PhD Doctoral Thesis

MEASUREMENT POSSIBILITIES OF EQUITY PORTFOLIO RISKS

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1. BACKGROUND, OBJECTIVES AND HYPOTHESES OF THE RESEARCH

Life always gives us decisions to make. We make hundreds of small decisions every day and we face even bigger decisions in our lives. In fact, life is a chain, a process of these smaller and bigger decisions. Consequently, our decisions play a vital role in what turn our lives might take. Decision-making is the willing acceptance of liability for the risk involved (RÓKUSFALVY, 2002).

I think it is clear from the above reasoning that risk is present everywhere and directly or indirectly has an influence on our decisions and thus on our lives. That is the reason why I consider risk a significant topic to discuss and that is why I have chosen to focus my thesis on questions related to risk. Obviously risk is a very broad notion. In today's modern, globalized world it is extremely important to understand, to measure and to predict the risk also in the financial area. In my thesis, therefore, I focus on financial risk.

From a financial point of view modelling and the quantification of risk play an important role. And because of this, mathematics naturally appears. It is not an easy task for financial mathematics, similarly to all interdisciplinary research areas, to harmonize theory and practice. There are many mathematical theories and models available in literature which can hardly or cannot be used in practice at all. There are also many instances of mathematical models which are used inaccurately or even wrongly. This, of course, can lead to misunderstanding, incorrect results and conclusions.

One of the main goal of my thesis is to harmonize the theory and practice as well as possible. I make an attempt to define and clarify mathematical concepts and models precisely and to apply them appropriately to specific stock price data. I do this by examining questions connected with topics like measuring the risk of portfolios or finding the optimal portfolio.

It cannot be argued that there is a need for measuring, modelling and mathematically describing the financial risk. It is also important that the results should be comparable in order to facilitate the decision-making process. Even though these questions are coming more and more into the focus of researchers and scientists, it remains unclear what the mathematical notion, the measure which describes and calculates the risk accurately could be. My goal is to examine both on the theoretical and practical level which one of the most frequently used risk measures measures the risk more accurately, hence which can be considered to be more suitable. In my thesis I consider five risk measures, namely the variance, the standard deviation, the semivariance, the Value at Risk (VaR) and the Expected Shortfall (ES). Using the so-called backtesting method I make a more detailed analysis on comparing the Value at Risk and the Expected Shortfall. Furthermore, I examine what is the effect on optimal portfolios if one uses the mean-ES optimization model instead of the traditional mean-variance portfolio optimization model.

For risk management it is a major challenge to model the risk in high dimensions and to understand and predict the extreme events (EMBRECHTS et al., 2005). For multidimensional structures I describe copula functions and I examine whether these copula functions can help to predict the risk more accurate. For the empirical study I considered seven stocks (listed on the Budapest Stock Exchange) price data over a ten years period. Because of applicability of certain models and comparability reasons, in most cases I use return data calculated from stock prices. Consequently, my further aim in the thesis is to examine whether a simple or logarithmic return should be used for calculations and whether the evaluation of results depends on the type of a return.

During my research I considered the following hypotheses:

H1. Using the simple or logarithmic return has no effect on the results of the analysis.

H2. Using the Expected Shortfall (ES) instead of the Value at Risk (VaR) gives a more accurate measure of the portfolio risk.

H3. Using the mean-ES portfolio optimization model instead of the traditional mean-variance optimization model changes the composition of the optimal portfolio.

H4. Using copula function can improve the accuracy of measuring the risk.

2. DATA AND METHODS USED

2.1. The Data

For the empirical calculations, I worked with Hungarian daily stock prices between 01.07.2005 and 29.06.2015. The data was downloaded from the Budapest Stock Exchange homepage (www.bet.hu). I focused on seven stocks, namely FHB, MOL, MTELEKOM, OTP, Pannergy, Raba and Richter and analysed them in the mentioned time interval. Missing values were filled by the previous day data. The dataset over these ten years consists of 2607 daily stock prices of these seven stocks. Because of comparability reasons, it is obvious that one uses instead of the prices the returns of an asset. I considered logarithmic returns in my thesis. To perform the analysis, I used the mathematical software R and the package Rmetrics. The basic statistics calculated from the logarithmic returns (expressed in percentages) are summarized in Table 2.1.

Table 2.1: Basic statistics calculated from the logarithmic return of seven stocks

	FHB	MOL	MTELEKOM	OTP	Pannergy	Raba	Richter
Minimum	-19,720%	-16,223%	-12,573%	-16,235%	-16,138%	-16,229%	-12,189%
1. quantile	-1,140%	-1,184%	-0,905%	-1,314%	-0,786%	-0,900%	-0,954%
Median	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%	0.000%
Mean	-0,025%	-0,008%	-0,031%	-0,009%	0,000%	0,021%	0,012%
3. quantile	0,970%	1,176%	0,860%	1,351%	0,645%	0,852%	0,961%
Maximum	20,891%	14,027%	11,680%	20,916%	13,961%	24,701%	9,074%
SD	2,385%	2,268%	1,701%	2,686%	1,931%	2,131%	1,821%
Skewness	0,248	0,153	-0,535	-0,059	0,294	0,689	-0,105
Kurtosis	8,860	5,966	6,379	6,128	12,359	15,446	3,533
Range	40,611%	30,25%	24,253%	37,151%	30,099%	40,93%	21,263%
IQR	2,11%	2,36%	1,765%	2,665%	1,431%	1,752%	1,915%

Source: Own calculation

Most theories assume normality of return distributions. In practice this normality is rarely realized, which can lead to incorrect conclusions. After examining the basic statistics, skewness, kurtosis, several test results (Shapiro-Wilk and Kolmogorow-Smirnov), plots and figures (histogram, Q-Q plot), I found that the logarithmic returns of the stocks do not follow normal distributions as many mathematical models in finance assume.

2.1.1. The Return

As I already mentioned, I use in the calculations instead of the prices the returns of a stock. Returns can be calculated in different ways. The two, most often used returns are the so-called simple and logarithmic returns.

The simple return of an asset at time t is defined by:

$$R_t^S := \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1, \tag{2.1}$$

where P_t is the price of the asset at time t. The logarithmic return of an asset at time t is expressed by:

$$R_t^L := \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1 + R_t^S). \tag{2.2}$$

Similarly to the stock returns, one can calculate the return of a portfolio. Let us consider a portfolio which consist of n assets. The simple return of this portfolio is

$$R_t^S := \frac{S_t}{S_{t-1}} - 1 = \sum_{i=1}^{n} w_{t-1,i} R_{t,i}^S \tag{2.3}$$

while the logarithmic return is calculated by

$$R_t^L := \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln\left(1 + \sum_{i=1}^{n} w_{t-1,i} R_{t,i}^S\right) = \ln\left(\sum_{i=1}^{n} w_{t-1,i} \exp\left(R_{t,i}^L\right)\right). \tag{2.4}$$

In equations 2.3 and 2.4, S_t refers to the amount of money invested in the portfolio at time t, $w_{t,i}$ refers to the relative weights of the asset i at time t, $R_{t,i}^S$ and $R_{t,i}^L$ refer to the simple and the logarithmic returns of asset i at time t respectively.

It is important to add, that the relative weights change in time according to the asset prices. In case of an equally weighted portfolio the number of assets in the portfolio has determined at every time point. It is also important to highlight that the portfolio simple return is the sum of the weighted simple returns of its constituents. Unfortunately, the logarithmic return of a portfolio does not have a similar convenient property: the portfolio logarithmic return can be "only" approximated by the sum of the weighted logarithmic returns of the constituents of the considered portfolio.

2.2. Methods Used

Finance and mathematics go hand in hand. Mathematics provides tools and models for finance, while finance provides questions and problems to be solved for mathematicians. Therefore it is important that the two areas understand each other. In my thesis - within some of the topics of financial risk - I tried to combine the mathematical theory with the financial practice.

2.2.1. Risk Measures

Risk and risk measures have no unique definition and usage in literature . However, from a mathematical point of view, the goal is to express the risk by the simplest indicators, preferably with only one number. Risk measures are defined mappings from a set of financial assets or portfolio returns to the real numbers (GÁLL – PAP, 2010).

Mathematically we can formulate the risk as follows: Let $\mathcal{X}$ be a set of random variables (the profit/value of the set of corresponding portfolios or financial assets) over a probability space $(\Omega, \mathcal{F}, P)$. A function $\rho : \mathcal{X} \rightarrow \mathbb{R}$ is called risk measure. This means actually that measuring the risk is equivalent to establishing a correspondence between random variables and a real number. This correspondence without restrictions is a very broad concept. In order to get a "meaningful" definition of a risk measure, some properties and restrictions has to be satisfied. Naturally, there are many such restrictions. In literature (for example ARTZNER et al., 1999; RÜSCHENDORF, 2013; EMMER et al., 2015) most often used properties are the following (GÁLL – PAP (2010)):

Monotonicity
If a portfolio never has smaller values than another one, then it should not have larger risk than the other one:

$$X \leq Y \Rightarrow \rho(X) \geq \rho(Y), \qquad \forall X, Y \in \mathcal{X}. \tag{2.5}$$

Subadditivity
Subadditivity refers to the diversification effect. Merging two portfolios should not increase the riskiness:

$$\rho(X + Y) \leq \rho(X) + \rho(Y), \qquad \forall X, Y, X + Y \in \mathcal{X}. \tag{2.6}$$

Positive Homogeneity
If one multiplies the portfolio value but keeps the relative proportions then the risk of the portfolio should change proportionally to the value of it:

$$\rho(\lambda X) = \lambda \rho(X), \qquad \forall X, \lambda X \in \mathcal{X}, \forall \lambda \geq 0. \tag{2.7}$$

Translation Invariance
If one realizes an additional non-random, fixed cash flow then the risk should be reduced by exactly the sum of that cash flow:

$$\rho(X + a) = \rho(X) - a \qquad \forall X, X + a \in \mathcal{X}, \forall a \in \mathbb{R}. \tag{2.8}$$

A risk measure is called coherent, if it satisfies the above mentioned four properties, such that it is monotone, subadditive, positive homogeneous and translation invariant.

In order to measure the risk as accurate as possible, a coherent risk measure is required.

In my thesis, I consider the following risk measures in more detail:

<u>1. Variance and Standard Deviation</u>
Markowitz invented the variance as a risk measure and hence variance is one of the oldest risk measure. Standard deviation is the square root

of the variance. In this case one measures the risk associated to the return of a portfolio as the mean of the squared deviation of the asset returns from the portfolio mean (SZEGÖ, 2002):

$$\sigma^2(X) := \mathbb{E}(X - \mathbb{E}(X))^2. \tag{2.9}$$

Higher variance refers to higher risk (BACON, 2011).

2. Semivariance

Semivariance is a so-called down-side risk measure. This means that it considers only the values smaller than the expected value (expected return). (ALEXANDER, 2009):

$$SV(X) := \mathbb{E}((\min\{X - \mathbb{E}(X), 0\})^2). \tag{2.10}$$

3. Value at Risk (VaR)

In the recent years Value at Risk is the most common used risk measure. VaR is a number from which bigger loss can happen at the probability of smaller than or equal to α:

$$\text{VaR}_\alpha(X) := -\inf\{x \in \mathbb{R} | F_X(x) \geq \alpha\}. \tag{2.11}$$

4. Expected Shortfall (ES)

Value at Risk answers to the following question: What is the minimum potential loss that a portfolio can suffer in the worst $100 * \alpha\%$ cases. However, expected shortfall answers to the question, that what is the expected loss that a portfolio can suffer in the worst $\alpha * 100$ cases (ACERBI et al., 2001), thus expected shortfall is the expected value of the loss (profit) in the worst $100 * \alpha\%$ cases:

$$ES_\alpha(X) := \frac{1}{\alpha} \int_0^\alpha \text{VaR}_u(X) du. \tag{2.12}$$

2.2.2. Copula Functions

One of the main difficulty in the financial and economic sector is to estimate and to model multidimensional distributions. Mathematical theories and models assume in many cases normal or at least elliptical distributions of the data, although empirical data fulfilles this condition rarely (HÄRDLE et al., 2008). Additionally, there are dependency structures where the correlation coefficients are the same, but the structures themselves are completely different. Moreover, the well known correlation coefficients, such that the Pearson correlation coefficient, the Kendall and the Spearman coefficients can be used to discover linear and rank correlations. Higher order and more complex dependency structures can be measured by the so-called copula functions.

An n-dimensional copula is an n-dimensional distribution function with uniform marginals on the unit interval [0, 1].

Sklar's theorem is the basic theorem in the topic of the applications of copula theory. The theorem states, that a multivariate cumulative distribution function can be expressed in terms of its marginals and a dependence structure between them. And this dependence structure is the copula. The theorem also shows how to construct new n-dimensional cumulative distribution functions.

In my thesis, I consider in more detail the Gauss (or normal) and Student-t copulas from the implicit copula family; the Gumbel, the Clayton and the Frank copulas from the explicit copula family. The density function of these copulas in case of two variables are plotted in Figure 2.1.

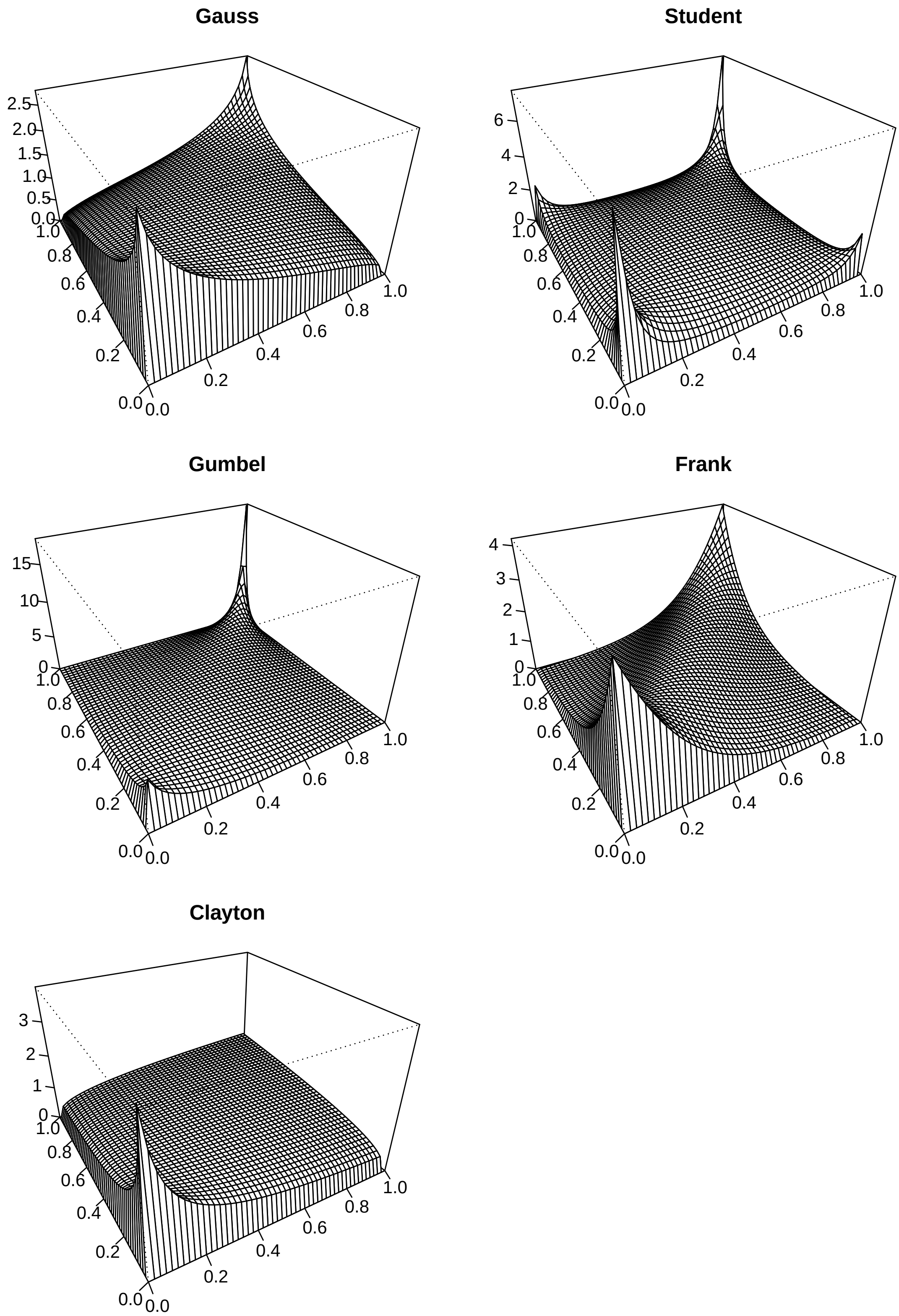

Figure 2.1: Density functions of Gauss, Student, Gumbel, Frank and Clayton copulas

Source: Own edition

3. MAIN CONCLUSIONS OF THE THESIS

3.1. Coherent Risk Measures

Measuring the risk accurately requires the use of a coherent risk measure. Among the examined risk measures, variance (Equation 2.9) does not satisfy any of the necessary properties. Such that, it is for instance not subadditive and non-subadditivity implies that portfolio diversification, namely, that diversification should not increase the risk of a portfolio. An additional problem with the variance is that it does not distinguish between profit and loss. It can be used for measuring the risk only if the distribution of the random variable (profit, returns, etc.) belongs to the class of elliptical distributions (EFTEKHARI et al., 2000). The standard deviation (unlike variance) is subadditive and positive homogeneous, but it is neither monotone nor translation invariant. And similarly to the variance it does not distinguish between profit and loss. It can be proven that the semivariance (Equation2.10) is translation invariant but it is not monotone, not positive homogeneous and not subadditive. Non-subadditivity implies that portfolio diversification may lead to an increase of risk. And this is in contrast with the Markowitz portfolio theory, in which the subadditive property is essential: no new investment increases risk. Semivariance is a downside risk measure, so it is not symmetric. From this point of view it can measure risk more accurate than variance or standard deviation, but it satisfies only one property of the four conditions. So since the semivariance is not coherent, it is not recommended to used it as a risk measure.

Value at Risk (Equation 2.11) determines the risk of a portfolio in a single number and as a currency, in contrast to the standard deviation which describes an interval. Hence VaR can be easily interpreted and values can be easily compared among different portfolios (ACERBI et al., 2001). Similarly to the semivariance, VaR has also the benefit of being a downside risk measure. However, VaR is heavily criticized because of its negative features. One of the main problem with VaR is that it does not measure losses exceeding VaR (as a value), in spite of that knowing and modelling extreme events would be important. An additional problem with VaR is that it does not satisfy subadditivity and hence does not incorporate the diversification assumption. Furthermore, at different confidence levels, it can provide conflicting results (SZEGÖ, 2002). Above all, non-convexity of VaR makes it impossible to use in optimization problems (WOZABAL, 2008). VaR can be used for measuring the risk only in the case of elliptical distributions. In the non-elliptical world (most of the data used in practice is not elliptical), VaR may give misleading results.

In contrast to the above mentioned risk measures, Expected Shortfall (Equation 2.12) is monotone, subadditive, positive homogeneous and translation invariant, that is, ES is a coherent risk measure. Nevertheless, it has also a drawback, since the existence of ES requires the assumption of $\mathbb{E}((X)^-) < \infty$, while VaR always exists.

To measure a risk as accurate as possible I was looking for a risk measure which is monotone, subadditive, positive homogeneous and translation invariant, thus coherent. In Table 3.1 I summarized the properties of the variance, the standard deviation, the semivariance, the Value at Risk and the Expected Shortfall. One can see, that considering the most common used risk measures, only ES coherent.

Table 3.1: Properties of some known risk measures

	Monotonicity	Subadditivity	Positive Homogeneity	Translation Invariance
Variance	-	-	-	-
Standard Deviation	-	✓	✓	-
Semivariance	-	-	-	✓
VaR	✓	-	✓	✓
ES	✓	✓	✓	✓

Source: Own edition

3.2. Simple vs. Logarithmic Return

It can be proven (for example in the case of stocks and portfolios) that if the simple return is near to zero, it is very comparable to the logarithmic return. This raises the question whether the used return-type (i.e. simple or log return) has an effect on the calculations and thus on the results. To answer to this question I considered the so-called riskiness order. I calculated the risk (variance, standard deviation, semivariance, VaR and ES) of all the considered stocks using simple (Equation 2.1) and logarithmic returns (Equation 2.2). On the first place in the riskiness order stands the most risky stock with respect to the fixed risk measure. Finally I compared these orders. A similar method was used in the case of portfolios. I considered seven portfolios, each of which consist of six distinguishing stocks (I left away one of the seven stocks) and I calculated the risk of all these portfolios using simple (Equation 2.3) and logarithmic returns (Equation 2.4) in order to generate and compare the riskiness orders. The results of the calculations show different riskiness orders depending on the return type used in case of both stocks and portfolios eventhough the two return-type values are very similar. These different riskiness orders suggest different decisions in the same situation depending on the used return type. Results show also clearly that not only the type of return (simple or logarithmic) or the given risk measure can play a major role in creating the riskiness order but also the chosen alpha level and this may change the decision.

3.3. Backtesting

It can be proven that the variance, the standard deviation, the semivariance, and even the most often used VaR are not suitable for measuring the risk, since none of them satisfy the four expected properties on a coherent risk measure. However, there exists a coherent risk measure: the expected shortfall. I compared the two most common used risk measures, i.e. VaR and ES using so-called backtesting methods. I examined which risk measure can be considered to be "better". I call a risk measure better if one can measure the risk more accurately, meaning, if the difference between the values it estimates and the actual values is smaller.

For the backtesting method, one needs to organize the data into so-called time windows. In literature and practice one considers as time windows typically discrete time intervals of containing 250 business days, i.e. one year ($T = 250$). This means that the first time window lasts from day 1 to day 250. The second time window lasts from day 2 to day 251, etc. Using the data in the first time window one can calculate the risk, which is a prediction for the next day i.e. for the day 251. If the return is available also on the day 251 then the estimated value based on the first time window can be compared with this return value. Now, one can calculate the risk using the data in the second time window, which is a prediction for the next day, i.e. for the day 252. If the return available also on the day 252 then the estimated value can be compared, and so on, using all the available time windows. This method is called historical simulation, because it uses historical data and based on the assumption that the past describes, predicts the future.

Backtesting VaR means that one can calculate the VaR for all the above described time windows, which are estimates for the next days. After that one can compare the estimated and the real values and count how many times the VaR underestimated the risk, i.e. whether the return is smaller than (-1)-times the estimated VaR value for the considered day. According to the definition of the value at risk (Equation 2.11), the probability that VaR underestimates the risk equals the confidence level, i.e. equals α. Based on this, and considering the realization of the random variables, one can estimate α as follows:

$$\widehat{\alpha} = \frac{1}{M-1} \sum_{m=1}^{M-1} \mathbf{1}_{\{r_T^{m+1} < -\,\mathrm{VaR}_\alpha^m\}}, \tag{3.1}$$

where M denotes the number of time windows, $T = 250$ is the length of one time window, r_t^m is the return at time t, $t = 1, \ldots, T$, and time window m, $m = 1, \ldots, M$. VaR_α^m is the VaR for time window m at the level of α, and $\mathbf{1}_{\{.\}}$ is the indicator function of the set $\{.\}$. The close the estimated alpha value to the real alpha value is, the more accurate the VaR estimates the risk.

Many people were convinced that ES is not backtestable because of its lack of a mathematical property (called elicitability). New research seem to solve this problem, since it is shown that expected shortfall is in fact backtestable. The following expression can be deduced from the definition of the expected shortfall for estimating α using realization of random variables:

$$\widehat{\alpha} = -\frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{m=1}^{M}\frac{r_t^m \mathbf{1}_{\{r_t^m < -\mathrm{VaR}_\alpha^m\}}}{ES_\alpha^m}, \tag{3.2}$$

where ES_α^m denotes the ES for time window m. The closer the estimated risk is to the real risk the more accurate the ES predicted the risk and the closer the estimated and the real alpha values are to each other.

One can see the results of the historical backtesting in Table 3.2 in case of VaR and ES, at the level of $0,5\%$, 1%, 2%, $2,5\%$ and 5%. I examined the absolute errors at different α levels and also the higher dimensional relative errors. Considering these relative and absolute errors, one can say that expected shortfall measures the risk more accurate than value at risk.

Table 3.2: Result of historical backtesting

	VaR		**ES**	
α	$\widehat{\alpha}$	$\lvert\widehat{\alpha}-\alpha\rvert$	$\widehat{\alpha}$	$\lvert\widehat{\alpha}-\alpha\rvert$
0,5%	0,89%	0,39%	0,41%	0,09%
1%	1,27%	0,27%	0,83%	0,17%
2%	1,99%	0,01%	1,68%	0,32%
2,5%	2,72%	0,22%	2,42%	0,08%
5%	5,17%	0,17%	4,85%	0,15%

Source: Own calculation

3.4. Portfolio Optimization

After I showed in theory and in practice (using historical backtesting) that ES is more appropriate risk measure than VaR, I investigated portfolio optimization. I compared the traditional Markowitz mean-variance portfolio optimization model with a similar model which uses expected shortfall for measuring the risk (mean-expected shortfall portfolio optimization model).

Modern portfolio theory (MPT) (was introduced by Markowitz) considers the return of an asset as a random variable and the return of a portfolio as a weighted combination of the constituent assets' returns. Moreover, MPT assumes that the return of an asset follows a normal or elliptical distribution, that there are no transaction fees, that investors are rational and risk averse, i.e. between two portfolios which offer the same expected return they choose the less risky one, and that there and assets can be divided freely (for example one can buy 0.5869 units of a stock). The goal of portfolio optimization is to minimize the risk for a given target return. In the Markowitz model, return and risk are estimated by the sample mean and the sample variance of the asset returns (WÜRTZ et al., 2009).

In the case of mean-ES portfolio optimisation, the goal is the same as it was discussed in the mean-variance case: minimize the risk for a given target return. In the mean-ES model, the return and the risk are estimated by the sample mean and the ES respectively. In contrast to the mean-variance portfolio optimization, the set of assets are no longer restricted to have an elliptical distribution (WÜRTZ et al., 2009).

Therefore, the question is whether we would get different optimal portfolio compositions using the ES instead of the variance for measuring the risk. In order to answer to this question, I calculated the weights on the efficient frontier in the case of the mean-variance and the mean-ES optimization at the level of $\alpha = 5\%$ and $\alpha = 1\%$. Using in section 2.1 introduced seven stocks, the portfolio which has the maximum Sharpe-ratio offers a return of 0.02%. Thus, I considered three different target returns between 0% and 0,02% (0% <return< 0,02%) and examined the weights in the optimal portfolios. The results are shown in Figure 3.1.

In Figure 3.1 one can see that there are differences between the optimal portfolios using one or the other method. If we fix the target return at

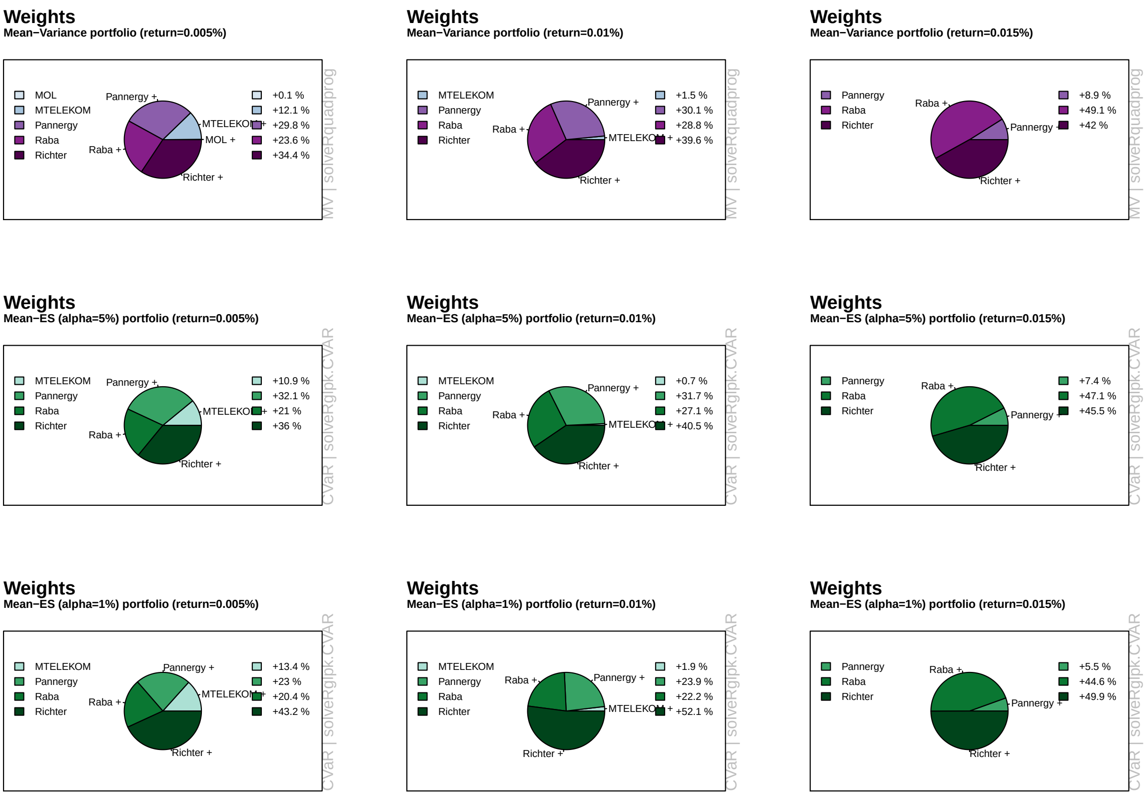

Figure 3.1: Optimal portfolio weights on the efficient frontier for different fixed expected returns

Source: Own calculation

0,015% or at 0,01% then the optimal portfolio consists of the same stocks, but they have different weights. The optimal portfolio with fixed 0,015% return consists of three stocks: Pannergy, Raba, Richter (but with different weights), while the optimal portfolio with fixed 0,01% return consists of four stocks: Richter, Pannergy, Raba and MTELEKOM (but also with different weights). In these specific cases – because of the differences in the weights – we would decide differently concerning the composition of the portfolio. There is an even a bigger difference for example in the case of fixed 0.005% target return, because in addition to the different weights also the compositions of the portfolios are different. The model which minimizes the variance suggests five stocks in the optimal portfolio: Richter, Pannergy, Raba, MTELEKOM, MOL. Based on the minimal $ES_{0,05}$ and $ES_{0,01}$ model the optimal portfolio consists of four stocks. Thus MOL is not in the optimal portfolio. This example shows that the two models suggest not only different weights, but also different compositions in the optimal portfolio.

3.5. The Copula Function

As it was shown already diversification plays an important rule to reduce the risk of a portfolio. In literature and in practice also known risk measure, which supports the diversification is the expected shortfall. Nevertheless, in history, one had to face several financial crises. One reason for these crises may be that this risk-reducing effect of the diversification can, however, be cancelled by the same behaviour of the risk factors in the portfolio. Diversification can be worthless if all or some of the stocks in the portfolio react on an event with the same extreme behaviour. Such negative events occur more often than we would think in the case of elliptical distributions (BENEDEK et al., 2002).

In order to avoid large losses and to provide more accurate modelling, it is necessary to understand and measure the dependence of the random variables, i.e. the relationships of the assets in the portfolio. Analysing the pairwise plots of the stocks' logarithmic returns (Figure 3.2) one can see that the stocks are not independent, there is some correlation between every pair.

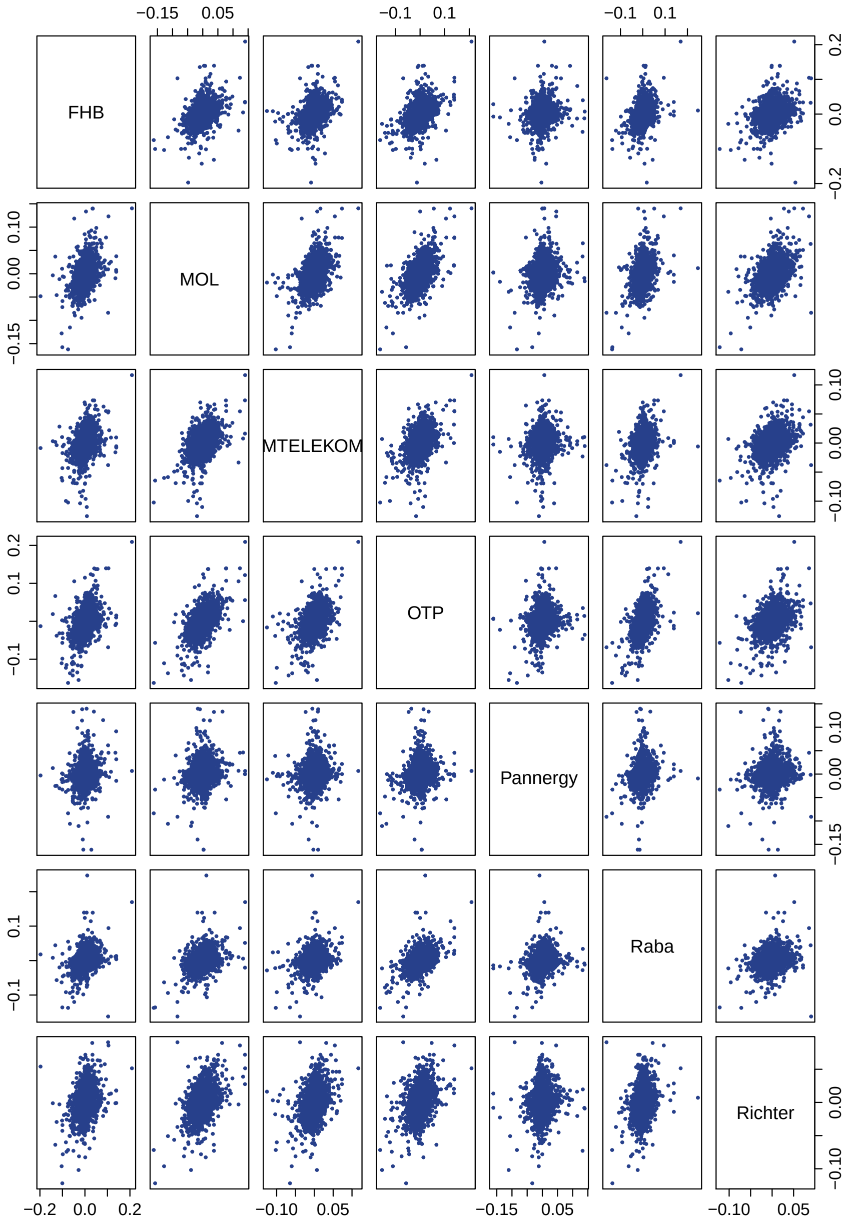

Figure 3.2: Pairwise scatterplots of the stocks' logarithmic returns
Source: own edition

Pairwise linear correlation can be measured by the Pearson correlation coefficient, while rank correlation can be measured by the Kendall and Spearman coefficients. The correlation coefficients are positive in all the cases, which confirms the assumption based on the pairwise plot, that there is some correlation between every stock pairs. Based on the above mentioned three correlation coefficients, the strongest correlation was measured between OTP and MOL, while Pannergy is almost independent from all the other stocks.

It is important to analyse the correlation between stocks in a portfolio in order to measure the risk of this portfolio. Nevertheless, a more accurate risk measure requires more than measuring pairwise and/or linear correlations. Higher order and more complex dependency structures can be measured by copula functions. I applied backtesting method (introduced in section 3.3) to show that by using copula functions for measuring the risk leads to more accurate riak estimates. But instead of using historical data I used data generated by Monte Carlo simulation based on copula functions.

Steps for copula-based Monte Carlo simulation of estimating portfolio risk (XU – CHEN, 2012) are:

1. Select a copula model (Gauss, Student, Archimedean etc.)
2. Select marginal distributions and estimate the parameters of them
3. Transform the original data into the domain of copula function
4. Fit the copula model to the transformed data and estimate the parameters of this copula function
5. Use the estimated copula function to generate random variables from the associated joint probability density function
6. Invert the generated random variables by using the inverse function of the marginals
7. Calculate the portfolio loss/profit based on the simulated data
8. Repeat steps 1-7 many times to be confident that the simulation distribution is sufficiently close to the "true" distribution
9. Calculate VaR and ES based on the distribution of the simulated portfolio returns

I compared the most commonly used copula functions in practice, namely the normal (Gauss) and Student-t copulas from the implicit copula family and the Gumbel, the Clayton and the Frank copulas from the explicit copula family. (The density functions of these copulas are plotted in figure 2.1). I had to leave away the Gumbel copula, because it only applies to positively correlated data and this positive correlation has to hold for all the time windows. It turned out that in the first time window the Kendall's tau between Raba and Pannergy equals to -0.004 and thus the Gumbel copula could not be considered further in the analysis.

I estimated the distribution of the stocks by the empirical distribution function. These distributions are the marginal distributions. For estimating the parameters of the chosen copula, I used the so-called canonical maximum likelihood method, because this method is based on the empirical marginal distribution functions. The theorem of Sklar says that the marginal distributions and the copula function defines the joint distribution function. Having such a joint distribution function one can simulate the logarithmic return of the stocks in the portfolio. Using these simulated data one can apply the backtesting method for the VaR (Equation 3.1) and for the ES (Equation 3.2) too.

In Table 3.3 one can see the results of the copula based Monte Carlo backtesting for VaR and ES at the level of 5%, 2.5% and 1%. The numbers in Table 3.3 show the differences between the estimated and the real (original) alpha values, since this determines the accuracy of measuring the risk. The result in Table 3.3 shows, that among these

Table 3.3: Results of copula based Monte Carlo backtesting and historical backtesting

	VaR			ES		
alpha / model	**5%**	**2,5%**	**1%**	**5%**	**2,5%**	**1%**
Gauss	0,7749%	0,8121%	0,5287%	0,0125%	0,0133%	0,0142%
Clayton	0,6900%	0,4299%	0,1890%	0,0120%	0,0128%	0,0138%
Frank	1,4112%	1,4490%	0,9958%	0,0128%	0,0135%	0,0144%
Student	0,9023%	0,8121%	0,4013%	0,0123%	0,0130%	0,0139%
historical	0,1761%	0,2153%	0,2728%	0,0149%	0,0775%	0,1712%

Source: Own calculation

four copulas at each alpha level, I found that the Clayton copula best estimates the risk in both the VaR and ES cases, while the worst result was given by the Frank copula. In the financial area the Student copula is preferred over normal copula, because the Student copula has a more fat tail than the normal copula and that is why it is more suitable for modelling extreme events. In most cases, data are not normally distributed and it is not so easy to model extreme events. They occur usually more often than one would think in the case of assuming normally distributed data. The results of my study also supports this. It can be seen, that the Gauss copula was better only in the case of VaR at the level of 5%. In the tails and in the case of ES, Student copula measured the risk better than the normal copula.

If I compare VaR with ES, it is evident that ES considered to be a better risk measure in the case of all copulas, at all the alpha levels. This suggests again to use ES instead of VaR.

It is a further important question whether with these copula models one can simulate the reality better than using the historical simulation (see Table 3.3). The results shows, that in the case of VaR at the level of 5% and 2.5% the historical simulation predicted the risk better than the copula based Monte Carlo method. However, with the Clayton copula at the level of $\alpha = 1\%$ I could simulate the reality better than using historical simulation. This means that in this situation the Clayton copula – which was the best in my study from the considered four copulas – could measure the extreme events and so that the risk better than the historical simulation.

In the case of ES the results are much more straightforward than in the case of VaR. All the four copula models, at all the alpha levels predicted the risk better than using the historical simulation.

4. NEW AND NOVEL RESULTS OF THE RESEARCH

1. I summarized the theoretical and practical information of some known risk measures. I considered five risk measures: the standard deviation, the variance, the semivariance, the Value at Risk (VaR) and the expected shortfall. It was proven that only the ES is coherent, so from the considered risk measures only ES is suggested to be used for measuring risks. In addition, I even showed how to calculate these risk measures using realization of the random variables.

2. I clarified the definitions, the notions and the differences as well as the connections between the simple and logarithmic returns in the case of stocks and portfolios. In a small empirical study, by examining the effect of the used returns type on the riskiness order, I have proved that the type of the return has an influence on the results of the study.

3. I clarified the description of the historical backtesting for VaR and ES. For a long time it has been thought that ES is not backtestable, I find a novel result backtesting ES for specific stock data. Using this backtesting method I showed that ES can be considered to be a more adequate risk measure than VaR. In the analysis I used data having non-normal and non-elliptical distributions.

4. In an empirical study I showed that using the mean-ES model instead of the traditional mean-variance model for optimizing portfolios can change the composition of the optimal portfolio.

5. I have highlighted how important it is to discover and to model the dependence structure of the assets in the portfolio. In higher dimension it is possible with the use of copula functions. With the help of copula functions I could investigate the dependence structure in seven dimensions. Using the copula based Monte Carlo simulation together with the backtesting method, I examined which

copula function can be better in practice. This simulation can be taken as a proof of the fact that copula functions can improve the accuracy of risk computations.

5. PRACTICAL AND THEORETICAL APPLICABILITY OF RESULTS

The goal of my thesis was to examine the ways with which the risk could be measured and modelled more accurately. One of the most difficult tasks proved to be harmonizing theory and practice. Modelling the reality as precisely as possible plays a decisive role in the decision. Forecasting the future can lead to the correct decisions, which process requires the consideration of different options and possibilities.

To be able to weigh the different options, it is essential for the models and results to be comparable. For example, one cannot compare the prices of the stocks because the prices move on different scales. Calculations and conclusions using these data are only possible if they are measured on the same scale. That is the reason why one uses the simple or the logarithmic returns instead of the prices. It can be shown that simple and logarithmic return data – calculated from the same price data – are usually (if the simple returns are close to zero) close to each other. Therefore the question arises whether the return type used has any influence on the results. The analysis of the riskiness orders clearly showed that the result of the calculation, i. e. the riskiness order can depend on the type of data used. The results would lead to different decisions in the same situation. That is the reason why I consider it of vital importance to use the same type of return within one study and one must be aware of the possible instabilities when comparing return results. With this I would like to highlight the fact that using mathematical tools and models must be treated very carefully in practice. It is important to describe the applied model clearly, mathematically correctly and it must be understood before using it in the analysis.

Additionally, it is remarkable that not only the return type used but also the chosen α level has influence on the riskiness order. My study shows different riskiness order at different α levels. This can lead to a choice of another decision.

In the financial sector, in regard to decision making, it is also important to measure the risk as accurately as possible. Based on theoretical calculations and empirical studies I discovered that all the risk measures considered have properties which can lead to false conclusions. VaR is still today the most commonly used risk measure, in spite of the fact that it is heavily criticized (because it does not measure losses exceeding VaR and it does not support diversification). The Expected Shortfall satisfies all the required properties, thus ES is a coherent risk measure, although it is not so often used. Using the historical backtesting I compared VaR and ES at five different alpha levels. VaR was better only at one alpha level, in all the other cases ES measured the risk more accurately. Considering the theoretical and the empirical results, I think, that with the expected shortfall, risk can be measured more precisely and in my opinion the usage of ES in the future should be emphasised and encouraged, for example in the area of portfolio optimization. If we compare optimal portfolios in the case of using the traditional mean-variance optimization model with the case of using mean-ES optimization model, one can see, that depending on the fixed expected return, the chosen alpha level and the risk measure the optimal portfolios have different compositions. And, of course, this has an influence on the decision for the optimal portfolio.

As for measuring the risk, it is important to examine the relationship as well as the dependence structure of the assets in the portfolio in question. The risk-reducing effect of the diversification becomes meaningless if all or some of the stocks in the portfolio react on an event with the same extreme behaviour. I believe that precise modelling requires more than linear and/or pairwise correlation measures. Copula functions can be used to discover higher dimensional and non-linear correlations. After comparing the results of the copula-based simulation with the historical simulation, it was shown that the copula based method can improve the accuracy of measuring the risk.

It should be noted, however, that the conditions of applicability of a model must always be considered. (For example, applying geometric

Brownian motion to simulate stock prices would give misleading results applied on non-normal logarithmic returns; or VaR can only be used for elliptically distributed data to measure the risk, and most of the data used in practice is not elliptical.) Failure or omission to do so can lead to misleading results which can have serious consequences.

6. LIST OF PUBLICATIONS

1. **Miskolczi, P.**: A történeti és a kopula függvényen alapuló kockázatszámítás összehasonlítása. Pénzügyi Szemle. "közlésre elfogadva", 2018
2. **Miskolczi, P.**: Note on simple and logarithmic return. *Apstract 11* (1-2), 127-136, 2017
3. **Miskolczi, P.**: A Value at Risk és az Expected Shortfall összehasonlítása történeti szimuláció segítségével. *SZIGMA*, XLVII (3-4), 139-160, 2016
4. **Miskolczi, P.**: Differences between mean-variance and mean-cvar portfolio optimization models. *Anal. Univ. Oradea. Ştiinţ. econ 25 (1), 548-557.*, 2016.
5. **Miskolczi, P.**, Tarnóczi, T.: Risk Analysis with Financial Ratios. In: 3rd International PhD Students Conference : New Economic Challenges, Masaryk University, Brno, 238-246, 2011.

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List of publications related to the dissertation

Articles, studies (4)

1. **Miskolczi, P.**: A történeti és a kopula függvényen alapuló kockázatszámítás összehasonlítása.
   Pénzügyi Szemle "közlésre elfogadva", [1-20], 2018. ISSN: 0031-496X.

2. **Miskolczi, P.**: Note on simple and logarithmic return.
   Apstract 11 (1-2), 127-136, 2017. ISSN: 1789-221X.
   DOI: http://dx.doi.org/10.19041/APSTRACT/2017/1-2/16

3. **Miskolczi, P.**: A Value at Risk és az Expected Shortfall összehasonlítása történeti szimuláció segítségével.
   Szigma 47 (3-4), 139-160, 2016. ISSN: 0039-8128.

4. **Miskolczi, P.**: Differences between mean-variance and mean-cvar portfolio optimization models.
   Analele Universităţii din Oradea. Ştiinţe economice = Annals of University of Oradea. Economic science 25 (1), 548-557, 2016. ISSN: 1222-569X.

Conference presentations (1)

5. **Miskolczi, P.**, Tarnóczi, T.: Risk analysis with financial ratios.
   In: 3rd International PhD Students Conference : New Economic Challenges, Masaryk University, Brno, 238-246, 2011.

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List of other publications

Articles, studies (1)

6. Tóth, C. T., **Miskolczi, P.**, Csubák, M.: Parlagfű kivonat hatásának in vitro vizsgálata Monilinia laxa ellen.
   Agrártudományi közlemények = Acta agraria Debreceniensis 47, 117-120, 2012. ISSN: 1587-1282.

The Candidate's publication data submitted to the iDEa Tudóstér have been validated by DEENK on the basis of Web of Science, Scopus and Journal Citation Report (Impact Factor) databases.

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