



Semi-analytical analysis of transport mechanism in hybrid nanomaterial dynamics with oxytactic microbes using Xue model

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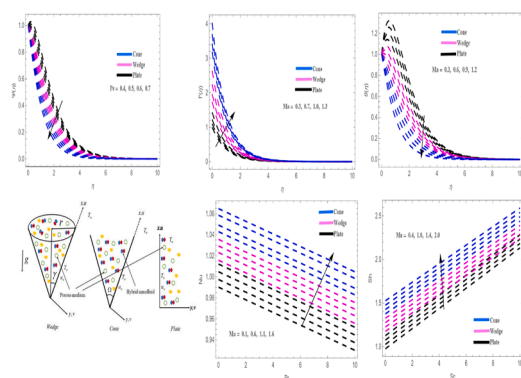
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GRAPHICAL ABSTRACT



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ABSTRACT

This study presents a comparative semi-analytical examination of Marangoni-driven mixed convective flow of a hybrid nanofluid containing oxytactic microbes over three different vertical geometries: cone, wedge and plate. The model integrates thermal radiation, viscous dissipation,

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Soret and dufour effects
Marangoni convection

Dufour-Soret effects, cross-diffusion effects, and magnetohydrodynamics within a single computational framework. The Xue thermal conductivity model characterizes the hybrid nanofluid's thermophysical properties with two different NPs dispersed in the base fluid. Results demonstrate that the Marangoni number significantly enhances velocity profiles and improves both heat and mass transmission rates across all geometries. The plate geometry exhibits superior performance due to uniform surface characteristics compared to the curved cone and wedge surfaces. The magnetic parameter slightly suppresses heat transfer through Lorentz forces. The thermal radiation promotes cooling by reducing thermal boundary layer thickness. Soret and Dufour effects considerably influence concentration and temperature distributions. Microbes' distributions are strongly affected by Peclet number and bioconvection Schmidt numbers for both gyrotactic and oxytactic types. This study provides critical insights for microfluidic bio-devices, thermal management systems, biomedical engineering, and renewable energy technologies requiring controlled transportation of heat, mass, and motile microbes. The velocity profile increases when the Marangoni convection parameter is increased, but the temperature and concentration profiles decrease.

Introduction

Two or more dissimilar nanoparticles are dispersed into a traditional improper fluid to create hybrid nanofluids. In comparison to single-particle nanofluids, the mixing of several nanoparticles results in synergistic effects that greatly improve thermal conductivity, mass transfer capabilities, viscosity properties, and overall transport performance. These enhanced thermo-physical characteristics have led to hybrid nanofluids' extensive use in high-performance engineering utilizations such as heat exchangers, solar energy collectors, nuclear cooling systems, medicinal devices, power generators, and other sophisticated thermal management systems. However, the growing industry demand for nanomaterials for improved technological growth has encouraged researchers to give intentions in this era. The thermal examination of the hyperbolic tangent model is used to study a hybrid nanofluid's MHD Darcy-Forchheimer flow via a stretched sheet with quadratic heat radiation and Richardson number [1]. Gradient descent-based analysis of mass transmission rate of HNF over rectangular and trapezoidal fins was investigated for sustainable energy systems [2]. The ideal melting heat effect for industrial heat transport uses in the course of an elastically deformed, dusty Boger HNF (hybrid nanofluid) was observed [3]. Statistical and numerical techniques were used to study the HNF radiative flow on a porous convective moving surface with the generation of heat [4]. Heat radiation, viscous dissipation, and entropy creation are all involved in a pulsed HNF flow [5]. The study of magneto-hydrodynamic (MHD) nanofluid flows has attracted considerable research interest due to their extensive uses in energy systems, thermal management, and industrial cooling procedures [6]. The incorporation of hybrid nanoparticles into conventional base fluids has been shown to significantly enhance thermal conductivity and heat transmission rates, particularly in rotating systems where Coriolis and centrifugal forces play a critical role in flow dynamics [7]. These rotating configurations are commonly encountered in turbomachinery, chemical processing equipment and geothermal extraction, making their thermal characterization essential for system optimization [8]. Recent investigations have extended the classical NF paradigm to include motile microbes, which introduce bioconvection effects that further influence heat and mass transportation in bio-nanofluidic systems relevant to biomedical devices and enzyme reactors [9]. The simultaneous properties of radiation, inclined magnetic fields, and porous media on these complex flows remain insufficiently characterized, particularly in the context of hybrid vehicle driving techniques where thermal management is critical [10]. Many similar studies are discussed in [11–15].

The steady upward flow of bacteria that are 5–10% denser than water is known as bioconvection. When microorganisms propel themselves forward and upsurge the density of the base fluid in a certain direction, bioconvection takes place in the flow. Additionally, there are a number of motile microorganisms based on impellent, including gyrotactic, oxytactic, and negative gravitational microbes. Motile microbes develop the solutal and mass transport rate of the species in interruption; this has beneficial consumptions in the domains of medical liquid products transportation, polymer sheets, biotechnology and chemical processing. The impact of Darcy–Brinkman–Forchheimer and MHD over a porous Riga plate [16], having the movement of thermo-microbes and Oxytactic bacteria in nanofluid flow. CCHF in rotating frame having microbes within nanofluid using Killer-Box method and compared to artificial neural network analysis is investigate [17]. Comparative studies of mixed convection heat transmission under heat radiation effects with porous medium flow over dual stretched surfaces have further highlighted the importance of radiative parameters in determining overall thermal performance [18]. Beyond engineering applications, similar transport phenomena govern biomedical contexts such as targeted drug delivery and disease modeling, where neural network approaches have been successfully applied to predict complex biological spread dynamics [19]. The stretchable surfaces with chemical reactions present additional challenges due to the non-linear coupling between velocity, temperature, and concentration fields, warranting further investigation to establish robust predictive frameworks [20]. Non-similar analysis has been employed to examine bioconvection MHD micropolar NF flow over stretching surfaces, incorporating the influences of Soret and Dufour effects [21]. Artificial intelligence-driven neural network approaches have been developed for heat transmission optimization in squeezed microfluidic flow, demonstrating the potential of ML (machine learning) techniques in thermal system design [22].

A statistical analysis of cross nanofluid [23] having microbes with Magneto -bioconvection over a lubricious surface. Arrhenius activation energy in a porous cylinder [24] with microbes in magneto-Sutterby-nanofluid. The influence of hall and heat generation with living and dissipative HNF over moving wedge on MHD flow is discussed [25,26] explored the magnetohydrodynamic

bioconvective flow Of Maxwell fluid: influence of suction, microorganism Decay and thermal stratification parameter. Arrhenius activation energy in a porous cylinder [27] with microbes in magneto-Sutterby-nanofluid. The microorganism's transportation under electro-magnetohydrodynamic forcing and heat radiation with Soret effect in Casson THNF flow is analyzed [28]. The radiated micropolar NF flow with modified Cattaneo-Christov analysis for bioconvection phenomenon [29]. The hybrid numerical of MHD nanofluids with motile organisms [30] and compared the result to artificial neural network modeling, that is an emerging technique among fluid researchers [31] explored the polymer solution carrying microbes near a perforated Riga plate. They wrapped up in a Darcy–Forchheimer medium involving Arrhenius kinetics. Their findings uniquely showed that the local Nusselt number rises by increasing porosity parameter due to enhanced fluid permeability [32–35]. A flexible cylinder with Arrhenius kinetics in [36] in inspection the chemically reactive bioconvective flow with Cross-nano-material conveying oxytactic microbes. In this study, the entropy generation rate is minimized at moderate values of the magnetic parameter, offering an optimal operating condition for energy-efficient manufacturing processes. Radiative-dissipative properties on bioconvective MHD flow have been studied in Eyring-Powell THNFs, providing insights into the coupled transport phenomena in complex fluid systems [37]. The absolute sensitivity of energy transmission in lithium-ion battery electrode coatings with HNF has been analysed, contributing to improved battery thermal management [38]. Cross-thermal streamline patterns and heat transmission in Eyring-Powell NFs have been examined using neural network approaches with uncertainty analysis [39].

The Dufour and Soret properties play a central role in describing how heat and mass transportation interact within fluid mixtures composed of multiple components. The investigation for nanofluid flow [40] while accounting for combined Dufour and Soret contributions. The squeezed MHD convective flow of HNF [41] with combined effect of thermo-diffusive and chemical reaction using ANN has been investigated. Comparative analysis of NFs has been conducted with Dufour and Soret influence using ANN approaches [42]. The performance of magnetic and heat radiation NPs has been enhanced for couple stress THNFs, with insinuations for biomedical uses [43]. Artificial neural network analysis has been employed to study entropy-optimized flow of Jeffrey NF invoking the Cattaneo-Christov theory, demonstrating the effectiveness of machine learning approaches in predicting thermal behavior [44]. Modified solutal and thermal have been investigated through convective flow of Reiner-Rivlin material, providing insights into non-Fourier transport phenomena [45]. Double-diffusive convective EG (entropy generation) in rheological NMs with non-Fourier Cattaneo-Christov fluxes has been analyzed for thermal energy storage systems, highlighting the role of coupled transport mechanisms [46].

Marangoni convection, interacts with van der Waals pressure to regulate stability and heat transfer, arises from surface tension gradients due to temperature or composition shifts, driving fluid from low to high tension regions. There are many industrial applications for Marangoni convection in thin-film coating, electric recharging batteries, and microgravity applications. Recent studies reveal that electrophoresis and thermophoresis intensify these currents and modify heat transfer. Emerging research also shows Marangoni flows enable defect-free organic semiconductor production via gravity-assisted molecular alignment [47] analyzed Bio-convective flow in the occurrence of CBCs (convective boundary conditions) in Casson fluid over a sheet. MC (Marangoni convection) is inspected [48] in ethylene glycol base HNF flow a porous medium. The study of Marangoni convection HNF flow across a surface were explored [49].

The novelty of this study lies in the first-time incorporation of multiple advanced transport mechanisms in a single computational framework for HNF flow over three distinct geometries (vertical cone, wedge, and plate). Specifically, this work uniquely incorporates:

- Dual microorganism bioconvection by simultaneously modelling oxytactic microorganisms, capturing their distinct swimming behaviors and collective impacts on momentum and mass transfer.
- Comparative geometric analysis across cone, wedge, and plate surfaces under identical thermal, solutal, and biological boundary conditions, providing direct insight into how surface curvature and radial flow affect bioconvective transport.
- Marangoni convection under the impact of Dufour and Soret in a hybrid nanofluid.
- Integration of heat radiation, magnetic field and viscous dissipation effects within a hybrid nanofluid containing two different nanoparticles, enhancing both thermal and mass transport while considering realistic energy losses and field interactions.

This comprehensive model offers new insights into the optimization of bio-thermal systems in microfluidic devices, biomedical engineering, and renewable energy applications, where multi-physics coupling and geometry-specific performance are critical.

Governing equations

Consider the influence of incompressible flow of HNF (hybrid nanofluid) over the surface of a cone, wedge, and plate with oxytactic microorganisms.

The generalized governing equations are given below:

$$\nabla \cdot \mathbf{V} = 0. \quad (1)$$

$$\rho_{nf}(\mathbf{V} \cdot \nabla \mathbf{V}) = \nabla \cdot \mathbf{P} + \mathbf{P},$$

$$P = g\beta_T(T - T_\infty)\cos\omega - \frac{\sigma_{hnf}}{\rho_{hnf}}B_0^2u. \quad (2)$$

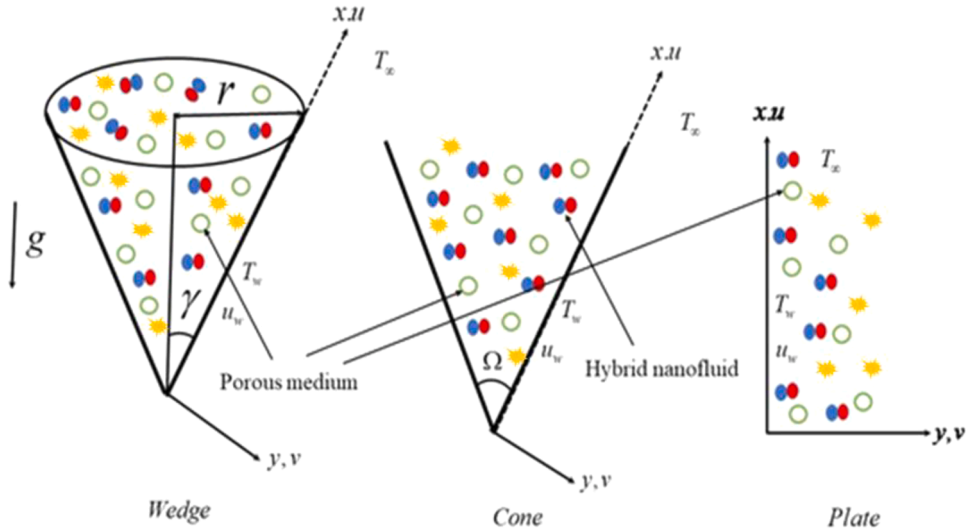


Fig. 1. Flow description.

$$(\rho C_p)_{\text{nf}}(V \cdot \nabla T) = \frac{D_T}{T_\infty} \nabla^2 C + \frac{16\sigma^* T_\infty^3}{3k^* \rho C_p} \nabla^2 T + Q, \quad (3)$$

$$Q = \frac{\mu_{\text{hnf}}}{(\rho C_p)_{\text{hnf}}} \left(\frac{\partial u}{\partial y} \right)^2,$$

$$(V \cdot \nabla)C = D_B \nabla^2 C + \nabla^2 T, \quad (4)$$

$$V \cdot \nabla N + \frac{bW_c}{\Delta C} (N \nabla^2 C + \nabla C \cdot \nabla N) = D_m \nabla^2 N, \quad (5)$$

$$V \cdot \nabla K = D_s \nabla^2 K - \frac{\gamma(N - N_\infty)}{\Delta K}. \quad (6)$$

The cone, in this problem, is characterized by half-angle ω and radius r , see Fig. 1. The plate, wedge and cone surfaces are kept at a constant concentration C_w , temperature T_w , K_w oxytactic microorganisms and microbe density N_w . These surface values are higher than their corresponding ambient conditions, i.e., $T_w > T_\infty$, $C_w > C_\infty$, and $N_w > N_\infty$, where T_∞ , C_∞ , K_∞ and N_∞ , represent the uniform thermal, solutal, and microbe density far away from the surface. The governing equations for the current problem are mention below (cited from: [50–54]).

$$\frac{\partial(r^m u)}{\partial x} + \frac{\partial(r^m v)}{\partial y} = 0, \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{\text{hnf}} \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty)\cos\omega - \frac{\sigma_{\text{hnf}} B_0^2 u}{\rho_{\text{hnf}}}, \quad (8)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{\text{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 C}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k^* (\rho C_p)_{\text{hnf}}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{\text{hnf}}}{(\rho C_p)_{\text{hnf}}} \left(\frac{\partial u}{\partial y} \right)^2, \quad (9)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (10)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_m \frac{\partial^2 N}{\partial y^2} - \frac{bW_m}{K_w - K_\infty} \left[\frac{\partial}{\partial y} \left(N \frac{\partial K}{\partial y} \right) \right], \quad (11)$$

$$u \frac{\partial K}{\partial x} + v \frac{\partial K}{\partial y} = D_s \frac{\partial^2 K}{\partial y^2} - \frac{\gamma(N - N_\infty)}{\Delta K}. \quad (12)$$

Boundary conditions [50–54]:

$$u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad N = N_w, \quad K = K_w, \quad y = 0 \quad (13)$$

$$\mu_{mf} \frac{\partial u}{\partial y} = -\sigma_T \frac{\partial T}{\partial x} - \sigma_C \frac{\partial C}{\partial x} \quad (14)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N = N_\infty, \quad K = K_\infty, \quad y \rightarrow \infty \quad (15)$$

Similarity Transformations

$$u = U_w f'(\eta), \quad v = -\frac{v_f}{x} \sqrt{Re_x} \left(\frac{m+1}{2} f(\eta) + \frac{m-1}{2} \eta f'(\eta) \right), \quad (16)$$

$$\eta = \frac{y}{x} \sqrt{Re_x}, \quad Re_x = \frac{U_w x}{\nu_f}, \quad (17)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \Psi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \quad \Theta(\eta) = \frac{K - K_\infty}{K_w - K_\infty}. \quad (18)$$

Transformed governing equations (ODEs)

$$A_1 f''' + A_2 \left(\frac{m+1}{2} f f'' - \left(\frac{m-1}{2} \right) (f')^2 \right) + A_4 (\lambda \theta) \cos \omega - A_3 M f' = 0 \quad (19)$$

$$\frac{1}{Pr} \left(A_5 + \frac{4}{3} Rd \right) \theta'' + A_6 \left(\frac{m+1}{2} f \theta' \right) + Du \phi'' + Ec A_1 (f'')^2 = 0 \quad (20)$$

$$\frac{1}{Sc} \phi'' + \frac{m+1}{2} f \phi' + Sr \theta'' = 0 \quad (21)$$

$$Sb(f \Psi') \frac{m+1}{2} + \Psi'' - Pe(\{\Omega + \Psi\} \Theta'' + \Theta' \Psi') = 0, \quad (22)$$

$$\Theta'' + Se(f \Theta') - A Se \Psi = 0. \quad (23)$$

Transformed boundary conditions

$$\theta = 1, \quad \phi = 1, \quad \Theta = 1, f = 0, \quad \Psi = 1, \quad \eta = 0, \quad (24)$$

$$f'' A_1 = -Mn(1 + Ma) \quad \eta = 0, \quad (25)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0, \quad \Theta \rightarrow 0, \quad \Psi \rightarrow 0, \quad \eta \rightarrow \infty. \quad (26)$$

Symbol	Name	Definition
Ma	Marangoni number	$\frac{\sigma_T \Delta T L}{\mu_f \alpha_f}$
M	Magnetic Parameter	$\frac{\sigma B_0^2 L}{\rho f U_w}$
Rd	Radiation Parameter	$\frac{4\sigma^* T_\infty^3}{k^* k_f}$
Ec	Eckert number	$\frac{U_w^2}{C_p \Delta T}$
Du	Dufour number	$\frac{D_T \Delta C}{\alpha_f \Delta T}$
Sr	Soret number	$\frac{D_T \Delta T}{T_\infty D_B \Delta C}$
Pr	Prandtl number	$\frac{\nu_f}{\alpha_f}$
Sc	Schmidt number	$\frac{\nu_f}{D_B}$
Sb	microorganisms Schmid number	$\frac{\nu_f}{D_m}$
Se	Oxytactic Schmidt number	$\frac{\nu_f}{D_c}$
Pe	Gyrotactic Peclet number	$\frac{b W_c}{D_m}$
λ	Thermal buoyancy parameter	$\frac{Gr_x}{Re_x^2}$
Φ_1, Φ_2	Nanoparticle volume fractions	—
m	geometry parameter	0(plate), 1(wedge / cone)
ω	Cone half-angle	—

Table 1

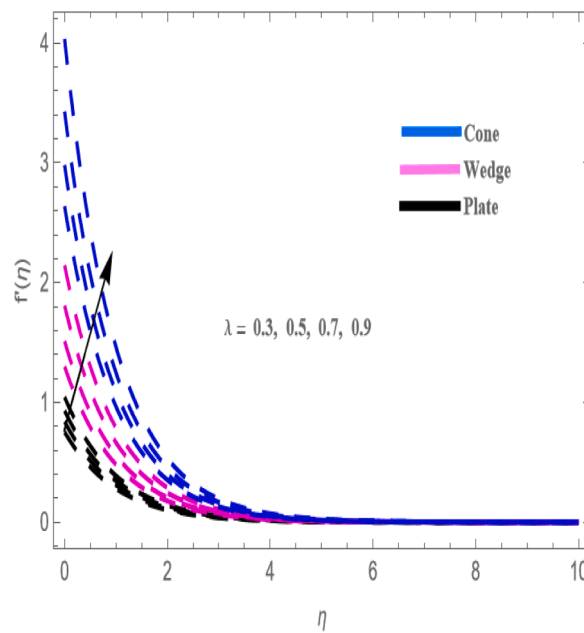
Thermophysical presence of the base liquid and NPs [56,57].

Properties	SiO ₂	MoS ₂	water
ρ (kg/m ³)	2200	5060	997.1
C_p (J/kgk)	703	397.21	4147
k (W/mk)	1.4	904.4	0.613
σ (Ωm) ⁻¹	5.6×10^{-5}	2.09×10^{-4}	0.05

Table 2

Thermos-physical properties of HNF [56,57].

Characteristics	HNF
μ_{hnf}	$A_1 = \frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \Phi_1)^{2.5} (1 - \Phi_2)^{2.5}}$
ρ_{hnf}	$A_2 = \frac{\rho_{hnf}}{\rho_f} = \left[(1 - \Phi_2) + \frac{\rho_{s_2}}{\rho_f} \Phi_2 \left\{ (1 - \Phi_1) + \Phi_1 \frac{\rho_{s_1}}{\rho_f} \right\} \right]$
σ_{hnf}	$A_3 = \frac{\sigma_{hnf}}{\sigma_{nf}} \times \frac{\sigma_{nf}}{\sigma_f} = \frac{\sigma_{hnf}}{\sigma_{bf}} = \left[\frac{-\Phi_2 (\sigma_{nf} - \sigma_{s_2}) + \sigma_{s_2} + \sigma_{nf}}{+\sigma_{nf} + \sigma_{s_2} + (\sigma_{nf} - \sigma_{s_2}) \Phi_2} \right] \times \left[\frac{-\Phi_1 (\sigma_f - \sigma_{s_1}) + \sigma_{s_1} + \sigma_f}{\Phi_1 (\sigma_f - \sigma_{s_1}) + \sigma_{s_1} + \sigma_f} \right]$
$(\rho c_p)_{hnf}$	$A_6 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} = \left[\left\{ (1 - \Phi_1) + \Phi_1 \frac{(\rho c_p)_{s_1}}{(\rho c_p)_f} \right\} \Phi_2 \frac{(\rho c_p)_{s_2}}{(\rho c_p)_f} + (1 - \Phi_2) \right]$

**Fig. 2.** Impact of λ on $f'(\eta)$.

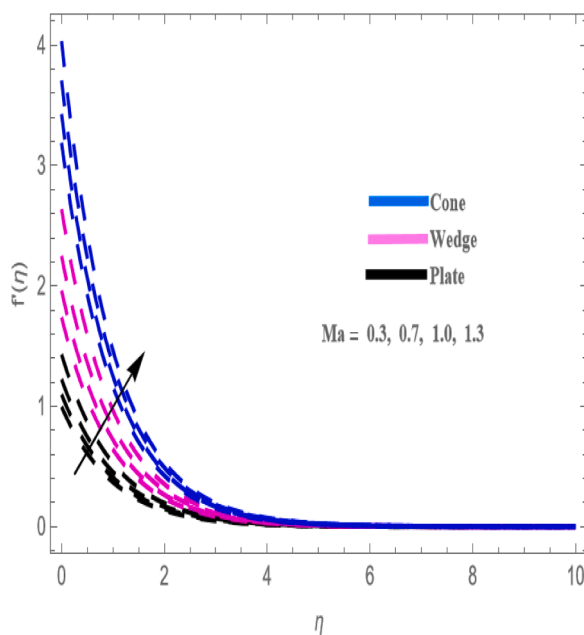
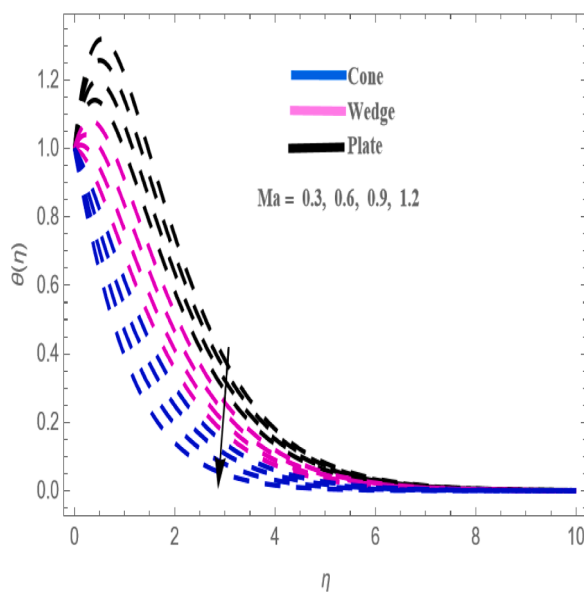
$$Cf = \frac{2}{A_1} f''(0) \quad (27)$$

$$Nu = - \left(A_5 + \frac{4}{3} Rd \right) \theta'(0) \quad (28)$$

$$Sh = -\phi'(0) \quad (29)$$

$$Nn = -\Psi'(0), \quad Nk = -\Theta'(0) \quad (30)$$

The Xue thermal conductivity model is expressed as follow [55].

Fig. 3. Influence of Ma on $f'(\eta)$.Fig. 4. Outcomes of Ma on $\theta(\eta)$.

$$\frac{k_{hnf}}{k_{nf}} = \frac{1 - \Phi_2 + \left(\frac{k_2}{k_2 - k_{nf}}\right) 2\Phi_2 \ln \frac{k_2 + k_{nf}}{2k_{nf}}}{1 - \Phi_2 + \left(\frac{k_{nf}}{k_2 - k_{nf}}\right) 2\Phi_2 \ln \frac{k_2 + k_{nf}}{2k_{nf}}}, \quad (31)$$

$$\frac{k_{nf}}{k_f} = \frac{1 - \Phi_1 + \left(\frac{k_3}{k_3 - k_f}\right) 2\Phi_1 \ln \frac{k_3 + k_f}{2k_f}}{1 - \Phi_1 + \left(\frac{k_f}{k_3 - k_f}\right) 2\Phi_1 \ln \frac{k_3 + k_f}{2k_f}}. \quad (32)$$

Table 1 demonstration the thermophysical properties of based fluid and NPs. Table 2 display the thermophysical properties of HNF.

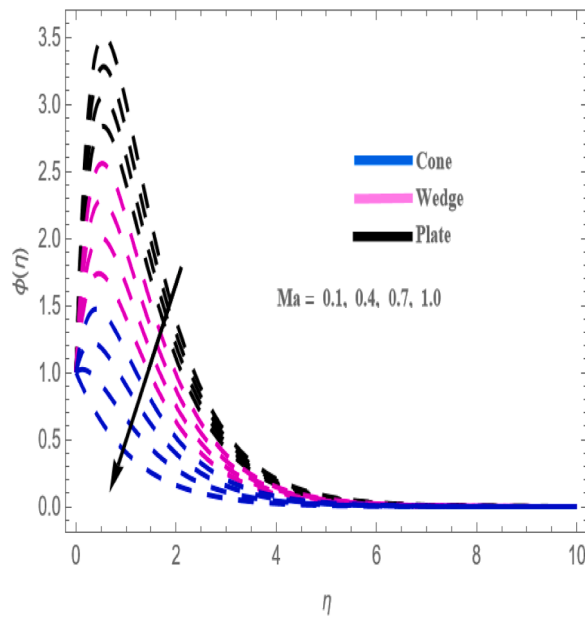


Fig. 5. Impact of Ma on $\phi(\eta)$.

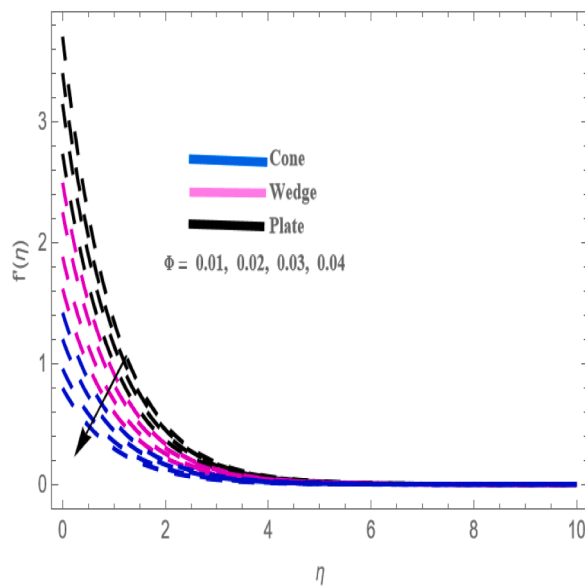


Fig. 6. Effect of Φ on $f'(\eta)$.

Methodology

The Homotopy Analysis Method (HAM) is a semi-analytical technique used to find approximate solutions to linear and nonlinear differential equations. HAM does not require small parameters and includes a convergence-control parameter to assure the solution's accuracy and convergence. Because of its versatility and effectiveness, HAM is frequently used in fluid mechanics, heat transfer, nonlinear dynamics, and other engineering and scientific applications.

Graphical discussion

The analysis of vertical cone, wedge, and plate geometries also yields useful information for developing heat exchangers, solar collectors, and aerodynamic surfaces with increased thermal performance. Fig. 2 demonstrates the variational behavior of the

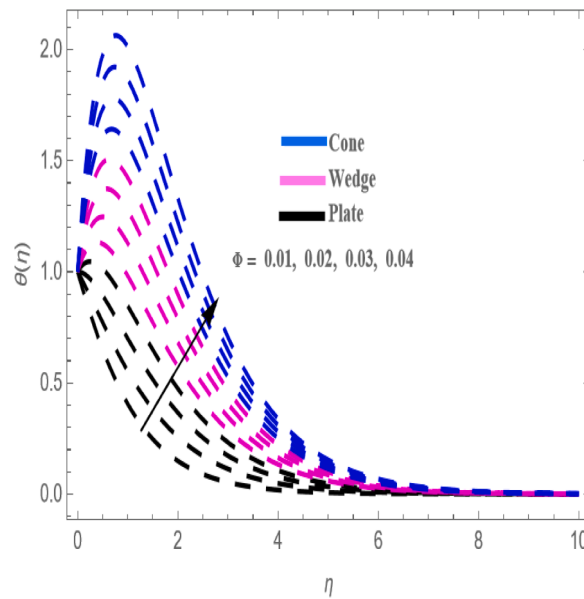


Fig. 7. Influence of Φ on $\theta(\eta)$.

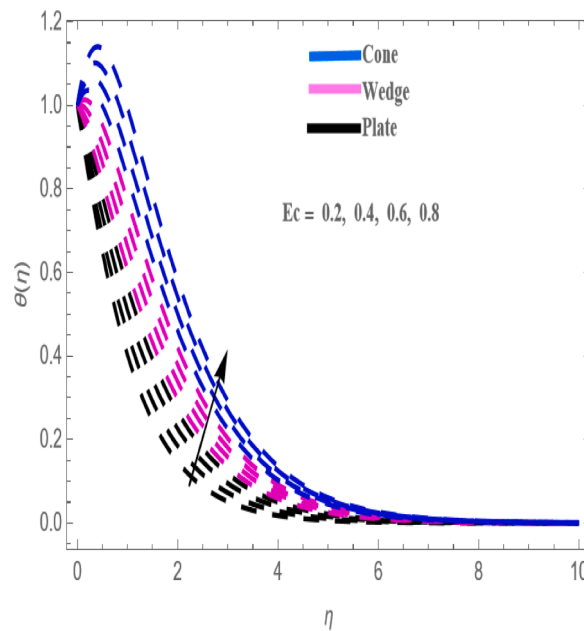


Fig. 8. Effect of Ec on $\theta(\eta)$.

buoyancy factor on the velocity field for cone, wedge, and plate geometries. A rise in λ enhances the velocity profile across all three geometries. The plate geometry exhibits the highest velocity enhancement, followed by the wedge and cone, due to differences in surface curvature and radial flow effects that modify the momentum boundary layer development. Fig. 3 unfolds the performance of the Marangoni number on the velocity profile. Growing Ma significantly enhances the velocity near the surface for all geometries. The Marangoni effect creates surface tension gradients that induce additional tangential stresses, pulling the fluid along the surface and increasing its velocity. This effect is most pronounced in the plate geometry, where the uniform surface allows for stronger Marangoni-driven flow compared to the curved surfaces of the cone and wedge. Fig. 4 presents the outcomes of the Marangoni number on the $\theta(\eta)$. Increasing Ma reduces the temperature profile across all geometries. The Marangoni-induced flow enhances convective heat transfer, resulting in more efficient cooling and a thinner TBL. The plate exhibits the most substantial temperature reduction, benefiting from uniform Marangoni stresses. Fig. 5 reveals the effect of the Marangoni number on the concentration TBL. Increasing Ma values reduce

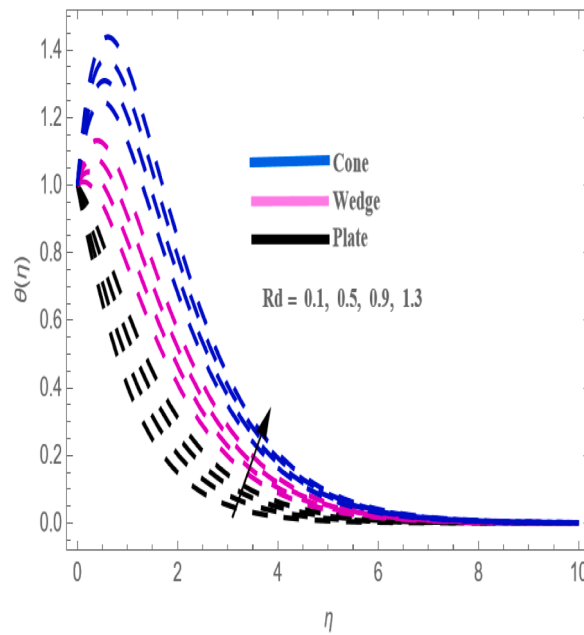


Fig. 9. Effect of Rd on $\theta(\eta)$.

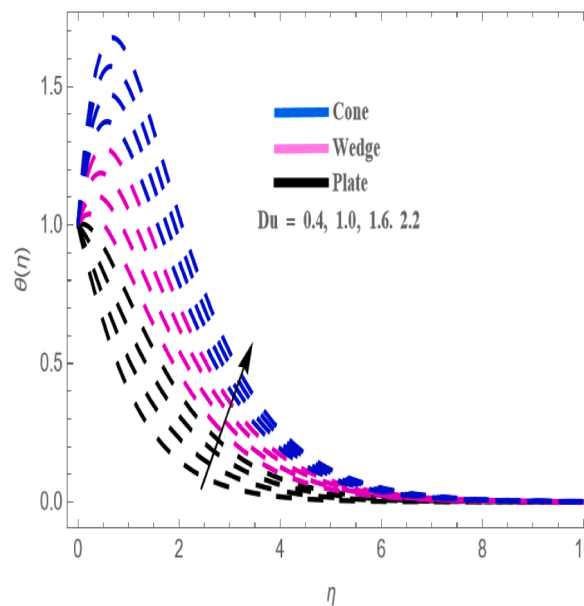


Fig. 10. Effects of Du on $\theta(\eta)$.

the concentration profile. Marangoni convection enhances mass transmission away from the boundary wall. The plate shows the strongest reduction due to uniform surface tension gradients. Fig. 6 demonstrates the declining behavior of velocity profile against the increasing values of nanoparticle volume fraction. As Φ increases, the velocity profile declines slightly. This reduction occurs because higher nanoparticle concentrations increase the viscosity of the HNF. It creates greater resistance to flow. The cone geometry shows the most significant velocity reduction due to its curved surface, which amplifies viscous effects compared to the plate and wedge.

Fig. 7 displays how NPs volume fraction disturbs the energy field. Growing Φ values increase the energy profile. This is attributed to the greater thermal conductivity of the HNF, which improves heat conduction within the fluid. The cone geometry shows the highest temperature elevation due to its larger surface area and curved geometry, which promote better heat absorption and retention. Fig. 8 examines the impact of the Eckert number on the thermal field for fluids. Growing Ec raises the temperature profile noticeably. The Eckert number represents viscous dissipation effects; higher values indicate more mechanical energy converted to thermal energy

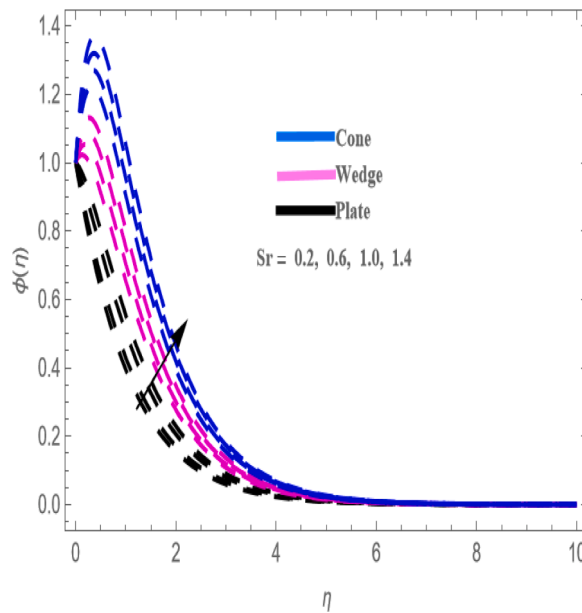


Fig. 11. Effects of Sr on $\phi(\eta)$.

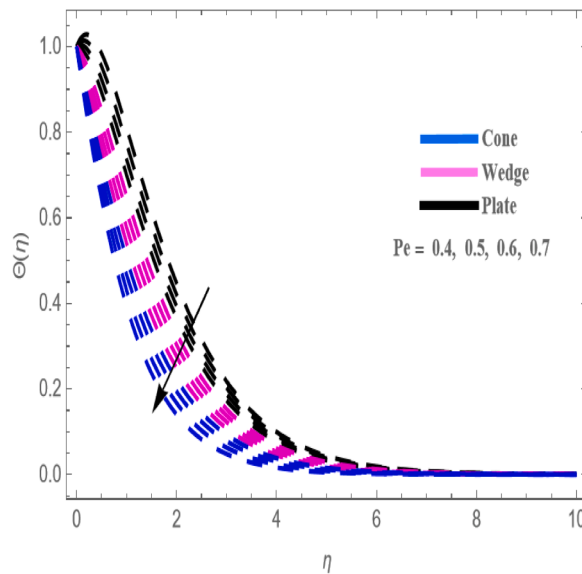


Fig. 12. Effects of Pe on $\psi(\eta)$.

through fluid friction, thereby growing fluid temperature. This outcome is most prominent in the wedge geometry, where flow acceleration enhances viscous heating.

Fig. 9 shows the variational behavior of the radiation factor on the thermal field. As Rd increases, the temperature field decreases. Thermal radiation facilitates additional energy loss from the system, enhancing cooling and reducing the TBL thickness. The plate benefits most from radiative cooling due to its direct exposure compared to the curved cone and wedge surfaces. Fig. 10 displays the influence of the Dufour number on the energy boundary layer. Increasing Du elevates the temperature profile. The Dufour effect represents cross-diffusion where concentration gradients induce heat flux; higher Du values enhance this energy transport, raising fluid temperature. This cross-diffusive heating is most effective in the cone geometry. Fig. 11 displays the impact of the Sr on the mass transfer boundary layer profile. Growing Sr elevates the profile. The Soret effect (thermos diffusion) drives mass flux due to temperature gradients; higher Sr values enhance this phenomenon, increasing solute concentration near the surface. This effect is most pronounced in the cone geometry, where temperature gradients are steepest.

Fig. 12 displays the consequence of the Pe on the gyrotactic microbes profile. Increasing Pe enhances the microorganism profile. It

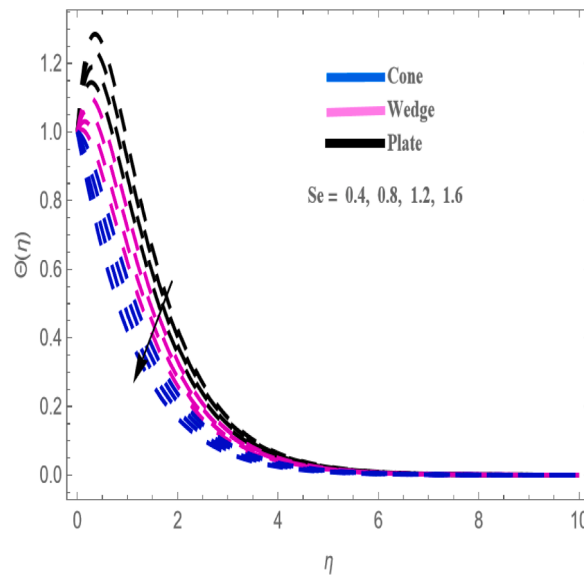


Fig. 13. Effects of Se on $\Theta(\eta)$.

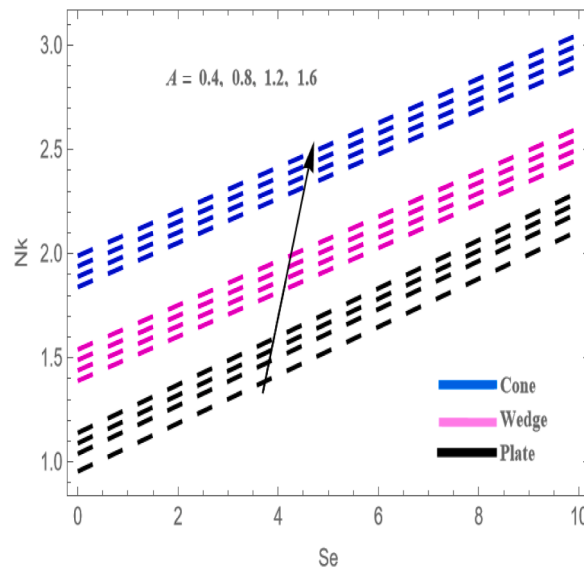


Fig. 14. Effects of A on Nk profile.

characterizes the ratio of advective to diffusive transportation; higher values indicate stronger swimming-driven advection, which increases microorganism concentration near the surface. This is particularly evident in the plate geometry, where flow conditions favor gyrotactic accumulation. Fig. 13 presents the influence of parameter Se on Oxytactic microorganism's profile. Increasing Se reduces the Oxytactic microorganisms profile, indicating enhanced thermal dissipation or entropy effects that promote cooling. This parameter captures complex thermodynamic interactions that improve heat transfer efficiency, particularly in the wedge geometry. Fig. 14 displays the behavior of parameter A on the Nk profile (microorganism density metric). Increasing A reduces the Nk profile, suggesting that this parameter may represent inhibition factors or environmental stresses that limit microorganism growth or accumulation. The cone geometry shows the most sensitivity to changes in A .

Fig. 15 presents the behavior of Peclet number on the Nm field (another microorganism density measure). Similar to Fig. 12, higher Pe values increase microorganism density due to enhanced advective transport. Fig. 16 reveals the consequence of the Dufour number on the Nu . Increasing Du enhances Nu , indicating improved heat transfer rates. The Dufour effect contributes additional energy transport through cross-diffusion, boosting convective heat transfer. Fig. 17 demonstrates the outcome of the Sr on the Sherwood number. Higher Sr values increase Sh , reflecting enhanced mass transfer rates. The Soret effect improves solute transport through

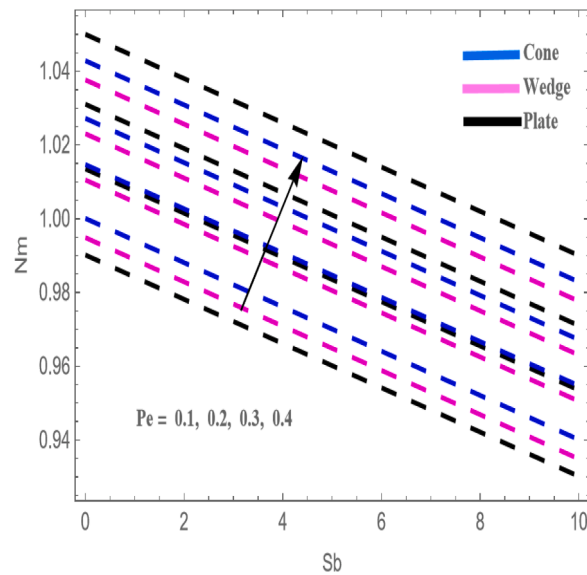


Fig. 15. Effects of Pe on Nm profile.

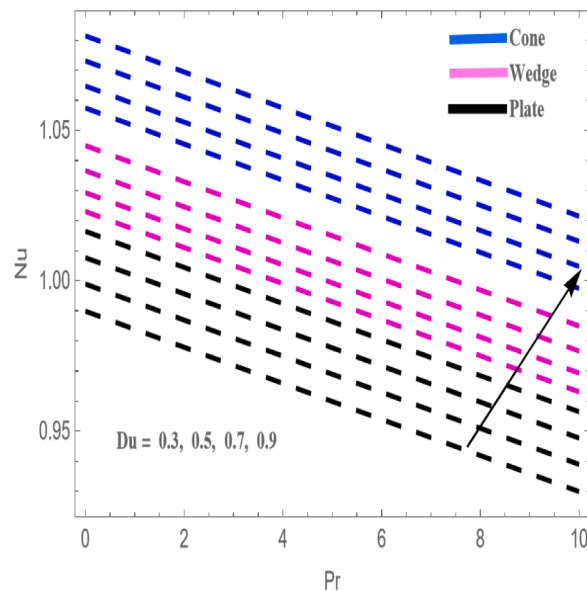


Fig. 16. Effects of Du on Nu .

thermos diffusion, particularly effective in the cone geometry where temperature gradients are maximized. Fig. 18 illustrates how the Marangoni number affects the Nusselt number. Growing Ma significantly enhances Nu across all geometries. Marangoni convection strengthens surface-driven flow, improving convective heat transfer. Fig. 19 grants the impact of the Ma on Sh . Similar to heat transmission, increasing Ma improves Sh by enhancing convective mass transfer through Marangoni-driven flow. The wedge geometry demonstrates the most substantial mass transfer enhancement. Fig. 20 demonstrates the outcome of Φ on the Nusselt number Nu . Higher Φ values increase Nu due to enhanced thermal conductivity of the HNF. The cone geometry benefits most from nanoparticle addition, as its curved surface maximizes conductive heat transfer enhancement. Fig. 21 demonstrates the influence of the M on Nu . Increasing M reduces Nu slightly. The MF (magnetic field) creates Lorentz forces that oppose fluid motion, reducing convective heat transmission rates. This suppression effect is most noticeable in the plate geometry, where flow is more susceptible to magnetic damping. Table 3 shows how the present research's findings on local heat transfer rates and skin friction relate to those of a previous study. The objective of the two outcomes is clear.

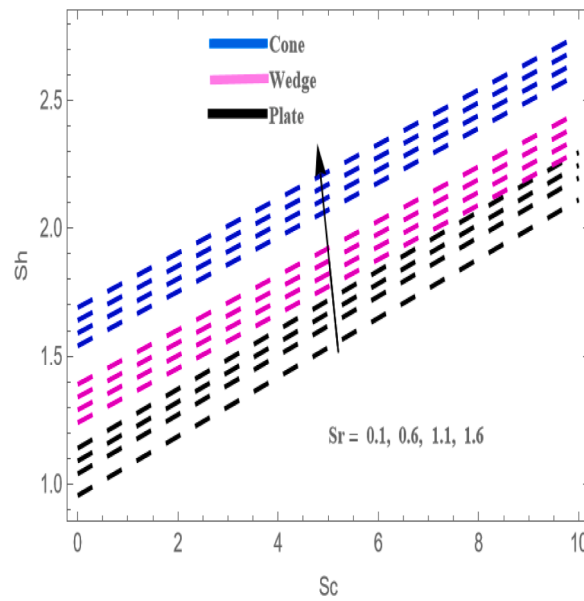


Fig. 17. Effects of Sr on Sh .

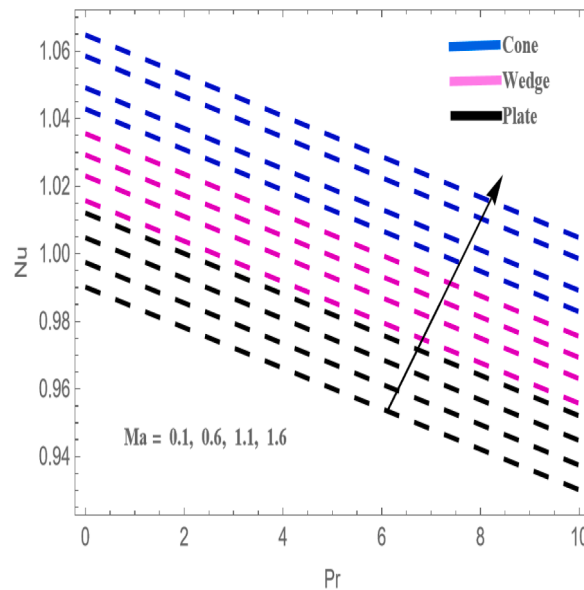


Fig. 18. Outcomes of Ma on Nu .

Conclusions

Among the geometries, the plate consistently demonstrated heat and mass transmission performance compared to the cone and wedge. This work is attributed to its uniform surface and boundary layer development. Key findings are summarized as follows:

- The Marangoni number significantly enhances the velocity profile and improves both Nusselt and Sherwood numbers across all geometries due to surface tension-driven flow.
- Growing the nanoparticle volume fraction elevates the temperature profile and boosts the Nusselt number, confirming the thermal conductivity enhancement offered by hybrid nanofluids.
- The magnetic parameter slightly reduces heat transfer by opposing fluid motion through Lorentz forces.
- Thermal radiation promotes cooling by reducing the TBL (thermal boundary layer) thickness.

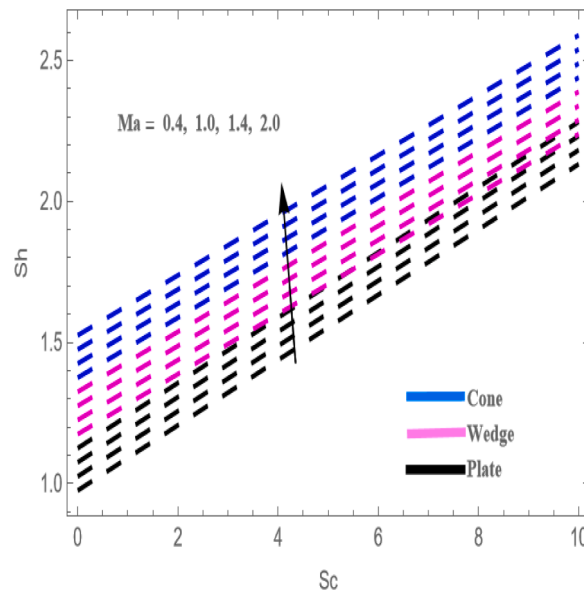


Fig. 19. Influence of Ma on Sh .

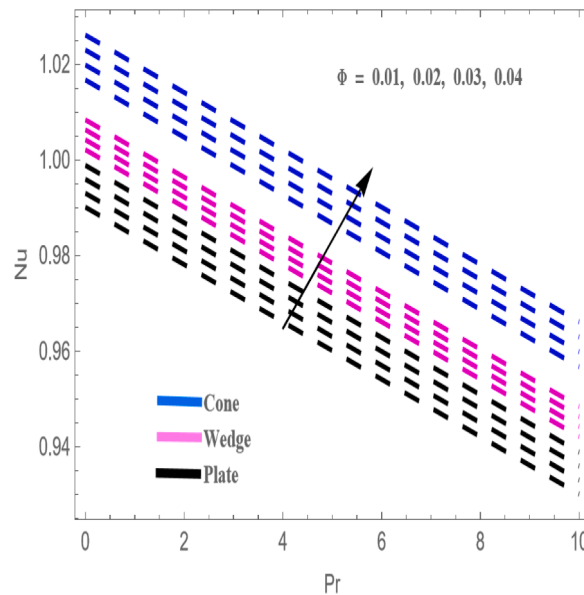


Fig. 20. Impact of Φ on Nu .

- The Soret and Dufour effects noticeably influence concentration and temperature distributions, demonstrating the importance of cross-diffusion in coupled heat and mass transmission.
- Microorganism distributions are affected by the Peclet number and bioconvection Schmidt number. It highlights the role of bioconvection in transport processes.

This study provides valuable insights for applications in bio-microfluidic systems, thermal engineering, and biotechnology where controlled transport of heat, mass, and microorganisms is essential.

Future work may explore unsteady flows, entropy analysis, and different nanoparticle combinations to further optimize system performance.

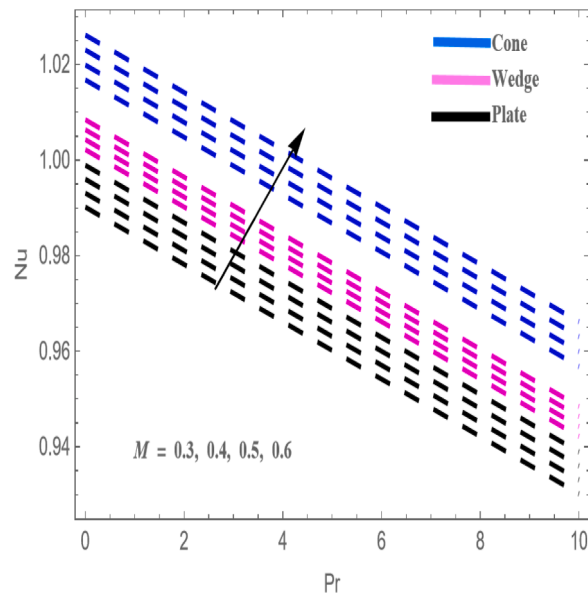


Fig. 21. Outcomes of M on Nu .

Table 3
Comparison of existing result and keeping in all new parameter zero.

M	Cf		Nu	
	Previous result [58]	Present results	Previous result [58]	Present results
1	0.9463919	0.9463930	0.4641856	0.4641877
2	0.8752615	0.8752629	0.4322101	0.4322128
3	0.8177847	0.8177856	0.4060915	0.4060930

CRediT authorship contribution statement

Munawar Abbas: Supervision, Methodology, Conceptualization. **Saba Liaqat:** Methodology, Formal analysis. **Muhammad Azhar Iqbal:** Methodology, Formal analysis. **Ilkhom Khaydarov:** Writing – review & editing. **Ali Hasan Ali:** Writing – review & editing. **Mohamed Abbas El-Naggar:** Methodology, Formal analysis.

Declaration of competing interest

The authors have no relevant financial or non-financial interests to disclose.

Data availability

The authors do not have permission to share data.

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