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Effect of the loading condition on the statistics of crackling noise accompanying the failure of porous rocks

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We test the hypothesis that loading conditions affect the statistical features of crackling noise accompanying the failure of porous rocks by performing discrete element simulations of the tensile failure of numerical rock samples and comparing the results to those of compressive simulations of the same specimens. Cylindrical samples are constructed by sedimenting randomly sized spherical particles connected by beam elements representing the cementation of granules. Under a slowly increasing external tensile load, the cohesive contacts between particles break in bursts whose size fluctuates over a broad range. Close to failure breaking avalanches are found to localize on a highly stressed region where the catastrophic avalanche gets triggered and the specimen falls apart into two pieces along a spanning crack. The fracture plane has a random position and orientation falling most likely close to the center of the specimen perpendicular to the load direction. In spite of the strongly different strengths and spatial structure of damage of tensile and compressive failure of numerical porous rocks, our calculations revealed that the size, energy, and duration of crackling avalanches, and the waiting time between consecutive events all obey scale free statistics with power law exponents which agree within the error bars in the two loading cases.

1. Introduction

Rocks experience complex loading conditions including tension, compression, and shear during their geological history. Deformation is accompanied by the release of elastic energy from micro-cracking events, which can be registered in the form of acoustic waves [1]. Acoustic emissions (AE) are the primary source of information about the microscopic processes [2–5] of fracturing providing us with valuable data about the temporal and spatial evolution of the ensemble of micro-cracks with the ability to discriminate between micro-cracking events dominated either by local tension or shear [6]. Further refinement of the experimentally available information on fracturing solids notably on the evolution of the spatial structure of damage can be achieved by synchrotron imaging of live tests [7,8]. The acoustic ‘crackling noise’ generated during the compressive failure of rocks in laboratory experiments has been found to exhibit scale-free statistics similar to earthquakes, which addressed the universality of cracking phenomena across a broad range of length scales [9–14]. In spite of the large amount of experimental and theoretical efforts, the effect of the loading conditions on the statistical features of crackling bursts generating acoustic emissions still remained an open problem of significant importance. Due to the attenuation of materials and the not ideal placement of sensors on specimens, usually only a small fraction of cracking events produce notable acoustic signals [1], which limits the experimentally available information. To obtain a deeper understanding of the statistics and dynamics of crackling noise computer simulations of discrete models of heterogeneous materials are indispensable. Large scale simulations of the failure process of lattices of electric fuses [15–17], springs [18], fiber bundles [19,20], and cohesive granular materials [21] all revealed a high degree of robustness of the statistics of bursts of local failures against the amount of materials disorder, but to date no systematic studies have been performed at different loading conditions.

In order to understand how the loading condition affects the intermittent evolution of the fracture process and the spatial structure of damage in porous rocks, here we use discrete element simulations to analyze the statistical and dynamical features of crackling noise emerging during the tensile failure of a realistic model rock and compare the results to the outcomes of simulations obtained under compressive loading of the same samples. We demonstrate that under uniaxial tensile loading the numerical rock samples have a quasi-brittle behaviour where the fluctuating ultimate strength and the strain where cracking sets on are both described by Weibull distributions. Simulations revealed that as the specimen is slowly elongated fracturing proceeds in bursts of micro-cracking events which have a scale free statistics, i.e. the size, duration, and energy released by breaking avalanches are all power law distributed with a finite size cutoff. The beginning of the failure process was found to be dominated by the disordered micro-structure of the material which gives rise to random nucleation of small sized cracks all over the sample. Approaching failure, breaking avalanches tend to localize and merge into a sharply defined fracture plane along which the specimen falls apart. With a careful numerical analysis we give a quantitative characterization of the fluctuating sharpness, orientation, and position of the fracture plane. We compare the results to the outcomes of the simulations of uniaxial compressive failure of the same specimens [21,22]. In spite of the substantial differences of the spatial structure of damage in the two cases, for the statistics of avalanche quantities the same qualitative trends are obtained and also the value of the exponents fall rather close to each other, which confirms the robustness of the statistics of crackling noise with respect to the loading condition.

2. Discrete element Model of porous rocks

To study crackling bursts generated during the tensile failure of porous rocks we use a discrete element model which has been introduced recently as a generic modelling framework for heterogeneous materials with a porous micro-structure [21–23]. In the model cylindrical shape specimens are considered with diameter D and height H having the aspect ratio $H/D = 2.3$ typical for experimental setups with geo-materials [6]. The disordered micro-structure of

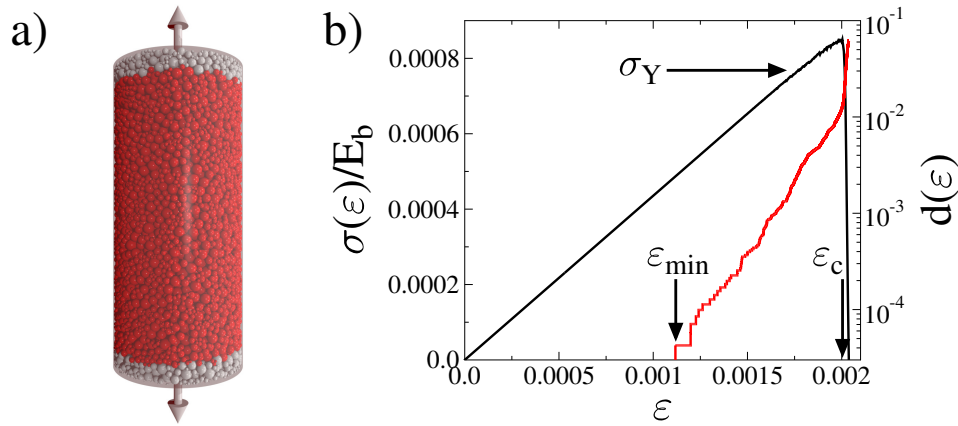


Figure 1. (a) Setup of the numerical experiments. DEM simulations were performed by slowly elongating cylindrical specimens composed of a random packing of spherical particles. A few particle layers (highlighted in grey) were clamped at the top and bottom of the specimen which were then slowly moved against each other along the cylinder axis. (b) Constitutive curve $\sigma(\epsilon)$ (black) of the system together with the accumulated damage $d(\epsilon)$ (red) in a single sample. The stress σ is scaled with the Young modulus of beams E_b . The arrows indicate the strain values where cracking starts ϵ_{min} , where final breakdown occurs ϵ_c , and the Yield stress σ_Y , where the first discernible deviation from linearity of the $\sigma(\epsilon)$ curve occurs.

sedimentary rocks is captured by sedimenting spherical particles in the cylinder with a random radius R sampled from a log-normal distribution commonly found for granules of sedimentary rocks [2]. To represent the cementation between granules, a Delaunay triangulation is performed with the center of spheres in three dimensions, and the particle centers are connected by beam elements along the edges of the triangles. As the specimen deforms under an externally exposed loading, beams can suffer elongation, compression, shear, and bending exerting forces and torques on the particles. Beams break when they get over-stressed during the dynamics of the system, which leads to the formation of micro-cracks. We use a physical breaking criterion which takes into account that beam breaking is caused by stretching and bending

$$\left(\frac{\epsilon_{ij}}{\epsilon_{th}}\right)^2 + \frac{\max(|\Theta_i|, |\Theta_j|)}{\Theta_{th}} \geq 1, \quad (2.1)$$

where ϵ_{ij} is the longitudinal deformation of the beam between particles i and j , while Θ_i and Θ_j denote the bending angles at the two beam ends. The breaking parameters ϵ_{th} and Θ_{th} , which control the relative importance of the two breaking modes, have fixed values for all the beams. When particles not connected by beam come into contact, an elastic restoring force is applied according to the Hertz contact law [24].

Tensile loading of the cylindrical samples with deformation control was performed by clamping a few boundary layers of particles at the bottom and top of the cylinder, which were then slowly moved against each other along the cylinder axis. The time evolution of the particle system was generated by solving the equation of motion of the particles by means of a 5th order Predictor-Corrector scheme [25] taking into account the boundary and loading conditions. The breaking criterion Eq. (2.1) is evaluated in each iteration step and those beams which fulfill the condition are removed from the sample. As a result of consecutive beam breakings, cracks are formed and the sample eventually breaks apart. The initial sample and the loading condition are illustrated in Fig. 1(a). The model has been successfully used to study the failure process of porous rocks under compressive loading where it could reproduce the experimental findings on the spatial structure of damage and on the intermittent breaking activity accompanying the failure process [21–23,26].

In the present simulations we considered cylindrical samples comprising on average $N \approx 20,000$ particles under tensile load. We then compare the results from previously published simulations on uniaxial compressive loading. Unless stated otherwise, the results presented here refer to the tensile load case. The diameter D of the base circle of the sample is $D \approx 87 \langle R \rangle$, where $\langle R \rangle$ denotes the average diameter of the spherical particles. Averaging was performed over $K = 1000$ simulations with samples of different random arrangements of the particles. For more details of the modelling approach, see Refs. [22,23,26].

3. Quasi-brittle response

We performed computer simulations of the uniaxial tensile loading of cylindrical specimens monitoring their macroscopic response and the underlying microscopic process of fracturing. To characterize the mechanical response of the sample under deformation controlled loading, we measured the force F needed to maintain the deformation as the clamped boundary particle layers were slowly moved along the cylinder axis. The macroscopic stress σ and strain ε of the sample were obtained as $\sigma = F/A$ and $\varepsilon = \Delta H/H$, where $A = D^2\pi/4$ is the initial cross section of the cylinder, and ΔH is the elongation of the sample. As the sample is slowly elongated beams, representing cohesive particle contacts, gradually break and form cracks which lead to global failure when the specimen falls apart into two large pieces. The simulation stops when the force F acting on the boundary layers drops down to zero. Figure 1(b) presents the constitutive curve $\sigma(\varepsilon)$ for a single sample, i.e. the stress-strain curve following the evolution of the system up to final breakdown. The system has a linearly elastic behaviour over a broad range of ε and non-linearity is only observed in the vicinity of the maximum beyond which the stress σ rapidly falls to zero. The overall strength of the sample can be characterized by the position ε_c and value σ_c of the maximum of the $\sigma(\varepsilon)$ curve, which define the critical strain and the critical stress of the system, respectively.

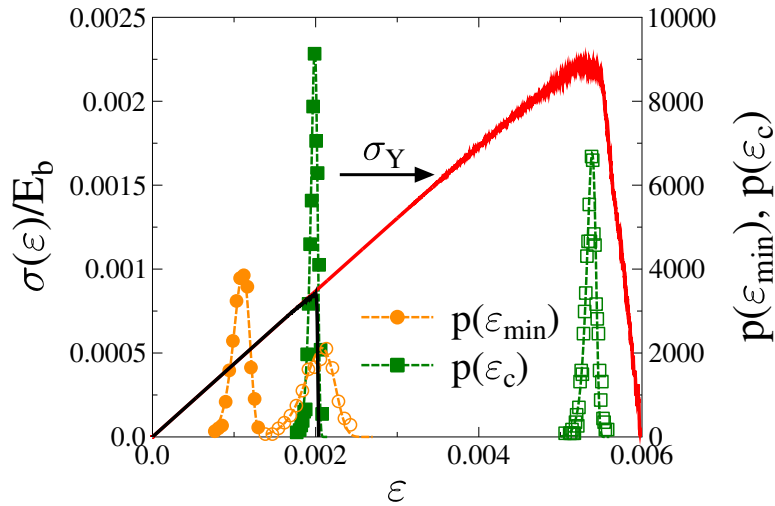


Figure 2. Comparison of the constitutive curves $\sigma(\varepsilon)$ of the same sample measured under uniaxial compression (red) and elongation (black). The stress σ is scaled with the Young modulus of the beams E_b . The probability distributions $p(\varepsilon_{min})$ and $p(\varepsilon_c)$ of the strain of crack initiation ε_{min} (circle) and of global failure ε_c (square) are also presented for both loading cases. Filled and open symbols are used for the tensile and compressive results, respectively. The dashed lines represent fits of the distributions with the Weibull distribution Eq. (3.1). The horizontal arrow indicates the Yield stress σ_Y of the compressive case.

The state of damage can be quantified by the fraction of broken contacts $d = N_{br}/N_B$, where N_{br} and N_B denote the number of broken beams and the total number of intact ones in the initial state, respectively. Comparing the damage curve $d(\varepsilon)$ to the evolution of the mechanical response $\sigma(\varepsilon)$, it can be seen in Fig. 1(b) that non-linearity of $\sigma(\varepsilon)$ emerges when beams start to break and the growing non-linearity is caused by the acceleration of damage accumulation as the critical point is approached. The relatively weak non-linearity and the sudden drop-down of the stress at global failure imply a quasi-brittle response of the model rock. The tensile and compressive responses of the same specimen are compared in Fig. 2, where all the model parameters had the same values in the simulations, only the direction of the motion of the boundary particle layers were opposite to each other. The two $\sigma(\varepsilon)$ curves fall on the top of each other, which confirms that the effective Young modulus Y_{eff} , i.e. the slope of the linear regime of $\sigma(\varepsilon)$, does not depend on the loading condition. Careful fitting of the constitutive curves yields $Y_{eff}/E_b \approx 0.22$, where E_b denotes the Young modulus of the beam elements of cohesive interparticle contacts. However, the ultimate strength ε_c and σ_c , where global failure occurs proved to be significantly higher under compression due to the stabilizing effect of closing cracks, primarily perpendicular to the direction of maximum principal stress. Note that the drop down of the stress at global failure is less abrupt for compression and failure is preceded by a stronger non-linearity than for tensile loading implying the higher degree of brittleness of the sample in the tensile case.

The disordered micro-structure of the sample has the consequence that the strain of the onset of cracking ε_{min} , furthermore, the critical strain ε_c and stress σ_c of ultimate failure are stochastic quantities with sample-to-sample fluctuations. To give a measure of these fluctuations we determined the probability distributions $p(\varepsilon_{min})$ and $p(\varepsilon_c)$ for both tensile and compressive loading, which are also included in Fig. 2. The relative variances have the values $var(\varepsilon_{min})/\langle \varepsilon_{min} \rangle \approx 0.09$ and $var(\varepsilon_c)/\langle \varepsilon_c \rangle \approx 0.025$ (tension), $var(\varepsilon_{min})/\langle \varepsilon_{min} \rangle \approx 0.105$ and $var(\varepsilon_c)/\langle \varepsilon_c \rangle \approx 0.014$ (compression), which indicate that both quantities ε_{min} and ε_c have mild fluctuations at the system size N considered. Numerical analysis revealed that for both loading cases the probability distributions $p(\varepsilon_{min})$ and $p(\varepsilon_c)$ can be well fitted by the Weibull distribution

$$p(x) = \frac{m}{\lambda} \left(\frac{x}{\lambda} \right)^{m-1} e^{-(x/\lambda)^m}, \quad (3.1)$$

with two parameters, where λ sets the scale of the values of x , while the exponent m controls the shape of the distribution. Best fits presented in Fig. 2 were obtained with Eq. (3.1) using the parameter values $\lambda = 0.0012$, $m = 11$ for ε_{min} , and $\lambda = 0.00204$, $m = 50$ for ε_c , and $\lambda = 0.00209$, $m = 12$ for ε_{min} , and $\lambda = 0.0054$, $m = 99$ for ε_c , for tensile and compressive loading, respectively. The critical load σ_c was found to obey the same statistics with similarly high Weibull exponents both for tension and compression. The high Weibull exponents are consistent with the mild fluctuations quantified by the relative variances above. The modal values of the strain of crack initiation ε_{min} and ultimate failure ε_c are 0.00112, 0.00199, and 0.00219, 0.00536 for tensile and compressive loading, respectively. It follows from the numerical analysis that the average strength of the sample is about 2.7 times higher under compression than under tensile loading although the relative fluctuations of the strength values are nearly the same in agreement with experimental findings [27]. To characterize the degree of brittleness of the sample, we determined the ratio of the Yield stress σ_Y , i.e. the stress where the first discernible deviation occurs from the linear behavior, and of the critical stress σ_c of ultimate failure. Comparing the average ratios $\langle \sigma_Y/\sigma_c \rangle$ obtained for tension $\langle \sigma_Y/\sigma_c \rangle \approx 0.89$ and for compression $\langle \sigma_Y/\sigma_c \rangle \approx 0.62$, a significantly higher degree of brittleness of the samples can be inferred under tension.

4. Scale-free bursting activity

In spite of the smooth accumulation of damage $d(\varepsilon)$ (see Fig. 1(b)), simulations revealed that damaging of the sample proceeds in intermittent bursts of beam breakings, which can be considered as the counterparts of acoustic emission sources in outbreaks in real experiments.

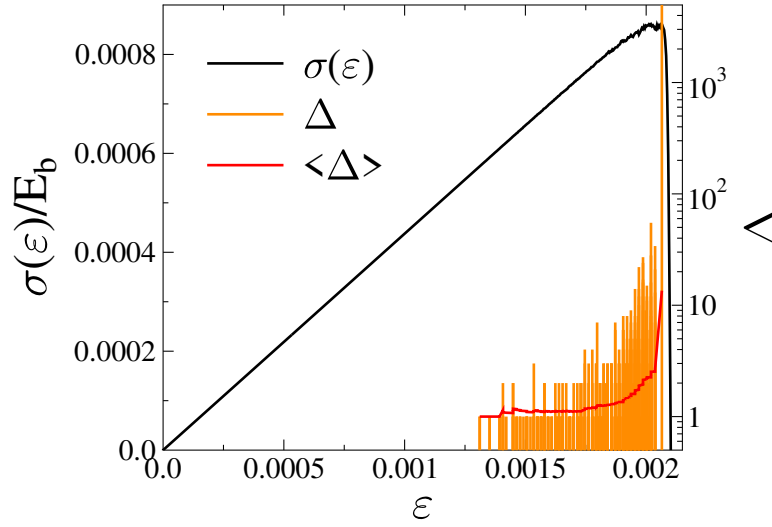


Figure 3. Sequence of breaking avalanches of beams along with the constitutive curve $\sigma(\varepsilon)$ of a representative sample. The height of the bars represent the size Δ_i of the avalanches $i = 1, \dots, n_b$. The continuous red line indicates the moving average of the avalanche size $\langle \Delta \rangle$ obtained by averaging over 51 consecutive events.

The reason is that the breaking of a beam is followed by the redistribution of stress in the sample, which can relax but also increase the load on beams in certain regions of the local neighborhood triggering additional failure events, which are again followed by load redistribution. Through such sequences of breaking-load redistribution steps a single beam breaking may trigger an entire avalanche of breaking events which either stops when all the beams can sustain the local load inside the sample or leads to catastrophic failure of the system. The time scale t_c of load redistribution is controlled by the speed of elastic waves of the sample. In order to identify avalanches of local breaking events, we follow the techniques developed in Refs. [21,22]: in the simulations we record the time t_j of all individual beam breakings and assume that those breaking events which follow each other within the correlation time $t_{j+1} - t_j < t_c$ belong to the same trail of breakings. Single avalanches determined by the algorithm are characterized by their size Δ , duration T , spatial position $\vec{r}$, and energy E dissipated during the avalanche. The burst size Δ is obtained as the total number of beams breaking in the bursts, which is proportional to the newly created free surface inside the sample. The spatial position $\vec{r}$ of an avalanche of size Δ is calculated as the center of mass position of the cloud of beams

$$\vec{r} = \frac{\sum_{j=1}^{\Delta} \vec{r}_j^b}{\Delta}, \quad (4.1)$$

where $\vec{r}_j^b$ ($j = 1, \dots, \Delta$) denotes the position vector of the center of beams breaking in the avalanche. The burst duration T is calculated as the time difference between the last and the first breakings of the avalanche

$$T = t_{last} - t_{first}, \quad (4.2)$$

while the dissipated energy E is obtained as the sum of the elastic energies stored in the deformation of beams at the time of their breaking.

The sequence of bursts is illustrated for a failure process in Fig. 3, where the size of bursts Δ_i ($i = 1, \dots, n_b$), represented by the height of the bars, is plotted at the strain where the bursts occurred. At the beginning of fracturing bursts comprise only a few broken beams, however, as global failure is approached the bursts have an increasing size Δ apart from fluctuations, and

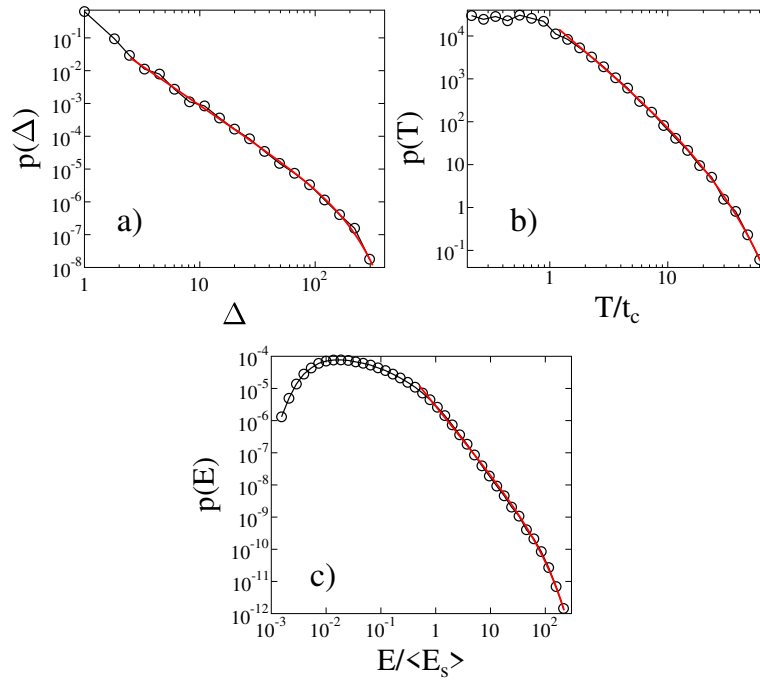


Figure 4. Probability distribution of the size $p(\Delta)$ (a), duration $p(T)$ (b), and dissipated energy $p(E)$ (c) of bursts taking into account all the events which occurred before catastrophic collapse (up to the maximum of the constitutive curve $\sigma(\varepsilon)$). The continuous red lines represent fits with the functional form Eq. (4.3). The duration T and the energy E are made dimensionless by division with the correlation time t_c and the average energy $\langle E_s \rangle$ dissipated by a single beam breaking, respectively.

they follow each other after smaller and smaller strain increments indicating the acceleration of the failure process. Due to the inherent disorder of the structure of the sample the fluctuating burst size Δ covers a broad range, where the largest burst is always the last one, i.e. the catastrophic avalanche during which a macro-crack is formed spanning the entire sample. To quantify the statistics of the fluctuating quantities we determined the probability distributions of the size $p(\Delta)$, duration $p(T)$, and energy $p(E)$ of burst considering all events except for the catastrophic one. It can be observed on the double logarithmic plots of Fig. 4 that for sufficiently large values of Δ , T , and E the three distributions can be well approximated by a power law functional form followed by an exponential cutoff due to the finite sample size

$$p(x) \sim x^{-\tau} \exp(-x/x_0). \quad (4.3)$$

Here x_0 denotes the characteristic value of the avalanche quantity controlling the cutoff. The value of the power law exponent τ was obtained by fitting the curves in Figs. 4(a), (b), and (c) as $\tau_\Delta = 2.4 \pm 0.11$, $\tau_T = 2.25 \pm 0.08$, and $\tau_E = 2.11 \pm 0.05$ for the distributions of the burst size $p(\Delta)$, duration $p(T)$, and dissipated energy $p(E)$, respectively. In the case of the dissipated energy, the local maximum of $p(E)$ is caused by the energy distribution of single breaking beams.

Of course, the exponents τ_Δ , τ_T , and τ_E are not independent of each other, avalanche quantities are usually positively correlated because avalanches of larger size typically have a longer duration and dissipate a higher amount of energy. To give a quantitative characterization of this correlation we calculated the average duration $\langle T \rangle$ and energy $\langle E \rangle$ of bursts of a given size Δ . Figure 5(a) demonstrates that for sufficiently large burst sizes the correlation of the three

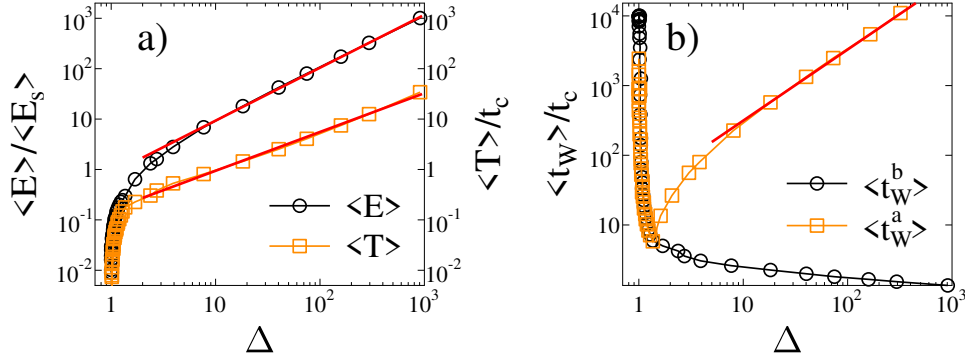


Figure 5. (a) Average energy $\langle E \rangle$ and duration $\langle T \rangle$ of avalanches of size Δ . The straight lines represent power laws of exponent $\nu_E = 1.01$ and $\nu_T = 0.77$. (b) Average value of the waiting time elapsed before $\langle t_W^b \rangle$ and after $\langle t_W^a \rangle$ avalanches of size Δ . Power law correlation is revealed between $\langle t_W^a \rangle$ and Δ with the correlation exponent $\nu_W = 1.02$.

quantities can be well described by power laws

$$\begin{aligned} \langle T \rangle &\sim \Delta^{\nu_T}, \\ \langle E \rangle &\sim \Delta^{\nu_E}, \end{aligned} \quad (4.4)$$

where best fit of the curves was obtained with the exponents $\nu_T = 0.770 \pm 0.025$ and $\nu_E = 1.01 \pm 0.02$. Using the correlation exponents ν_T and ν_E , we can establish relations between the exponents of the probability distributions

$$\begin{aligned} \tau_T &= (\tau_\Delta + \nu_T - 1)/\nu_T, \\ \tau_E &= (\tau_\Delta + \nu_E - 1)/\nu_E. \end{aligned} \quad (4.5)$$

Substituting the numerical values obtained by fitting a reasonable agreement is provided by Eqs. (4.5).

For the time evolution of the intermittent avalanche sequence, it is crucial to study the statistics of waiting times t_W , i.e. the duration of silent periods between consecutive bursts. Figure 6 demonstrates that the probability distribution of waiting times $p(t_W)$ has also a scale-free behaviour well-described by the functional form Eq. (4.3). The exponent τ_W of the waiting time distribution $p(t_W)$ has a relatively high value $\tau_W = 1.81 \pm 0.04$ which implies that large waiting times are relatively rare in the sequence. Since avalanches are driven by the gradual redistribution of load over the intact beams in the solid, a correlation may arise between the size of avalanches and the waiting time until the next avalanche is triggered. Avalanches release stress in their close vicinity so that it can be expected that after a larger avalanche one has to wait longer to build up again the stress field and initiate the next avalanche under the slowly increasing deformation controlled loading. To quantify this effect we determined the average value of the waiting time elapsed before $\langle t_W^b \rangle$ and after $\langle t_W^a \rangle$ avalanches of a given size Δ . Figure 5(b) shows that $\langle t_W^b \rangle$ rapidly converges to the vicinity of t_c , which implies no correlation between t_W^b and the size of the following burst Δ . However, $\langle t_W^a \rangle$ exhibits a power law increase with Δ

$$\langle t_W^a \rangle \sim \Delta^{\nu_W}, \quad (4.6)$$

which shows that it takes longer to initiate the next avalanche after a larger one. The exponent ν_W proved to have the value $\nu_W = 1.02 \pm 0.02$.

Computer simulations of uniaxial compression of the same specimens performed in Refs. [21, 22] revealed the same type of scale-free statistics of avalanche characteristics. Table 1 summarizes the value of the exponents of the probability distribution of the size, energy, and duration of

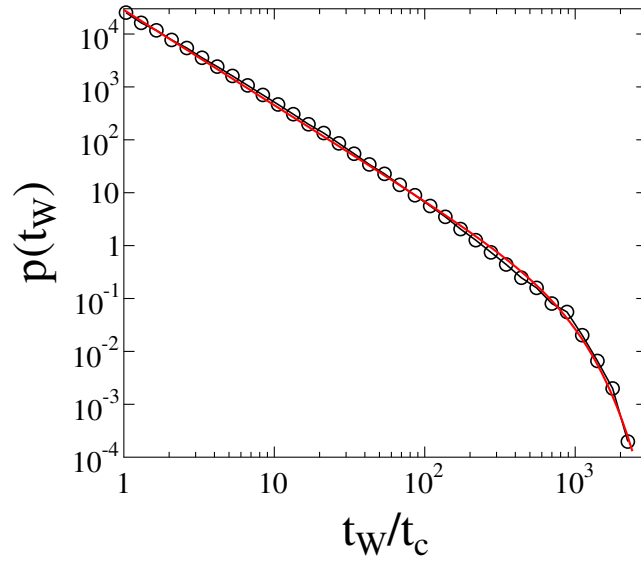


Figure 6. Probability distribution of waiting times $p(t_W)$ between consecutive avalanches. The red line represents the best fit obtained with Eq. (4.3).

Table 1. Power law exponents characterizing the statistics of breaking avalanches obtained under strain controlled uniaxial tensile and compressive loading. Results of compression simulations are taken from Refs. [21,22].

Exponent	Notation	Tension	Compression
Avalanche size	τ_Δ	2.40 ± 0.11	2.22 ± 0.12
Avalanche duration	τ_T	2.25 ± 0.08	2.4 ± 0.13
Avalanche energy	τ_E	2.11 ± 0.05	2.02 ± 0.06
Waiting time	τ_W	1.81 ± 0.04	2.0 ± 0.06
Size-Duration	ν_T	0.770 ± 0.025	0.8 ± 0.02
Size-Energy	ν_E	1.01 ± 0.02	1.15 ± 0.03
Size-Waiting time	ν_W	1.02 ± 0.02	1.37 ± 0.05

cracking avalanches, furthermore, the correlation exponents of the three quantities along with the waiting time exponents for both tension and compression simulations. Due to the higher strength and stability of the sample observed under compression, the system can tolerate a larger number of bursts n_b which can grow to larger sizes without becoming catastrophic. Our analysis revealed that the cutoffs of the distributions of avalanche quantities, i.e. the average of the largest avalanche size $\langle \Delta_{max} \rangle$, energy $\langle E_{max} \rangle$, and duration $\langle T_{max} \rangle$, and the average number of avalanches $\langle n_b \rangle$ are 5 to 8 times larger under compression than under tension. From Table 1 the corresponding exponents of tension and compression simulations agree with each other within the error bars except for the correlation exponents ν_E and ν_W which are smaller for tension.

5. Spatial structure of damage

In spite of the robustness of the statistics of crackling noise, computer simulations revealed substantial differences between the spatial structure of the damage of tensile and compressive failure. As the specimen is slowly elongated first the weakest beams break, which results in random crack nucleation scattered all over the sample in an uncorrelated manner as illustrated by Fig. 7(a). The early cracks have a small size, comprising only a few broken bonds, however,

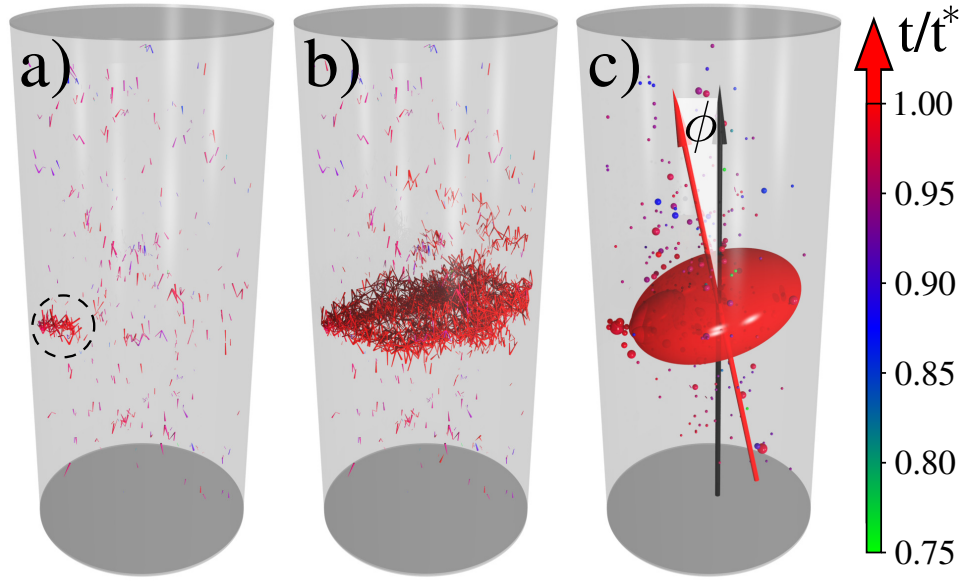


Figure 7. Spatial structure of damage in the specimen. (a) Broken beams of avalanches which occurred before the catastrophic event. The circle highlights the region where the last 5 avalanches occurred before the catastrophic one, this is also the region where the catastrophic avalanche was initiated. (b) All the broken beams including also the ones of the catastrophic avalanche. (c) The oblate ellipsoid obtained for the point cloud of the center of the broken beams of the catastrophic event. Avalanches occurred before the catastrophic one are indicated by spheres with a radius proportional to the avalanche size. The color of the spheres indicates the time t when the avalanche occurred according to the color scale on the right, where t is scaled with the critical time t^* of failure. Angle ϕ enclosed by the shortest axis of the ellipsoid with the load direction is also highlighted.

as fracturing proceeds the size of avalanches and the resulting cracks increase in size and their spatial appearance gets also correlated. In Fig. 7(a) the spacial region where micro-cracks of the last five avalanches occurred before the catastrophic one is highlighted by a circle. A localization of damage can be inferred inside the circle where in turn also the catastrophic avalanche gets initiated. In order to characterize the spatial sequence of avalanches we determined the average distance $\langle |\Delta \vec{r}_{i,i+1}| \rangle$ of consecutive events, where $\Delta \vec{r}_{i,i+1} = \vec{r}_{i+1} - \vec{r}_i$ is the relative position of two bursts with positions $\vec{r}_i$ and $\vec{r}_{i+1}$ following each other in the event sequence. In Figure 8 the average distance $\langle |\Delta \vec{r}_{i,i+1}| \rangle$ is re-scaled with the diameter D of the cylindrical sample so that the value $\langle |\Delta \vec{r}_{i,i+1}| \rangle / D \approx 0.45$ of the ratio indicates the random dispersion of consecutive events over the entire sample. This behaviour is characteristic for the beginning of the fracture process until avalanches have a relatively low average size $\langle \Delta \rangle$. However, when $\langle \Delta \rangle$ starts to increase in the vicinity of global failure, the distance of consecutive avalanches gets gradually reduced indicating the emergence of spatial correlations in agreement with the spatial distribution of damage in Figs. 7(a, b). Note that the high values of $\langle |\Delta \vec{r}_{i,i+1}| \rangle / D$ observed below the average crack initiation strain $\langle \varepsilon_{min} \rangle$ are caused by large sample to sample fluctuations.

Computer simulations revealed that failure of the sample occurs when avalanches get localized in space and the broken beams form a fracture plane which spans the sample nearly perpendicular to the load direction (see Fig. 7(b) for illustration). The macroscopic fracture is formed by a cloud of micro-cracks (broken beams) which accumulate during the localized avalanches. To characterize the spatial extension and shape of this cloud, and hence, the sharpness of the fracture plane we determined the moment of inertia matrix I of the set of the middle points of the broken beams. The three eigenvalues of I , i.e. the principal moments of inertia A , B , and C give a measure of the extensions and shape of the point cloud. Calculations revealed that two eigenvalues have

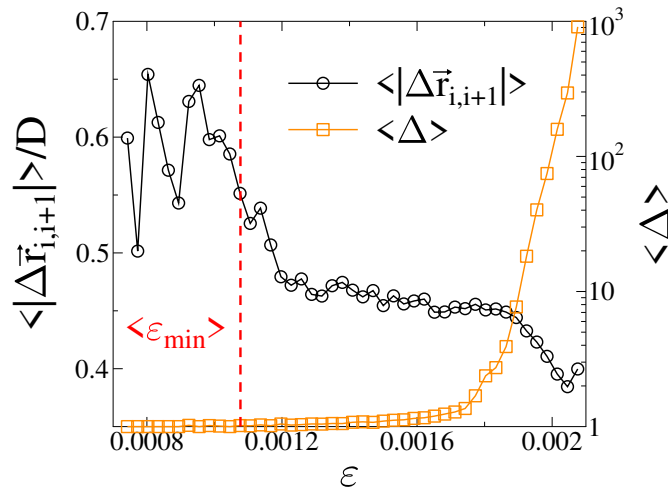


Figure 8. Average distance $\langle |\Delta \vec{r}_{i,i+1}| \rangle$ of consecutive avalanches together with the average avalanche size $\langle \Delta \rangle$ as function of ε . The vertical dashed line indicates the average value of the strain $\langle \varepsilon_{min} \rangle$, where micro-cracking starts.

nearly the same magnitude $A \approx B$, while the third one is significantly lower $C < A$. These relative magnitudes imply that the shape of the point cloud can be approximated by an oblate ellipsoid. As an example, Fig. 7(c) presents the ellipsoid obtained by this analysis where the three axis are directed along the eigenvectors of I and the lengths of the axis are proportional to the corresponding eigenvalues $C < B < A$. The center of the ellipsoid is positioned to the location of the center of mass of the point cloud of broken beams of the catastrophic avalanche.

To obtain a deeper quantitative insight into the shape and orientation of the fracture plane we determined the angle ϕ enclosed by the eigenvector of the moment of inertia matrix I corresponding to the smallest eigenvalue C and the direction of the external load (see Fig. 7(c) for illustration). With this definition ϕ is always positive, zero implies a normal parallel to the stress direction and $\pi/2$ perpendicular to it. The distribution $p(\phi)$ on Fig. 9(a) has its mode ϕ_m at the angle $\phi_m/(\pi/2) \approx 0.12$ and the average value $\langle \phi/(\pi/2) \rangle \approx 0.22$. The results imply that apart from some fluctuations the direction of the eigenvector is sub-parallel to the load direction. For a continuum we would expect $\phi = 0$, but this is a local minimum in the distribution on Fig. 9(a). The sub-parallel orientation of the mean and mode indicate material disorder (fluctuations) and the local dynamics (including interactions) have a discernible effect on the outcome. How sharply the fracture plane is defined can be characterized by the aspect ratio of the oblate ellipsoid, i.e. by the ratio of the smallest and largest eigenvalues C/A of the moment of inertia matrix I . It can be observed in Fig. 9(c) that the value of C/A fluctuates, however, its distribution has a relatively sharp peak at $C/A \approx 0.55$, which indicates that ellipsoids like the one presented in Fig. 7(c) are typical for the tensile failure of the uniaxially loaded cylindrical sample. Simulations showed that this behaviour emerges because the fracture path is composed of several planar segments which are somewhat shifted with respect to each other along the load direction. During the final catastrophic avalanche these segments merge and form the spanning crack, which is not planar, again inconsistent with the continuum solution. The position of the crack plane z along the cylinder axis, approximated as the z coordinate of the center of mass of the catastrophic avalanche Eq. (4.1), is randomly selected during the fracture process. This is illustrated in Fig. 9(b) where the distribution $p(z)$ has a broad maximum centered at $z/H \approx 0.5$.

Computer simulations in Refs. [21–23,26] showed that for uniaxial compression the spatial structure of damage undergoes a similar overall evolution, i.e. early stages of the fracture process are characterized by the random nucleation of small sized micro-cracks scattered over the entire

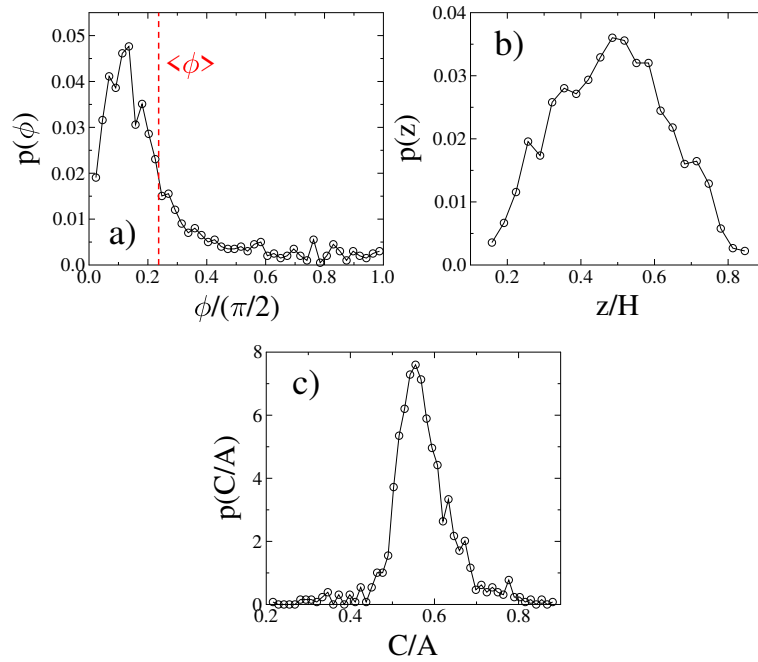


Figure 9. (a) Probability distribution $p(\phi)$ of the angle ϕ between the normal of the plane of the oblate ellipsoid, representing the fracture plane, and the load direction. The value of the average angle $\langle \phi \rangle$ is highlighted by the vertical dashed line. (b) Probability distribution $p(z)$ of the position z of the fracture plane along the vertical axis of the cylinder. (c) Probability distribution of the parameter C/A characterizing the sharpness of the fracture plane.

sample in an uncorrelated manner. As the system gradually approaches failure, spatial correlation of subsequent bursts emerges which leads to localization of damage. However, instead of a fracture plane, localization leads to the formation of an extended damage band [21–23,26]. Inside the band a large number of avalanches concentrate which leads to the complete fragmentation of the material with a power law distribution of fragment sizes [22,23]. Large scale simulations in Ref. [23] revealed that the orientation of the damage band is determined by the internal friction coefficient of the material in agreement with experiments.

6. Discussion and Conclusion

The fracture of heterogeneous materials is accompanied by the emission of crackling noise generated by avalanches of local breaking activity. Acoustic emission measurements provide a deep insight into the dynamics of fracturing addressing even the possibility of forecasting the imminent catastrophic failure of loaded systems. Understanding the statistical features of cracking avalanches and the spatial structure of damage, furthermore, their dependence on the loading conditions, is essential both for the acoustic monitoring of engineering constructions and for field measurements on natural catastrophes such as earthquakes and landslides. Here we performed a computational study of breaking avalanches generated during the fracture of porous rocks under tensile loading. Comparing the results to the corresponding outcomes of compression experiments obtained with the same specimens we wanted to clarify how the loading condition affects crackling noise in geomaterials.

Computer simulations revealed that the overall response of the specimens is significantly more brittle under tensile loading than under compression, which is quantified by the higher ratio of the yield stress to the ultimate strength in the tensile case. This means that the stress-strain curve has an overall linear character, non-linearity is only observed in the close vicinity of global failure. The

critical strain where failure occurs proved to be only about the third of the compressive strength of the same specimens. The fluctuating strength obeys a Weibull distribution with a rather large value of the shape parameter confirming the mild fluctuations of strength values at the sample size considered. On the micro-scale fracture of the specimen proceeds in bursts which are driven by the gradual redistribution of load in the local neighborhood of micro-cracks. Compared to the case of compressive failure, microcracking sets on earlier at a lower strain and the specimen can tolerate a lower amount of accumulated damage where a catastrophic avalanche destroys the entire sample. Avalanches, identified as correlated trails of consecutive micro-cracking events, have a fluctuating size with a growing average as failure is approached. Our calculations revealed that under tensile loading a smaller number of avalanches occurs spanning a narrower range of size, duration, and energy than during compression. This implies that under tensile loading the sample is more prone to failure in the sense that avalanches are more unstable leading to more sudden catastrophic collapse.

DEM simulations have the advantage that in addition to revealing the dynamics of fracturing they provide direct access to the spatial structure of damage. Our calculations showed that the beginning of the fracture process is dominated by the structural disorder giving rise to randomly dispersed small-sized avalanches all over the specimen. Spatial localization of avalanches is only observed in the close vicinity of macroscopic failure in such a way that a spatial region randomly emerges where the load concentration generated by breaking avalanches triggers further avalanches and eventually leads to the emergence of a catastrophic avalanche. As a consequence, a sharply defined fracture plane is formed along which the specimen falls apart into two large pieces. Contrary to compressive failure [21,22,26] no damage zone emerges where the material gets crushed. The fracture plane has a random position and orientation falling most likely in the center of the cylindrical specimen perpendicular to the load direction and oriented so that the normal to the best fit ellipsoid is sub-parallel to the continuum solution of zero degrees. The continuum solution is a local minimum in the distribution of this angle for our simulations.

Single avalanches are characterized by their size, duration, and energy dissipated during the avalanche. All quantities are found asymptotically to follow power law distributions with an exponential cutoff. Avalanches of larger size typically have a longer duration and a higher energy, which is described by a power law form of their correlation. Comparison of the avalanche exponents obtained under tension and compression revealed the robustness of the statistics of avalanche quantities. In spite of the substantially different micro-structure of damage and overall strength, there is no systematic difference of the statistics of breaking bursts between the two loading cases: The total number of avalanches and the range spanned by the avalanche size, duration, and energy proved to be higher under compression but the power law exponents of tension and compression agree within the error bars. Our simulation results confirm the high degree of robustness of the statistical features of crackling noise with respect to the external loading conditions.

In contrast, the loading configuration has a strong effect on the local failure mode of cohesive contacts inside the sample. The failure criterion Eq. (2.1) captures that the stretching and bending (shear) of a beam contribute to its breaking. The failure mode of a beam can be called tension or shear dominated if the first or second term of Eq. (2.1) is greater than the other one. The fraction of beams n_t and n_s of the tension and shear failure modes is plotted for a single sample in Fig. 10 as function of strain ε together with the constitutive curve for both loading cases. For uniaxial tensile loading (Fig. 10(a)) almost all the beams fail due to local tension, and shear dominated failure only occurs in the close vicinity of final breakdown. Under uniaxial compression (Fig. 10(b)) the early uncorrelated cracking is dominated by tension, however, as fracture proceeds shear induced breaking more often occurs and it becomes dominating around the final localization in the damage band. The results are in agreement with experimental findings on fracture processes of porous rocks where the tensile and shear type of cracking could be discriminated [6]. Our results imply that these details of the local failure mode do not affect the overall statistics of trails

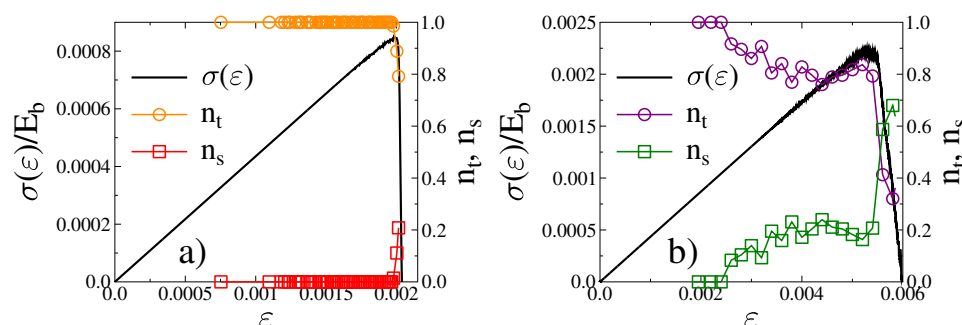


Figure 10. The fraction of beams failing dominantly under tension n_t and shear n_s are plotted as a function of strain ε for a single sample along with the corresponding constitutive curve $\sigma(\varepsilon)$ for uniaxial tension (a) and compression (b). Note that $n_t(\varepsilon) + n_s(\varepsilon) = 1$ holds for any ε .

of breakings, however, they play a decisive role in the temporal evolution of the burst sequence, which will be explored in a forthcoming publication.

Conflict of interest declaration

We declare we have no competing interests.

Data accessibility

All data are accessible publicly at Dryad under the following link: <https://doi.org/10.5061/dryad.nzs7h44wt> [28].

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