling frameworks that can model system behaviour under different input scenarios and quantify the effects of trade-offs between competing policy goals [4, 5, 12].

It is important to distinguish between three main modelling approaches in the literature: macroeconomic models, dynamic microsimulation, and dynamic optimisation. Early research on the topic consisted primarily of deterministic macroeconomic simulations, and the pay-as-you-go (PAYG) pension system was studied from the perspective of fiscal sustainability [4, 5, 12]. These studies quantified the effects of various fiscal and political decisions,

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as well as demographic changes. However, the handling of feedback across multiple periods and trade-offs between social and fiscal objectives was limited due to the complex, multidimensional nature of the pension system [1, 2, 10]. In contrast, dynamic stochastic general equilibrium (DSGE) and overlapping generations (OLG) models use a stochastic macroeconomic approach. The OECD and EU's examination of pension adequacy and inequality effects [19] can be classified as dynamic microsimulation, but their implementation requires large amounts of microdata and computational capacity [6, 8]. The third approach is dynamic optimisation modelling, which is also related to the present study. It provides an optimal decision path that minimizes the differences between economic and social goals using a predefined objective function [3, 10]. This method occupies a distinct position in the international literature: it bridges the transparency of deterministic simulations and the analytical rigor of dynamic optimisation [17, 18]. In line with the framework and priorities set out in the [20] Pensions at a Glance and the [21] Ageing Report, pension system reforms can only be considered sustainable in the long term if, in addition to fiscal considerations, intergenerational fairness and social risk-sharing are also taken into account [15], 2019; [13].

This paper offers a solution to this shortcoming by presenting a dynamic optimisation model designed to model and improve the trajectory of a public pension system over a finite period. It is based on Bellman's principle of optimality [3, 7]. The model includes seven decision variables: retirement age for men and women, contribution rate, gross average wage, early retirement rate, share of active pensioners, and employment rate. Evaluates policy outcomes using an objective function that penalises deviations from a predefined baseline year (2024). Each variable is optimised within a set of legal, demographic and economic constraints, resulting in an optimal decision path that minimises aggregate economic and social cost.

The model was implemented using a discrete-time backward induction algorithm. The system's population dynamics are estimated from official Romanian statistics (2010–2024), forming a deterministic basis for state transitions across age and employment groups. Legal constraints, including contribution rate bounds and statutory retirement age thresholds, are explicitly encoded to ensure policy feasibility.

The model and its corresponding computational framework can provide answers to important policy questions such as: What are the social and economic trade-offs involved in preserving the current retirement age? Can this objective be realistically sustained under existing or adjusted contribution rates? Is it feasible to maintain both a stable contribution rate and the current retirement threshold within an optimised policy configuration? To what extent can the projected 2024 pension system deficit be mitigated if deficit reduction is prioritized? What outcomes emerge when equal weight is assigned to all decision criteria? Furthermore, the model offers information on the types of interventions required to operationalize each scenario. In addressing these and related questions, the framework serves as a tool to evaluate the implications of alternative pension policy strategies under varying priority structures.

If no corrective measures are implemented relative to the 2024 baseline state of the Romanian public pension system, the deficit is projected to evolve, as illustrated in Fig. 1. By 2032, it will reach a local maximum, exceeding 1.5 times the 2024 level in real terms. Moreover, by the end of the projection period in 2037, it will remain above this 1.5-fold threshold. The objective of this research is to identify policy configurations that yield more favourable fiscal outcomes relative to this baseline scenario.

Although this study focuses empirically on the Romanian public pension system, the model can be adapted to any country operating a pay-as-you-go scheme and facing similar structural challenges. By merging demographic modelling, dynamic optimisation, and computational

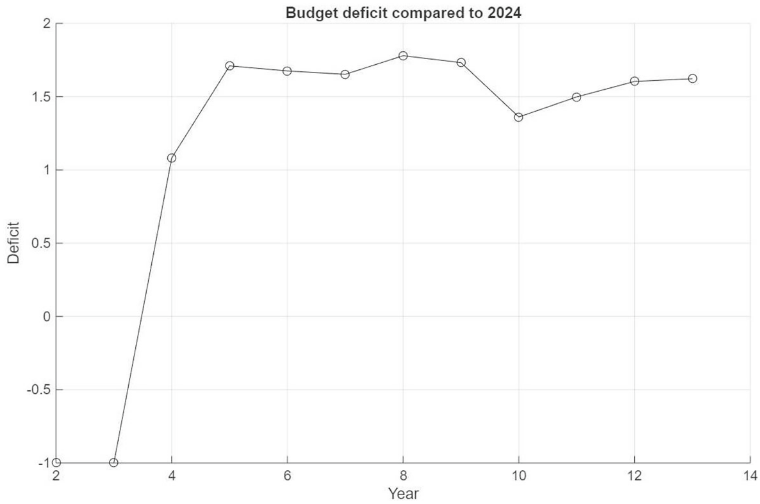

Fig. 1 Projected deficit of Romanian public pension system

policy analysis, the paper contributes a replicable and extensible framework for long-term pension reform evaluation.

The structure of the paper is as follows. It starts with a methodological overview that describes the applied optimisation framework, presents the demographic module, and describes the dynamics of the pension system. Then the algorithm created for the implementation is explained. Several scenarios that may have practical implications for economic policy are examined in the results section. In the end, a summary of the study's main conclusions and recommendations for further research directions is included.

2 Methodology

This section presents the structure of the dynamic optimisation model of the pension system. The model integrates demographic forecasting, fiscal balance dynamics, and annual policy optimisation using a deterministic, discrete-time dynamic programming approach.

The model optimises a set of annually adjusted decision variables over the 2024–2036 period:

- arw_n : retirement age for women
- arm_n : retirement age for men
- T_n : contribution level
- $\overline{S}_n$: gross national average wage
- e_n : early retirement rate
- k_n : the proportion of active pensioners
- u_n : employment rate

All variables are subjected to political and legal constraints. These are optimised based on the priorities of decision-makers, which are reflected in a weighted objective function.

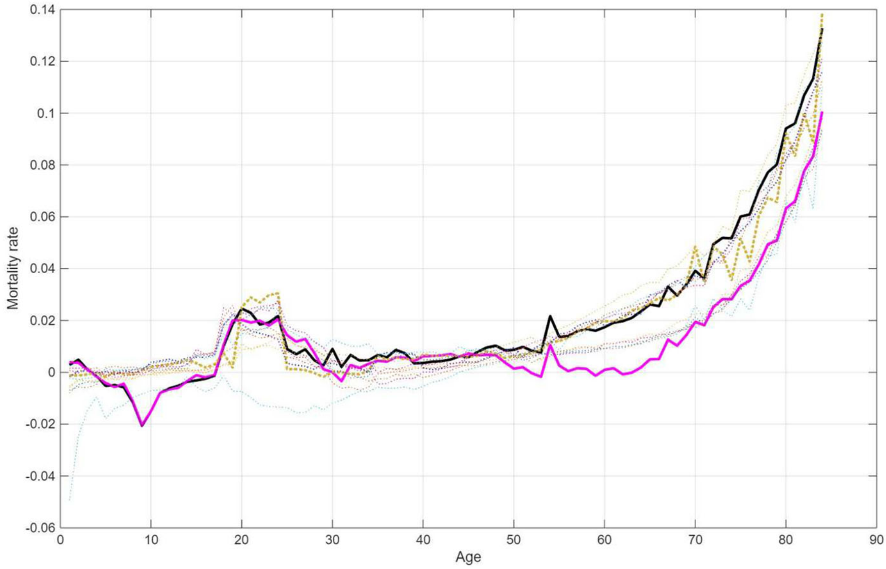

Fig. 2 The Romanian mortality rate

2.1 Demographic Module

To forecast long-term system dynamics, the model integrates a cohort-based demographic projection using age- and sex- specific population data obtained from Romanian National Institute of Statistics (INSSE Institutul Național de Statistică) for the years 2010–2023. The annual evolution of the population is derived based on observed mortality rates, which are further used to generate a deterministic transition mechanism.

Figure 2 illustrates the age-specific mortality rates $MR_{a,n,g}$ where a denotes age, n the year, and $g \in \{m, w\}$ the gender. Shaded curves represent raw mortality across cohorts, while bold lines indicate the averaged trajectories $AMR_{a,n,g}$ for men and women, respectively. A local maximum is observed around age 20, which reflects the impact of emigration. Thus, migration effects are incorporated into the model, and the mortality rate includes the full extent of population change.

In Fig. 2, the bold purple line represents women, while the bold black line represents men. The "negative mortality rate" results from population decline adjusted for migration effects. In the model, migration is incorporated into the "mortality" (transition loss) rate, an accounting convention designed to capture net cohort attrition, not actual mortality.

Population matrices $Pop_{a,n,g}$ are maintained separately for males and females, with age classes indexed by rows and years by columns. Inter-period transitions are computed using the transition loss rate (TLR), capturing the year-to-year relative decline in each cohort:

$$MR_{a,n,g} = \frac{\left(Pop_{a,n,g} - Pop_{a+1,n+1,g}\right)}{Pop_{a,n,g}} \tag{1}$$

Average mortality rates are computed over the historical period as:

$$AMR_{a,g} = \frac{1}{12} \sum_{n=2010}^{2023} MR_{a,n,g} \tag{2}$$

These average rates are then used to project future population developments: they are treated as elastic

$$Pop_{a+1,n+1,g} = Pop_{a,n,g} * \left(1 - AMR_{a,g}\right) \tag{3}$$

The total population in year n is a vector composed of three functional subgroups:

$$Pop_{n,g} = YP_{n,g} + UP_{n,g} + OP_{n,g} \tag{4}$$

where:

$$YP_{n,g} = \sum_{a=0}^{18} Pop_{a,n,g} \tag{5}$$

represents the underage population,

$$UP_{n,g} = \sum_{a=18}^{ar_n} Pop_{a,n,g} \tag{6}$$

refers to working-age individuals.

$$OP_n = \sum_{a=arm_n}^{a\_\max_{n,m}} Pop_{a,n,m} + \sum_{a=arw_n}^{a\_\max_{n,w}} Pop_{a,n,w} \tag{7}$$

OP_n is the retired population, disaggregated by gender and retirement age thresholds. The arm_n and arw_n denote the statutory retirement ages for men and women, respectively, restricted within the following bounds: 65$\leq arm_n \leq$ 70; 63$\leq arw_n \leq$ 68. The term a_max refers to the maximum life expectancy for both men and women.

2.2 Financial Balance and Pension System Dynamics

The model assumes a public pension system. This implies that current contributors finance current beneficiaries. The system's fiscal position in year n is defined by the difference between total revenue REV_n and expenditure EXP_n:

$$BAL_n = REV_n - EXP_n. \tag{8}$$

Annual revenue is computed based on the number of contributors C_n , their average income $\overline{S}_n$(with $\overline{S}_1$=7567 RON in 2024 and a constraint 5000 $\leq \overline{S}_n \leq$ 12000, for the Romanian case), and the statutory contribution rate T (with $T = 20.25\%$ regarded to Romania and a constraint $15\% \leq T_n \leq 30\%$):

$$REV_n = T_n * \overline{S}_n * C_n * 12. \tag{9}$$

The number of contributors is a function of the labour force and active pensioners:

$$C_n = LF_n * u_n + k_n * OP_n,$$

where LF_n denotes the available labour force, u_n the employment rate (with an initial value of $u_1 = 64\%$ in Romania, and a constraint $60\% \leq u_n \leq 68\%$), and k_n the share of pensioners still active in the labour market (constraint $0\% \leq k_n \leq 30\%$), $\wp_n$ refers to the working-age population (for the Romanian case $\wp_1 = 8364000$). The labour force is determined as:

$$LF_n = \wp_n * (1 - ds_n - er_n) + NM_n. \tag{10}$$

Here, ds_n denotes the disability rate, er_n the early retirement rate, and NM_n the net migration in flow of working-age individuals.

Both u_n and NM_n are treated as elastic with respect to changes in the contribution rate T_n and gross wage $\overline{S}_n$, following the structure:

$$\frac{u_n - u_{n-1}}{u_{n-1}} = \left(\frac{\overline{S}_n - \overline{S}_{n-1}}{\overline{S}_{n-1}} * eu_{\overline{s_n}} + \frac{T_n - T_{n-1}}{T_{n-1}} * em_{T_n}\right), \tag{11}$$

$$\frac{NM_n - NM_{n-1}}{NM_{n-1}} = \left(\frac{\overline{S}_n - \overline{S}_{n-1}}{\overline{S}_{n-1}} * em_{\overline{s_n}} + \frac{T_n - T_{n-1}}{T_{n-1}} * em_{T_n}\right), \tag{12}$$

where $eu_{\overline{s_n}}$, euT_{T_n}, $em_{\overline{s_n}}$, em_{T_n} denote the elasticities of employment and migration with respect to wage and contribution levels, respectively.

There is no single universal value for these elasticity parameters, as their magnitude varies across multiple dimensions. Based on existing empirical studies, the own-wage elasticity of labour demand typically falls within the interval [−0.072%, −0.446%] [9]. Reductions in payroll taxes are estimated to increase employment by [0.05%, 0.2%] [16]. An 1% increase in wages is associated with an average 0.76% rise in immigration flows, whereas accession to the Schengen Agreement reduced immigration from non-EU countries [14]. For migration elasticity with respect to contributions, no European consensus exists. Therefore, small absolute values is [0, 0.1%] are used.

Total expenditures EXP are calculated based on the number of retirees P_n and the average pension $\overline{PP_n}$ by

$$EXP_n = P_n * \overline{PP}_n * 12. \tag{13}$$

The number of beneficiaries is estimated as

$$P_n = OP_n * rr_n, \tag{14}$$

where rr_n is the old-age retirement rate among the eligible elderly population.

Pension indexation follows the legal provisions of the Romanian pension law (Law 263/2010). Until 2030, pension increases are based on inflation and a diminishing share of real wage growth. From 2031 onwards, adjustments are made based solely on inflation:

$$\overline{PP}_n = \begin{cases} \overline{PP}_{n-1} * \left[i_n + 1 + \left(\left(\frac{S_n}{S_{n-1}} - 1\right) * r\right)\right], & \text{if } n \leq 2030, \\ \overline{PP}_n = \overline{PP}_{n-1} * [i_n + 1], & \text{if } n > 2030, \end{cases} \tag{15}$$

where $r = 0,3 - (0,05 * (n - 2024))$ if $n \in [2024, 2030]$.

3 Optimisation Framework

Dynamic programming is a mathematical optimisation method. It refers to simplifying a complex problem by recursively breaking it down into simpler subproblems. While some decision problems cannot be decomposed in this way, decision spanning multiple points in time can often be recursively decomposed. Bellman called this the "optimality principle" [3]. The proposed model for the pension system is formulated as a dynamic optimisation problem, addressed using backward dynamic programming. If subproblems can be recursively embedded in larger problems, so that dynamic programming methods can be applied, then there is a relationship between the value of the larger problem and the values of the sub-problems. In the optimisation literature, this relationship is called the Bellman equation. The algorithm starts at the terminal year n and iteratively moves in reverse to the initial period, determining the optimal trajectory of key decision variables:$arw_n,\ arm_n,\ T_n,\ \overline{S}_n,\ e_n,\ k_n$ and u_n.The optimisation objective is to minimise a weighted objective function that captures the normalised deviation of each variable from its reference level in 2024.

A decision matrix A is constructed, in which rows correspond to time periods and columns to normalised policy variables. The influence of each variable is scaled by a set of weights $M,\ W,\ B,\ C,\ D,\ E$ and G that sum to one. These weight parameters reflect the priorities of the decision-maker. Normalisation ensures comparability across variables by expressing all deviations in relation to their 2024 baseline values, where the first row of the table is set to zero:

	M	W	B	C	D	E	G
A_1	0	0	0	0	0	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_n	$\frac{arm_n-arm_1}{arn_1}$	$\frac{arw_n-arw_1}{arw_1}$	$\frac{T_n-T_1}{T_1}$	$\frac{\overline{S}_n-\overline{S}_1}{\overline{S}_1}$	$\frac{BAL_n-BAL_1}{BAL_1}$	$\frac{er_n-er_1}{er_1}$	$\frac{u_n-u_1}{u_1}$

This framework helps the model to assess and rank several choices based on their alignment with predefined policy preferences.

The objective function for year n is defined as:

$$F_n = M\frac{arm_n - arm_1}{arn_1} + W\frac{arw_n - arw_1}{arw_1} + B\frac{T_n - T_1}{T_1} + C\frac{\overline{S}_n - \overline{S}_1}{\overline{S}_1} + D\frac{BAL_n - BAL_1}{BAL_1} + E\frac{er_n - er_1}{er_1} + G\frac{u_n - u_1}{u_1} \tag{16}$$

The model incorporates a comprehensive set of constraints to ensure realistic and feasible policy recommendations:

$$\begin{cases} f(1) & 65 \le arm_n \le 70 \\ f(2) & 63 \le arw_n \le 70 \\ f(3) & 15\% \le T_n \le 30\% \\ f(4) & 5000 \le \overline{S}_n \le 12000 \\ f(5) & 0 \le BAL_n \le 0.5 \cdot BAL_1 \\ f(6) & 0 \le er_n \le 3\% \\ f(7) & 60 \le u_n \le 68\% \end{cases}$$

We denote by FS_n the set of possible parameters

$$FS_n = \left\{p_n = \left(arm_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n\right) : \right. \tag{17}$$

$$\left. \text{the parameters that satisfy conditions } f(i),\ i = \overline{1, 7}.\right\} \tag{18}$$

The goal is to solve the following optimisation problem:

$$\begin{cases} F_1\left(arm_1, arw_1, T_1, \overline{S}_1, Pop_1, er_1, u_1\right) \to \min \\ BAL_n \le 0.5 \cdot BAL_1 \end{cases} \tag{19}$$

State transitions are governed by:

$$\begin{cases} F_n = F_n\left(arn_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n\right) \\ BAL_n = BAL_n\left(arn_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n\right) \end{cases} \tag{20}$$

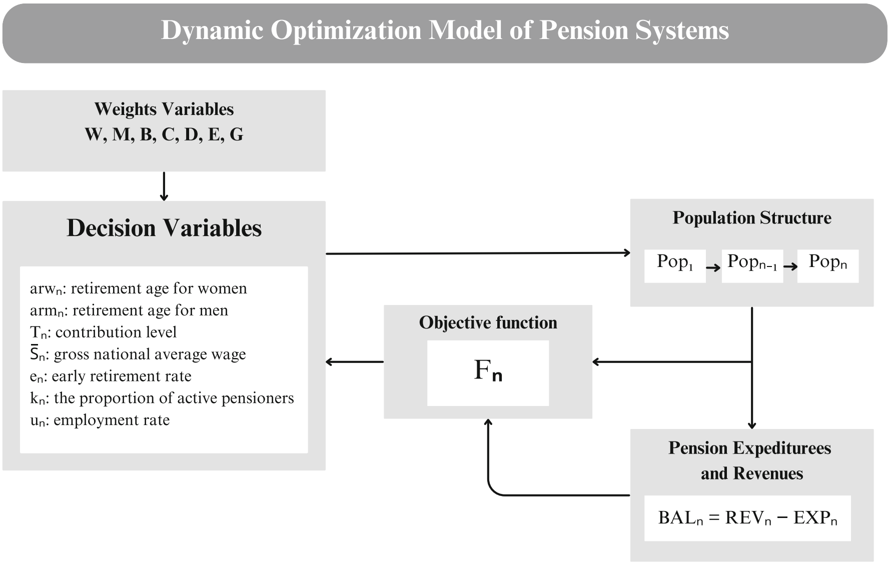

Fig. 3 Graphical summary of the model structure

The optimal dynamics are obtained by

$$F_{n-1}^{*}\left(p_{n-1}^{*}\right)=\min_{p_n\in FS_n}\left\{F_n\left(p_n\right)+F_n^{*}\left(p_n^{*}\right)\right\}, \tag{21}$$

where the superscript "*" denotes the optimal values from period n to the end of the period. $F_n^{*}\left(p_n^{*}\right)$ is the minimum of the objective function for the stages from step n to the end of the era. This is Bellman's equation, which is at the core of dynamic programming theory. This equation is associated with a set of optimal policy functions p_n^{*} and p_{n-1}^{*} for each period n, which are defined by

$$\left\{p_{n-1}^{*}, p_n^{*}\right\}\in Arg\min_{p_n\in FS_n}\left\{F_n\left(p_n\right)+F_n^{*}\left(p_n^{*}\right)\right\}.$$

Figure 3 provides a graphical summary of the model structure.

The objective function ensures a trade-off between social protection and fiscal discipline by penalising deviations from 2024 baseline values in key variables such as retirement age, contribution rate, wages, deficit, early retirement, and employment. Assigned weights capture policy priorities, balancing demographic alignment with economic realisation.

The Appendix A provides a tabular summary of the formulas and relationships presented in the methodology section.

4 Algorithm

The algorithm for the optimisation task assigned to the pension scheme (19) is as follows.

Algorithm 1 Required Parameters and Calculation of Deficit in 2024

Require: M, W, B, C, D, E, G ▷ Parameters of the objective function
Require: N ▷ Year index: 1 = 2024, 2 = 2025, ...
Require: Early retirement rate parameters:
$ermin = 0$, $ermax = 0.03$, $in\_er = 0.01$
Require: Employment rate parameters:
$umin = 0.60$, $umax = 0.68$, $in\_uu = 0.64$
Require: Share of active pensioners:
$kmin = 0$, $kmax = 0.3$, $in\_kk = 0.03$
Require: Retirement age intervals:
Men: $arminM = 65$, $armaxM = 70$
Women: $arminW = 63$, $armaxW = 70$
Require: Contribution rate:
$Tmin = 0.15$, $Tmax = 0.3$, $in\_T = 0.2025$
Require: Gross average wage:
$Smin = 5000$, $Smax = 12000$, $in\_S = 7567$
Require: Step values:
$st\_er = 0.01$, $st\_uu = 0.01$, $st\_T = 0.02$, $st\_S = 100$, $st\_kk = 0.01$
Require: $PopM, PopW$ ▷ Population tables by age and year (2014–2023)
Require: $GAW2024 = 7567$ ▷ Gross average wage in 2024
Require: $BALCondition = 0.5$ ▷ Allowed deficit
Require: $arminM = 65$, $arminW = 63$ ▷ Retirement ages in 2024
Require: $prop\_active = 0.823$ ▷ Proportion of active working-age population
Require: $A\_P = 2244$ ▷ Average pension in 2024

Step 1: Demographic and Population Initialization
Calculate annual population change and mortality rates (see formulas (2))
Compute initial population of the dynamic system (see formulas (4), (7))

Step 2: Initialize Parameters
$T(1) \leftarrow in\_T$
$S(1) \leftarrow in\_S$
$Popaktiv(1) \leftarrow 10\,162\,926$ ▷ Working-age population in 2024
$Poppension(1) \leftarrow 4\,711\,597$ ▷ Pension-age population in 2024

Step 3: Calculate Active Contributors
$Pop2024 \leftarrow (Popaktiv(1) * prop\_active) * in\_uu + in\_kk * Poppension(1)$

Step 4: Revenue and Expenditure
$REV \leftarrow 12 * T(1) * S(1) * Pop2024$ ▷ Annual pension scheme revenue
$EXP \leftarrow 12 * Poppension(1) * A\_P$ ▷ Annual pension scheme expenditure

Step 5: Calculate Deficit
$BAL(1) \leftarrow REV - EXP$ ▷ Deficit or surplus in 2024

Algorithm 2 Determination of Possible Solution Set of Parameters (17)

```
OPTPar ← []                                              ▷ Initialize the set of possible solutions
kk ← in_kk                                               ▷ Initial share of active pensioners
for er = ermin to ermax step st_er do
    for uu = umin to umax step st_uu do
        for k = 2 to N do
            for arM = arminM to armaxM do
                for arW = arminW to armaxW do
                    Compute population of the dynamic system using formulas:
                      (2), (4), (7)
                    Poppension(k)                        ▷ Pension-age population at year k
                    Popactive(k)                         ▷ Working-age population at year k
                    for T = Tmin to Tmax step st_T do
                        for S = Smin to Smax step st_S do
                            Pop2 ← (Popactive(k) ∗ prop_active) ∗ uu
                                             +kk ∗ Poppension(k)
                            REV ← 12 ∗ T ∗ S ∗ Pop2      ▷ Annual revenue
                            sn1 ← S/Smin − 1
                            annualgrowth ← max (0, [0.5 − 0.05 ∗ (k + 1)] ∗ sn1)
                            EXP ← 12 ∗ Poppension(k) ∗ A_P ∗ (1 + annualgrowth)
                                                         ▷ Annual expenditure
                            BAL(k) ← REV − EXP           ▷ Deficit or surplus at year k
                            if BAL(k) ≥ BAL(1) − BALCondition ∗ BAL(1) then
                                Par ← [k, arF, arN, T, S, Poppension(k),
                                    Pop2, Popactive(k), BAL(k), er, uu]
                                Append Par to OPTPar
                            end if
                        end for
                    end for
                end for
            end for
        end for
    end for
end for
```

Algorithm 3 Determination of the Optimal Solution of the Dynamic System Using the Bellman Equation (21)

```
Require:
  Initial values:
initial2024 ← [1, in_arM, in_arF, in_T, in_S, Poppension(1),
        Pop2024, Popactive(1), BAL(1), in_er, in_uu]
  ▷ Initial values for Romania in 2024:
  ▷ initial2024 ← [1, 65, 63, 0.2025, 7567, 3778091, 5611544, 10438566, 1447566905, 0.03, 0.64]
Penalty function:
F ← [OPTPar(:, 1),
    M ∗ (OPTPar(:, 2) − in_arM)/in_arM+
    W ∗ (OPTPar(:, 3) − in_arF)/in_arF+
    B ∗ (OPTPar(:, 4) − T(1))/T(1)+
    C ∗ (OPTPar(:, 5) − in_S)/in_S+
    D ∗ (OPTPar(:, 9) − OPTPar(1, 9))/OPTPar(1, 9)+
    E ∗ (OPTPar(:, 10) − in_er)/in_er+
    G ∗ (OPTPar(:, 11) − OPTPar(1, 11))/OPTPar(1, 11),
  OPTPar(:, 2), OPTPar(:, 3), OPTPar(:, 10), OPTPar(:, 11)]
Initialization:
[sF1, sF2] ← size(F)
Smin ← 1000 ∗ ones(1, N)
Smin(N + 1) ← 0
J ← zeros(1, N)
J(1) ← 1
armaxM1 ← armaxM
armaxW1 ← armaxW
Backward iteration using Bellman recursion:
for i = sF2 to 1 step −1 do
  for j = N to 1 step −1 do
    if F(i, 1) == j then
      if F(i, 3) ≤ armaxM1 and F(i, 4) ≤ armaxW1 and
            F(i, 3) ≥ armaxM1 − 1 and F(i, 4) ≥ armaxW1 − 1 and
            (Pop2024 − OPTPar(i, 8) ∗ prop_active∗
                              OPTPar(i, 11))/OPTPar(i, 6) ≥ 0 then
        if F(i, 2) + Smin(j + 1) < Smin(j) then
          Smin(j) ← F(i, 2) + Smin(j + 1)
          J(j) ← i
          armaxM1 ← F(i, 3)
          armaxW1 ← F(i, 4)
        end if
      end if
    end if
  end for
end for
Assigning optimal values:
OPTPar(J, [1, 2, 3])
Plotting:
Generate Figure 5 showing the behaviour of the dynamic system.
```

5 Results

This paper presents a dynamic pension model by using demographic data from 2010 to 2023. The population is annually divided in underage, working-age, and elderly groups based on retirement age. The model employs backward induction from 2024, computing feasible

The Economic Application of the Algorithm
Optimal Scenarios

SCENARIO NAME	DESCRIPTION	THE PRIORITY WEIGHTS						
		M	W	B	C	D	E	G
1. Maintaining Social Welfare at the Current Retirement Age	Preserves 2024-level social welfare and current retirement age, focusing on the most vulnerable groups.	0,35	0,35	0,05	0,05	0,05	0,05	0,05
2. Maintaining Broad Social Welfare	Minimises welfare deterioration while considering retirement age and employee burden.	0,25	0,25	0,125	0,07	0,088	0,15	0,07
3. Ensuring Economic Sustainability	Focuses on minimizing the pension system deficit, prioritising financial sustainability.	0,05	0,05	0,05	0,05	0,7	0,05	0,05
4. Equal Priority Distribution	Applies equal weights to all criteria, ensuring a balanced policy approach.	0,143	0,143	0,143	0,143	0,143	0,143	0,143

Fig. 4 The economic application of the algorithm optimal scenarios

trajectories based on optimisation problem (19), and selects the optimal path by minimising a weighted objective function via Bellman's equation (21). In other words, solving the Bellman equation is just a matter of finding the fixed point of the Bellman equation. From the perspective of the pension system, this means that if the system parameters are configured such that solutions to the Bellman equation exist-i.e., the penalty function is minimized-then the system will remain in this equilibrium state. In the extension phase of this research, our objective is to investigate the stability of these fixed points. Let T be operations involved in the computation of the Bellman's equation. To resolve this equation is equivalent to finding the fixed point of operator T, where $F^*_{n-1} = TF^*_n$.

Results are illustrated in Fig. 5 across nine scenario-specific figures, covering key indicators such as retirement ages, contribution rates, wages, employment and pensioner numbers, pension deficit (real 2024 terms), early retirement, active pensioners, and employment rate. Feasible solutions appear as black dots; optimal values as solid black circles.

The economic applicability of the model is presented in the article [11] entitled "Dynamic Scenario Analyses of the Public Pension System", which is to be published in an economics-oriented scientific journal. The paper outlines four optimal scenarios, each reflecting distinct pension policy priorities through a multi-criteria decision-making model.

Figure 4 illustrates four optimal scenarios, also showing the priority weights. A detailed economic description and analysis of these scenarios is presented in the above-mentioned publication.

In the following we introduce an additional scenario that was not included in the previously discussed analysis. The outcomes of this scenario are subsequently compared with the baseline projection of the pension system's financial balance in the absence of policy intervention, as outlined in the introductory section of the study.

This scenario aims to balance long-term sustainability with intergenerational fairness by moderately adjusting the retirement age and contribution rates. The priority weights are as follows: $M = 0.25$; $W = 0.15$; $B = 0.15$; $C = 0.05$; $D = 0.30$; $E = 0.05$; $G = 0.05$.

The dynamics of the Adaptive Integrational Equity Scenario is presented in the Fig. 5.

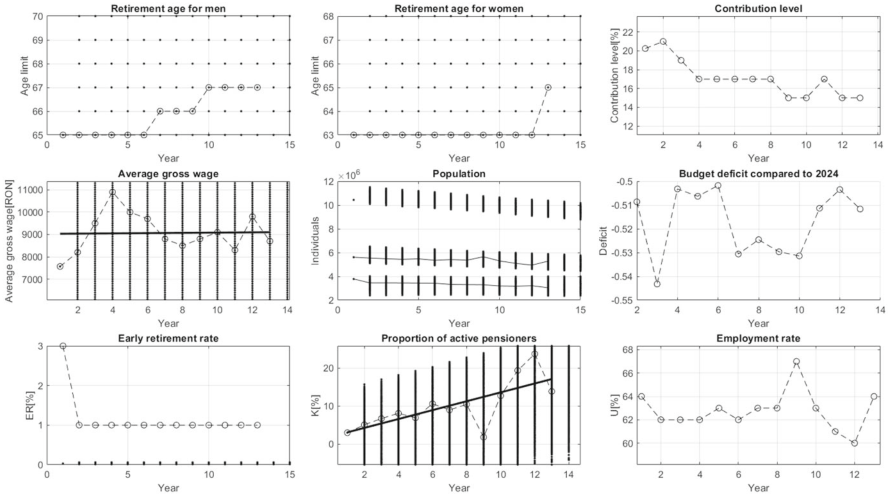

Fig. 5 Adaptive integrational equity scenario

6 Conclusions

This study introduces a deterministic dynamic optimisation framework for scenario-based modelling of publicly funded pension systems. The model aims to optimise annual policy decisions across seven key system parameters: retirement age for women and men, contribution rate, national average wage, early retirement rate, proportion of active pensioners, and employment rate based on predefined policy priority weights. The objective is to minimise deviations from a 2024 reference baseline over a multi-year horizon.

A key element of the model is a cost-based objective function that quantifies suboptimality via normalised deviations from baseline values. Optimisation is performed using backward induction in accordance with Bellman's principle of optimality. Demographic evolution is modelled deterministically using age-and gender-specific historical data, while legal and fiscal constraints are explicitly embedded to ensure the feasibility of policy trajectories.

By changing the weights of priorities, the model enables decision-makers to explore alternative policy directions in line with different strategic preferences. These optimised trajectories are visualised to support interpretability and facilitate policy evaluation under competing objectives. Compared to the no-intervention baseline presented in the introductory section, the optimised trajectories reveal significant improvements in fiscal balance, demonstrating the model's capacity to support more efficient long-term planning. In the scenarios presented in the optimal solutions, the deficit level of 2024 is reduced by half, while in the no-intervention scenario, it doubles in real terms by the end of the period.

The main limitation of this study is that the model relies on deterministic demographic projections, without explicitly incorporating uncertainty factors. In addition, the elasticity parameters for employment and migration have been estimated in a simplified form. Although the modular structure of the model allows for adaptation to other countries, the current version is primarily based on Romanian legal and institutional frameworks. International adaptation would require additional calibration and country-specific parameterization. At this stage, the model should be viewed as a prototype framework demonstrating methodological feasibility rather than a comprehensive policy support tool.

Potential future enhancements include the integration of stochastic modelling elements, rule-based adaptive policies, and labour market feedback mechanisms. Such extensions would further strengthen the model's applicability as a decision support tool for dynamic, long-term pension system design. The integration of sensitivity analyses would allow for a more rigorous assessment of the model's robustness and its responsiveness to variations in input parameters.

Appendix A Methodology

See Figs. 6, 7, 8.

Methodology

DECISION VARIABLES	NOTATIONS	CONSTRAINTS
	arw_n: retirement age for women	$63 \le arw_n \le 68$
	arm_n: retirement age for men	$65 \le arm_n \le 70$
	T_n: contribution level	$15\% \le T_n \le 30\%$
	$\bar{S}_n$: gross national average wage	$5000 \le S_n \le 12000$
	e_n: early retirement rate	$0 \le e_n \le 3\%$
	k_n: the proportion of active pensioners	$0 \le k_n \le 30\%$
	u_n: employment rate	$60\% \le u_n \le 68\%$

1. DEMOGRAPHIC MODULE

EQUATION NAME	EQUALITY	DESCRIPTION
Mortality Rate	$MR_{a,n,g} = \frac{(Pop_{a,n,g} - Pop_{a+1,n+1,g})}{Pop_{a,n,g}}$	Inter-period transitions are computed using the transition loss rate (TLR), capturing the year-to-year relative decline in each cohort.
Average Mortality Rate	$AMR_{a,g} = \frac{1}{12} \sum_{n=2010}^{2023} MR_{a,n,g}$	Mortality rates are averaged over the 2010–2021 period to smooth yearly fluctuations and generate stable long-run estimates.
Population Forecast	$Pop_{a+1,n+1,g} = Pop_{a,n,g} * (1 - AMR_{a,g})$	Average rates are used to project future population developments, assuming a deterministic transition mechanism.
Total Population	$Pop_{n,g} = YP_{n,g} + UP_{n,g} + OP_{n,g}$	The total population in year *n* is a vector composed of three functional subgroups: underage (YP), working-age (UP), and retired (OP).
Underage population	$YP_{n,g} = \sum_{a=0}^{18} Pop_{a,n,g}$	Represents the underage population, all individuals aged 0–18 in year n.
Working-Age Population	$UP_{n,g} = \sum_{a=18}^{ar_n} Pop_{a,n,g}$	Refers to individuals aged 19 up to the statutory retirement age in year *n*.
Retired Population	$OP_n = \sum_{a=arm_n}^{a\_max_{n,m}} Pop_{a,n,m} + \sum_{a=arw_n}^{a\_max_{n,w}} Pop_{a,n,w}$	Disaggregated by gender and retirement thresholds, this includes all individuals above retirement age up to maximum life expectancy.

Fig. 6 Decision variables

Methodology

2. FINANCIAL BALANCE AND PENSION SYSTEM DYNAMICS

EQUATION NAME	EQUALITY	DESCRIPTION
Fiscal Balance	$BAL_n = REV_n - EXP_n$	The system's fiscal position is defined by the difference between total revenue and total expenditure in year n.
Annual Revenue	$REV_n = T_n * \overline{S}_n * C_n * 12$	Annual revenue is computed based on the number of contributors, their average gross wage, and the statutory contribution rate, multiplied by 12 months.
Contributors	$C_n = LF_n * u_n + k_n * OP_n$	The number of contributors includes both employed individuals and the share of active pensioners, weighted by their respective rates.
Labor Force	$LF_n = \wp_n * (1 - ds_n - er_n) + NM_n$	The labour force is adjusted by subtracting disability and early retirement rates from the working-age population and adding net migration inflows.
Employment Elasticity	$\frac{u_n - u_{n-1}}{u_{n-1}} = \left(\frac{\overline{S}_n - \overline{S}_{n-1}}{\overline{S}_{n-1}} * eu_{\overline{s_n}} + \frac{T_n - T_{n-1}}{T_{n-1}} * em_{T_n}\right)$	The employment rate is elastic with respect to changes in the average wage and contribution rate.
Migration Elasticity	$\frac{NM_n - NM_{n-1}}{NM_{n-1}} = \left(\frac{\overline{S}_n - \overline{S}_{n-1}}{\overline{S}_{n-1}} * em_{\overline{s_n}} + \frac{T_n - T_{n-1}}{T_{n-1}} * em_{T_n}\right)$	Net migration of working-age individuals is influenced by wage growth and contribution changes.
Total Expenditure	$EXP_n = P_n * \overline{PP}_n * 12$	Total pension expenditure is calculated as the product of the number of beneficiaries, the average pension, and 12 months.
Beneficiaries	$P_n = OP_n * rr_n$	The number of pension beneficiaries is estimated as the retired population multiplied by the old-age retirement rate.
Pension Indexation	$\overline{PP}_n = \begin{cases} \overline{PP}_{n-1} * \left[i_n + 1 + \left(\left(\frac{S_n}{S_{n-1}} - 1\right) * r\right)\right], & \text{if } n \leq 2030, \\ \overline{PP}_n = \overline{PP}_{n-1} * [i_n + 1], & \text{if } n > 2030, \end{cases}$ where $r = 0,3 - (0,05 * (n - 2024))$ if $n \in [2024; 2030]$	Pension increases are based on inflation and a decreasing share of real wage growth, following Law 263/2010. From 2031 onwards, pensions are indexed only to inflation.

Fig. 7 Financial balance and pension system dynamics

Methodology

Weights Parameters of the Decision Variables	M, W, B, C, D, E, G

3. OPTIMISATION FRAMEWORK

EQUATION NAME	EQUALITY	DESCRIPTION
Objective Function	$F_n = M\frac{arm_n - arm_1}{arn_1} + W\frac{arw_n - arw_1}{arw_1} + B\frac{T_n - T_1}{T_1} + C\frac{\overline{S}_n - \overline{S}_1}{\overline{S}_1} + D\frac{BAL_n - BAL_1}{BAL_1} + E\frac{er_n - er_1}{er_1} + G\frac{u_n - u_1}{u_1}$	Captures the weighted deviation of key policy variables from their 2024 reference levels, reflecting decision-maker priorities.
Constraint Set	$\begin{cases} f(1) & 65 \le arm_n \le 70 \\ f(2) & 63 \le arw_n \le 70 \\ f(3) & 15\% \le T_n \le 30\% \\ f(4) & 5000 \le \overline{S}_n \le 12000 \\ f(5) & 0 \le BAL_n \le 0.5 \cdot BAL_1 \\ f(6) & 0 \le er_n \le 3\% \\ f(7) & 60 \le u_n \le 68\% \end{cases}$	Ensure that all decision variables remain within politically and legally feasible bounds.
Feasible Solution Set	$FS_n = \{p_n = (arm_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n):$ the parameters that satisfy conditions $f(i)\ ,\ i = \overline{1,7}.\}$	Denotes the set of all parameter combinations in year n that satisfy the full constraint system f(1)–f(7).
Static Optimisation Problem	$\begin{cases} F_1(arm_1, arw_1, T_1, \overline{S}_1, Pop_1, er_1, u_1) \to \min \\ BAL_n \le 0.5 \cdot BAL_1 \end{cases}$	The objective is to minimise deviations from baseline in the first year, while maintaining fiscal balance within acceptable limits.
State-Transition Equations	$\begin{cases} F_n = F_n(arn_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n) \\ BAL_n = BAL_n(arn_n, arw_n, T_n, \overline{S}_n, Pop_n, er_n, u_n) \end{cases}$	Define how both the objective function and fiscal balance evolve over time as a function of current decisions.
Bellman Recursion (Dynamic Optimisation)	$F^*_{n-1}(p^*_{n-1}) = \min_{p_n \in FS_n} \{F_n(p_n) + F^*_n(p^*_n)\}$	Represents the recursive structure of the optimisation: each stage minimises the sum of its own cost and the future optimal value.
Optimal Policy Path	$\{p^*_{n-1}, p^*_n\} \in Arg \min_{p_n \in FS_n} \{F_n(p_n) + F^*_n(p^*_n)\}$	The Bellman equation generates a sequence of optimal policies by backward induction, starting from the final year.

Fig. 8 Weights parameters

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest. This manuscript does not contain any studies involving human participants or animals performed by any of the authors.

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