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The Impact of an Active Learning – Based Intervention for Indonesian Prospective Mathematics Teachers' Pedagogical and Mathematical Skills

Thesis for the Degree of Doctor of Philosophy (PhD)

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UNIVERSITY OF DEBRECEN

Doctoral Council of Natural Sciences and Information Technology

Doctoral School of Mathematical and Computational Sciences

Debrecen, 2023

Hereby I declare that I prepared this thesis within the Doctoral Council of Natural Sciences and Information Technology, Doctoral School of Mathematical and Computational Sciences, University of Debrecen in order to obtain a PhD Degree in Natural Sciences at the University of Debrecen.
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Dissertation submitted in partial fulfilment of the requirements for the doctoral (PhD) degree in Mathematics and Computer Science

Written by Linda Devi Fitriana certified as Master of Education

Prepared in the framework of the doctoral school of Mathematical and Computational Sciences - University of Debrecen
(Didactics – teaching of mathematical and computational sciences program)

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The date of the dissertation defence: ……………… 2023

Acknowledgement

I would like to sincerely thank my supervisor, Dr. Zoltán Kovács, for his infinite patience, care, and trust throughout my doctoral study. He gives me fruitful advice, beneficial suggestions, and insightful feedbacks while also making me passionate about mathematics teaching and learning, thus inspiring me in this field. It is an honor to study under his supervision.

I would like to extend my gratitude to the members of my dissertation committee, Dr. Eszter Herendiné Kónya and Dr. Antonín Jančařík, for their constructive feedback, which has significantly contributed to the final form of this work.

I am indebted to the participants who generously dedicated their time and commitment to this research.

Furthermore, I sincerely thank the members of the Doctoral School of Mathematical and Computational Sciences and my Indonesian lecturers for their professional support, and the Hungarian government through the Stipendium Hungaricum program for the study support.

The last but most important, I am immensely grateful to my husband, who was simultaneously preparing his thesis, all my parents, siblings, and friends. Their tremendous support and unconditional love mean the world to me.

I dedicate this dissertation to my beloved mother, who is no longer in this world, but her love eternally empowers me.

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Supplementary material and data:

The problem-posing and problem-solving products and the transcripts are available at

https://drive.google.com/drive/folders/1R9ZTcWPE6mb4FczfAxc6YNqMa3xARisg?usp=sharing

1. Introduction

Despite having improved the quality of education over the last decade (Tobias et al., 2014; World Bank, 2020), Indonesia continues to face some educational challenges. While the education system is being strengthened to boost learning, there is a demand to recover lost learning, given that most students still need to reach the national academic targets (World Bank, 2020). Furthermore, the international comparative study report on educational achievement, i.e., PISA, reveals Indonesian mathematics mean score under the average mathematics mean score of the participating countries (OECD, 2019). Those facts indicate the need for enhancement in mathematics teaching and learning.

In response to the challenge, the curriculum has been updated several times to overcome those problems and to cope with changing times which are increasingly complex and challenging. The current curriculum's cornerstone places students at the center of learning to collaborate and share ideas and allows teachers to be more flexible in structuring relevant learning for students (Kepmendikbudristek, 2022). In turn, it will lead to the emergence of a mathematically rich learning environment.

A mathematically rich learning environment is widely regarded to improve learning quality. In this environment, the students' role should not be passive listeners and note takers, but rather participating in learning activities that are cognitively demanding, such as discussion, review, and evaluation. In the meantime, the teachers' role is to encourage interaction, collaboration, and reflection. As stated by Hurst et al., (2013) and Telhaj (2018), social interactions within classrooms have a positive effect on learning outcomes. Thus, the lesson should encourage social interactions, facilitate the emergence of emotional connections, and accommodate the development of self-confidence (Caine et al., 2004). Attempts to maximize learning can be accomplished by providing students opportunities to explore by relying on their prior knowledge and experience and not being afraid to make mistakes (Sullivan et al., 1997). According to Gogus (2012), the abovementioned components are the principle of active learning with its subset, collaborative learning.

One approach that has emerged to play a role in active learning is problem posing (Ellerton, 2013). Compared to its companion, i.e., problem solving, it still acquires scant attention in school mathematics (Ellerton, 2013; Kilpatrick, 1987; Lee, 2012; Silver, 1994; Stoyanova & Ellerton, 1996; Van Harpen & Sriraman, 2013), including those in Indonesia (Putra et al., 2020). In most cases, students are focused on problem solving with little or even no opportunity to formulate the problem, resulting in a gradual acceptance of problems created by others are the only ones that need to be solved. While according to Kilpatrick (1987), many real-life problems must be created or discovered by problem solvers who provide the initial formulation of the problem. The problem is not given, but human beings construct it in the attempt to make sense of intricate and puzzling situations (Schon, 1979).

In school, some students who engage in mathematics decide to become mathematics teachers. In most cases, as they have grown accustomed to being given mathematical problems from books and the internet to solve without the opportunity to pose problems, it is not surprising that later they pass on their experiences. They might continue to teach their class in the same manner, only solving without formulating problems. Based on their previous experience, they regard this learning sequence as an effective way to teach and learn mathematics. Even if that happens, they should not rely solely on problems from books or the internet but should also create their own that might be more challenging and relevant to their students' personal experiences.

With a focus on prospective teachers, a cross-national study was carried out to gain a deeper insight into Indonesia's educational challenges (Fitriana et al., 2022). The obtained results denote the tendency of Indonesian prospective teachers to pose simple rather than challenging mathematical problems, most of which are arithmetic in nature, and thus tend to utilize an arithmetic rather than an algebraic approach when solving their self-proposed problems.

The algebraic approach is a type of mathematical reasoning related to symbolization (Schoenfeld, 2008). Acquainting prospective teachers to the algebraic approach will be a beneficial endeavor. In posing or solving mathematical problems, utilizing an algebraic approach will allow them to generalize relationships and complete procedures in a general way (Freudenthal, 1977). Furthermore, broadening their perspectives to be more familiar with an algebraic approach does not necessarily preclude them from using an arithmetic approach, given that algebra also allows arithmetic to be performed (Britt & Irwin, 2008).

Taking into account (1) the findings of the cross-national study above, (2) the current curriculum's cornerstone that places students at the center of learning and encourages relevant learning, and (3) some studies that confirm active learning improves academic performance (Rotgans & Schmidt, 2011) and persistence in learning (Bédard et al., 2012), this Ph.D. thesis presents and analyzes an intervention for Indonesian prospective mathematics teachers in which active learning principles are promoted through collaborative learning, and the topics are directed toward problem posing and algebraic thinking.

Intervention participants are Indonesian prospective mathematics teachers from private and public universities. The instructor is the author of this dissertation. In the intervention, participants get acquainted with problem posing, pose and solve problems individually and collaboratively, and discuss the proposed problems to get a better version. They also implement their proposed problems to the school students in a classroom setting at the beginning, middle, and end of the intervention, considering that doing so relates to the classroom activity they will face in the future. Their teaching implementations were video-recorded and followed by self-reflection and peer feedback. Implementing a self-proposed problem is a novelty that fills the research gap in the nonexistence of observing the implementation of self-proposed problems. The research scope explicitly mentioned in Ellerton (2013) affirms this point.

Collaborative discussion on the proposed problems and peer feedback activities were purposefully arranged to stimulate their critical manifestations. Given teachers' limited knowledge and skills in using active learning techniques due to their personal experience (Gogus, 2012), the activities outlined above aim to provide them with empirical experience that will enrich their approach, perspective, and practice in mathematics teaching.

In summary, this research is expected to contribute to understanding and overcoming the educational challenges in Indonesia and to support the current curriculum implementation through a series of activities designed to develop Indonesian prospective mathematics teachers' pedagogical and mathematical skills. Figure 1 illustrates the current challenge and the contribution of this research.

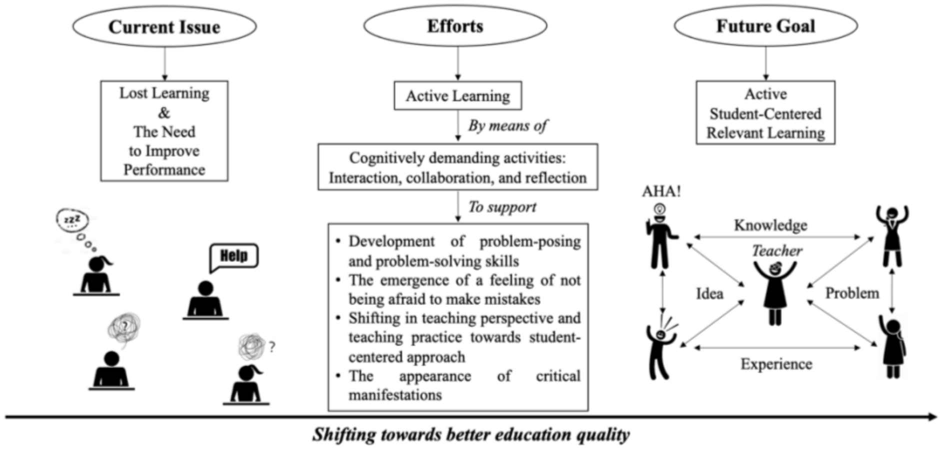


Figure 1. Indonesia's current educational challenge and the contribution of this research

2. Indonesian Educational Background

In Indonesia, education institutions (whether public or private) are managed by two ministries: the Ministry of Education, Culture, Research, and Technology and the Ministry of Religious Affairs (OECD, 2015; Rosser, 2018). Since 1994, the Indonesian government has mandated a nine-year basic education program for all citizens and made several updates to the curriculum since 1947, and the most recent was introduced in 2022.

In mathematics, the curriculums implemented before 1975 belong to the pre-modern mathematics curriculum, as students were directed to memorize rather than comprehend mathematical concepts (Mailizar et al., 2014). As awareness among Indonesian scholars of the need to improve mathematics teaching and learning in schools has flourished, more attention has been placed on developing understanding rather than memorization and calculation skills, and emphasizing student-centered learning since the implementation of the 1975 curriculum (Mukminin et al., 2019). These efforts were continuously promoted through the 1984 curriculum, which addressed the implementation of active learning in all schools (Bjork, 2005; Mailizar et al., 2014; Wahyudin & Suwirta, 2017). In 1994, there began to be greater attention toward problem solving to develop students' reasoning skills, which had not previously been explicitly stated in the curriculum (Mailizar et al., 2014). As for curriculum reform, some challenges remained, such as the unsatisfactory implementation of active learning principles (Mailizar et al., 2014), the persistence of traditional teaching as the most dominant approach implemented by teachers (Fauzan, 2002; Rachman, 2019), teachers' tendency to dictate formulas and procedures to solve problems rather than asking questions (Fauzan, 2002), and students fear of mathematics (Tanujaya et al., 2017).

To support the vision of Indonesian education, including addressing the gap between targets and achievements, and as part of the efforts to recover learning in the post-Covid-19 pandemic, the current curriculum was developed as a more flexible curriculum framework that upholds the principle of relevant learning and focuses on developing students' character and competencies. Official textbooks issued by the Ministry of Education, Culture, Research, and Technology contain several essential features, such as an overview and the rationale of the topic to foster students' interest and motivation in learning the main idea, triggering questions that lead to understanding the topic, mind maps, hints, and several guiding activities (let us remember, let us explore, let us think critically, let us think creatively, let us try, let us discuss, let us reflect, let us collaborate, and let us use technology). Problem solving is continuously emphasized and integrated into the guiding activities.

Unlike active learning and problem solving, problem posing has yet to be explicitly stated in the curriculum. On the one hand, there is a growing interest in incorporating problem posing into schools and teacher training programs (Christidamayani & Kristanto, 2020; Hasanah et al., 2017; Masriyah et al., 2018). On the other hand, although problem posing is a popular topic among Indonesian scholars, its presence in schools is still sporadic. In teacher training programs, it is generally incorporated into the problem-solving subject.

3. Research Questions

The main research question is: Is the active learning-based intervention successful in helping Indonesian prospective mathematics teachers to develop their pedagogical and mathematical skills?

To provide a more explicit domain for pedagogical and mathematical skills, the main research question is broken down into several questions as follows:

1. How do Indonesian prospective mathematics teachers perform in problem-posing throughout the active learning-based intervention?

   This question is followed up by categorizing the problem-posing products based on Fitriana's framework (Fitriana et al., 2022) and analyzing the complexity of the problem-posing task based on Kontorovich's framework (Kontorovich et al., 2012).
2. How do Indonesian prospective mathematics teachers perform in problem-solving throughout the active learning-based intervention?

This question is followed up by categorizing the problem-solving products into blind, correct, and incorrect solutions (along with the errors made). The performance will be analyzed according to Schoenfeld's framework (Schoenfeld, 1985).

3. How do the Indonesian prospective mathematics teachers' critical manifestations look throughout the active learning-based intervention?
   This question is followed up by categorizing the manifestations during the lessons and teaching reflections based on Fitriana's framework (Fitriana, 2022a).
4. How do the Indonesian prospective mathematics teachers' teaching perspectives shift throughout the active learning-based intervention?
   This question is followed up by comparing the results of the teaching perspectives questionnaire in Fitriana (2022b) at the beginning and the end of the intervention.
5. How do the Indonesian prospective mathematics teachers' teaching implementations shift throughout the active learning-based intervention?
   This question is followed up by comparing the approaches of the teaching implementations based on Fitriana's framework (Fitriana, 2022b) at the beginning and the end of the intervention.

4. Theoretical Background

4.1 Active Learning

Numerous researchers are acquainted with active learning. Though different authors have different interpretations, it is not reasonable to present universally accepted definitions of active learning, but it is reasonable to present commonly accepted definitions. A shared understanding of what constitutes active learning leads to learners actively participating in the learning process. In the point of view of Prince (2004), active learning is any instructional method that involves students in the learning process. It enables learners to participate, take responsibility, and connect ideas through analysis, synthesis, and evaluation during teaching and learning activities (Gogus, 2012). Besides doing those meaningful activities, active learning also encourages learners to think about what they are doing (Bonwell & Eison, 1991). Thus, students play their role as an agency for their learning (Lombardi et al., 2021), in contrast to the traditional lecture in which students passively listen to the instructor (Freeman et al., 2014; Prince, 2004).

In general, and particularly in mathematics, teaching cannot be about the teacher cramming mathematical knowledge into the students' heads but rather about interacting with students as they engage with mathematical ideas (Sullivan, 2011). For prospective teachers, it tends to be arduous to move their pedagogical disposition toward teaching related to active learning, especially if they have not typically experienced mathematics taught in this manner (Litster et al., 2020). As a promising attempt, they can incorporate active learning by utilizing open tasks to promote reasoning and problem solving, and emphasize reasoning, thinking, and active interaction with mathematics.

Given that direct experience and interaction with intellectual, social, and physical environments will result in the most lasting learning (Edwards et al., 2014), Edwards (2015) proposed an active learning framework to describe and plan for different types of active learning instruction, which consists of:

- Intellectual activity

  Active learning entails students engaging with the lesson intellectually by utilizing critical thinking or higher-order thinking (a broader term that includes critical thinking, problem solving, creative thinking, and decision making). According to Lewis and Smith (1993), higher-order thinking happens when an individual takes new information and information retained in memory and interrelates, rearranges, and expands this information to attain a goal or find possible answers in puzzling situations. Thus, the emphasis on intellectual activity implies that students should think beyond memorization or basic comprehension, such as applying, analyzing, evaluating, and creating (Anderson & Krathwohl, 2001).

- Social activity

  Active learning should bring students socially active such as discussing a particular topic with partners, working on a project in small groups, or having a whole class discussion.

- Physical activity

  Students should physically move during the lesson for experiential learning, manipulation, and experimentation, among other things.

In practice, students can be intellectually active and socially or physically active at the same time, as illustrated in Figure 2. It is possible for an instructional method to only fit into one category or fall into more than one category simultaneously. Nevertheless, it should be noted that instructional activities should involve not only active learning but also must be purposeful. The goal is to encourage students to think critically rather than memorize the knowledge, and this can be aided by collaborative work. The collaboration can be as simple as having partners discuss a topic.

As such, allowing students to collaborate is a crucial component of the active classroom.

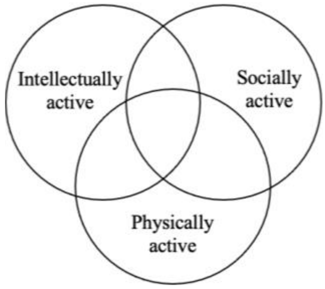


Figure 2. Active learning framework by Edwards (2015)

The principle of the active learning framework highlights at least two notable notions: critical thinking and collaborative learning. Sternberg (1986, p. 3) defined critical thinking as "the mental processes, strategies, and representations people use to solve problems, make decisions, and learn new concepts." The mental process comprises three intertwined elements: analyze, evaluate, and improve thinking (Paul & Elder, 2014). As previously stated, active learning fosters critical thinking. Therefore, it is possible to identify students' manifestations that appeared during the lesson, whether critical or not. Furthermore, according to Lahann and Lambdin (2014, p. 75), "collaborative learning involves a team of students who learn through working together to share ideas, solve a problem, or accomplish a common goal." The heart of collaborative learning is student interaction rather than learning as a solitary activity (Prince, 2004). Gogus (2012) mentioned it as a subset of active learning. Thus, collaborative learning is a practical method to implement active learning.

4.2 Problem

The term "problem" has attracted much interest from researchers to express it. Polya (1962) stated that having a problem implies consciously seeking an appropriate course of action to achieve a clearly defined but not immediately attainable goal. While in the viewpoint of Schoenfeld (1985), a problem refers to a task that is intellectually (rather than computationally) difficult for the person attempting to solve it. Those preceding statements implicitly assert that when attempting to solve a problem, the solver must face obstacles that are challenging to overcome, even to recognize.

Another analogous thought comes from Krulik and Rudnick (1989), who defined problem as a situation that an individual or group must solve but does not see a clear path to resolving. The term problem is prone to be used interchangeably with exercise. Even though both are situations, the use of the terms should not be mixed up. An exercise requires drill and practice to reinforce a previously learned skill or algorithm to solve, whereas a problem requires analysis, synthesis, and control of previously learned knowledge to solve. A problem becomes an exercise, which Polya (1962) referred to as a routine problem, when the solver immediately recognizes and knows the correct process for solving it.

For any reason, an individual must perceive the existence of a problem to consider it. An example from Lénárd (1978):

> Egy másik egyetemi hallgató a fenti kérdésre egyszerűen azt válaszolta: „nem tudom”. Mivel ezzel a válasszal egyúttal abba is hagyott minden gondolkodási tevékenységet, a kérdés számára nem jelentett problémát. Nem tett ugyanis semmi erőfeszítést a probléma megoldása, akadályok leküzdése érdekében. (p. 39)
>
> Given a question, a university student simply replied: “I don’t know”. Since this answer also meant that the student had stopped thinking, the question was

not a problem for them as they did not make any effort to solve the problem or to overcome any obstacles[1] (Lénárd, 1978).

It implies that a situation or a question is not a problem for an individual if he or she does not accept the challenge to solve it. In addition, the problem is a subjective notion (Krulik & Rudnick, 1989; Lénárd, 1978; Polya, 1962; Yeo, 2017). What was once a problem for children later becomes an exercise and, eventually, simply a question as they continue their mathematical training. Once the situation has been modeled or can be easily solved using previously learned algorithms, it is no longer considered a problem, given that being confronted with a problem means the individual is confronted with something he or she is unfamiliar with. Likewise, what is a problem for one person may not be a problem for another since everyone has a different mathematical background, which Lénárd (1978) regards as a rich and varied body of knowledge acquired through experience and learning.

Along with his belief that the nature of the problem may indicate the nature of the solution, Polya (1962) divided problems into two categories: problem to find and problem to prove.

- Problem to find

  Concerning this problem, there are three principal components, i.e., the unknown, the condition, and the data. The goal is to find a specific object, locate the unknown of the problem, and satisfy the condition of the problem that connects the unknown to the problem's data. To solve the problem, we must first understand it. To understand it, we must first recognize the unknown, the data, and the condition. That is why when

[1] The supervisor approved the translation.

solving a problem, it is strongly advised to pay close attention to those three main components.

An example that is likely to be a problem for 7th graders: What is the sum of the first n natural numbers in closed form?

- Problem to prove

  A mathematical proposition, in its most frequent form (but not always), consists of or can be split into a hypothesis (the first part begins with "If") and a conclusion (the second part begins with "then"). Those two are the principal components of the problem to prove. The goal of the problem is to determine whether the mathematical proposition is true or false, to prove or disprove it. To prove it, we must establish a logical link between the hypothesis and the conclusion. To disprove the proposition, we must reveal that the hypothesis does not imply the conclusion, for instance, through a counterexample. Therefore, those two principal components deserve special attention.

  An example that is likely to be a problem for 7th graders: For $n \in N$, prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{n}$.

4.3 Problem Solving

Global educational policies, some researchers based on their theoretical and empirical arguments, and even society have asserted various purposes of mathematics teaching. Among these, the common goal is to familiarize learners with the nature of mathematical thinking (Mason et al., 2010; Polya, 1962; Schoenfeld, 1985; Sullivan et al., 1997; Tanner & Jones, 1997). To achieve this goal, one of and perhaps the most prominent tool is problem solving (Krulik & Rudnick, 1989; Polya, 1962; Schoenfeld, 1985).

According to Polya (1962), problem solving means an attempt to overcome adversity and impediment to achieving a goal that is not immediately attainable. It is the process by which the problem solver applies previously acquired knowledge, skills, and understanding to meet the demands of an unacquainted situation (Krulik & Rudnick, 1989). The process commences with the initial confrontation and wraps up when the answer is obtained and reconsidered in light of the initial circumstances. Teachers should know the process, both theoretically and practically, so that they can recognize what factors contribute to the success or failure of a problem-solving attempt and thus support students in improving their problem-solving skills (Schoenfeld, 1985).

There are various models of the problem-solving process. In the last century, the Gestalt psychology trend promoted the process of problem solving. One comes from Wallas (1926), who described a four-phase process that can overlap each other. The process is apparent in the work of Hadamard (1945), who examined several mathematicians and physicists when solving a problem.

- Preparation

  In this phase, the problem solver consciously establishes the mental groundwork and gathers the resources and information that lead to an idea. The activities include researching, planning, and getting into the proper frame of mind and deliberation.

- Incubation

  At this point, the problem solver is unconsciously thinking about the problem, no direct effort is being made to solve the problem. There are two distinct aspects:

- The negative fact if the solver does not consciously think about the problem during incubation
- The positive fact if a series of unconscious and involuntary mental events may occur during incubation

● Illumination

This is the “AHA” moment when the idea comes suddenly and concisely.

● Verification

During this phase, the idea is tested for validity and reduced to its exact form. This phase is similar to the preparation phase, with fully conscious effort as those in control of preparation.

The preceding describes the general problem-solving process. Thus, the following session will concentrate on mathematical problem solving. One model was developed by a well-known Hungarian-born mathematician, Polya (1945), who also proposed a four-step process that consists of:

● Understanding the problem

This step entails identifying the unknown, the data, the condition, and the possibility of satisfying the condition, drawing a figure, and introducing appropriate notation.

● Devising a plan

The goal of this step is to obtain a plan for the solution by identifying the connection between the data and the unknown and considering auxiliary problems if an immediate connection cannot be identified.

● Carrying out the plan

This step comprises working on the plan, checking each step to see if it is correct, and proving it if so.

● Looking back

Looking back not only constitutes checking the obtained result but also the argument, looking for alternate ways to derive the result, and considering whether the result and method can be applied to other problems.

Another model of the mathematical problem-solving process comes from Mason et al. (2010), who emphasized the dynamic nature of the problem-solving process. Phases of working in mathematical problem solving encompass entry, attack, and review phases as shown in Figure 3.

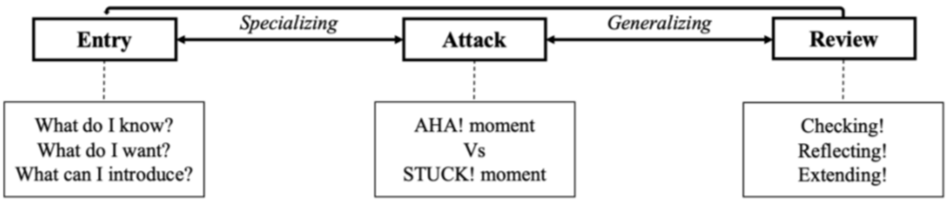


Figure 3. The dynamic problem-solving process by Mason et al. (2010)

● Entry

This phase takes place when an individual encounters the problem and ends when he or she begins attempting to solve it. This phase essentially corresponds to a Polya's step, namely, understanding the problem. This phase results in the solver holding what he or she knows, desires, and introduces.

● Attack

The attack phase involves more specializing and generalizing as the solver struggles with the problem. If an idea comes to the solver's mind, then it leads to an AHA moment; otherwise, it leads to a STUCK moment in which the solver must go back to the entry phase and start over.

- Review

  This stage must be completed before leaving the problem. It is time to examine the work. If an error or inadequacy occurs, it may result in an attack or even the entry phase. The processes and challenges are reflected which account for metacognitive activity. If checking confirms the solution, an extended act, such as posing a new problem by generalizing, specializing, or modifying the condition, can be performed. Thus, the whole process starts again. Roundly, this phase is parallel to the final stage of Polya, which is looking back.

Table 1 outlines how the above problem-solving phases align with each other.

Table 1. The comparation of problem-solving phases

Wallas (1926) and Hadamard (1945)	Polya (1945)	Mason et al. (2010)
Preparation	Understanding the problem	Entry
	Devising a plan	Attack
Incubation & Illumination	Carrying out the plan	
Verification	Looking back	Review

To analyze success and failure in mathematical problem solving, Schoenfeld (1985) introduced a framework which includes several aspects of complex intellectual activity:

- Cognitive resources

  This is an individual's mathematical knowledge, a collection of facts and procedures. It denotes what the person knows and how it is accessed for use.

- Heuristics

  Heuristic refers to general strategies for advancing in difficult situations when solving a mathematical problem. Here are some heuristics that can be utilized (Tiong et al., 2005): act it out, use a diagram/model, use guess-and-check, make a systematic list, look for patterns, work backwards, use before-after concept, make suppositions, restate the problem in another way, simplify the problem, solve part of the problem, think of a related problem, and use equations.

- Control

  Problem-solving performance is determined not only by what one knows, but also by the efficiency with which individuals apply the knowledge. Competent decision making can help ensure success even if one has few resources to begin with, whereas poor decision making can ensure failure even if one has access to a large pool of resources. Control implies that the solvers should devote time to cognitive tasks such as analyzing, planning, implementing, or verifying a solution. In verifying the solution, some errors may be found which can be factual, procedural, conceptual, and careless errors (Brown et al., 2016). The last error is not due to a lack of knowledge or skill, but simply being tired or distracted while solving the problem. In this step, when the problem solver realizes the error, the finding of the error is part of the control. On the contrary, if the problem solver does not recognize the error and the solution remains wrong, the error is part of cognitive resource deficiency.

- Belief systems

  This component refers to the psychological contexts in which people perform mathematics. This can ultimately determine their orientation to the problem, even spurring their subconscious access to tools and techniques that they consider or are potentially relevant and useful for solving problems.

4.4 Problem Posing

Problem posing has received tremendous attention from several researchers. Some argue that problem posing is critical to the discipline of mathematics and the nature of mathematical thinking (Bonotto & Santo, 2015) and view it as an essential component of mathematics education reform (Cai, 1998; Crespo, 2003; Ellerton, 1986; Silver, 1994; Silver & Cai, 1996). Thus, it should be in the heart of the curriculum (Ellerton, 2013). Through problem posing, there is an opportunity to formulate mathematical tasks from open situations, as life does not provide us with ready-made mathematical problems like those in the textbook but in the form of situations. As Varga (1987) stated:

> ... kellenek amellett nyitott szituációk is, amelyekben a tanulók ismerik fel és fogalmazzák meg a matematikai feladatot, gyakran többfélét. Ez a megszerzett matematikai ismeretek alkalmazását is segíti. Az élet nem tankönyv-nyelven megfogalmazott matematikai feladatok elé állít minket, hanem szituációk elé. (p. 29)
>
> There should be open-ended situations in teaching mathematics, in which students recognize and formulate the mathematical problem, more often different ones. Life does not present us with textbook-style mathematical problems, but rather situations[2] (Varga, 1987).

Nevertheless, regardless of its allure, researchers' interpretation of problem posing is not homogeneous (English, 2020; Koichu, 2020; Papadopoulos et al., 2021). In the early 1990s, Silver (1994) outlined problem posing as an activity to generate new problems and reformulate given problems. It conforms to Duncker's (1945) interpretation. On the

[2] The supervisor approved the translation.

other hand, Stoyanova and Ellerton (1996, p.518) defined it as a “process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems.”

It is also worth seeing the viewpoint of Cai and Hwang (2020) that through problem posing, we refer to a group of activities involving or assisting teachers and students in formulating (or reformulating) and expressing a problem or task based on a specific context (problem context or problem situation). As various approaches to the meaning of problem posing are possible to identify, Papadopoulos et al. (2021) grouped the definition of problem posing in the existing literature into five:

- Generating new problems

  Example: The student may observe the divisors of various numbers and notice that the number of divisors varies. This observation can lead to looking at integers with three and two divisors and determining their relationship. Further, he or she may analyze some examples of prime numbers and factorization of numbers into prime numbers and ask if the relationship between a number and its divisor is a function. Finally, a new problem about the fundamental theorem of arithmetic may arise: can an integer larger than 1 be presented as a product of prime numbers in only one way? (Kilpatrick, 1987)

- Reformulating existing or given problems

  Example: Students are given two alternatives to make a cloth-drying rack. The clotheslines are made of string strung between two parallel supports or two crossbars. Given the length of the outer side and the distance between neighboring lines, students are instructed to determine the length of the clothesline required for each option. Both models seem realistic but can be questionable for the students as both models need to include the clothesline needed to tie the rope to the support. Those models are simplified. The students may reformulate a second model that includes an additional clothesline to tie. (Kilpatrick, 1987)

- Both generating new and/or reformulating given problems

  The example of this category refers to the previous two problem posing activities.

- Raising questions and viewing old questions from a new angle

  This category appears to be similar to reformulating a problem. This category focuses on the question of the problem, but in reformulating the problem, the attention is on the data set. The data in the previous case is the correctness of the number, which indicates the length of the clothesline. This category can also be linked to Brown and Walter's (1983) use of the "what-if-not" strategy. The use of the strategy can be seen in Kovács (2017).

- An act of modeling

  An act of modeling implies transforming the problem's natural language representation into a mathematical language representation. This category is exemplified when students examine a list of menus that includes the product, the price, and the ingredients. They should create an order based on the structure of the receipt, such as quantity and price. Finally, they must calculate the total amount to be paid.

Those are not strict categories. A problem-posing situation does not always belong to a single category but may fall into more than one category, depending on how it is interpreted. Another important note, we should distinguish the term "new problems" from the point of view of students and mathematicians. In the classroom context, a new problem is not necessarily a new problem from the teacher's or mathematician's point of view, but it is a new problem for the students. The students might formulate a problem they have never encountered before, but the teachers already know it.

Starting from a problem-solving perspective, Kontorovich et al. (2012) introduced a framework for dealing with the complexities of problem posing. The framework consists of the following components: task organization, knowledge base, problem-posing heuristics and schemes, group dynamics and interactions, and individual considerations of aptness.

- Task organization refers to the didactic decision when planning a problem-posing activity, such as whether to manage the class to work individually or in small groups, whether to intervene in the student's actions, and whether to discuss the problems in a whole class setting.
- The knowledge base constitutes mathematical facts, definitions, mathematical discourse and writing competencies, and prototypical problems used by the poser. Prototypical problems refer to problems typically or commonly encountered by problem poser. The knowledge base can be investigated by looking at the mathematical validity of the problems posed.
- Problem-posing heuristics serve as guidance for generating new problems. An example comes from Brown and Walter (2004), who proposed the "What-if-not" strategy. This strategy guides the poser to generate a new problem by modifying the initial conditions or the initial goals of the original problem through several steps: (1) choosing a starting point, (2) listing attributes of the original problem, (3) posing what-if-not condition for the original attributes, (4) posing

mathematical questions according to the modified attributes, and (5) analyzing the proposed problem.

- Group dynamics and interactions refer to the social nature process that occurs when a group works on a problem-posing task. The process may include the normalization, conformity, and innovation. During normalization, group members progressively and mutually agree on a common frame of reference. Within the conformity process, the group majority or the most assertive member pressures deviating individuals to follow. In the process of conformity, the majority of the group or the most assertive members pressure the deviant individual to conform. Finally, innovation occurs after negotiations among group members to resolve the conflict have taken place. Members play various roles during those processes, such as generating an idea and mediating.
- Individual aptness considerations are interpretations of a problem-posing task's explicit and implicit requirements. This aspect includes the suitability of the problems posed to the poser, potential evaluators, potential solvers, and group members.

Meanwhile, Stoyanova and Ellerton (1996) categorized problem-posing situations into free, semi-structured, and structured. The situation is free when the instructor asks students to formulate a problem based on the given, either contrived or naturalistic situation. It is semi-structured when the instructor gives students an open situation and encourages them to explore the structure and complete it by applying their cognitive resources. At last, the situation is considered structured when the instructor encourages students to explore a particular problem, the structure of the solution, and possible relations between the problem statement and the solution idea, and finally asks students to generate a new problem based on the previous problem. In most cases, identifying semi-structured situations appears to be challenging as they can overlap with free or structured situations. The following are some examples of free and structured problem-posing situations.

- Free situation

  How much time do you need to leave the house before class starts in order to arrive in time? Pose a mathematical task based on the situation. (Halmos & Varga, 1978)

  The situation is given but not in detail. The students have the opportunity to define the problem and describe detail circumstances.

- Structured situation

  Presume the time on a circular clockface is exactly 12 noon. Assuming the clock is working properly, how many minutes (to one decimal place) will it take for the minute and hour hands to point in the same direction again? Formulate a new task with a similar structure to the problem. (Ellerton, 2013)

  It requires students to identify the structure and then apply it in a different context.

In those two situations, the starting point and/or the goal are open. Thus, it corresponds to what Pehkonen (1997) referred to as open problems. In a free situation, both the starting point and the goal are open. In a structured situation, the starting point is closed while the goal is open.

The role of problem posing in higher education, particularly in teacher preparation programs, has a long history of research. It has been recognized in mathematics teaching and acknowledged as a principal skill in mathematics teachers' professional development (Osana & Pelczer, 2015). So far, there have been a number of studies revealing the incorporation of problem posing in the school setting. One comes from da Ponte & Henriques (2013), who reported investigation activities, including problem posing in a university course. The activities were designed to pique students' curiosity, prompt them to pose questions, and foster their investigation skills. As a result, they went through mathematical processes supplemented by asking questions, formulating conjectures, and testing them.

In a mathematics teacher preparation program, Crespo (2003) introduced a non-traditional mathematical problem into her class and asked prospective teachers to investigate it. Then, prospective teachers and school students worked together in problem-posing activity. The prospective teacher wrote a letter in which he or she proposed a mathematical problem for school students, and the students responded with feedback on the proposed problem. Getting acquainted with this activity, the prospective teachers considered it a particularly profound experience. Another effort to enhance problem posing in the mathematics teacher preparation program was carried out by Tichá and Hošpesová (2013). They realized that students, who enrolled in the program, typically have a naive belief about mathematics and the nature of mathematical thinking. In the end, problem posing with shared reflection results in a more positive shift in their beliefs.

Given that problem posing can be a tool in teaching mathematics (Cai & Hwang, 2020), Kovács (2017) implemented problem posing by asking prospective teachers to explore a textbook problem more deeply by using the "what-if-not" strategy, and it resulted in the generation of some new promising problems. Nevertheless, it is worth noting that implementing problem posing in the classroom can result in several phenomena, such as the emergence of unexpected pedagogical and mathematical situations, the appearance of many student proposals with potential deadlocks, and the presence of a complex interplay between not understanding and understanding mathematical concepts (Kovács & Kónya, 2021). Thus, it might be a critical issue for the teachers as they should handle the unexpected situation. Mason (2015) refers to it as knowing to act, not later but now.

Some research also revealed that some students pose non-mathematical, unsolvable, irrelevant to the given situation, and ill-formulated problems (Cai et al., 2015; Chapman, 2012). Students might need more time to get used to posing problems and improving their skills. While recognizing this circumstance, some studies attempted to investigate the tasks generated by students from various backgrounds and, as a result, proposed some evaluation methods for the problem-posing products (see Cai (1998) and Tabach & Friedlander (2013).

Crespo (2015) identified two main categories for evaluating students' problem-posing products: empowered and disempowered, each with subcategories.

- Disempowered problem

  Closed (The proposed task is a story problem that requires quick translation or a calculation exercise); *simplified* (The task proposed by narrowing down the mathematical scope of the original problem); and *blind* (The task with underestimated mathematical complexity).

- Empowered problem

  Open (The task necessitates problem solvers to explain their work and communicate their ideas); *mathematically challenging* (The task brings new ideas, challenges the solvers' understanding, or promotes their thinking); *mathematically interesting* (The proposed task is generated by applying aesthetic criteria of mathematics such as surprise, novelty, simplicity, and fruitfulness), and *socially relevant* (The task entails using mathematics to understand and address social issues).

Several authors evaluated problem-posing products from a creativity perspective, i.e., fluency, flexibility, and originality (See Leikin & Elgrably, 2020; Tabach & Friedlander, 2015), while Kovács (2020) proposed several terms to categorize problem-posing products, namely: *blind* (an unsolvable task), *inadequate* (a task with a single answer or a clear goal), *exercise* (a task which leads to an arithmetic solution or can be solved by using simple calculation), and *problem* which is divided into a simplified problem (a task with a mathematical scope narrowed down from the original problem), analogous problem (a task utilizing the same methods and strategies as the original problem), and flexible problem (a task with a different mathematical structure than the one in the original problem).

4.5 Interconnection between Problem Posing, Problem Solving, and Active Learning

Problem posing is not a separate notion, but rather a companion to problem solving (Bonotto & Santo, 2015; S. I. Brown & Walter, 2004; English, 1997; Kilpatrick, 1987; Polya, 1945; Silver, 1994). The activity can be aided by problem solving (Bonotto & Santo, 2015) and, thus, might occur prior to, throughout, or after problem solving (Silver, 1994).

- Problem posing prior to problem solving refers to the generation of a new problem based on a given or realistic situation by which the proposed problem will be solved afterward.

- Problem posing throughout problem solving means posing a subgoal, related, similar, or simpler problem, which is more attainable as a bridge to solving the main problem. It belongs to a successive reformulation of the original problem. This strategy has been suggested by Polya (1945) in the step of "Devising a plan," addressed by Lénárd (1978) as paving the way to a solution, and mentioned by Schoenfeld (1985) in the "Heuristics" aspect.
- Problem posing after problem solving denotes generating a new problem by modifying the condition of the previously solved problem. It refers to the "Looking back" step by Polya (1945) with guiding advice to investigate whether the result or the method can be applied to other problems. Later, Ernest (1991) highlighted problem posing as a distinctive feature of mathematical investigation. The occurrence of problem posing following problem solving is also mentioned by Radnainé-Szendrei (1988),

  > Egy-egy problémával még a megoldás kialakítása után is „eljátszunk". Több oldalról való vizsgálatával, az általánosítási lehetőségek utáni kutatással az a cél, hogy az újonnan megszerzett tudás mélyebben épüljön a megoldó korábbi ismeretei közé, és az olvasó teljesebb képet kapjon a matematika műveléséről. (p. 3)
  >
  > After finding a solution, even we can "play" with the problem by looking at it from various angles and looking for generalization possibilities. This allows newly acquired knowledge to be integrated into prior knowledge while also providing a comprehensive picture of how we do mathematics[3] (Radnainé-Szendrei, 1988).

[3] The supervisor approved the translation.

Recognizing the two intertwined notions, some research groups integrated problem posing and problem solving into their studies. The designed intervention substantially influenced the originality of the problems posed by the students, their problem solving abilities, and their beliefs and attitudes toward problem posing and problem solving (Chen et al., 2015). The students' problem-posing and problem-solving performance increased when the pre-test and post-test results were compared (Kopparla et al., 2019).

In addition, problem posing and problem solving have been attributed by several scholars to active learning. To investigate the feasibility of integrating problem-posing activities into the curriculum in parallel with problem-solving activities, Ellerton (2013) proposed an active learning framework demonstrating how and where the combined activities might fit into the teaching plan. Classroom and prevailing student actions that exhibit passive versus active student involvement are clearly laid out. Passive student involvement and active teacher involvement are demonstrated when the teacher models examples and the students listen, imitate, and memorize. In contrast, active student involvement and teacher facilitation are demonstrated when the class discusses and solves problems posed by students and the students critique, question, and clarify. In a recent study, problem posing highlighted as a method of active learning that needs to be addressed alongside problem solving (Polat & Özkaya, 2023).

4.6 Teaching Perspectives

Views on mathematics, learning, and teaching contribute to one's conception of the mathematics course's instruction (Chamberlin, 2013). These three elements are contained in what Ernest (1989) referred to as belief, which involves the conception of the nature of mathematics, the nature of teaching and learning mathematics, and the principle of education. There are three schools of thought on the nature of mathematics:

- The instrumentalist view

  Mathematics is a beneficial but disjointed set of facts, rules, and abilities.

- The Platonist view

  Mathematics is a unified but static body of knowledge that is discovered rather than created and comprised of interconnected structures and truths.

- The problem-solving view

  Mathematics is not a finished product, but rather a constantly evolving field of human inquiry that is always open to revision.

In his study, Rott (2019) connected the instrumentalist view to what Dionné (1984) refers to as the traditional perception of mathematics. In addition, Ernest (1989) perceived the conception of teaching and learning mathematics as a mental model since it is a collection of ideas, which may include past memories, on which the teacher "models" his or her behavior. Pehkonen (1994) amplified this assertion by stating that teachers' prior experiences in mathematics teaching and learning greatly influence their teaching behavior through models. He categorized the conception of teaching mathematics into either narrow, instrumental, and basic skills or broader, creative, and exploratory; meanwhile, the perspective on learning mathematics can be either passive reception of knowledge or active reception of knowledge. In their study of beliefs in mathematics and teaching practices, Saadati et al. (2021) reported that most traditional beliefs held by the teacher are associated with teacher-centered learning.

In summary, beliefs about mathematics are reflected in teachers' perspectives of mathematics teaching and learning, and thus in their practices (Thompson, 1984).

By paying attention to beliefs in mathematics and conceptions of teaching and learning mathematics in general, the author of this dissertation proposes two possible perspectives about the style in teaching mathematical problem-solving as follows (see Table 2).

Table 2. Teachers' perspectives in teaching mathematical problem solving

Beliefs in mathematics (Ernest, 1989)	**Conception in learning mathematics**	**Style in teaching mathematics**	**Perspectives in teaching mathematical problem solving**
The instrumentalist view	- Learners passively construct knowledge (Marshman & Goos, 2018) - Learners memorize and use the formula correctly (Safrudiannur & Rott, 2019)	Teachers demonstrate how to use the formula correctly by giving some examples (Safrudiannur & Rott, 2019)	Teaching problem solving as transferring knowledge
The Platonist view	- Learners actively construct knowledge (Marshman & Goos, 2018) - Learners understand the concepts by underlying the formula from the teacher's explanation (Safrudiannur & Rott, 2019)	Teachers explain concepts related to how to get or to prove the formula (Safrudiannur & Rott, 2019)	Teaching problem solving as facilitating students to construct knowledge by themselves
The problem-solving view	- Learners autonomously explore their own interests (Beswick, 2012)	Teachers let students discover the formula in their own ways (Safrudiannur & Rott, 2019)	

	- Learners draw logical conclusions to deduce the formula (Safrudiannur & Rott, 2019)		

4.7 Teaching Implementations

Along with the educational reform, there is a growing interest in implementing active learning due to its numerous advantages. As active learning places students at the center of learning, it enables classroom activities to evolve into critical discussions. Critical discussions are initiated by particular forms of talk that promote a deep understanding of concepts and robust reasoning. Sohmer et al. (2009) classified teachers' various forms of talk into four categories.

- Recitation

  By assuming special authority to ask questions and evaluate students' responses, the teacher completely controls the content and direction of the conversation. Students are cast as seekers of the correct answers that the teacher desires.

- Stop-and-talk (partner talk)

  The teacher poses a pointed question to the students and instructs them to discuss it with at least one partner. Students actively participate in reflecting and contributing. During small group discussions, the teacher selects crucial student voices to be discussed by all class members.

- Student presentation and group critique

  The teacher instructs the student to present his or her work in front of the class, with follow-up questions proposed by the other students or the teacher. The student who is presenting is positioned as the expert in his or her work.

- Whole-group 'position-driven' discussion

  The teacher facilitates a discussion on a single problem or question with multiple answers, allowing students to present reasonable arguments. This type of discussion encourages students to participate actively by proposing ideas and listening to one another even before they are fully competent in what they discuss.

The last three talk forms lead to the implementation of active learning which can be supported by appropriate teaching behavior. Focusing on teaching implementation on problem solving, Rott (2019) classified teachers' behavior into closely managed, neutral, and emphasizing strategies. In each step of the problem solving by Polya (1945), the characteristics of each behavior are as follows.

- Understanding the problem
  - Closely managed: The teacher demonstrates the problem formulation.
  - Neutral: The teacher provides no comments about the problems and does not respond to the students' questions.
  - Emphasizing strategies: The teacher provides hints but does not explain the problem.
- Devising a plan
  - Closely managed: The teacher tells the students on the proper approach to take.
  - Neutral: The teacher provides no guidelines or strategic support.
  - Emphasizing strategies: The teacher proposes (ideally) different approaches and encourages students to pursue their own ideas.

- Carrying out the plan
    - Closely managed: The teacher provides students with specific content-related guidance, often early in the process.
    - Neutral: The teacher provides almost or no assistance and does not respond to students' problems-related questions.
    - Emphasizing strategies: The teacher provides staggered assistance in the form of motivation, feedback, general strategies, or specific strategies related to content.
- Looking back
    - Closely managed: The teacher only focuses on the result, perhaps one (arithmetic) approach is presented.
    - Neutral: The teacher presents various approaches but does not explicitly highlight strategic ideas or the differences between approaches.
    - Emphasizing strategies: The teacher emphasizes approaches and strategies, he or she may present the numerical results as well, but it is considered secondary in importance.

The classification refers to the distinction between teachers who are controllers and those who are facilitators. According to the characteristics, the closely managed behavior represents a teacher-centered approach in which the teacher performs as a controller. The other two behaviors are more in line with the teacher's role as a facilitator, which aims to stimulate mathematically rich and precious discussions in the classroom.

Paying attention to the characteristics of each form of teacher's talk and behavior will disclose the link between those two components. For instance, recitation complements the closely managed behavior, whereas the other forms of talk sustain neutral and emphasizing strategies behaviors. Accordingly, the teaching behavior can be detected by the tendency to use a particular form of instruction, whether it leads to a productive talk or not. Finally, analyzing the teacher's form of instruction and behavior will cue whether the lesson appertains as active learning or not.

5. Research Methods

5.1 Background Information

This study is action research conducted in the form of an intervention. Action research has been growing in popularity, especially among practitioner-researchers, and it leads to professional development (Koshy, 2005). Accordingly, following the characteristic of action research, the researcher also acted as the instructor.

This research involves prospective teachers from two universities representing two types of universities in Indonesia. In addition, the prospective teachers from each university have different mathematical backgrounds. Initially, the intervention was held separately for participants from private and public universities. The class with private university students was the first cycle, while the class with public university students was the second cycle. In some points, the second cycle was the modification of the first cycle, as some scholars emphasized that action research involves spiral activities of planning, acting, observing, and reflecting on the process and the outcome of the practice (Elliott, 1991; Koshy, 2005; MacIntyre, 2012; O'Leary, 2004).

At the final stage of the intervention, one prospective teacher from the private university left the university program. Thus, the class was integrated for students from both universities to discuss the last three meetings. Given that students from each university have different mathematical backgrounds, the class unification encouraged the students from the private university becoming more active. The timetable of the intervention is depicted in Figure 4.

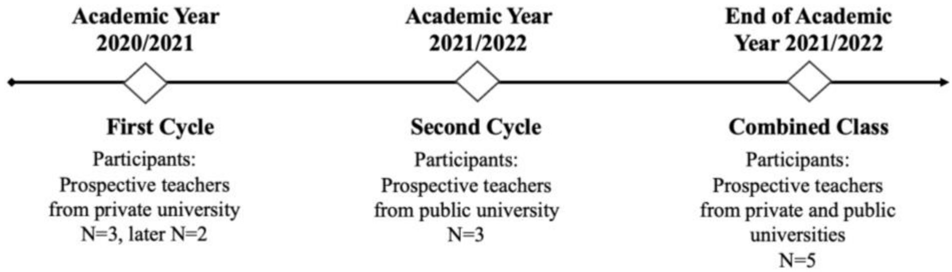


Figure 4. The intervention's flow

As previously mentioned, active learning involves intellectual, social, and physical activities (Edwards, 2015). The main objective of this study is to develop the pedagogical and mathematical skills of the participants through active learning. Thus, the activity during the intervention contained posing and discussing mathematical problems collaboratively to encourage intellectual and social activities. In addition, the participants also implemented their self-proposed tasks with the school students (hereafter referred to as "pupils" to to distinguish them from university students as the research participants), that is regarded as a form of cognitive and social activities. In class, they deal with cognitive activity as they may have to control an unpredictable situation or question proposed by the students. Furthermore, classroom activities also involve interaction with students. Thus, it falls into a social activity.

They implemented their tasks in the preliminary, middle, and closing parts. As the intervention was also concerned with the pedagogical aspects, the participants reflected and shared their implementations with their peers during the class meeting after watching each other's video recordings of their teaching implementations.

5.2 Participants

As mentioned above, there are two participating groups in this research. One group comes from a private university which consists of three prospective teachers (AM, KK, and VI) in the early part of the intervention and later consists of two prospective teachers (AM and KK) as VI left the university program, while the other group comes from a public university which consists of three prospective teachers (AI, TK, and AF) until the end of the intervention. Thus, five prospective teachers participated in the entire intervention.

The prospective teachers from private and public universities have different level of mathematical background. Those from public university have passed a very demanding selection procedure that is held concurrently by all public universities or by the respective public university. In general, becoming public university students is their primary aim, and they are graduates of flagship high schools. Therefore, the public university students are assumed to have a stronger mathematical background than the private university students.

All participants are between the ages of 18 and 21. They are in the same batch participating in a 4-year mathematics teacher training program for grades 7-12. Private university students began attending the intervention in the academic year 2020/2021 when they were in the second year of the program, while public university students began attending in the academic year 2021/2022 when they were in the third year of the program. At the end of the intervention, when the class is combined, all students are in their third year of study.

27 private and 24 public university students were offered to join the intervention. Some of them handled a non-formal course with pupils, which aimed to strengthen the lesson provided at school. As the latter activity contributes to the intervention's program, namely, implementing self-proposed problems in a classroom environment, these students were given priority in the selection process. In the end, six students interested in joining the intervention voluntarily and who handled the non-formal course were selected as the participants. Before the intervention, both private and public university students were unfamiliar with the problem-posing approach.

5.3 Data Collection

The data in this study consists of problem-posing performances, problem-solving performances, critical manifestations, perspectives in teaching mathematical problem solving, and teaching styles. Problem-posing and problem-solving performances came from the activity when the participants were assigned to pose a problem based on the given situation, solve, and discuss it with peers. In certain cases, interviews were conducted to clarify the proposed task or the solution. Critical manifestations were identified through a series of video-recorded lessons and text-based discussions. For analysis, all conversations were transcribed. To catch their perspectives in teaching mathematical problem solving, a questionnaire (see Figure 5) was generated by referring to the characteristics of teaching mathematical problem-solving styles by Rott (2019). Ultimately, the participants were directed to implement their self-proposed tasks to the pupils to obtain data on teaching styles, and the classes were video recorded.

Choose one of the options!

Understanding the problem

- ❑ The teacher should make sure everyone understands what to do.
- ❑ The teacher should not comment on the text of the task and let the students interpret it themselves.
- ❑ The teacher should not interpret the task but gives a guiding question.

Devising a plan

- ❑ The teacher should tell pupils which approach will lead to the right solution.
- ❑ Teachers should not make any comments but provide opportunities for students to investigate appropriate strategies.
- ❑ The teacher should mention some applicable approaches but encourages students to follow their own ideas.

Carrying out the plan

- ❑ From the first steps, the teacher should steer the students on the right direction.
- ❑ The teacher should not give any help regarding the solution method but provide opportunities for students to work according to their line of thinking
- ❑ The teacher should support students gradually, according to their needs, just to the extent necessary.

Looking back

- ❑ The teacher should direct the students to check the numerical results (numbers) as the most important thing.
- ❑ Teachers should direct students to show the variety of strategies they use without clearly discussing the differences one by one.
- ❑ Teachers should direct students to point out the variety of strategies they use (as the most important thing) and clearly discuss the differences one by one, while numerical results (numbers) are the second most important.

In teaching mathematical problem solving, the teacher should clearly show their students how to solve the problem.

Disagree ○ ○ ○ ○ ○ Agree

Students learn better when they find their own way to solve the problem.

Disagree ○ ○ ○ ○ ○ Agree

The teacher should provide opportunities for students to solve the problem in their own way, before showing them other (effective) ways to solve the problem.

Disagree ○ ○ ○ ○ ○ Agree

Figure 5. Questionnaire on teaching mathematical problem-solving perspectives (Fitriana, 2022)

Preliminary and follow-up tests were carried out to see whether there was a change in the problem-posing and problem-solving products as well as the problem-solving instruction. In the preliminary and follow-up tests, the participants were asked to fill out the questionnaire, pose a problem, solve it, and implement it to the pupils. As the participants from the private university did not fill out the questionnaire prior to the intervention, at the end of the intervention they were instructed to reflect on whether there was a shift in their conception of teaching mathematical problem solving and convey it into the questionnaire. The prospective teacher from the private university, who did not fully participate until the end of the intervention (VI), was directed to fill out the questionnaire just after her last meeting to catch her final perspective in teaching mathematical problem solving. In order to track the category of the problem-posing products and problem-solving performances, the participants were encouraged to pose a task and solve it during the preliminary, middle, and follow-up tests. Meanwhile, they were not required to solve the task during the lesson as the main aim was on collaborative problem posing.

5.4 Research Trajectory

In determining the topic of the intervention, two conditions are taken into account:

1. None of the participants had experience with problem posing before the intervention.
2. Referring to the cross-national study results, Indonesian prospective teachers tend to pose mathematical task which are arithmetic in nature and tend to utilize arithmetic rather than an algebraic approach when solving their self-proposed problems (Fitriana et al., 2022).

Thus, the topics (see Table 3) are meant to offer them a variety of problem-posing situations and to encourage the emergence of algebraic thinking.

Table 3. Research trajectory

Preliminary test: Individual problem posing and teaching implementation
Problem-posing type: generating new problems. Instruction:

March 2022						
Mon	Tue	Wed	Thu	Fri	Sat	Sun
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

1. Discover at least three patterns in this month’s calendar.
2. Explain those patterns.
3. Create a problem based on one of the above patterns and solve it.
4. Implement the problem with your pupils.

Topic 1: Teaching reflection and giving feedback to each other

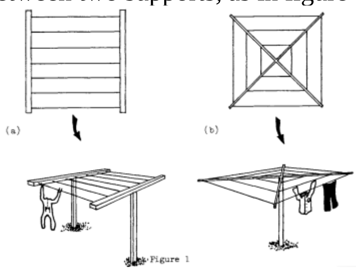
Before the meeting, the participants observe the video recording of their peers’ classes.
Instruction during the meeting:
1. Please reflect your teaching implementation.
2. Please give feedbacks for your peers’ teaching implementations.

Topic 2: Introduction to problem posing (Theoretical discussion)

The instructor introduces the existing definition of problem posing, explains when problem posing may occur, and presents several problem-posing situations.
Follow-up questions:
Have you understood what problem posing is, its occurance, and its situations?
What are your thoughts on those things?
Source: Papadopoulos et al., 2021; Silver, 1994; Stoyanova & Ellerton, 1996

Topic 3: Collaborative problem posing

Problem-posing type: reformulating already existing or given problems.
- Preliminary discussion:
  “Farel and Vian have 8 books altogether. Farel has 5 books. How many books does Vian have?”
  Can you give me your answer and the reason?
  Does anyone have a different interpretation?
  What do you think about this problem?
  Do you have any idea how to rephrase the problem to make the semantic relations more explicit?
- Main discussion:
  We want to make a cloth-drying rack for the backyard. We can fix it so that the clothes lines are strung between two supports, as in figure (a) or (b).



  How many meter of clothesline would we have to get for each of these options if the outer square measures 1.8 meter on a side and the separation between adjacent lines is 0.3 meter?
  What is the length of the clothesline needed for each pattern?
  Which pattern do you prefer to choose? Give me your reason!
  Does it make sense? What to consider?
  Is there another better solution?
  From the options, which one will you choose?
Source: adapted from Kilpatrick (1987)

Topic 4: Collaborative problem posing

Problem-posing type: raising questions and viewing old questions from a new angle.

Instruction:
Connect the midpoints of two opposing sides of the parallelogram with arbitrary points on the other two sides. Show that the area of the quadrilateral defined by the four connecting segments is half of the area of the parallelogram!
What are given in the problem? What if not them?
Determine the new situation!
What is the new statement or condition you have?
Can you pose a new problem according to the new condition?
Source: Kovács (2017)

Topic 5: Individual problem posing and collaborative improvement of the problems

Problem-posing type: an act of modeling.
Instruction:
Please generate problem(s) by using this situation: Adit, Kevin, and Leo took turns driving home from a trip. Leo drove 80 km more than Kevin. Kevin droves twice as many km as Adit. Adit drove 50 km.
Source: adapted from Silver and Cai (1996)

Middle test: Individual problem posing, collaborative improvement of the problems, and teaching implementation

Problem-posing type: generating new problems.
Instruction:

Dad just bought a padlock for the fence.

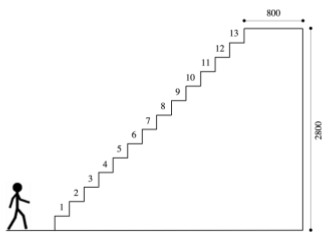
1. Please pose a problem based on the situation above and solve it.
2. Please discuss the problems proposed by your peers and improve them collaboratively if needed.
3. Implement the problem with your pupils.

Topic 6: Teaching reflection and giving feedback to each other

Before the meeting, the participants observe the video recording of their peers' classes.
Instruction during the meeting:
1. Please reflect your teaching implementation.
2. Please give feedbacks for your peers' teaching implementations.

Topic 7: Individual problem posing and collaborative improvement of the problems

Problem-posing type: generating new problems.
Instruction:

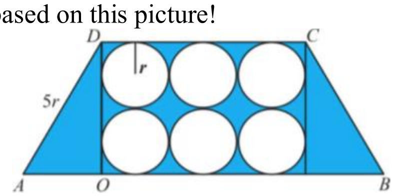


Please pose as many problems as you like based on the picture above!
Generate the rule and solve your problem!

Do you realize the pattern?
Can you generalize it?

Topic 8: Individual problem posing and collaborative improvement of the problems

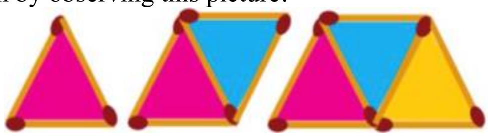
Problem-posing type: generating new problems.
Instruction:
Please pose a problem based on this picture!



Source of the picture: Indonesian mathematics textbook

Topic 9: Individual problem posing and collaborative improvement of the problems

Problem-posing type: generating new problems.
Instruction:
Please pose a problem by observing this picture!



Source of the picture: Indonesian mathematics textbook

Topic 10: Pattern recognition and generalization

Instruction:

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$

What comes next?
Find as many ways as you can to prove the equation right!
My friend thought of a number and this kind of sequence in her mind. But I don't know the number. Can you help me to figure it out?
(The number in her mind referred to the serial number of the equation. E.g., what will be the tenth equation?)
Source: John Mason's lecture, Debrecen, 5th April 2022

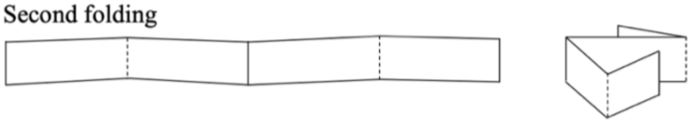
Topic 11: Pattern recognition and generalization

- Peliminary discussion:
  According to legend, when the famous Greek scientist, Archimedes, discovered an important law in a bathtub, he shouted the word "Eureka" loudly enough so that the entire city can hear it. The phrase translates as "I found it."
  I have a slot machine. It requires two inputs and gives one output. I hold the rule and put it upside down on the board. Your task is to find out the rule, how the machine works. I give you two inputs and one output. You try to guess the rule by mentioning the three numbers. I will tell you whether the rule on the board fits your numbers or not. If someone is sure of the rules, please shout, "Eureka"!
  Source: Eureka sequences in Mason et al. (2010, p. 91)
- Main discussion:
  Fold a sheet of paper into two equal parts. How many fold lines and sections are formed?
  Do the second folding, and so on. How many fold lines and sections are formed?

Illustration:

First folding

Second folding



What is the rule? Can you explain it?

Folding	0	1	2	3	4	5	…	n
$S_{(n)}$	1	2	4	8	16	32	…	2^n
$C_{(n)}$	0	1	3	7	15	31	…	2^n-1

$S_{(n)}$: the number of the formed sections; $C_{(n)}$: the number of the folding lines

Follow-up test: Individual problem posing, collaborative improvement of the problems, and teaching implementation

Problem-posing type: generating new problems.

Instruction:

1. Please pose a problem based on the paper folding situation, and it must contain the compulsory word "mouse."
2. Please discuss the problems proposed by your peers and improve them collaboratively if needed.
3. Implement the problem with your pupils.

The reasons for selecting the topic are as follows: to acquaint participants with problem posing (topic 1), to give opportunity to reflect and improve teaching practice (topic 2, topic 6), to provide a warming-up problem-posing activities (topic 3, topic 4), to give opportunity to practice posing a problem individually and improve it (topic 5, 7, 8, and 9), to encourage the appearance of algebraic approach (topic 7, 9, 10, and 11), and to see if problems not typically found in textbooks may arise (topic 8 and 9). All topics were deemed applicable to various grade levels and represented several problem-posing situations. In addition, the compulsory word "mouse" was given to encourage the emergence of relevant problems to the students, as in Indonesia, the mouse and its holes are easy to find in villages.

5.5 Evaluation

5.5.1 Problem-posing performance

The problem-posing products generated in the preliminary, middle, and follow-up tests and during the lesson are classified according to the framework in Fitriana et al. (2022), see Figure 6.

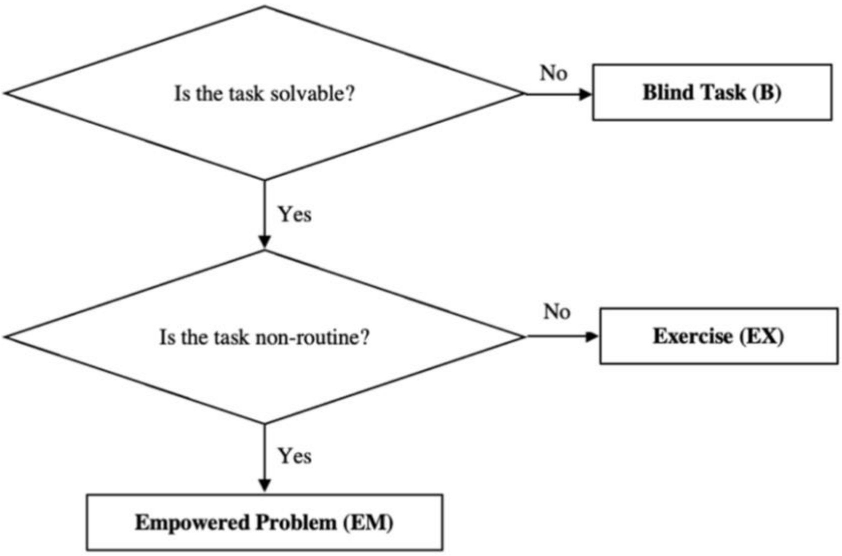


Figure 6. Coding chart flow of the problem-posing products

The product belongs to a blind task if it cannot be solved or there is insufficient data to solve it. If the product can be solved, but it is a routine task or a simple word problem that can be solved by using basic calculations, then it is labeled an exercise. An empowered problem is a product that is mathematically challenging, stimulates creativity, or allows solvers to explore as many distinctive ways as possible and communicate their ideas. The framework by Kontorovich et al. (2012) will be referred to explain the participants' performance. The characteristics of the tasks they propose, the heuristics they use, and their roles during the discussion will be presented to give an overview of how they performed in each framework component.

5.5.2 Problem-solving performance

The problem-solving products will be grouped into correct, incorrect, and blind solutions. Blind solution refers to the solution of the blind problem-posing products. The problem-solving performance will be analyzed according to Schoenfeld's (1985) framework for analyzing success and failure in mathematical problem-solving. The first three aspects (cognitive resources, heuristics, and control) will be explored, while the fourth (belief systems) will be excluded due to limitations in exploring it.

- Cognitive resources (consist of intuitions, informal knowledge, the possibility, facts, and procedures possessed by the solver)
  Does the solver have sufficient knowledge, facts, or procedures to solve the self-proposed problem?
- Heuristics (encompass drawing figures, using a rule of thumbs, introducing suitable notation, exploiting related problems, reformulating problems, working backward or forwards, testing and verifying procedures, decomposing, recombining, generalizing, and specializing, etc.)
  What heuristic does the solver use? Does it lead to success in solving the self-posed problem?
- Control (involves planning, monitoring, assessing, making decisions, and conscious metacognitive acts)
  Does the solver choose the facts, procedures, and heuristics efficiently?

If the solution is incorrect, the solver is considered not to have properly controlled the problem-solving process. The type of error performed by the solver will be noted.

5.5.3 Critical manifestations

The manifestations that appeared during the discussion (both in lessons and oral reflections) will be classified into critical thinking or non-critical thinking manifestations based on the diagram in Figure 7 and analyzed qualitatively.

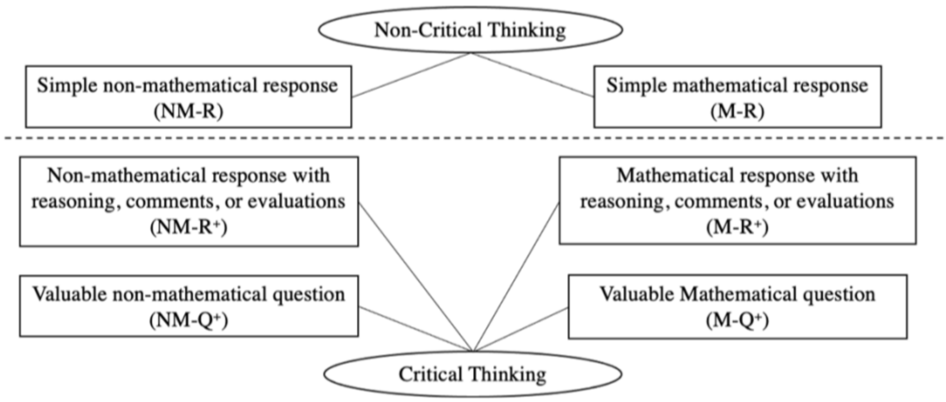


Figure 7. Manifestations which belong to critical or non-critical thinking

5.5.4 Teaching perspectives

Perspectives in teaching mathematical problem solving will be categorized into teaching as transferring knowledge or teaching as facilitating students to construct knowledge by themselves. In the questionnaire (see Figure 8), the options in each problem-solving step indicate the teaching behavior in order, i.e., closely managed, neutral, and emphasizing strategies. The first option (the closely managed) is categorized as teaching as transferring knowledge. In contrast, the second and the third options reflect neutral and emphasizing strategies subsequently, which are categorized as teaching as facilitating students to construct knowledge by themselves. In addition, the last three statements presented in a Likert scale are used to get additional information on which teaching behavior and, thus, the perspective that the participants best correspond (see Figure 5 or 14). Finally, the results of the questionnaire at the beginning and the end of the intervention will be compared and analyzed in terms of changes.

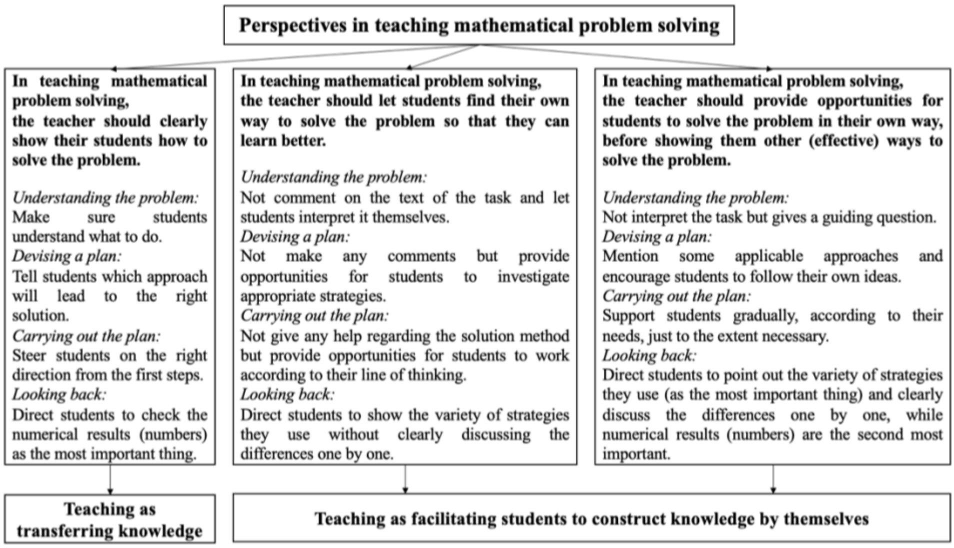


Figure 8. Perspectives in teaching mathematical problem solving

5.5.5 Teaching implementations

The teaching practice will be analyzed based on the forms of talk by Sohmer et al. (2009) and the teaching behaviors by Rott (2019), then categorized it into traditional approach or active leaning as follows (see Figure 9).

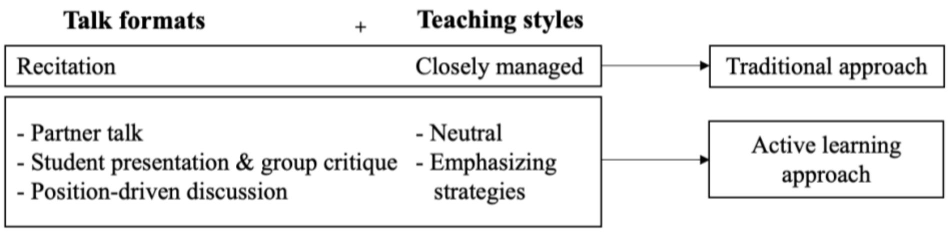


Figure 9. The category of teaching practice

6. Results and Discussions

6.1 Problem-Posing Performance

6.1.1 Result

Table 4. Problem-posing products in the preliminary test[4]

Participant	Task code	Preliminary test		
		B	EX	EM
AM	P.1		✓	
KK	P.2			✓
VI	P.3		✓	
AI	P.4			✓
TK	P.5	✓		
AF	P.6			✓
Sum		1	2	3

Table 5. Problem-posing products in the middle test and follow-up test[5]

Participant	Middle test								Follow-up test							
	Before discussion				After discussion				Before discussion				After discussion			
	Task code	B	EX	EM	Task code	B	EX	EM	Task code	B	EX	EM	Task code	B	EX	EM
AM	M.1		✓		M.2		✓		F.1	✓			F.2			✓
	M.3		✓		M.4		✓									
KK	M.5	✓			M.6		✓		F.3	✓			F.4		✓	
	M.7		✓		M.8		✓									
VI	M.9		✓		M.10		✓									
	M.11		✓		M.12		✓									
	M.13			✓	M.14			✓								
AI	M.15			✓	M.16			✓	F.5			✓	F.6			✓
	M.17			✓	M.18			✓								
TK	M.19		✓		M.20		✓		F.7		✓		F.8		✓	
	M.21		✓		M.22		✓		F.9		✓		F.10		✓	
AF	M.23	✓			M.24	✓			F.11		✓		F.12			✓
													F.13			✓
													F.14			✓
													F.15			✓
Sum		2	7	3		1	8	3		2	3	1		-	3	6

[4] B: Blind task EX: Exercise EM: Empowered problem

[5] The highlighted parts indicate the changes in the category of the problem-posing products before and after the discussion.

Table 6. Problem-posing products during the lesson[6]

Participant	Topic	Before discussion				After discussion			
		Task code	B	EX	EM	Task code	B	EX	EM
AM	Topic 5	L.1	✓			L.2		✓	
KK		L.3	✓			L.4		✓	
VI		L.5		✓		L.6		✓	
		L.7		✓		L.8		✓	
		L.9			✓	L.10			✓
		L.11			✓	L.12			✓
		L.13		✓		L.14		✓	
AI		L.15			✓	L.16			✓
		L.17			✓	L.18			✓
TK		L.19		✓		L.20		✓	
AF		L.21			✓	L.22			✓
AM	Topic 8	L.23		✓		L.24		✓	
AI		L.25			✓	L.26			✓
TK		L.27		✓		L.28		✓	
		L.29		✓		L.30		✓	
AF		L.31			✓	L.32			✓
Summary	Before discussion								
	B			EX			EM		
	7			14			12		

Participant	Topic	Before discussion				After discussion			
		Task code	B	EX	EM	Task code	B	EX	EM
AM	Topic 9	L.33		✓		L.34		✓	
KK		L.35		✓		L.36		✓	
		L.37		✓		L.38		✓	
AI		L.39	✓			L.40			✓
TK		L.41	✓			L.42			✓
		L.43	✓			L.44			✓
		L.45	✓			L.46			✓
AF		L.47		✓		L.48		✓	
AM	Topic 10	L.49		✓		L.50		✓	
		L.51		✓		L.52		✓	
KK		L.53			✓	L.54			✓
		L.55			✓	L.56			✓
		L.57		✓		L.58		✓	
AI		L.59			✓	L.60			✓
						L.61			✓
		L.62			✓	L.63			✓
		L.64			✓	L.65			✓
TK		L.66	✓			L.67	✓		
Summary	After discussion								
	B			EX			EM		
	1			16			17		

6.1.2 Discussion

In order to provide a comprehensive overview of the problem-posing peformance, the framework developed by Kontorovich et al. (2012) will be used to analyze the situation (See page 23 and 47).

1. Task organization

   The participants were offered a situation or a particular object to observe. Then, based on it, they were instructed to generate a challenging or interesting mathematical task for pupils, especially 7th to 12th graders.

   The preliminary test was intended to get an initial overview of the participants' performance, while the following two tests were follow-up tests. In all tests, they should write down the task. Notably, there was

[6] B: Blind task EX: Exercise EM: Empowered problem

an online peer meeting after the task submission in the middle test (only among public university participants) and the follow-up test (all participants) to allow participants to listen and consider their peers' viewpoints, followed by revising the task individually.

The difference in treatment on the middle test was due to the nature of action research. The instructor reflected on the first round (by which the participants were from the private university) and considered it essential to hold a peer meeting in the second round (by which the participants were from the public university). While during the lesson, participants proposed the task individually, either in text or orally, and then improved it with peers in an online meeting to encourage collaborative work.

The dual-task design (by which the participants pose a task and revise the first version in some cases) and peer discussion have advantageous effects on the quality improvement of the task, at least in 15 cases (29%) of the overall tasks proposed during the discussion of several topics (see Table 5: the after discussion shaded cells). First, the formulation of the revised task is better than in the previous version. Thus, the meaning is clear to comprehend. Unrealistic and unsolvable tasks were modified to make more sense and be solvable. The context becomes more appropriate and closer to pupils' lives. Finally, the tasks not only contain near or far generalizations but also include the final step of generalization. The abovementioned cases apply only to the task for which the category was changed. However, there were several tasks in which the category remained the same (i.e., the original task was an empowered problem, as was the revised version), but the formulation and situation were better than in the first submission.

2. Knowledge base

In the preliminary test, several mathematical terms appeared, such as arithmetic sequence, modular arithmetic, and multiple of a particular number. While the other two participants put specific examples to state the sum of n consecutive numbers horizontally, vertically, or diagonally (see corpus P.1, P.2): *"n:(5, 6, 7, 8, 9), S=35", "n:(3, 10, 17, 24, 31), S=85", n:(1, 7, 13, 19), S=40"*, one participant expressed it algebraically through generalization (P.4): *"The sum of dates in one row (one week) is 7 times the number on Thursday. Suppose the number on Thursday is n, so* $(n-3) + (n-2) + (n-1) + n + (n+1) + (n+2) + (n+3) = n+n+n+n+n+n+n = 7n$*".*

Five of the six proposed tasks were solvable, categorized into exercises and empowered problems. One of the exercises revealed that the poser used knowledge of a prototypical task, namely finding the sum of the first four terms of an arithmetic sequence (P.3). The last task was deemed a blind task as the formulation was unclear and arduous to comprehend (P.5). Due to the absence of peer discussion in the preliminary test, the interview was conducted to clarify the poser's intention, which revealed that it was a simple question about the multiple of six: *"If the sequence starts from 12 not 6, i. e., 12, 18, 24, …. Can we say the sequence belongs to the multiple of 6-number pattern?".*

In the middle test, most participants engaged a combinatorial topic. The tasks were those that are often found, such as "how many codes can be made." The difference is that exercises were elementary combinatorial tasks similar to those commonly found in the textbook, but those which were empowered problems came with a more complicated condition. In one of the empowered problems, the poser provided an interesting and mathematically challenging situation by embedding palindromes and probabilities in a combinatorial task (M.15). Following a discussion with the instructor, the participant who proposed a blind task (M.5), which was initially unrealistic, revised it into an exercise (M.6).

In the follow-up test, the participants used some units of measurement, such as time, length, mass, speed, and volume, since the first submission. They also used a variety of mathematical topics, i.e., exponential (eight products of thirteen, namely F.2, F.4, F.5, F.6, F.11, F.12, F.13, F.15), probability (three products, F.11. F.13, F.15), and direct proportion (three products, F.4, F.7, F.9), and one person presented a solution that appertained to near generalization (F.5):

> It makes a burrow 6 meters long from the ground. In each 1 meter long, there is a branch. By using the sigma rule:
>
> $$2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 127$$
>
> $$127 \times \frac{1}{2}\text{hour} = 63{,}5 \text{ hours}$$
>
> $$63{,}5 \text{ hours} : \ 24 \text{ hours/day} = 2{,}6\ (1/30) \text{ days}$$
>
> Thus, the mouse cannot find a suitable home location to stay within 2,5 days.

According to Rivera (2013), near generalization tasks last up to stage 9, whereas far generalization tasks begin at stage 10 and go up. Thus, the task above may lead to far generalization if the mouse makes a burrow of at least 10 meters long from the ground.

Moreover, one participant posed a task in which the situations collided (F.1) and another participant who intended to employ a direct proportion could not provide appropriate data to conclude (F.3).

There are three mouse traps, as shown in the picture, which contain food bait of different types and weights as follows:

• 1st trap: 0,5 ounces of cheese

• 2nd trap: 150 grams of carrots

• 3rd trap: 350 grams of cucumber

Each mouse trap has a trapping time speed of 4 seconds with a bait weight of 1 ounce. If the mouse wants to eat cucumber slices, with a duration of 6 seconds to eat every 1,5 ounces. Determine whether the mouse will be caught in the 3rd trap or not. (F.1)

Each trap will close within 4 seconds after the mouse steps in or when the bait weighs 1 ounce. The task is solvable if the mouse directly steps into the third trap. However, the situation collides if the mouse steps into each trap. The bait weighs in the first trap less than 1 ounce, meaning the first trap had been closed. The student should have noticed the decided situation that the weight of the bait must be more than 1 ounce.

In the second submission, after peer discussion, the participants provided better formulations of the task than those in the first submission. Given that the task in the first submission contained near and may lead to a far generalization step as F.5, the task in the second submission also included the final step of generalization as follows.

There was an experiment using a mouse. The mouse is put in the tunnel. At each end of the tunnel there is a box connecting two branches. A bread is placed in one of the boxes after the third intersection.

a. How many possibilities to put bread? (F.12)

   The number of branches: $2^3 = 8$.

b. What is the probability that the mouse enters the box containing the bread? (F.13)

The probability that the mouse gets the box containing the bread: $\frac{1}{8}$.

c. How many tunnels can the researcher create? (F.14)

Infinite.

d. How many possibilities to put the bread after a certain intersection and the probability that the mouse enters a box containing bread? (F.15)

The number of branches: $2^n, n = 0,1,2,3, \ldots$. The probability: $\frac{1}{2^n}$.

Either the task in the first submission (F.5) or the second submission (F.15) plays its role for pupils. Allowing pupils to deal with at least one near generalization task as F.5 will assist them in formulating an initial abduction that they can easily inductively verify. To obtain the outcome of the near generalization task, pupils may count, construct a diagram, or make a table. Meanwhile, to obtain the result of the final step of generalization, such as in task F.15, solvers might have repeated successes in verifying the correctness of all far generalization items. It will allow them to smoothly progress to the encapsulation and generalization justification phases, where the final expression of the generalization comes up (Rivera, 2013).

In addition, the participant who proposed a task involving direct proportionality revised the data to reach a conclusion, though it remains questionable whether a direct proportionality is a good model to reflect the relationship between the mice's weight and running speed (F.4).

During the lesson, the participants not only continued using some units of measurements but also wrote several terms related to 2-D and 3-D figures, such as area, volume, triangle, circle, circumference, ratio, diameter, isosceles trapezoid, rectangle, right angle, equilateral triangle, etc. Moreover, they also brought up congruence (L.41), Pythagorean triple numbers (L.41), patterning (L.25, L.31, L.59), and generalization into the tasks (L.61). "Determine the area," "prove that," and "find a specific term of the sequence" remained the existing instructions typically found in textbook tasks.

In the discussion, the task of finding a certain term of the sequence was expanded by another participant into a generalization, namely finding the n^{th} term of the sequence. Some participants proposed promising tasks, but it should be noted that the formulations require further explanation. Otherwise, the task would be obscure to the solvers. For instance, the given object in topic 8 was a 2-D figure, two participants expanded it into a 3-D figure and formulated some tasks related to the 3-D figure without giving detailed explanations (L.39, L.43. L.45). As a result, the tasks were initially labeled as blind. During the discussion, the posers explained their intentions and all participants collaborated to modify the existing tasks.

Summarizing the result of the problem-posing products, Table 7 indicates the characteristics of the proposed task in each category. Besides the fact that participants could formulate some tasks with complex situations that require reasoning and lead to generalization, it cannot be denied that they also proposed tasks with vague formulations. Phrasing appears to be a challenge in problem-posing, which has been noted in Ellerton (2013). Thus, it is worth noting to highlight the role of peer discussion on participants' knowledge base in problem-posing. Through peer discussion, participants improved the task formulation, moved from a specific to a general case, and proposed tasks that covered all generalization processes (see Rivera, 2013).

Table 7. Characteristics of the proposed tasks

Blind	Exercise	Empowered problem
The formulation is vague Task: P.5, L.1, L.39, L.43, L.45	*Require simple calculation* Task: P.1, L.5, F.11, L.2, L.4, L.6, L.7, L.8, L.13, L.14, L.19, L.20, L.23, L.24, L.27, L.28, L.29, L.30, L.47, L.48, L.49, L.50, L.51, L.52.	*Require more complex calculation* Task: L.31, L.32, L.40, L.44, L46,
Not belong to a mathematical topic Task: M.23, M.24.	*Entail the direct application of a formula* Task: P.3, L.33, L.34, L.35, L.36, L.37, L.38, L.57, L.58.	*Require reasoning* Task: P.4, P.6, L.9, L.10, L.11, L.12, L.15, L.16, L.21, L.22, L.42, L.62, L.63, L.64, L.65.
Unreasonable Task: M.5.	*Elementary combinatorial task* Task: M.1, M.2, M.3, M.4, M.9, M.10, M.11, M.12, M.19, M.20, M.21, M.22,	*More complex combinatorial task* Task: M.13, M.14, M.15, M.16, M.17, M.18,
Unsolvable Task: F.1, F.3, L.66, L.67.	*Simple patterning* Task: M.6, F.4, F.7, F.8,	*More complex patterning and lead to generalization* Task: P.2, F.2, F6, F.12, F13, F.14, F15, L.25, L.26, L.53, L.54, L.55, L.56, L.59, L.60, L.61
The terms used are not appropriate Task: L.3, L.41.	*Simple logic* Task: M.7, M.8, F.9, F.10,	*Combine several mathematical topics* Task: F.5, L.17, L.18

3. Problem-posing heuristics and schemes

The preliminary task encouraged the participants to *identify the givens*, namely some numerical patterns in the calendar, as the starting point to pose a problem. They all chose to *accept the givens* as they were. These two heuristics persisted in all steps of the intervention, accompanied by the emergence of some other heuristics. The following are notable instances of the appearance of strategies that might be heuristics for participants. Within a single task, strategies might overlap or be combined with one another.

- Symmetry

  In topic 5, information regarding the distance traveled by each person was provided. KK switched the original condition and goal. In her proposed task, she put the provided quantity not as the distance but rather the speed and included the duration at which they drove to ask who drove the farthest. The following is the task after being revised collaboratively since the first version needed to be clearer regarding the measurement and the unit.

Adit, Kevin, and Leo drove to their respective homes. Leo was driving at a speed of 80 km/h faster than Kevin, in 2 hours. Kevin drove with the speed at twice as fast as Adit, in 1 hour and Adit drove for 50 km in 45 minutes. So who had the farthest travel distance? (L.4)

In topic 9, the visible information was: to form one triangle, three matchsticks are required; to form two triangles, five matchsticks are required; to form three triangles, seven matchsticks are required, KK and AI posed the invers tasks, i.e., asking for the numbers of triangles that can be formed if there are 17 and 100 matchsticks (L.55, L.62).

- Constraint manipulation
  - Numerical variation

    The given object was a padlock with possible numbers ranging from 0 to 9 for each digit. AM posed the tasks by narrowing the range from 1 to 5 (M.1, M.3). In other words, AM's strategy was simplifying the given.

    Dad just bought a padlock for the house fence. He will set a password on the padlock consisting of 5 different numbers (1, 2, 3, 4, 5). Determine how many codes can be set if the value of the password is less than 40.000! (M.1)
  - What-if-not

    In topic 5, AI put two different situations, if the driving duration is the same for each person and what if not the same but according to the given ratio, to ask the fastest driver (L.15, L.17).

    a. Who is the fastest driver if they take turns driving for the same duration? (L.15)
    b. Who is the fastest driver if they take turns driving with the ratio of Adit, Kevin, and Leo's driving duration is 2:3:6? (L.17)

In the middle test, the given object was a padlock containing a five-digit password. KK and VI posed the task not with five-digit but with four- and three-digit passwords consecutively (M.5, M.6, M.9, M.11, M.13). While the given object in topic 8 was a 2-D figure, AI and TK transformed it into a 3-D figure and posed the tasks associated with it (L.39, L.41, L.43, L.45).

- Goal manipulation

The provided calendar in the preliminary test was November, and AF proposed the task by expanding it into the following month, namely December. She asked not about the date in November but in December, while the initial assumptions regarding the pattern in December are accepted with no change (P.6).

> On what day will be the Sunday of the next two weeks which is in December 2021? (P.6)

- Targeting a particular solution

Given an August calendar, VI focused on the diagonal numbers 2, 8, 14, …. She mentioned the arithmetic sequence and asked for the sum of the first four terms (P.3). It seems that she intended to encourage the solver to utilize the ready-made formula to determine the sum of the first n terms of an arithmetic sequence, as highlighted in her solution.

> Given an arithmetic sequence 2, 8, 14, ... What is the sum of the first 4 terms of the sequence? (P.3)

- Generalization

Given the sequence of the triangle formed from matchsticks, AI posed a task to ask the number of matchsticks in the 10^{th} term (L.59). Starting from the task proposed by AI, TK suggested asking the number of matchsticks in the n^{th} term, which indicated a generalization (L.61).

How many matchsticks are needed for the 10^{th} term? (L.59)

How many matchsticks are needed for the n^{th} term? (L.61)

In the follow-up test, AF first asked about the possibility and probability of putting the bread after the third intersection (F.12). In the following task, she asked about the possibility and probability of putting the bread after a particular intersection without giving a specific number of the intersection order (F.15).

- Specification

In the follow-up test, the problem-posing task was based on the folding paper activity, with the number of sections and folding line 2^n and 2^{n-1}, respectively. AM posed the task by focusing on 2^3 (F.2), and AF posed by emphasizing 2^7 and 2^3 (F.11, F.12).

In Kalitengah village, there is a mouse which gives birth and produces its first offspring of 8 tails. If each of those mice produces 8 more mice and this continues for the next generation, then in what offspring will the mice reach a population of 160? (F.2)

Note: Kalitengah is the name of the village where her pupils stay.

Moreover, the radius of the circle in topic 8 was stated as r. In formulating the tasks, KK and AF also gave specific numbers for r (L.35, L.36, L.47). In topic 9, AM and KK indicated the side length of the triangle (L.49, L.57), whereas the original situation did not.

- Focusing on an interesting part

Given a composite figure, several participants proposed a task by focusing on an interesting figure. Even though the figure contained triangles, circles, a trapezoid, and a rectangle, the tasks proposed by AM (L.34) and KK were concerned with the circle (L.36, L.38), while the task submitted by TK concentrated on the triangle (L.42). Those tasks could be solved by merely focusing on the intended figure without consideration on the others.

> Based on the picture, determine the total circumference of the 6 circles! (L.34)
>
> Prove that triangle DOA has sides that satisfy the Pythagorean triple numbers! (L.42)

- Introducing an additional condition

Besides modifying the number of digits in the padlock, VI established additional situations from simple to more complicated to ask the number of passwords that can be set., i.e., if the value of the code is even; what if not only the value of the code is even, but also each digit is different; and what if not only the value of the code is even, but also not more than 600 and each digit is different (M.9, M.11, M.13). Meanwhile, the other situation set by AI was that the password should form a palindrome by which only two digits contained the same number (M.15).

> Celsa's father bought a padlock for the fence of the house. Celsa was asked by her father to set a password on the padlock. Her father requested that the password should be a palindrome number and there are only two digits had the same numbers. After setting the password, Celsa asked her father to guess the password that had been made according to her father's request. Do you think that Celsa's father can guess the password set by Celsa correctly? If so, what is the reason? (M.15)

In topic 5, VI put some additional situations, i.e., if they traveled for a specific duration; what if not only that but they also got stuck for 30 minutes due to traffic jams, to ask whether they could get home on time or not and the speed to increase (L.9, L.11). AF, like VI, included the driving duration in asking for the average speed of each driver (L.21). In topic 7, AF also mentioned the length of the last flat part to ask the perimeter and the area of the figure (L.31). It was followed by AI, who put the weight of five boxes of matchsticks (without the boxes) and ten matchsticks to determine whether making 90 triangles with a box of matchsticks is possible (L.64).

- Chaining

This is a particular method to organize the task. This method was utilized by KK and AI during the middle test (M.7, M.8, M.15, M.17). The solver must answer the first question before moving on to the second. AF also used this strategy in the follow-up test (F.12, F.13, F.14, F.15). She first requested the number of possible places for the bread, then the probability that the mouse would enter the box containing the bread. To answer the second question, the solver must be aware of the possible outcomes, namely the number of places where the bread could be placed.

a. Dad just bought a padlock for the house fence and already set the password at the shop. After he arrived home, when he intended to open the padlock, he only remembered that he had previously set the password based on numbers with a certain pattern, namely 4, 7, 10, and unfortunately, he forgot the next number. If so, what is supposed to be the fourth number that Dad thought? (M.7)
b. When successfully open the padlock, Dad concluded that the first digit: correct, the second digit: correct, the third digit: one number wrong, the fourth digit: one number correct. The rules are connected to each other. Then, what is the padlock code? (M.8)

The participants used a variety of strategies when formulating the tasks. Through peer discussion, they can see how various strategies were utilized. In addition, by observing the task proposed by utilizing strategies different with what they used, it would lead them to broaden their perspectives in posing a task. It should be noted that no specific strategy was prescribed. Though constraint manipulation, particularly what-if-noting, was presented during the topic 4 lesson, it appeared to be one of the participants' preferred strategies. Table 8 summarizes the occurence of heuritics in the proposed tasks.

Table 8. Heuristics used in the proposed tasks

Symmetry	**Constraint manipulation: Numerical variation**	**Constraint manipulation: What-if-not**
Task: L.4, L.55, L.56, L.62, L.63P.1, P.2.	Task: M.1, M.2, M.3, M.4,	Task: L.15, L.17, L.18, L.27, L.28, L.40, L.41, L.42, L.43, L.44, L.45, L.46, M.5, M.6, M.9, M.10, M.11,M.12, M.13, M.14, M.21, M.22,
Goal manipulation	**Targeting a particular solution**	**Generalization**
Task: P.6.	Task: P.3, L.29, L.30.	Task: L.59, L.61, F.12, F.15.
Specification	**Focusing on an interesting part**	**Introducing an additional condition**
Task: F.2, F.11, F.12, L.35, L.36, L.47, L.48, L.49, L.50, L.51, L.52, L.57, L.58, P.4,	Task: L.31, L.32, L.33, L.34, L.35, L.36, L.37, L.38, L.42, F.4, F.5, F.6,	Task: M.9, M.11, M.13, M.15, M.19, M.20, L.2, L.4, L.7, L.8, L.9, L.10, L.11, L.15, L.16, L.21, L.22, L.25, L.26, L.31, L.64, L.65.
Chaining		
Task: M.7, M.8, M.15, M.16, M.17, M.18, F.12, F.13, F.14, F.15, L.59, L.60, L.61		

4. Group dynamics and interactions

This section presents the social nature processes when participants discuss their proposed tasks. As mentioned in Wit (2018) and applied by Kontorovich et al. (2012), there are three primary modalities of cognitive tuning toward a commonly shared frame of reference when a group works on a task. Though the task organization in this study differs from the one in Kontorovich et al. (2012), the three modalities, which consist of normalization, conformity, and innovation, will be adapted and defined specifically for this study. The task in Kontorovich et al.

(2012) was collaborative problem-posing in a group, whereas the task in this study is the discussion task aimed at improving the proposed task.

The emergence of three modalities is admitted during the discussion of the proposed task. The normalization process emerged when participants gradually and mutually converged on an agreed view of the proposed task. During the process, some members defended what they agreed upon and convinced other members to follow what all agreed. This process may be followed by conformity. On the other hand, the conflict occurred when the deviant member persisted and introduced another point of view. This attempt to settle the conflict in conciliation between group members led them to innovation, i.e., a new version of the proposed task.

In addition to the appearance of the three modalities, each participant played distinct functional roles in the discussion processes. Their roles can be determined by analyzing their actions and interactions (Leikin, 2005). Taking into account several actions that appeared in the research by Kontorovich et al. (2012) and paying particular attention to the discussion process in this study, the following are some notable roles that emerged during the discussion and might overlap. *Initiator* (the person who starts the discussion, either by asking a question, commenting on the proposed task, or presenting his/her point of view); *Clarifier* (the person who makes a statement or a situation more clearly comprehensible by asking a question or explaining the situation); *Challenger* (the person who expresses an opposing viewpoint to the previously mentioned perspective); *Settler* (the person who acts as a harmonizer and finds a mid-way of opposing perspectives); *Modification provider* (the person who proposes a new formulation of the discussed task); *Solution provider* (the person who presents the solution of the discussed task); *Supporter* (the person who agrees on other's point of view with additional personal statement); *Follower* (the person who agrees on other's point of view without giving any further comments); *Viewer* (the person who just sees and listens to what the discussion is about).

Among several discussions on various topics, the following three are presented to provide an overview of the discussions in the different study groups and when both groups discussed the proposed tasks together.

- Topic 5 (private university group)

  AM's original task:

  At first Leo drove 10 km and at 2 km Leo took turns with Kevin, Kevin drove in 80 km. Then they rested for 1 hour in a rest area. After that they continued their trip with Adit driving for 50 km at a constant speed of 70 km/h then Adit alternated with Leo and at intervals of 2 hours at a speed of 70 km/h alternated with Kevin at a speed of 50 km/h and 20 km. How long did Kevin drive before taking turns with Leo and who was the last driver? (L.1)

  VI commented on AM's original task that the formulation was confusing, followed by KK inquiring if her interpretation was right. As KK's interpretation differed from her intention, AM clarified what she meant and VI proposed a sentence modification. The next sentence was clear for them but not the following sentences. VI again expressed her opinion that the sentence was complicated, confirmed by KK. Discussing the last sentence, AM mentioned that the given situation should be incorporated as the part of the task to make the task solvable. After accepting the discussed version, they discussed the solution to ensure the task was solvable.

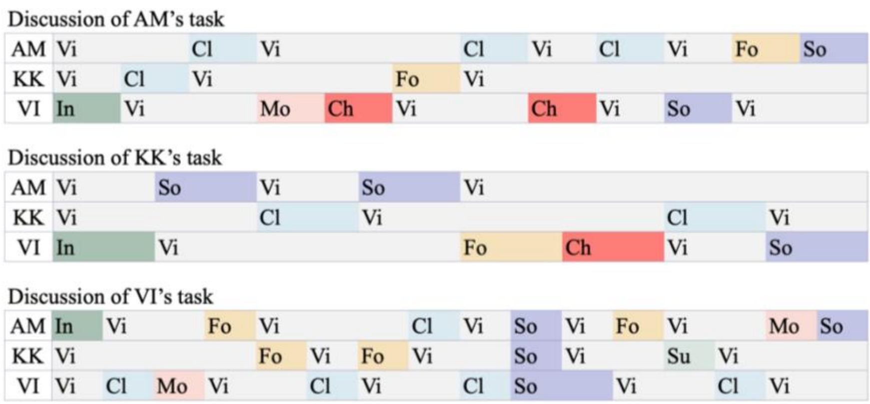
Discussing KK’s task (L.3), VI directly focused on how to solve the task. In the mid of solving the task, she realized that the comparison did not match with the measurement unit, *“(It should be) farther not faster”*. This is followed by KK, who emphasized, *“The time should be distance. The farthest distance”*. As previously noted, KK utilized a symmetry strategy. After noticing that KK did not use the given situation as it was, VI said the task was not directly related to the given information.

The last discussion was about VI’s tasks (L.5, L.7, L.9, L.11, L.11). She commented on her task that additional information about the speed is needed. Again, her peers reinforced her thought with KK emphasized the task whould be confusing without additional information. AM appeared as the arbiter of the situation they were discussing. She suggested putting an additional phrase that the task is the continuation of the previous tasks.

The normalization process began when VI shared her thoughts on each task proposed by each group member. The conformity proceeded smoothly as they continued with sentence formulation and feedback on each other’s ideas.

During the discussion, VI was an influential member. She was the one who always started the conversation. She acted not only as a challenger, as she was the one who initially criticized the proposed tasks but also as a settler, as she sought a solution by suggesting a sentence modification. KK served as a clarifier in the discussion of AM’s and her tasks, asking the poser’s intention or explaining her intention in her proposed task. She constantly reinforced VI's point of view by stating that the task's formulation was perplexing to her. Meanwhile, AM only confirmed VI's critics without giving any personal consideration. She was active during the discussion of KK's task solution and her task because she needed to clarify her intention. However, for the most part, she went with the flow. In addition, AM's role was that of a settler, as she reached a compromise during the discussion of VI's task. Figure 10 demonstrates the participants' roles in this meeting. The tasks proposed by AM and KK were improved from blind tasks to exercises, while those proposed by VI remained the same, as an exercise or an empowered problem.

Figure 10. Timeline roles in topic 5 discussion[7]



- Topic 8 (public university group)

Firstly, the participants discussed the AF's proposal (L.47): "If r=7 cm, what is the area of the blue part?". The normalization process started. Though it was a simple task with a clear formulation, TK immediately provided the solution in a numerical form, and AI

[7] In: Initiator Cl: Clarifier Ch: Challenger Se: Settler Mo: Modification provider So: Solution provider Su: Supporter Fo: Follower Vi: Viewer

followed it with a different expression containing π. TK explained her steps to finding the solution, and AI did it afterward. Their explanations revealed that they used different approaches to achieve the same result. This response was justified by AF, who also stated that her approach was the same as that of TK.

Next, they discussed TK's proposal (L.41, L.43, L.45). AF started the conversation by interpreting the given figure as an aerial view of the milk storage room. AI then stated that he could not understand point c, as did AF, who later stated the same. The conformity began from this step. TK clarified that she converted the given figure from two to three dimensions. The current figure should be a trapezoidal prism with six drums in the shape of cylinders in it, with the prism and the cylinder having the same height. In addition, the blue section represented the cold water inside the prism but outside the drum. As AI and AF had not fully comprehended AF's intent, it took time for them to inquire and for TK to respond several times. AI attempted to sketch the intended figure in the middle of the clarification process. Finally, TK suggested moving on to discuss AI's proposal as she wanted to improve the clarity of her task.

TK initiated the discussion on AI's task (L.39) by clarifying AI's intention. AF also inquired, asking AI if he intended a bottle rack in the shape of a trapezoidal prism, with the bottle fitting into the holes. Several more inquiries and responses followed. AI clarified that the sticker would cover the whole part, including the bottom part of the rack but not the hole. Instead of thinking about the sticker lining the rack, AF told the members she was thinking about the sticker that usually lines the bottles. According to her, the student would also interpret it that way. As is customary, a sticker with the brand's name covers the soy sauce bottle. They continued to clarify what AI intended, including the given size. In the middle of the clarification, AI proposed a sentence adjustment and stated that the figure should be 3-D.

TK's visual image of a soy sauce bottle | AI's visual image of a soy sauce bottle | The shape of the soy sauce bottle by consensus

Figure 11. The shape of the soy sauce bottle

TK and AI had different imaginations of the shape of the soy sauce bottle (see Figure 11). TK thought the pupils were forced to visualize the soy sauce bottle as AI intended. They could not guarantee that would happen as there are several shapes of the bottle, which affect the solution of the task. AI disagreed with TK, believing that the diameter of the hole in the rack could be determined by the largest diameter of the bottle, which is usually at the bottom. In addition, the bottle shape would not affect the solution. Aside from supporting AI, AF acted as a settler who suggested including the figure of the intended bottle in the task. Moreover, because the rack is typically in the shape of a cube or a cuboid in the market, the "unique rack" should be mentioned.

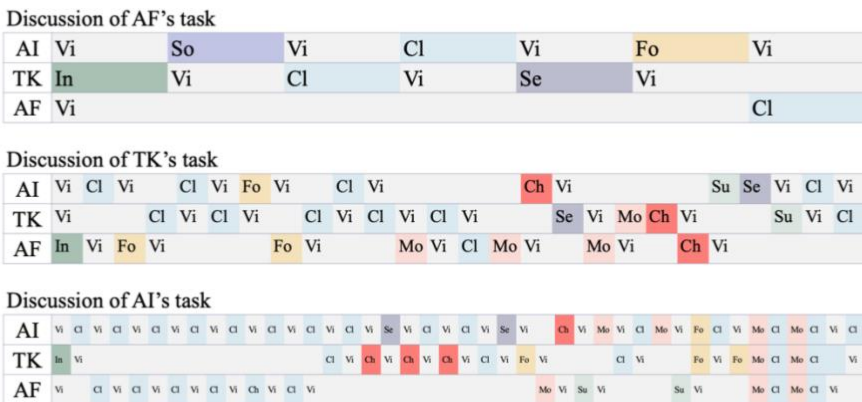
The width of the sticker was specified in the task. AI was concerned that the students would misinterpret the sentence about the size of the rolled sticker. The pupils might interpret the width as the length of the sticker. Again, AF came as a settler. She suggested putting the roll sticker's figure alongside the size without any explanation in the text to avoid repetition. TK agreed with this idea. They continued to ponder the task formulation and contributed to its improvement. Positioning as a data provider, TK searched on the internet for the actual bottle size to ensure that the context was realistic. Another consideration appeared from AI that the height of the rack should be less than the height of the bottle so that it is easier to take out the bottle from the rack. He then defined the rack's height in a form containing r.

They continued discussing TK's proposals in the following section. Before the meeting, TK made some changes to the formulation. AI questioned the modified formulation during the meeting, and AF explained TK's intention. Following that, all participants worked together to revise the formulation. AF tried several times to propose the formulation or her point of view, but AI disagreed and explained his point of view. Occasionally, TK supported AF or AI, along with providing additional views. Innovation emerged when they found a middle ground for their thoughts by modifying the figure and the task formulation.

The idea about the storage room for the milk came from TK's imagination, as small Indonesian retailers commonly put canned and bottled drinks in a box with ice or cold water. To broaden their insight, the instructor, under the supervisor's guidance, showed them the figure of an ice house that was commonly used in several European countries prior to the invention of the refrigerator.

In light of the overall outcome of the discussion, the task proposed by AF remained an exercise, whereas the four blind tasks proposed by AI and AF became empowered problems. All three participants had balanced roles. Their roles shifted as the object under discussion changed. During the discussion of AF's task, there were no challengers. The challengers appeared with opposing viewpoints when discussing AI and TK's tasks. Figure 12 depicts the timeline roles of the participants that contributed to the considerable changes.

Figure 12. Timeline roles in topic 8 discussion[8]



- Follow-up test (all participants)

First and foremost, they discussed TK's proposal. KK was curious about TK's solution because she had a different solution. She explained how she obtained the answer, followed by TK's clarification. The process of normalization started from here. TK discovered her error while explaining her solution and stated that the

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correct answer should be the one proposed by KK. TK reviewed her task and concluded that the last dose was too massive, and the mouse would die. AI agreed with TK and thought the task was unreasonable because it took too long for the mouse to faint, which could affect its metabolism as well.

AI then interrogated TK's imagination about how the scientist administered the dose to the mouse. TK clarified that they could assume the last dose was the accumulation and that the mouse could be reinjected whenever the effect of the given dose wore off. It remained unreasonable for AI. There should be a reason to anesthetize the mouse, such as a treatment or clinical action. "What a pity mouse," he said if the mouse awakens, feels pain, and gets anesthetized again. The conformity happened as TK tried to lead the group members to assume as she did.

At last, AI remarked that TK might give it to her students only for calculation practice, but the situation was unreasonable. As a solution, TK planned to keep giving it to the students with an additional question: what the students thought about the task, including their thoughts on giving the dose and the effect on the mouse. This idea appeared as an innovation that AF supported. In this discussion, AM had not joined the meeting, KK actively participated at the beginning, and AF appeared at the end, supporting the additional question to force students to explain their thought. Meanwhile, AI and TK engaged in debates because they held opposing viewpoints.

The task of AI was the topic of the following discussion. TK began the discussion by clarifying the situation and criticizing the absence of a time for the mouse to return to the previous branch to create another side of the main branch. At the same time, TK played her role as an initiator, a clarifier, and a challenger. AI responded that he had not considered it and might be ignored because the mouse moved for such a short time. Instead, he considered the overall duration of the mouse digging burrows in a time that he feared would be excessively long without rest. The mouse might have rested for 2 hours after 6 hours of digging, but it may have remained unreasonable. It must have an impact only on the final answer rather than on the number pattern.

According to TK's logic, when digging burrows, the mouse may only make the hole deeper into one of the two branches due to the soil moisture. Again, TK appeared as a challenger. She inquired on the forum if the duration would differ from the previous condition if the mouse dug the burrows evenly on both main branches. Furthermore, TK inquired about the likelihood of colliding burrows. However, AI clarified that it was also possible that it would not occur because all burrows are in three dimensions, not two. In this discussion, AI and TK remained the most prominent participants. In contrast to the previous discussion, in which AI appeared as a strong challenger with a point of view opposing the poser's (TK), in this section, TK appeared as a strong challenger to the poser, namely AI. AM had not attended, while KK and AF were viewers seeing as they did not take part in the discussion.

They discussed KK's proposal the following day. AI questioned AF's rationale for the division by 15. KK carried out a trial and discovered that if each weight was divided by the travel time, the result could always be rounded to 15. TK reemphasized AI's question, asking AF's reasoning, and she was dissatisfied with the task due to the division by 5. AI also commented that if the solution followed the given data, the division results were inconsistent, only rounded results, and did not precisely form a pattern.

Following AI, TK questioned whether the context was also reasonable if the mouse held a competition, and they could not guarantee that the mouse would run continuously. KK responded to TK's opinion that the situation was possible because she had already considered and calculated the situation. TK suggested adding "an experiment situation" to make it more acceptable. As AM joined later, she clarified the reason behind the division by 15 again. Following that, TK and KK subsequently clarified. Furthermore, AI argued that the mouse's weight might be inversely proportional to its running speed rather than directly proportional. Thus, KK should include an assumption in the task. However, while TK supported AI's thought with her additional consideration, AF supported KK's opinion that smaller mouse will run faster. Similarly to the previous two sections, AI and TK were the most prominent participants in this discussion, playing more roles and being the most vocal in challenging the poser. Meanwhile, AM and KK served as clarifiers, with KK serving as a supporter.

During the discussion of AF's task, AI wondered if the students should fold an A4 paper to solve the task, and it was ascertained that the maximum number of folding is seven. AI appeared as an initiator, a clarifier, and a challenger at the same time. AF defended her task because she had already checked it. The conversation moved on to the infinite nature of paper-folding patterns. Further, AI stated that the betting context was inappropriate for the students. He also suggested slightly modifying the situation. The mouse should get the best bread in the box to win. TK agreed on the suggestion because the mouse might smell good food. KK and AM also agreed upon it. In this discussion, AI was the only challenger who criticized the proposed task. As the proposer, AF arrived as a clarifier who defended her task. AM, KK, and TK appeared as AI supporters, and TK's role was not as prominent as it had been in previous discussions.

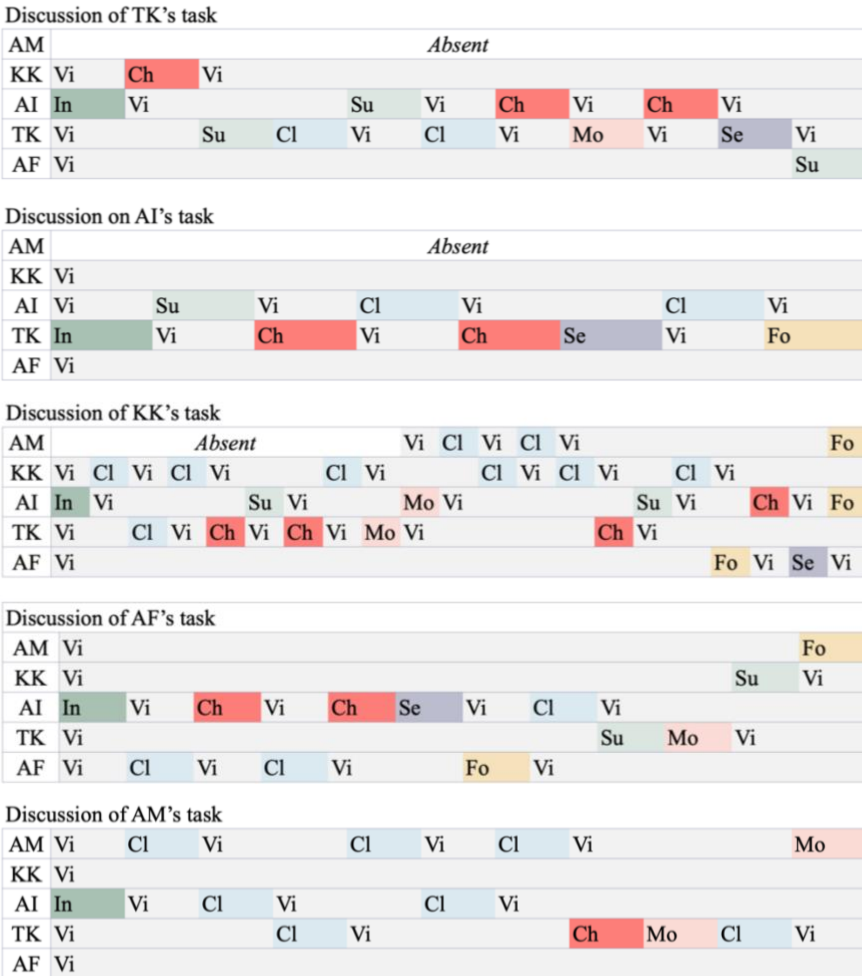
The last discussion was about AM's task. AI argued that the given time to catch the mouse was too long. TK supported this opinion by offering the possible duration. AM defended her task, claiming that it was unreasonable if the time limit was only one second, as AI mentioned. To ensure, AI checked the duration of the catching time on the internet. As the instructor realized that the given situations contradicted each other, AM explained that she intended to outwit the solver. In the meantime, TK argued that giving the catching time was unreasonable because the catcher should catch the mouse as soon as possible. Thus, according to her, it would be better to modify the situation. She proposed a different scenario where the mouse walks through a glass hall divided into chambers with bait inside. Nonetheless, the formulation was vague. In this discussion, TK again played a prominent role, namely as a challenger together with AI. These two participants dominated the conversation by taking on more roles. Aside from being challengers, they also played the same role as settlers, searching the information on the internet and proposing new scenarios to consider. Meanwhile, as the task proposer, AM served as a clarifier, while KK and AF only served as viewers because they did not contribute to this section.

To summarize the discussion, the dose in TK's task was quite large, which can have adverse effects on the mouse, whereas the mice conducting competition in KK's task are unreasonable. TK and KK intended to introduce a direct proportion by providing context to these two tasks. The context in TK's task may be acceptable, but as AI suggested, they should think about how the scientists administer the dose to the mouse. Meanwhile, the context of KK's task may need to be more relevant to the students. If the task structure perpetuates the notion that "everything is always linear", it will threaten the pupil's logic (Foster, 2013). In fact, real-world scenarios must be more complicated. As TK and AI stated, the mouse may not run continuously, and the relationship between mouse weight and run speed is only sometimes a direct proportion, not always, considering that the mouse may run fast if its weight falls within a specific range.

Other tasks proposed by AI, AF, and AM are also worthy of consideration. They examined the possibility that the mouse only dug the soil on one side of the main branches, the relationship between the number of participants and the number of sections of folded A4 paper, and the time it took to catch the mouse. The contexts appeared to be real-life for the poser at first glance, but upon closer inspection during the discussion, they were not (Boaler, 1993; Ward-Penny, 2010). Their talk led them to focus on Realistic Mathematics Education, which states that the context must be clear and imaginable to students (Hough & Gough, 2007). Simultaneously, participants gained practical experience in formulating a task and reconsidering the context of their proposed task.

Finally, the whole discussion was concluded with the decision to revise all proposed tasks in light of the discussion and to make them more closely related to the pattern of the paper-folding activity. Thus, the challenge remained that they must locate context within the zone of proximal relevance, which learners may be interested in and find fascinating (Watson & Mason, 2005). In the second submission, the task category underwent significant changes. AM and KK, who had previously proposed blind tasks, later revised them into an empowered problem and an exercise. The context in AM's revised taks contained the name of village where her pupils stay. AF changed her task from an exercise to four tasks that are classified as empowered problems. Finally, the AI and TK tasks remained the same: an empowered problem and an exercise. Figure 13 shows the timeline of the participants' roles that influenced the task category changes.

Figure 13. Timeline roles in the follow-up tests[9]



5. Individual considerations of aptness

This section outlines the participants' interpretations of the explicit and implicit requirements of the problem-posing tasks, observed during the discussion and from the problem-posing products. In some cases,

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their interpretation did not fully comply with the intended instructions or they had different interpretations from other participants.

The preliminary test instruction was to observe some patterns in the current calendar and pose a task based on one of the discovered patterns. The task proposed by VI (P.3) was no longer closely related to the calendar but to an arithmetic sequence derived from a diagonal pattern in the calendar.

> Given an arithmetic sequence 2, 8, 14, ... What is the sum of the first 4 terms of the sequence? (P.3)

In the middle test, the instruction was to observe the figure of a padlock with the password and pose a task with general situation "Dad just bough a padlock for the house fence". The task will be assigned to the poser's pupils. AF proposed a task (M.23) which is categorized as a blind task because of its nature as a mathematical puzzle unrelated to a specific mathematics topic. She did not entirely focus on the implicit requirement that the task must be related to the mathematics topics the pupils learn. In the follow-up tests, some participants initially misinterpreted the problem-posing task. The instruction was to reflect the folding paper activity and pose a task with a compulsory word "mouse". They proposed a task that only contained the compulsory word and related to the mathematics topic but no reference to the pattern discoved through the folding paper activity (see F.1, F.3, F.7. F.9, F.11).

Moreover, though a problem-posing situation involves interactions among several complex subsystems, there are various individual aptness considerations by which a specific type of aptness can be submerged in the others (Kontorovich et al., 2012). In their research, they discovered four types of aptness considerations. Three of them are presented in this study because no manifestations were identified that show consideration for how other people would evaluate the problem poser's skills and performances (aptness to the potential evaluator).

- Aptness to himself or herself as the poser

  This aptness relates to how satisfied the poser is with their proposed task. TK seemed displeased with her task formulation (M.19), knowing she said, "I think the story is too long." Meanwhile, AI was unsure of his task (L.25) though he stated, "My question is actually pointless, but I am pondering the staircase steps."

- Aptness to the pupils as the potential solvers

  In Kontorovich et al. (2012), this aptness is related to the poser's opinion of whether the proposed task is mathematically suitable for students. Though the intervention contained a discussion of the task before it is implemented, this study extends this aptness by incorporating not only the poser but also the group members' views.

  AI posed a task related to a palindrom (M.15) because he was sure students must recognize it from social media. Moreover, he had three comments on TK's task (M.19). Firstly, he was surprised when he realized the context of TK's task was about theft, but he thought it might be good, especially if there were many theft cases around the school, so students could relate and be motivated to set a complicated password. Second, the task was more challenging than the typical school task. Finally, he stated that the 5-digit information in the text would be redundant because it was also included in the figure. However, TK chose to keep it "because the student may think that the task is difficult and, thus, may not notice the 5-digit password".

Nonetheless, AI suggested that students be instructed to observe the figure and mention the information contained within it.

TK mentioned for which grade her task is suitable, “I think it is reasonable to give it to elementary or junior high school students.” Furthermore, AF said that her task would be appropriate for her students to practice because “Many of my students got confused when solving problems related to compound 2-D and 3-D figures.”

AM considered the modified formulation they discussed too simple for the students, saying that, “I prefer the previous version. The newest version looks so clear (for the solver) because the sentence on the question is usually deceiving.” It reflected what she believed about the formulation of mathematical tasks. On another occasion, AI was concerned that his task formulation would be misinterpreted by the students (L.39). According to him, students might think of the sticker’s width as the length of the sticker. When the sticker is unrolled, what they perceive as length could be its width.

In response to AI’s task (L.64), TK agreed on the given data about the matchstick weight, which must vary because she knew a stick was only half the normal size while the head was double. Furthermore, AF expressed her satisfaction by stating, “the pupils will be excited as the task is contextual,” followed by AI, who responded, “this type of task rarely appeared in the school exam but is beneficial to give to the pupils.”

- Aptness to their peers and the instructor as the group discussion members

  This aptness attributes to the poser's perceived notion of whether or not the group members will admit his or her idea. An example came from VI, who confirmed to the group members, "If it is still confusing (for you), what if we put the total distance by each person again?". VI also confirmed her idea to the instructor specifically, "What if like this, miss?" followed by KK, who asked after modifying her task orally, "What do you think, miss?"

  Discussing the proposed tasks in the middle test, they gave each other feedback and suggestions, including the wording and appropriate context for students, followed by individual revisions of their tasks. As a result, the tasks in the second submission were either superficially or deeply connected to the pattern discovered through the folding paper activity. For instance, the participant posed a task:

  > There is a bet using a mouse to determine the winner. The mouse is put in the tunnel. At the end of the tunnel, there are many boxes with bread in them. The owner of the box whose bread the mouse eats first is the winner. What is the probability of becoming a winner if the number of participants is equal to the number of sections formed by folded A4 paper as much as possible until it can no longer be folded? (F.11)

The poser received some comments from her peers. Betting as a context is inappropriate for students because the teacher is also responsible for equipping students with positive attitudes. In addition, even though the maximum number of folding A4 paper is seven, the sequence is essentially infinite. Thus, the poser revised the task to make it acceptable to the group members:

> There was an experiment using a mouse. The mouse is put in the tunnel. At each end of the tunnel there is a box connecting two branches. A bread is placed in one of the boxes after the third intersection. (1) How many possibilities to put bread? (2) What is the probability that the mouse enters the box containing the bread? (3) How many tunnels can the researcher create? (4) How many possibilities to put the bread after a certain intersection and the probability that the mouse enters a box containing bread? (F.12-F.15)

The context of the revised task became more appropriate for the students as it contained an experiment (positive attitude) rather than a bet (negative attitude), led to generalization, and became more deeply connected to the folding paper activity. In contrast, the previous version only had a superficial connection.

Finally, this section emphasizes that exercises were the most frequently generated tasks, followed by empowered problems and a few blind tasks. The dual-task design had a significant influence on improving the first proposed tasks. They used numerous mathematical topics and problem-posing heuristics when posing or revising the task. Furthermore, the discussion of the proposed tasks encouraged them to give feedback to each other, which occasionally led to divergent views and, at the same time, highlighted how they perceived the task's appropriateness for the students and group members.

6.2 Problem-Solving Performance

6.2.1 Result

Table 9. Problem-solving products in the preliminary, middle, and follow-up tests[10]

Participant	Preliminary test				Middle test								Follow-up test							
	Original work				Before discussion				After discussion				Before discussion				After discussion			
	Task code	B	I	C	Task code	B	I	C	Task code	B	I	C	Task code	B	I	C	Task code	B	I	C
AM	P.1			✓	M.1			✓	M.2			✓	F.1	✓			F.2			✓
					M.3		✓		M.4		✓									
KK	P.2			✓	M.5	✓			M.6			✓	F.3	✓			F.4			✓
					M.7			✓	M.8			✓								
VI	P.3		✓		M.9			✓	M.10			✓								
					M.11			✓	M.12			✓								
					M.13		✓		M.14		✓									
AI	P.4			✓	M.15			✓	M.16			✓	F.5			✓	F.6			✓
					M.17			✓	M.18			✓								
TK	P.5	✓			M.19			✓	M.20			✓	F.7		✓		F.8			✓
					M.21			✓	M.22			✓	F.9			✓	F.10			✓
AF	P.6			✓	M.23	✓			M.24	✓			F.11			✓	F.12			✓
																	F.13			✓
																	F.14			✓
																	F.15			✓
Sum		1	1	4		2	2	8		1	2	9		2	1	3		-	-	9

Table 10. Cognitive resources, heuristics, and errors

Participant	Task code	B/I/C	Cognitive resource (Have/doesn't have enough knowledge about…)	Heuristic	Error
AM	P.1	C	Direct proportion	Look for patterns	-
KK	P.2	C	Arithmetic operations	Make suppositions	-
VI	P.3	I	Arithmetic sequence	Look for patterns	Procedural or careless
AI	P.4	C	Modular arithmetic	Look for patterns	-
TK	P.5	B	-	-	-
AF	P.6	C	Arithmetic operations	Look for patterns	-
AM	M.1 & M.2	C	Fundamental counting principle	Solve part of the problem	-
AM	M.3 & M.4	I	Combination and fundamental counting principle	Solve part of the problem	Conceptual
KK	M.5 & M.6	B-C	Arithmetic sequence	Look for patterns	-
KK	M.7 & M.8	C-C	Combination and fundamental counting principle	Act it out	-
VI	M.9 & M.10	C-C	Fundamental counting principle	Solve part of the problem	-
VI	M.11 & M.12	C-C	Fundamental counting principle	Solve part of the problem	-
VI	M.13 & M.14	I-I	Fundamental counting principle	Solve part of the problem	Conceptual
AI	M.15 & M.16	C-C	Palindrome and fundamental counting principle	Solve part of the problem	-
AI	M.17 & M.18	C-C	Probability	Make a systematic list	-
TK	M.19 & M.20	C-C	Fundamental counting principle	Solve part of the problem	-

[10] B: Blind task I: Incorrect solution C: Correct solution

Participant	Task code	B/I/C	Cognitive resource (Have/doesn't have enough knowledge about…)	Heuristic	Error
TK	M.21 & M.22	C-C	Fundamental counting principle	Solve part of the problem	-
AF	M.23 & M.24	B-B	-	-	-
AM	F.1	B	-	-	-
AM	F.2	C	Exponential	Use a diagram/model	-
KK	F.3	B	-	-	-
KK	F.4	C	Exponential and direct proportion	Make a systematic list	-
AI	F.5	C	Exponential and sigma notation	Look for patterns	-
AI	F.6	C	Exponential and sigma notation	Look for patterns	-
TK	F.7 & F.8	I-C	Direct proportion	Make a systematic list	Procedural
TK	F.9 & F.10	C	Direct proportion	Make a systematic list	-
AF	F.11	C	Exponential and probability	Solve part of the problem	-
AF	F.12	C	Exponential	Use a diagram/model	-
AF	F.13	C	Probability	Solve part of the problem	-
AF	F.14	C	Exponential and generalization	Solve part of the problem	-
AF	F.15	C	Probability and generalization	Solve part of the problem	-

6.2.2 Discussion

The participants submitted the solution only when they first submitted the task. In the preliminary test, there was only one-time submission. There were no changes in the solutions before and after the mid-test discussion. The category of their problem-solving products is presented in Table 9.

KK's solution (M.5) shifted from blind to correct as she revised her task from blind to an exercise, but the solution was essentially the same as the first version. VI's solution remained incorrect because the discussion was only the first task to be implemented, and AF's solution remained blind because of the nature of her task, which is a mathematics puzzle. In the follow-up test, as some participants revised their tasks because the first version did not contain the pattern discovered from the folding paper activity, they proposed new tasks along with new solutions. AM and KK's solutions proceeded from blind to correct as they revised their tasks with new solutions, and TK's solution moved from incorrect to correct because KK noticed it and they discussed the correct solution. Notably, the solutions during the lesson were not coded because the emphasis was on formulating the task. Some tasks were submitted with the solutions, and some were without the solutions.

Most of the solutions provided were correct, indicating that the participants had sufficient knowledge and used relevant heuristics when solving the task (see Table 10). Nevertheless, it should be noted that several incorrect solutions also existed (see Table 10) due to some errors as defined in Brown et al. (2016). The errors in this case demonstrate the shortage of the solvers' cognitive resources as the solution remained incorrect.

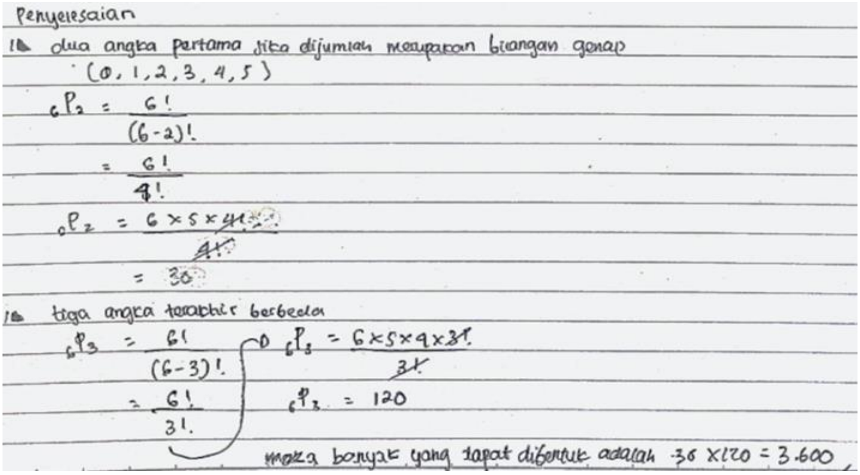
- Procedural error

  VI carried out an unordered completion procedure: $\frac{n}{2}(2a+(n-1)b)=\frac{4}{2}(2(2)+(4-1)6)=2(4+3)6=84$. (P.3)

  The participant might not realize how vital the parenthesis was or this case may also demonstrate the careless error if she did not notice that the parenthesis was supposed to be there.

- Conceptual error

  - AM put an incorrect binomial coefficient. Nevertheless, her task (determining the number of passwords by which the sum of the first two digits is even and the last three digits are different) was not related to the binomial theorem. Both the mathematical model and the calculation were incorrect.



Penyelesaian

1. dua angka pertama jika dijumlah merupakan bilangan genap
(0, 1, 2, 3, 4, 5)

$_6P_2 = \frac{6!}{(6-2)!}$

$= \frac{6!}{4!}$

$_6P_2 = \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}}$

$= 30$

2. tiga angka terakhir berbeda

$_6P_3 = \frac{6!}{(6-3)!}$

$= \frac{6!}{3!}$

$_6P_3 = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$

$_6P_3 = 120$

maka banyak yang dapat dibentuk adalah 36 x 120 = 3.600

In addition, she put 0 in the list of possible numbers that can be used to set the password. But in the task, she mentioned five numbers, i.e., 1, 2, 3, 4, and 5. (M.3)

- VI did not fully comprehend the fundamental counting principles. The inaccuracy in the placement of the possible numbers demonstrated this issue. (M.13)
- KK considered only one possible solution even though her task contained more than one possible solution. (M.7)
- TK misinterpreted the given data about the anaesthetic dose (in ml units) and the duration of fainting experienced by the mouse. Besides the mathematical model of the task does not exactly demonstrate the mathematical model in reality, which should not be a direct proportion; she also came to the incorrect conclusion about the relationship between those two variables. (F.7)

The last two cases demonstrate rational errors caused by monitoring failures when the solver was unaware that he or she was violating the problem situation. According to Ben-Zeev (1996), to correctly monitor the validity of a problem situation, someone must develop internal critics that signal when a number representation or rule violation occurs or when they encounter an unfamiliar situation that the system is unsure how to handle.

Considering the cross-national study result that Indonesian prospective teachers tend to utilize arithmetic rather than algebraic approach both in posing and solving problems (Fitriana et al., 2022), problem-solving performance in KK case will be described in detail to give a clear example as it obviously indicates the phenomenon. Besides the L pattern, we also refer to another task she implemented to her pupils, i.e., V pattern (Figure 14).

August 2020						
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
22	23	24	25	26	27	29
30	31					

August 2020						
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
22	23	24	25	26	27	29
30	31					

Figure 14. L and V patterns by KK

Given the sum of the sequence (S), KK divided the sum by five as the first step to find the solution. Since the result was a decimal, she rounded it up just to make it easier to calculate, then added one and eight for L and V patterns, respectively. Otherwise, just divided the sum by five, then added 1,2 and 8,4 for L and V patterns, respectively. The result would be the middle number of the sequence (n), while the remaining numbers would be placed by following the middle number (see Figure 15).

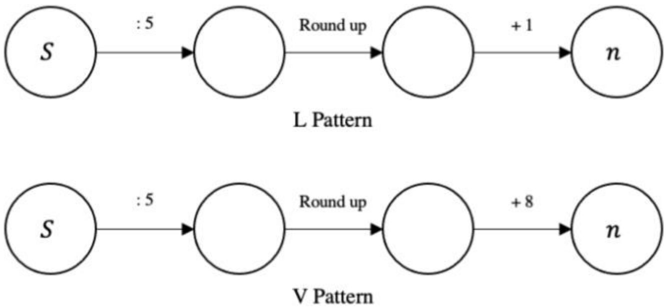


Figure 15. KK's rule

The following part explains KK's problem solving following the framework by Schoenfeld (1985, p.15): Before finding her own rule, she divided the sum by five, which worked for horizontal, vertical, and diagonal patterns with five consecutive numbers. Her intuition which led her to make a conjecture that the same rule might work for L and V patterns, along with the facts and algorithmic procedure, emerged as her *cognitive resources*. Unfortunately, this rule no longer applies to L and V patterns. She did *control* by laying out the plan and deciding on an alternative after getting stuck. Subsequently, she realized that the decimal digits obtained from dividing the sum by five were always the same, which she then connected to the middle number of the sequence to define a new rule. To explain the rule, she checked it by examining it only on three cases in each pattern. Albeit the *heuristic* appeared though she tested the rule, it is insufficient to prove its validity in all cases. The algebraic approach is tremendously helpful for proof.

The proof explanation: For the L pattern, the numbers can be defined as $(n-14)+(n-7)+n+(n+7)+(n+8)=5n-6$ as the sum. If it is divided by five, the result will be $n-1{,}2$. This is the reason why her rule works, need to add $1{,}2$ after dividing the sum by 5. It applies to V pattern, by which the sum is $5n-42$. If it is divided by five, the result will be $n-8{,}4$. Thus, it needs to add 8,4 after dividing the sum by 5 (see Figure 16).

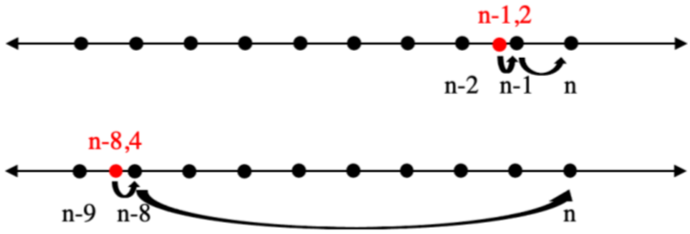


Figure 16. The proof of KK's rule

On the other hand, some of the emerging tasks promoted their perspectives into an algebraic approach to solving the task. The examples are as follows.

- How many ways can the person reach the top of the staircase if only allowed to take one, two, or three steps at a time?

  A solution from the discussion:

  $$U_n = U_{(n-1)} + U_{(n-2)} + U_{(n-3),}\ n \geq 4;\ U_1 = 1,\ U_2 = 2,\ U_3 = 4$$

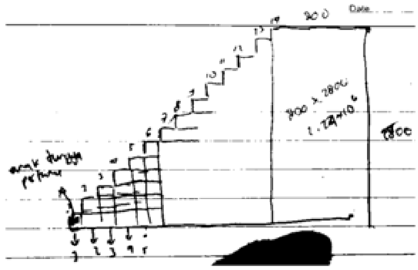
  Thus, $U_{14} = U_{(14-1)} + U_{(14-2)} + U_{(14-3)} = U_{13} + U_{12} + U_{11}$. There are $U_{13} + U_{12} + U_{11}$ to reach the top of the staircase according to the given provision.

- Based on the picture, given the height and width of each step are the same and the top of the figure (the last flat part) is 800 units of length, determine the perimeter and area of the figure!

  Solution from the participant:

The area (L):
It is known that the height of the staircase is 2800, so the height of each step is $\frac{2800}{14} = 200$ units of length.
Since it is known that the height and width of each step are the same, then the area of the first step is $200\times200 = 4\times10^4$ units of area.

Find the area of the staircase.
Based on the picture above, the area of the second step is twice the area of the first step, the area of the third step is three times the area of the first step, and so on.
Thus, the total area of the staircase is:
$L = (1 + 2 + 3 + \cdots + 13)\times L_{the\ first\ step}$
$L = 91\times4\times10^4$
$L = 3.64\times10^6$
Non-staircase area is $2800\times800 = 2.24\times10^6$
The total area: $(3.64\times10^6) + 2.24\times10^6 = 5.88\times10^6$ units of area.



The perimeter (P):
$2(3.4K + 2.8K) = 2(6.2K) = 12.4K$ units of length.

The discussion of the first task covered all the generalization processes presented by Rivera (2013), which started from concrete numerical experience and continued to a near or far member of the sequence until they reached the general term. The second task was also connected to generalization as it allowed them to find the general form for the area. However, it stopped until the 13th term, which is the step of finding the far member of the sequence. Nevertheless, it is a part of the generalization process as the PT started from 1, 2, 3, … and continued to the far member until the 13th term even though not reaching the final step, namely finding the general term.

In conclusion, this part points out that most solutions were correct, followed by incorrect and blind solutions due to their blind tasks. Most correct answers reveal they had adequate cognitive resources and controlled the process. They also worked with several heuristics. On the other hand, some of the incorrect solutions demonstrated a lack of cognitive resources or control over the process, as confirmed by some errors.

6.3 Critical Manifestations

6.3.1 Result

The following are excerpts from three meetings that are considered to represent the entire meeting. The first two are the meetings after the teaching implementation, i.e., one private university group meeting and one public university group meeting. Each meeting was attended by complete members of the group. The third meeting was about the folding paper activity attended by five participants from both groups who were present from the beginning to the end of the intervention. The first two meetings concerned manifestations related to teaching reflection and peer feedback, while the third meeting presented manifestations that appeared during the lesson.

- Meeting after teaching implementation (private university group)

  VI's teaching implementation

  VI: About my class, all the pupils understand. I feel it was too fast for me to explain the topic but when I asked the pupils "is it too fast?", they said no. But when I review the video, I do feel like it's too fast. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

  AM: VI's class was good. Perhaps, more attention should be paid to how the explanation is arranged to make it more structured. For instance, explaining the sum. You were directly dictating, "if you add them up (consecutive numbers horizontally in the calendar), it will be like this". It would be better to give a bridge explanation about the addition of several numbers (in general, not the numbers in the calendar) and how your strategy works. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

KK: Good, VI. Your response to your pupils is good. But for the second task (P.3), you only explained one task without giving another example. It's better if you give another one. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

AM's teaching implementation

AM: I only gave them one type of task, the sum of several numbers in a linear pattern. We did not cover some variations of the task, such as the L pattern discovered by KK. Also, I was often doubtful when giving answers to pupils' questions. I had the answer, but I'm afraid it's wrong. I was surprised they asked so many questions. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

KK: She just discussed one type of task, but the lesson was good. I like the way she communicated with her pupils. Her pupils were active, and they proposed several questions. Just less satisfied because she only implemented one type of task. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

VI: The class was good. What she told us is totally the same as what I wanted to say. Moreover, her voice needs to be louder, I think. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

KK's implementation

KK: My pupils tended to be silent and I was so nervous. (NM-R: Simple non-mathematical response)

VI: (For KK's class) The students tended to be silent but the rest was good. I also realized that my pupils were not as active as AM's pupils even though I had been trying to say, "take it easy, think of me as your study partner. If you have a question, please feel free to ask me". I can relate to KK's feelings. (NM-R^+: Non-mathematical response with reasoning, comment, or evaluation)

AM: The same note, that's good. KK, you have to be more confident. We are the same, we learn and practice together. When I looked at KK's video, it seemed like the lesson had to match her plan. You seemed unfree and constrained when teaching. Keep up your spirit, KK. (NM-R$^+$: Non-mathematical response with reasoning, comment, or evaluation)

- Meeting after teaching implementation (public university group)

AI's implementation

AI: I gave my pupils the multiplication table first, followed by the calendar table. I think it's better to give those tables reversely because the context of the calendar is closer to the pupils than the multiplication table, regardless that they have recognized the multiplication table since elementary school. Before implementing this lesson to these pupils, I gave the calendar table to 4th graders with only three numbers. They could find the pattern, even they realized that the sum of three consecutive numbers is three times the middle number. That's why I am thinking it's better even to directly give them the calendar because it makes them realize that mathematics is closely related to everyday life. Moreover, what made me happy is that it did not take too much time for my pupils to realize the rule from five numbers in a cross pattern to the plus pattern. (NM-R$^+$: Non-mathematical response with reasoning, comment, or evaluation)

TK: I am amazed with his pupils. As he gave his pupils 1, 4, 9, 16, they directly realized that it form a quadratic number pattern. Because your pupils are good at mathematics, you should level up the task so that it will encourage their critical thinking more and more. (NM-R$^+$: Non-mathematical response with reasoning, comment, or evaluation)

AF: I was surprised when he gave 1, 4, 9, and 16. His pupils immediately noticed n squared. So, what Thoif said is true that AI's pupils were really responsive and critical to the pattern being discussed. I agree that the task should be leveled up. (NM-R+: Non-mathematical response with reasoning, comment, or evaluation)

AF's implementation

AF: I directly gave my pupils the task and time to think about it. Their solution was right, but they utilized a different approach from mine. Then, I told them the other approaches, i.e., the number pattern and arithmetic sequence, which they realized after then. I am thinking. They did not notice the (feature of the vertical) pattern. Was it because I did not start my lesson with the number pattern as AI and TK did but directly gave them the task? (NM-Q+: Valuable non-mathematical question)

AI: In fact, I prefer your way because you gave the task right away, while I introduced the multiplication table first, which really took time. Though I applied it to elementary school students, some could understand, although with fewer numbers. (NM-R+: Non-mathematical response with reasoning, comment, or evaluation)

TK: Responding to AI and AF's teaching implementation, I think it's good in their own way, and they could keep their students active even though they must try to keep asking questions to their pupils. But it worked as their pupils were interested in their tasks. (NM-R+: Non-mathematical response with reasoning, comment, or evaluation)

TK's implementation

TK: I had one problem regarding the students because I invited students at different levels, one junior and two high school students. At first, I introduced simple patterns, such as natural numbers and multiples of two and three. I set it that way on purpose. When they got interested, I started giving the calendar table, which I found challenging then. My prediction was that they guessed the pattern for a long time, so I chose to explain first, but I was out of control, explaining too much. (NM-R$^+$: Non-mathematical response with reasoning, comment, or evaluation)

AI: Her decision to bring the calendar to the class was an excellent idea. I like it. (NM-R: Simple non-mathematical response)

AF: Yes, I appreciate her bringing the calendar. She taught her pupils step by step by asking questions and directing them. (NM-R: Simple non-mathematical response)

- Folding paper activity (All participants)

AI: This (the number of sections) follows a pattern, which is $2^n, n \geq 0, n \in N$. The sequence will be 1, 2, 4, 8, 16, and so on, while the number of folding lines follows $2^n - 1$. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

I: What mathematical topic do you think is closely related to this?

AM&TK: Sequences and series. (M-R: Simple mathematical response)

AF: I agree, sequences and series because it contained the number pattern. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

AI: It's a geometric sequence because we look for the nth term, not the sum up to the nth term. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

I: What made you notice that it's a geometric sequence?

TK: The ratio, it's constant. For example, $\frac{U_n}{U_{n-1}} = \frac{U_{n-1}}{U_{n-2}} = \frac{2}{1} = \frac{4}{2} = 2$. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

I: After observing this data (the number of sections and folding lines), what did you realize?

AI: 1+2, 3+4, 7+8. 7 comes from 3+4, 15 comes from 7+8.
7 can be expressed as 1+2+4. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

KK: (For $n = 4$), 15. (M-R: Simple mathematical response)

AI: 1+2+4. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

AF: +8. They form a pattern from here to here. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

AI: It means $g_n = \sum_{i=0}^{n} 2^i$. No, up to n-1. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

I: What can you say now?

AM: It's a series. (M-R: Simple mathematical response)

AI: Geometric series. (M-R: Simple mathematical response)

TK&AI: The ratio is 2. (M-R: Simple mathematical response)

TK: Well, $S_n = \frac{a(r^n-1)}{r-1}$. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

AI: Oh yeah, it will back to the first form we found. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

TK: $S_n = \frac{a(r^n-1)}{r-1} = \frac{1(2^n-1)}{1} = 2^n - 1$. (M-R$^+$: Mathematical response with reasoning, comment, or evaluation)

6.3.2 Discussion

In the private university group meeting, each participant reflected on their teaching implementation. The other two participants examined their practices, such as criticizing the class and their attitudes, while KK simply reviewed what happened in her class. On the other hand, she provided beneficial manifestations when discussing her peers' teaching implementation. For instance, she not only criticized but also suggested what her peers better to do next time. Another critical point, she appreciated and reinforced her peers' appropriate teaching behaviors. The other participants did the same. AM, in particular, also encouraged KK after criticizing her implementation.

The public university participants evaluated their teaching implementation by asserting the pleasant situation, the challenge, what should not be done, and questioning the reason for the pupils' performance. By reflecting on their peers' teaching videos, they expressed their amazement at the pupils and, by considering it, offered suggestions on what should be done as teachers. Like the private university participants, they also reinforced their peers' appropriate behavior. When one participant asked a question, another responded by reflecting on his implementation (see AI's response to KK's question). Moreover, even though her peers' response to TK's implementation was simple, it remained valuable as it contained reinforcement and appreciation.

Their discussions adhere to what Korthagen and Wubbels (1995) identified as critical features of teaching reflection, such as reviewing what occurred, why it occurred, what went wrong, and what they could have done differently. As such beneficial manifestations emerged during the meeting, this study admits the assertion of Nilsson (2008) that prospective teachers' reflection may lead to highlighting issues or situations that are important to them in shaping their understanding of their practice. It may also impact their understanding of the complex relationship between their teaching, which may foster their pedagogical content knowledge development.

The paper folding activity discussion also raised some essential critical manifestations. In responding to the instructor's question, they provided not only the answer but also the reason. When investigating and testing their conjecture, they collaboratively took the accessible approach that, according to Sternberg (1986), it belongs to critical thinking.

In summary, the entire discussion stimulates the emergence of critical manifestations, as presented in the discussion snippets above. These manifestations came in mathematical and non-mathematical statements and responses accompanied by reasoning, comments, or evaluations of peers' manifestations, all reflecting intellectual and social aspects of active learning (Edwards, 2015). In turn, these manifestations enable them to broaden their mathematical and pedagogical perspectives.

6.4 Teaching Perspectives

6.4.1 Result

Table 11. Perspectives in teaching mathematical problem solving

Phase	*Initial Perspectives*			*Final Perspectives*		
	Teaching as transferring knowledge	Teaching as facilitating students to construct knowledge by themselves		Teaching as transferring knowledge	Teaching as facilitating students to construct knowledge by themselves	
	Closely Managed	Neutral	Emphasizing Strategies	Closely Managed	Neutral	Emphasizing Strategies
Understanding	AM KK	VI AI	TK AF	TK	AM	AI KK VI AF
Planning	AM KK		VI AI TK AF	TK	AI	KK AM VI AF
Carrying out	AM KK	VI AI	TK AF	KK	VI	TK AI AM AF
Looking back	AM KK		VI AI TK AF	KK		VI TK AI AM AF

Approach		*Initial Perspectives*					*Final Perspectives*				
		Likert Scale (1 to 5)									
		•	• •	• • •	• • • •	• • • • •	•	• •	• • •	• • • •	• • • • •
Teaching as transferring knowledge	Closely Managed		AI AF	TK	AM KK	VI	AF	AM AI		KK VI TK	
Teaching as facilitating students to construct knowledge by themselves	Neutral		AM		KK	VI AI TK AF				AM KK	VI AI TK AF
	Emphasizing Strategies		AM	KK	AI	VI TK AF			AM	KK AI	VI TK AF

6.4.2 Discussion

The first part of Table 11 shows the changes in the participants' teaching perspectives at the beginning and end of the intervention in terms of each problem-solving step. While the second part shows their teaching style preference when teaching mathematical problem solving as a whole.

Two participants (AM and KK) had preliminary views on teaching problem solving as a form of knowledge transfer, reinforced by the results of the second part of the questionnaire, in which they gave four points for the closely managed style. They were adamant that teachers must ensure that all students understand what to do, tell students which approach will lead to the right solution, point students in the right direction as quickly as possible, and instruct students to check the numerical results as a primary concern.

Meanwhile, the rest (VI, AI, TK, AF), who constituted the majority, had a preliminary perspective in teaching mathematical problem solving as facilitating students to construct knowledge by themselves. They had differing views when teaching in the first and third stages. In understanding the problem, VI and AI thought that teachers should let students interpret the task by themselves without giving guiding questions, while TK and AF thought that teachers should give guiding questions. In carrying out the plan, their views varied on whether the teacher should let them work on their own without providing any assistance (VI, AI) or gradually support students according to their needs only to the extent necessary (TK, AF). All of them agreed on the planning and reviewing stages, namely that the teacher should mention some applicable approaches while encouraging students to follow their own ideas and, finally, direct students to review the variety of strategies they use as the most important thing rather than the numerical results. This group of students put five points for neutral and emphasizing strategies behaviors, demonstrating their views on teaching as facilitating.

Regarding how active they were during the intervention, the first two participants who saw teaching as transferring knowledge were less active than the other four, particularly during the preliminary meetings. Their behavior in class was consistent with their perspective on teaching mathematical problem solving. Thus, it is reasonable for them to position themselves as students in an environment where they believe the teacher has a decisive role and must guide students step by step to solve a problem.

The initial and final results show delicate (VI, AI, AF) and significant (AM, KK) shifts in their perspectives, more oriented toward teaching as facilitating students to construct knowledge by themselves, particularly emphasizing strategies in terms of each problem-solving step. An exception is in TK's case, as she shifted her perspective in the first two problem-solving steps towards teaching as transferring knowledge. This shift could be influenced by her teaching experience which might be challenging to position herself as a facilitator rather than clearly explaining content to show students how to solve the problem, as she reflected on her class,

> "My prediction was my students would guess the pattern for a long time, so I chose to explain it first, but I was out of control, explaining too much". (see p. 92)

As such, she might consider her first perspective applicable to ideal teaching conditions, i.e., when teachers are confident that students can work on problems independently. This case follows what Safrudiannur and Rott (2019) mentioned, that students' abilities impact the link between teachers' beliefs about mathematics and their teaching style. On the other hand, based on the Likert scale result, she put the maximum point on the teaching as facilitating and fewer points on the teaching as transferring knowledge. Thus, another hypothesis is that it might be a human error, such as carelessly filling in the questionnaire.

This section captures the shift in participants' perspectives on teaching mathematical problem solving. Their recent perspectives mostly led to teaching as a facilitator for students to construct their knowledge. In this case, the teachers can either assist students or not.

6.5 Teaching Implementations

6.5.1 Result

Table 12. Teaching implementations[11]

Participant	PS Step	Preliminary Test			Follow-up Test		
		Talk format	Teaching behavior	Teaching approach	Talk format	Teaching behavior	Teaching approach
AM	1	PDD	CM	Active Learning Approach	PDD	ES	Active Learning Approach
	2	PDD	N		PDD	N	
	3	PDD	N		PDD	ES	
	4	R & PDD	CM		PDD	ES	
KK	1	R	CM	Traditional Approach	PDD	N	Active Learning Approach
	2	R	CM		PDD	N	
	3	R	CM		PDD	N	
	4	R	CM		PDD & R	ES	
VI	1	R	N	Traditional Approach	R	CM	Traditional Approach
	2	R	CM		R	CM	
	3	R	CM		R	CM	
	4	-	-		PDD	ES	
AI	1	PDD	N	Active Learning Approach	PDD	N	Active Learning Approach
	2	PDD	CM		SP	ES	
	3	PDD	N		PDD	N	
	4	PDD	CM		PDD	ES	
TK	1	PDD	N	Active Learning Approach	PDD	N	Active Learning Approach
	2	-	-		PDD	N	
	3	PDD	N		PDD	N	
	4	R	CM		PDD	ES	
AF	1	PDD	N	Active Learning Approach	PDD	N	Active Learning Approach
	2	PDD	CM		PDD & SP	N	
	3	PDD	N		PDD & SP	N	
	4	PDD	ES		PDD	CM	

[11] R: Recitation SP: Student presentation PDD: Position-driven discussion
CM: Closely managed N: Neutral ES: Emphasizing strategies

6.5.2 Discussion

As mentioned in the methodology part, the teaching implementation belongs to the traditional approach if the participant mainly utilized recitation and performed closely managed behavior. Suppose the participant primarily utilized partner talk, student presentation & group critique, or position-driven discussion supported by the teaching behavior to be neutral or emphasizing strategies. In that case, the teaching implementation is considered an active learning implementation.

In the preliminary test, some participants implemented an active learning approach and some others implemented a traditional approach (see Table 12). With the aim to help pupils comprehend the task, the participants who closely managed the class explained the task in a detailed way through a question-and-answer activity and provided some concrete examples of the solution. They told pupils how to approach the problem and directed them to reach the correct solution. Finally, they placed a strong emphasis on numerical results, either with or without emphasizing the approach. Neutral participants allowed pupils to decide and work on it individually before discussing the task with the entire class. Moreover, there was one participant who emphasized the strategy indicated by highlighting several approaches.

In the follow-up test, the majority of implementations resulted in approaching active learning. Nonetheless, the traditional approach persisted, as evidenced by the use of the recitation form of talk and closely managed teaching behavior. The recitation was a minority style of talk, with position-driven discussion being the majority. Furthermore, one participant employed a student presentation form of talk that was not present in the preliminary test.

This study highlights that teaching implementation and perspective only go hand in hand to a certain extent. In one case, the participant saw teaching as transferring knowledge, and the implementation was oriented toward the traditional approach by maintaining the classroom situation as planned. In the other cases, the participants viewed teaching as a means of assisting students in constructing knowledge, but when teaching, they directed students excessively because they were out of control.

The facts above imply that the factor influencing teaching implementation, especially in the first career, is not merely the teacher's perspective on teaching. Other factors need to be considered, such as role models and intrinsic, altruistic, and extrinsic motivation (Gore et al., 2015). In detail, personal satisfaction, an interest in teaching, and a love of the professions belong to intrinsic motivation; altruistic motivation includes service to others, the community, and the country; extrinsic motivation covers salary, job guarantees, and working conditions. Some participants' expressed dissatisfaction with their implementation because they invited students from different grades, taught in the last hour of school, and explained too much to their students are evidence of other factors. Moreover, the teaching experience aspect must be addressed, given that participants were in teacher training programs with less experience than in-service teachers. Undeniably, experience influences a person's teaching style as he or she gradually gains confidence and certainty in himself or herself.

This session sheds light on the shift in the approach used by the participants. In the last teaching practice, most of them abandoned the recitation and closely managed style. Instead, they engaged students more, brought position-driven discussions and student presentations into the classroom, and chose between not giving or providing assistance where necessary.

7. Conclusions

The main objective of the intervention was to support the implementation of the current Indonesian curriculum by assisting prospective mathematics teachers in developing their mathematical and pedagogical skills. The results indicated that the intervention contributed positively to developing those skills. Given the broad nature of mathematical and pedagogical aspects, it is reasonable to break down the conclusions based on the research questions as follows.

RQ 1: How do Indonesian prospective mathematics teachers' perform in problem-posing throughout the active learning-based intervention?

The proposed tasks that fall into the exercises category appeared the most. The dual-task design and peer discussion had advantageous effects on the quality improvement of the task. The first submitted tasks, considered either exercise or blind because unrealistic and unsolvable, were modified to make more sense and be solvable with a better formulation. More specifically, one out of six participants (AI, see Table 13) tended to pose empowered problems, even from the beginning of the intervention.

Table 13. Personal account of problem-posing performance

Participant	Personal account
AM	She proposed mostly exercises from the preliminary test. She also proposed blind tasks twice, one of which is in the follow-up test. In this test, she modified the blind task into an empowered problem, indicating her first step toward improvement.
KK	She posed an empowered problem in the preliminary test but, in subsequent performances, did not show consistency, given that she also posed exercises and blind tasks besides empowered problems.

VI	She made significant improvements, as in the preliminary test she posed an exercise, and in later performances she posed exercises and empowered problems with no blind task.
AI	He was the participant who persistently posed empowered problems from the beginning to the end of the intervention. Although he had proposed a blind task once, he modified it to an empowered problem.
TK	In the beginning, she proposed a blind task. In the subsequent performances, she mainly proposed exercises, and there were some blind tasks. Her progress can be traced when she modified three blind tasks into empowered problems.
AF	In the preliminary test, she generated an empowered problem. She sought to maintain her performance, evidenced by posing mainly empowered problems. Although she posed a blind task once and some exercises, in the end, she demonstrated her capability in modifying an exercise into some empowered problems.

RQ 2: How do Indonesian prospective mathematics teachers perform in problem-solving throughout the active learning-based intervention?

Most of the solutions were correct, meaning the participants had appropriate cognitive resources, controlled the process, and utilized suitable heuristics when solving their proposed tasks. The use of an algebraic approach is visible in their solutions to the problems discussed during the lesson and the solutions to their proposed tasks, indicating that the organized topics fostered the algebraic approach to emerge. On the other hand, several incorrect solutions also existed because of the procedural or conceptual errors, or a careless problem-solving process. One out of six participants (VI, see Table 14) performed errors in two cases showing both procedural and conceptual errors.

Table 14. Personal account of problem-solving performance

Participant	Personal account
AM	Of the tasks she proposed, she gave the incorrect solution only once. At the end, as she modified her proposed task from a blind task to an empowered problem, she also provided the correct solution.
KK	In the preliminary test, she gave the correct solution to the task she proposed. In the following tests, in addition to submitting an exercise with a correct solution, she also submitted blind tasks but revised them to be solvable tasks with correct solutions.
VI	As a result of a procedural error, she gave an incorrect solution in the preliminary test. In the following test, in addition to giving correct

	solutions to her proposed tasks, she also gave an inappropriate solution due to a conceptual error.
AI	He consistently provided correct solutions to the solvable tasks he proposed, from the beginning to the end of the intervention.
TK	Because of the blind task in the initial test, she arrived at a blind solution. In subsequent performances, she mainly offered correct solutions, except for a case where she made an error caused by a careless attention.
AF	From the start to the end of the intervention, she always provided correct solutions to the solvable tasks she proposed. The exception was in one case, when she submitted a blind task whose solution was also classified as blind.

RQ 3: How do the Indonesian prospective mathematics teachers' critical manifestations look throughout the active learning-based intervention?

Critical manifestations appeared as mathematical or non-mathematical responses with reasoning, comments, or evaluations and valuable mathematical or non-mathematical questions. They evaluated their practice by highlighting key issues to shape their understanding of their practice while taking a practical approach to the task they discussed during the lesson. Throughout the intervention, all participants expressed their critical attitude (see Table 15).

Table 15. Personal account of critical manifestations

Participant	Personal account
AM	She demonstrated steady performance from the start to the end of the intervention.
KK	She got increasingly engaged once the groups unified and became more active in sharing her thoughts with the group.
VI	In most cases, she took her role as an initiator who triggered the direction of the discussion by asking questions or making comments. She played various roles during the discussion.
AI	Like VI, he took on various roles during the discussion and was mainly the initiator in the public university group.
TK	She occupied various roles and was frequently the challenger, voicing opposing views to those of her peers.
AF	Since the beginning, she has served in various roles, primarily as a supporter or follower. As the group merged, her involvement increased by taking on more clarifying, expressing her opposing viewpoints, and other duties.

RQ 4: How do the Indonesian prospective mathematics teachers' teaching perspectives change throughout the active learning-based intervention?

The preliminary and final results show a shift in their perspectives, becoming more oriented toward perspective teaching as assisting students in constructing knowledge, particularly emphasizing strategies. In particular, the two participants (AM and KK, see Table 16) who previously chose to be closely managed the class changed their preferred behavior into neutral or emphasizing strategies. In addition, one participant whose perspective was neutral and emphasizing strategies changed her view into closely managed for the first two problem-solving steps (TK, see Table 16). Her teaching experience might influence her perspective in this case.

Table 16. Personal account of teaching perspectives

Participant	Personal account
AM	Her teaching perspective shifted significantly from teaching as transferring knowledge (fully closely managed) to teaching as facilitating students to construct knowledge by themselves (being neutral and emphasizing strategies).
KK	Initially, she thought closely managed was an ideal teaching behavior in all problem-solving steps and, thus, perceived teaching as a means of transferring knowledge. In the end, she had a blended perspective on teaching as both transferring knowledge and facilitating students, as she determined emphasizing strategies as an ideal teaching behavior in the first two problem-solving steps.
VI	At first, she had a perspective on teaching as facilitating students to construct their knowledge, reinforced by her view of being neutral or emphasizing strategies in a balanced way. In the end, she maintained her perspective but emphasized more on emphasizing strategies.
AI	Like VI, from the beginning, he had the perspective of teaching as facilitating students to construct their knowledge by being neutral or emphasizing strategies in equal measure. Ultimately, he maintained his perspective by being less neutral and emphasizing strategies more.
TK	Early on, she had a strong view on teaching as facilitating students to construct knowledge by themselves. Lately, she had a combined perspective of teaching as transferring knowledge and facilitating students. She considered the first perspective is for the ideal pedagogical condition. According to her teaching experience, teachers

	must closely manage the class during the first two problem-solving steps, demonstrating a view of teaching as transferring knowledge.
AF	She maintained her teaching perspective from the beginning to the end of the intervention, i.e., facilitating students to construct knowledge by which the teacher should consistently emphasize the strategy.

RQ5: How do the Indonesian prospective mathematics teachers' teaching implementations look throughout the active learning-based intervention?

There was a gradual progression toward active learning in the participants' teaching implementation. Those whose implementation had been considered active learning since the beginning tended to maintain their styles, while others (KK and VI, see Table 17) attempted to interact more with their pupils by asking for their ideas on subsequent occasions. Besides the teaching perspective, other factors influence their teaching implementation, i.e., role models originating from the previous learning experience and intrinsic, altruistic, and extrinsic motivation.

Table 17. Personal account of teaching implementations

Participant	Personal account
AM	She had implemented an active learning approach since the beginning. There was a shift in her practice. The recitation and closely managed behavior no longer appeared in the last teaching.
KK	She tremendously changed her way of teaching. Initially, she implemented recitation and closely managed the class without allowing students to speak. Ultimately, she engaged her students through position-driven discussions with little recitation, being neutral and emphasizing strategies, demonstrating an active learning approach.
VI	She slightly reoriented her way of teaching. In the first practice, she applied memorization, closely managed, and sometimes became neutral, demonstrating a traditional approach. In the last practice, she started incorporating position-driven discussion and emphasizing strategies, moving towards an active learning approach.
AI	He had applied an active learning approach from the beginning. In his latest class, he used position-driven discussion and blended it with student presentations. Closely managed behavior had vanished, replaced with emphasizing strategies.
TK	She had implemented an active learning approach from her first practice. In her last class, she emphasized strategies and began to abandon the recitation and closely managed style as features associated with the traditional approach.

AF	She had practiced an active learning approach from her first teaching experience. In her last teaching, she included student presentations besides position-driven discussions and mainly chose to be neutral.

8. Pedagogical Implications and Concluding Remarks

As mentioned above, the intervention had some positive effects in helping participants to develop their pedagogical and mathematical skills, although the improvement that occurs in a participant does not have to be in all aspects. Some of the wealthy situations that this intervention has in its merits are noteworthy:

(1) The participants had an opportunity to practice formulating an empowered mathematical task. In some cases, they modified the contexts relevant to their pupils' lives.

(2) The participants had an opportunity to implement their self-proposed task, which they may have a personal approach to, as well as observe and evaluate their and their peers' implementation to reflect.

(3) The lesson and the meeting after teaching implementation directed the participants toward a critical discussion that is beneficial to broaden their insight into their pedagogical content knowledge.

Reflecting on the impact, this study suggests that teacher trainee programs should form study groups of prospective teachers with explicit instructions to do the abovementioned activities, not merely practice teaching together in their last year and report their teaching after several months of practicing. There should be problem-posing and discussion sections in the group accompanied by an instructor to make improvements during their teaching internship. Given that some (not all) universities provide courses to support prospective teachers during their teaching internships, such as lesson planning and learning module development, the above activities can be incorporated into these courses.

Finally, the following points are worth noting. This research was intended to test a model (active learning approach promoted through collaborative problem posing), whether it is successful in a small group, identify its benefits and challenges, and find promising areas for further research. Given that it involved a small number of prospective teachers and was conducted online, further research topics could be a similar model applied to in-service teachers or implemented in face-to-face meeting. In addition, the results could not be generalized due to the small number of participants. A scheme that allows the results to be generalized is to conduct research involving several small groups of students with a different instructor for each group.

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Attended Conference as Presenter

1. 13th Congress of the European Society for Research in Mathematics Education (CERME13)

   Presentation title:

   A promising path toward infinite improvement in mathematics teaching and learning.

   Budapest, Hungary. 10-14 July 2023.

2. Didactic Research Conference on Mathematics and Computer Science (MIDK)

   Presentation title:

   A promising path toward infinite improvement in mathematics teaching and learning.

   Oradea, Romania. 31 March - 2 April 2023.

3. International Conference on Mathematics and Science Education (ICoMSE)

   Presentation title:

   Generated worthwhile problem promoting algebraic thinking: from pattern exploration to generalization.

   Malang, Indonesia. 9-10 August 2022.

4. Conference on Contemporary Mathematics Education (CME)

   Presentation title:

   First experience with problem-posing: What can be done with a multiplication table?

   Gdańsk, Poland. 27-30 June 2022.

5. 12th Congress of the European Society for Research in Mathematics Education (CERME12)

   Presentation title:

   Exploring Indonesian prospective teachers' teaching belief and teaching practice.

   Bolzano, Italy. 2-6 February 2022.

6. Didactic Research Conference on Mathematics and Computer Science (MIDK)

   Presentation title:

   First experience with problem-posing: what can be done with a multiplication table?

   Baja, Hungary. 1-3 April 2022.

7. Conference on Contemporary Mathematics Education (CME)

   Presentation title:

   How do students with different personality types show their critical thinking when solving a mathematical problem?

   Gdańsk, Poland. 28-29 June 2021.

8. Didactic Research Conference on Mathematics and Computer Science (MIDK)

   Presentation title:

   Problem Solving: How Do Students with Different Personality Solve Mathematical Problem?

   Sárospatak, Hungary. 24-26 January 2020.

9. Science and Mathematics International Conference (SMIC)
   Presentation title:
   Translation among mathematical representations: How do moslem students with different gender perform the process?
   Jakarta, Indonesia. 2-4 November 2018.
10. International Conference on Science, Technology, Education, Arts, Culture, and Humanity (STEACH)
   Presentation title:
   Dynamism of open-ended problem solving: study on junior high school students' behavior based on Keirsey personality type
   Surabaya, Indonesia. 29 October 2018.
11. International Conference on Mathematics and Science Education (ICMScE)
   Presentation title:
   Student's critical-metacognition activitiy based on their personality type.
   Bandung, Indonesia. 5 May 2018.
12. Mathematics, Informatics, Science, and Education International Conference (MISEIC)
   Presentation title:
   Student's critical thinking in solving open-ended problems based on their personality type.
   Surabaya, Indonesia. 6 September 2017.

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Nyilvántartási szám: DEENK/251/2023.PL
Tárgy: PhD Publikációs Lista

Jelölt: Fitriana, Linda Devi
Doktori Iskola: Matematika- és Számítástudományok Doktori Iskola
MTMT azonosító: 10079926

A PhD értekezés alapjául szolgáló közlemények

Idegen nyelvű, külföldi könyvrészletek (1)

1. **Fitriana, L. D.**: First experience with problem-posing: What can be done with a multiplication table. In: Critical Thinking Practices in Mathematics Education and Beyond. Eds.: Bozena Maj-Tatsis; Konstantinos Tatsis, Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów, 137-146, 2022. ISBN: 9788382770131

Idegen nyelvű tudományos közlemények külföldi folyóiratban (1)

2. **Fitriana, L. D.**, Ekawati, R., Kovács, Z.: Perspectives on the problem-posing activity by prospective teachers: a cross-national study. *J. Math. Educ. 13* (1), 149-172, 2022. ISSN: 2087-8885. DOI: http://dx.doi.org/10.22342/jme.v13i1.pp149-172

Idegen nyelvű konferencia közlemények (2)

3. **Fitriana, L. D.**: Generated Worthwhile Problem Promoting Algebraic Thinking: From Pattern Exploration to Generalization. In: ICoMSE 2022 : 6th International Conference on Mathematics and Science Education Online Virtual Conference, 9-10 August 2022, [s.n.], "Accepted by publisher" [s.l.], 1-8, 2023.

4. **Fitriana, L. D.**: Exploring Indonesian prospective teachers' teaching belief and teaching practice. In: Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education. Eds.: Jeremy Hodgen; Eirini Geraniou; Giorgio Bolondi; Federica Ferretti, Free University of Bozen-Bolzano, Italy and ERME, Bozen, 3577-3584, 2022. ISBN: 9791221025378

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További közlemények

<u>Idegen nyelvű, külföldi könyvrészletek</u> (1)

5. **Fitriana, L. D.**: Problem solving: how do students with different personality types show their critical thinking when solving a mathematical problem?
   In: Critical thinking in mathematics : perspectives and challenges. Eds.: Bozena Maj-Tatsis; Konstantinos Tatsis, Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów, 153-163, 2021.
   ISBN: 9788379969036

<u>Idegen nyelvű tudományos közlemények külföldi folyóiratban</u> (2)

6. Fuad, Y., Ekawati, R., Sofro, A., **Fitriana, L. D.**: Investigating Covariational Reasoning: What Do Students Show when Solving Mathematical Problems?
   J. Phys. Conf. Ser. 1417 (1), 1-9, 2019. ISSN: 1742-6588.
   DOI: http://dx.doi.org/10.1088/1742-6596/1417/1/012061

7. **Fitriana, L. D.**, Fuad, Y., Ekawati, R.: Student's Critical Thinking in Solving Open-Ended Problems Based on Their Personality Type.
   J. Phys. Conf. Ser. 947, 1-8, 2018. ISSN: 1742-6588.
   DOI: http://dx.doi.org/10.1088/1742-6596/947/1/012007

<u>Idegen nyelvű konferencia közlemények</u> (1)

8. **Fitriana, L. D.**, Fuad, Y., Rosyidi, A. H.: Dynamism of Open-Ended Problem Solving: Study on Junior High School Students Behavior Based on Keirsey Personality Type.
   Advances in Social Science, Education and Humanities Research. 277, 1-5, 2019. ISSN: 2352-5398.
   DOI: https://doi.org/10.2991/steach-18.2019.1

A DEENK a Jelölt által az iDEa Tudóstérbe feltöltött adatok bibliográfiai és tudománymetriai ellenőrzését a tudományos adatbázisok és a Journal Citation Reports Impact Factor lista alapján elvégezte.

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Subject: PhD Publication List

Candidate: Linda Devi Fitriana
Doctoral School: Doctoral School of Mathematical and Computational Sciences
MTMT ID: 10079926

List of publications related to the dissertation

Foreign language international book chapters (1)

1. **Fitriana, L. D.**: First experience with problem-posing: What can be done with a multiplication table. In: Critical Thinking Practices in Mathematics Education and Beyond. Eds.: Bozena Maj-Tatsis; Konstantinos Tatsis, Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów, 137-146, 2022. ISBN: 9788382770131

Foreign language scientific articles in international journals (1)

2. **Fitriana, L. D.**, Ekawati, R., Kovács, Z.: Perspectives on the problem-posing activity by prospective teachers: a cross-national study.
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   ISBN: 9788379969036

Foreign language scientific articles in international journals (2)

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   DOI: https://doi.org/10.2991/steach-18.2019.1

The Candidate's publication data submitted to the iDEa Tudóstér have been validated by DEENK on the basis of the Journal Citation Report (Impact Factor) database.

14 June, 2023