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The impacts of the introduction of the function concept on students' skills

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Abstract

The concept of function is of fundamental importance to the learning of mathematics [7]. In the function concept development process the rule-followig and rule-recognition skills have an important role, that are necessary in the construction of value tables, which help children to figure out the relationship between quantities [4]. In this study, the skills mentioned above were tested among seventh grade students from Ukrainian and Hungarian schools, then consequences have been compared to results of previous studies [15]. We attempted to find out whether the introduction of this concept has an effect on the aforementioned skills.

Keywords: function concept, rule following and rule recognition skills, Hungarian and Ukrainian secondary education, comparison.

MSC: I23

1. Introduction

Over the past at least twenty years the concept of function has emerged as a unifying theme in mathematics curricula internationally. The notion of functions evolved from dependence relationships of real life phenomena to an abstract correspondence that is usually best describe in symbolic terms [13, 14].

In her study, Sierpinska [14] sets out the conditions of understanding the notion of function. These conditions illustrate that it takes time to reach a thorough understanding of the function concept. There is a long journey from the beginning to develop an understanding of the relationships between the elements of sets to the robust function concept. According to Kwari [10] the rule is an element of the function concept. Regarding the skills that can be linked to the function concept [10], possessing rule-recognition and rule-following skills (hereafter referred to as RR and RF) is exceptionally important in the period before providing the definition of function (preparation period) in order to be able to recognize function-like relations. However, studying this period in Hungarian [9] and Ukrainian mathematics curricula frameworks, we can state that the latter focuses less on developing skills discussed above [15].

Thus, the base of this paper is a previously conducted study among a group of 6^{th} grade students of a Hungarian school and a group of 6^{th} grade students of a Ukrainian school, where we tested their RF and RR skills before introducing them to the function concept. Taking the results of this study into consideration, the aim of this paper is to answer the following questions: (a) do the same students from Ukraine develop without targeted development in the aforementioned skills year by year; (b) what are the differences in the skills between the students of the two countries by the end of the 7^{th} form, after the introduction of the concept of function, does it have an impact on these skills.

2. Theoretical background

Mathematical definitions play an important role in the study of practically every area of mathematics ([5, 17, 18]).

The modern definition of function that frames this study is the Dirichlet-Bourbaki definition, which is "a correspondence between two nonempty sets that assigns to every element in the first set (the domain) exactly one element in the second set (the codomain)" (as cited in [6, p. 357]).

Vinner [20] drew attention to the distinction between the concept definition that mathematicians use to define a mathematical concept and the concept image which people generate in their mind. He also showed that most students use their own personal concept definition for the notion of function. Vinner and Dreyfus [6] categorized the students' definitions of a function into six categories, a refinement of the categorization by Vinner [20]: (A) correspondence (the Dirichlet-Bourbaki definition); (B) dependence relation (dependence between two variables); (C) rule (a function is a rule; a rule is expected to have some regularity, whereas a correspondence may be "arbitrary"); (D) operation (a function is an operation or manipulation); (E) formula (a function is a formula, an algebric expression, or an equation); and, (F) representation (graphical or symbolic representation) (as cited in [6, p. 360]).

Taking into account these categories, it can be highlighted that the function can be defined in various ways. Sierpinska [14] described the "worlds" that the study of functions should focus on: the world of changes or changing objects; the world of relationships or processes; and, the world of rules, patterns, and laws. According to Sierpinska [14] the difference between the rule and relationship is subtle because the rules, patterns and laws are simply well defined relationships. Relationships can be expressed verbally or using diagrams, tables, graphs or in symbols. A rule can be a verbal statement or a formula. It is possible for one to detect a relationship but fail to explicity state the rule. Finding rules, patterns and laws can be used as an entry point to the development of the function concept.

Among the skills that could be linked to the above listed "worlds", the RR and RF skills play a highly important role in recognising function-like relations, as well as in representing them in various ways during the concept-forming process. Because the concept of functions can be represented in a variety of ways (verbal, set diagram, function box, set of ordered pairs, table of values, graph, formula) [1] an important aspect of its understanding is the ability to use these multiple representations [11]. Thompson [16] claims that if students do not see something remain the same as they move among different representations then they see each representation as a topic to be learnt in isolation.

To summarize, the arguments above we can state that in order to form the function concept, it is crucial for the students to be able to: (1) recognize relations between cohesive elements if these relations are represented in different ways; (2) to express relations in various ways; (3) follow a rule, based on which elements are ordered to one another.

The aim of this study is to investigate whether the introduction of the function concept to 7^{th} grade students, after providing them with its definition and expanding its ways of different representations (e.g. arrow diagrams and graphs of functions are considered new ways of representations since the students of both groups are already aware of tables of values and formulas) have an impact on students' RF and RR skills.

3. Methodology

3.1. Sample

Participants were 24 seventh grade students (13-14 years old), with moderate abilities, in a school with Hungarian as the language of instruction in Ukraine and 20 students from the education system of Hungary (13-14 years old). When choosing our sample, we tried to balance between the two groups in a way that none of them are specialised classes in Mathematics, they study the subject in 4 hours per week and by the end of the sixth grade they acquire the same material. Based on their grades the students are on the same level of knowledge. In both schools they use the textbook supported by the Ministry of Education of the given country.

As the research was carried out in April, during the second semester of the seventh grade, students of both countries were already familiar with the natural numbers, fractions (common fractions and decimals), and had learned arithmetic operations with rational numbers, direct proportionality, equations. Also, the concept of function had been introduced to both groups in a more or less similar way and the level of acquisition had been tested before this research was conducted.

3.2. Background

In the Ukrainian and Hungarian (hereafter referred to as UA and HU) education system, function as a mathematical concept is defined at the seventh grade of the secondary school. In the lower classes, students are prepared with the use of different materials for the introduction of the function concept. Analysing the HU and UA curriculum and the textbooks for the fifth and sixth grade we can highlight that the major differences are in the requirements for developing RF and RR skills (in the lower classes in the UA education it does not exist at all) [15]. Prior researches (cf., studies cited above), however, suggest that they are necessary for the development of the function concept. Nonetheless, findings of previous studies prove that UA students participating in this research, at this stage of intellectual development (according to Piaget this is the transition from the concrete operations phase to the formal one) (as cited in [2, p. 48]), are able to recognise and follow a rule in order to assign cohesive elements without any targeted development by the end of 6^{th} grade, using only their previously acquired mathematical knowledge. However, argue in favour of a well-recognized rule either in a written form or with formulas remained a difficulty for them.

3.3. The questionnaire

A written test was used in order to investigate the RF and RR skills of students. Students worked independently and had 40 minutes to complete the test. The test contained six tasks that were based on the recognition and application of the relationship between the cohesive elements, as well as on the expression of the recognised rule, including as a formula. In other words, the base of these tasks was recognition or expression function-like connections between cohesive elements, represented by words, formulas, tables of values and graphs; without mentioning the word “function” in the instructions.

The cohesive element pairs did not clearly define a function, so more rules might be possible. In the instructions of the test, however, we tried to make it clear that we wanted students to find only one adequate rule. Some of the tasks were created by ourselves while the others were chosen from the literature. There were tasks that were included in the test from the previous year.

Task 1: Find a relationship between the x and y values of the columns and based on it, complete the table with the missing elements.

x	16	12	4	-8	20	4	0			
y	4	3	1					6	-3	-1

Write down the relationship: a) with words; b) with expression. Plot the pairs of values on a coordinate plane.

Task 1 targeted the recognition, application, expression of a relationship between cohesive elements using words and formula, and also the graphing of these ordered pairs on a coordinate plane. The filling in of the blank places of the tables assessed the following the rule. This table-filling task targeted the recognition and the application of a simple, one-step rule (e.g. determining y values based on x values and vice versa), furthermore, it provided us with the opportunity to study whether students were able to represent a rule that define the relationship between cohesive elements in various ways (such as by words, formulas, graphs).

Task 2: Find a relationship between the x and y values of the columns and based on it, complete the table with the missing elements.

x	*1*	*10*	*7*	*0*	*9*	*20*	*38*
y	*5*	*23*	*17*				

$y = \dots$

Write down the relationship: a) with words; b) with expression.

In Task 2 it was necessary to recognise some kind of relationship between the cohesive elements (x and y), hint to record the recognised relationship in the language of arithmetic, and express the relationship with a formula, in other words, the aim of this task was to find out, whether students were able to define a rule that determined the relationship between the elements. The "end product" (y value) should be found with the help of the given "raw material" (x value) according to the recognised rule (as cited in [3, p. 23]), while in the previous task knowing the "end product" and using the recognised rule, the "raw material" should be found, in this way, Task 1 served as a preparation for the use of inverse function. Unlike the previous task, students had to recognise a multi-step rule and represent it in different ways.

Task 3: Fill in the table according to the following rule: $y = \frac{x}{3} - 7$. Write down the rule in words.

x	*9*	*0*	*42*	*63*	*15*	*3*
y						

Task 3 was aimed at the interpretation and following of a predefined rule, given by a formula. The correct completion of the table indicated correct interpretation of the symbolic rule.

Task 4: 2 litres/second of water flows from a tap to a tank. How much water is in the tank at:

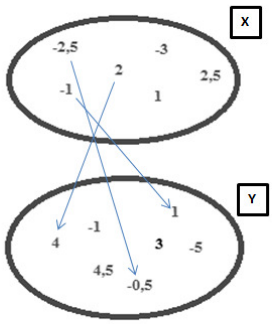
a) 1 s,
b) 2 s,
c) 5 s,
d) 10 s,
e) 16 s,
f) x s

later if the tank was empty at the beginning?

Illustrate the relationship between the amounts
I) in a table,
II) with a formula.
III) How much water is in the tank when, at the moment of opening the tap, the tank contains 3 litres of water? Use a formula to describe the connection between the values.

In Task 4, the rule was given verbally, in context. We take students' correct responses for parts (a) through (e) as an indication that the student had correctly interpreted the rule. Correct responses to exercise (f) indicated students' understanding of function (e.g. they could fill in the table with cohesive values based on a recognised rule since students were able to generalise the task, i.e. write down the rule using a formula). The aim of the third part of this task was to study if there is consistency between multi-step rulemaking and its description with a formula in cases where cohesive elements are provided verbally, in context and not in a table form (see Task 2).

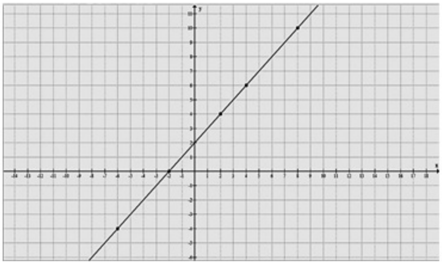
Task 5: In the figure below, several cohesive elements of two sets are connected with arrows. a) Find the assignment rule applicable. b) Connect the remaining cohesive elements using arrows.



In Task 5 cohesive elements were represented by arrows instead of a table of values. Students had to recognise the rule, based on which, the elements of set X were assigned to the elements of set Y. They were also required to argue in favour of the previously recognised rule using words. Part b) of this task was aimed at the application of the recognised rule.

The point of Task 6 was to find out whether students were able to transit from a mode of representation to another (convert data from a graph to a table) and whether they could read plotted points on a coordinate plane. The aim of part b) was to find out whether students could find a formula that describe the recognised rule (determination of assignment rules respresented by graph was not included in the HU and UA curricula frameworks before the beginning of seventh grade).

Task 6: Ordered pairs are graphed below.



a) Fill in the table with the pairs of values indicated with dots, and two additional ordered number pairs based upon your choice.

x								
y								

b) Write down the rule which recognized between the pairs of values with formula.

4. Results

4.1. Analysis of the students' answers

Task 1

18 UA students of 24 managed to fill in the table with correct values based upon the rule they recognised. 6 remaining students made the following mistake: they tried to apply different rules to each row of each column. The reason behind this could be that these students might find it problematic to represent or interpret a set of values in a table if these values are assigned to each other by a particular assignment rule. They might find it difficult even after introducing them to the concept of function. All these deficiencies were found despite the fact that they were able to read graphed points on a coordinate plane and to fill in the table with this set of ordered pairs.

10 UA students managed to argue in favour of a recognised rule using words, 9 of these students were able to use a formula for that. 14 students have found a rule that was not applicable to the table of values or just simply have skipped this part of the task. Hence, it is important to highlight that for these 14 students, the wording of thoughts and so describing a rule using words or formulas a problematic area.

17 students who found an applicable rule and followed it (e.g. filled in the table of values) could plot the points on a coordinate plane as well.

All of the 20 HU students managed to recognise an applicable rule. Following these rules, they could fill in the table of values correctly and also they could argue in favour of the recognised rule using formulas. 4 students either skipped the verbalization of the recognised rule or were inaccurate in representing the rule on a coordinate plane.

Several correct and similar answers of the UA and HU students:
“If we divide the x-values by 4, we get the y values.”
“x divided by four equals y, or x is four times y.”
“x is four times y.”
“$x = 4y; y = \frac{x}{4}$”

Task 2

Only 7 of 24 UA students managed to complete Task 2. These students could answer every part of this question, i.e. they could fill in the table based on the rule they recognised when pairing numbers and also they could word it and represent it by formula. Therefore, it seems that those students, who despite possessing the ability of one-step RR and RF, might have difficulties in the process of two-step RR and RF.

11 of the 20 HU students managed to recognise a rule between the cohesive elements. Also they could describe it with the help of words and formula. The table of values reflected the rule as well. There was only 1 student who did not fill in the table, however managed to answer correctly part a) as well as part b) of the question and thus followed an appropriate thought-process. Results suggest that similarly to the UA participants, there are students who have difficulties with recognising and following a two-step rule. On the other hand, if they manage to recognise a rule, they are able to represent and describe it in different ways.

Task 3

Task 3 can be considered successful, since 23 of the 24 UA students could complete this task correctly: they could fill in the table of values which allowed us to assume that they could interpret the given rule correctly. However, similarly to the previous tasks, some difficulties arose when students tried to describe it, since only 9 students were able to describe the explicite given rule verbally. Presumably, these students have problems with expressing their thoughts verbally.

All of the HU students performed well in interpreting the rule then, based upon it, they could fill in the table of values. There was only 1 student who, instead of describing the assignment rule, presented the procedure of his calculation (e.g. *“we have to find the common denominator so we can get the final result”).*

Task 4

18 UA students were able to make calculations with concrete values (a-e), however, they could not answer exercise f) correctly. 12 students managed to represent the values (a-f) in a table form but they could not describe the rule using a formula (Part II).

No more than 8 UA students could answer Part I and Part II of the task correctly. In addition, without knowing the elapsed time, they were able to generalise the rule, they could argue in favour of the rule using a formula. There were only 5 students who could solve Part III, i.e. write down the rule "$y = 2x + 3$" (letters used for variables may have varied). Those who could fill in the table with correct values based upon a rule they recognised and found an applicable formula as well (Task 2), were also included in this number. Similarly to the Task 2 where they had to deal with a set of related number pairs in a table, the majority of students found difficult with multi-step rulemaking in the case when they were provided with these values in an implicit way, embedded in text.

All of the 20 HU students could represent the connection between cohesive values in a table form. Similarly to the UA students trying to answer these exercises, also HU students could recognise a rule between related number pairs. In the case when the elapsed time was unknown (exercise f)), there were 9 students who either skipped this part of the question or their answers were incorrect. There were only 11 HU students, who were able to represent the ordered number pairs in a table form. However, when they had to represent the relationship (the recognised rule) by a formula, there were 19 students who could answer correctly. The various solutions of the two different, yet substantively identical questions (exercise f) and part II) suggest that it is problematic for the students to interpret the variable in a formula that indicates elapsed time in different ways. In other words, they have difficulties with interpreting the meaning of dependent and independent variables, in terms of the concept of function. There were 16 HU students who could solve Part III (wrote down the rule).

Task 5

In Task 5, a function was illustrated by arrow diagram; cohesive elements were indicated by arrows. Only 6 UA students of 24 could recognise the rule, based on which the elements of set X had been assigned to the elements of set Y. 5 students managed to phrase this assignment rule. This low ratio might be due to the "unpopularity" of teaching the function concept with the help of arrow diagrams. This fact is also supported by a research of Vinner [19] where he stated that a function, as taught in schools, is often identified with just one of its representations, either the symbolic or the graphical one. In this way, students are less capable of interpreting and understanding this type of representations of cohesive values. Most of the students attempted to solve this task but later on, they overlooked the cohesive elements given as an example and they assigned the remaining elements to each other using a different rule (e.g. they paired each non-negative value with their negative equivalents).

HU students managed to accomplish this task more effectively than UA students, although low success ratio occurred in here as well. 10 students recognised an applicable rule and managed to phrase it. There were 13 students who, in spite of the fact that they did not put the recognised assignment rule into words, the way they paired the remaining elements reflected correct judgements.

Overall, we can conclude that recognizing a connection between cohesive elements means difficulty for the students of both countries if these elements are represented by arrow diagram (which is a representation form of function).

Task 6

Reading plotted points on a coordinate plane required little effort from the students of both countries. All of the 20 HU and 22 UA students managed to fill in the table with this set of ordered number pairs and even could find two additional number pairs. It should be highlighted that most of the students could see none of these additional number pairs on the coordinate plane, since these pairs had not been graphed in the exercise. 8 UA and 16 HU students managed to describe a rule using formula; thus it can be stated that UA students can easily interpret a rule if it is described by a formula (see Task 3). On the other hand, as results of the previous tasks suggest (Task 1 and Task 2 for instance, where the function was represented by a table) these students have difficulties with describing a rule using symbols.

Analysis of the results of certain parts of the tasks

Figure 1 represents an analysis of the success ratio of certain parts of tasks that were aimed at assessing one-step RR and RF. We tried to answer the following questions: whether students will recognise and then follow a rule if cohesive elements are represented by different ways (by a table of values, by words, by graph or by using arrow diagram). We can see the results in the figure. The value of accuracy of arguing in favour of the recognised rule using words or formulas is not included (except for in the case of Task 6).

Comments:

Results of the following tasks were included when creating the diagram: Task 1 (filling in a table based on a recognised rule); Task 3 (filling in a table based on a predefined rule); Task 4, a)-e) (recognizing and following a rule embedded in text); Task 5, part b) (recognizing and following a rule illustrated by arrows, demonstrating how the function behaves by drawing missing arrows); Task 6, part b) (recognising an assignment rule that given by graph), since it reflects if students had recognised an assignment rule.

Clearly, the most problematic field for participants of both countries is recognizing and following a rule, based on which cohesive elements of sets are connected by arrows.

Figure 2 represents the success ratio of tasks that were aimed at assessing if students could recognise a multi-step rule and describe it with the use of formula (Task 2 and Task 4, part III).

Obviously, recognising and describing a multi-step rule with formulas proved to be a more difficult task than what we could experience in the case of a one-step rule. As seen in the Figure 2, recognizing and describing a multi-step rule using formula requires much more effort from UA students. Compering this to the results of one-step RR, the performance of both groups proved to be weaker in the case of multi-step RR and RF.

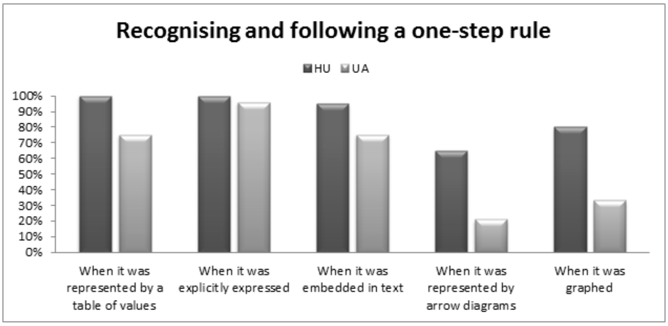


Figure 1

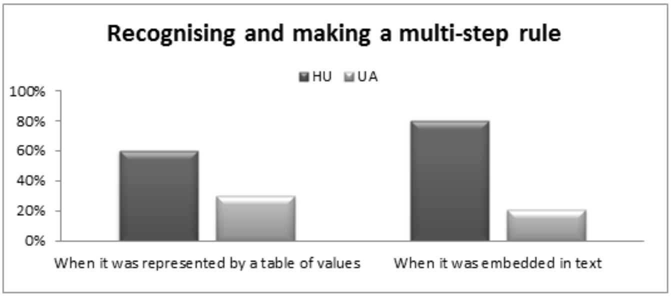


Figure 2

4.2. Do the students from Ukraine develop without targeted development in the RR and RF skills year by year?

As analysing the RF and RR skills of the same UA students who also had participated in the previous research (in 2014) it turned out that they were able to identify a rule between cohesive elements, however there were only 10 of 26 students who could desribe either a one-step or a multi-step rule using words, and there were only 2 who could argue in favour of these rules using formulas [15]. The aim of this study was to attempt to answer if the RR and RF skills of investigated UA students had developed after introducing them to the concept of function and its different ways of represantations.

The figures below represent the success ratio of UA students attempting to answer questions from the present and last year's assessment. These questions were either identical (Task 2 and Task 4, part I and II) or despite differing only in data sets and were the same types of exercises.

Figure 3 represents the success ratio of task from 2014 and 2015 that were used to assess the effectiveness of recognising and following a rule represented in different ways. The numbers in the brackets indicates the tasks which are used in current and previous investigations.

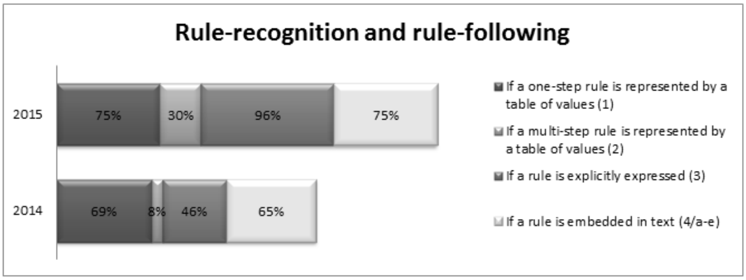


Figure 3

As it can be seen, greater development can be observed in the annual follow-up among investigated UA students, mainly in recognizing and interpreting a multi-step rule, also in following a rule that was explicitly expressed. Thus we can state that introducing the concept of function and its representations had an impact on the development of the students' RR and RF skills .

As it is represented in Figure 4, we could observe no development in arguing in favour of a rule using words; results are relatively stagnant. This allows us to come to the conclusion that the investigated UA students have difficulties with expressing their thoughts verbally. Many of the students, similarly to the previous research in 2014, skipped this part of the tasks, they did not even try to solve it.

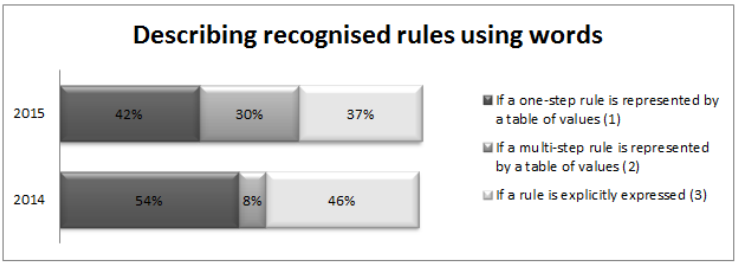


Figure 4

In describing the recognised rule using symbols some minor improvement occured as well (see Figure 5). As it can be seen, those students who were able to recognise a multi-step rule, could easily describe it using formula.

5. Conclusions

The focus of this study was to obtain information on the RR and RF skills of students in a 7^{th} grade of Hungarian and in a 7^{th} grade of Ukrainian schools,

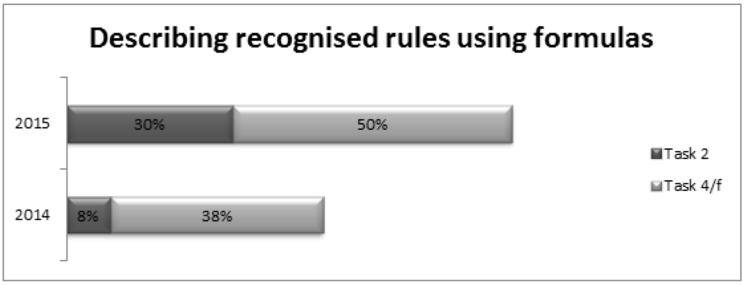


Figure 5

after introducing the concept of function, comparing the consequences to results of previous studies [15].

Considering the results of this current study, the following changes and deficiencies of the annual follow-up can be documented regarding the RF and RR skills of the students:

- Compared to previous studies, the majority of the 24 UA students could recognise and follow a one-step rule represented in different ways (in table form, by words, by graph).

- We could experience development in interpreting and following an explicitly expressed rule (see Figure 3).

Probably, that in the case of UA students, during the preparation process of the function concept, less attention was paid on developing RR and RF skills (before 7^{th} grade), the success ratio of the tasks done by UA students approached, in some cases reached (Task 1/representing, Task 3/a, Task 4/f, Task 6) the success ratio of the tasks done by HU students, whose learning/teaching process was more effectively assembled for improving RR and RF skills.

- Describing rules using words and/or symbols remained a slightly problematic area for the investigated UA students, similarly to last year.

This means, they have difficulties not only with wording their thoughts, but with the field of symbolism as well. Considering the representational complexity of the function concept [1], we can state that regarding the various representation forms of function, describing a function using either words (linguistic complexity) or formulas (mathematical complexity) requires a lot of effort. We could observe some minor development in recognising, following and describing a multi-step rule, but it is not significant (see Figure 3–5).

- Both of the investigated student groups experienced difficulties with recognising a rule in cases when it was represented by arrows (Task 5).

The recognition of functions represented in this way is problematic even for high school students who can define a function correctly [8]. Norman [12] explains this phenomenon with many mathematics teaching methods often favouring functions represented by graphs. This statement can be confirmed by what we have observed when students tried to convert different types of representations. Both UA and HU students needed little effort to convert data from a table to a graph and vice versa. Describing the function using words and/or symbols remained a difficulty for most of the UA student.

Considering the results it also should be highlighted that those UA and HU students who could describe a rule using words and symbols, demonstrated a similar way of thinking.

In conclusion, we can state that introducing the concept of function has an impact on the development of the RR and RF skills. However, results suggest that developing these skills would be useful to start even before introducing the function concept, since HU students performed better in this assessment, similarly to the previous year's.

Studying the characteristics of concept images of the investigated students in each country may be the subject of further researches.

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