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On Soft Closure Function in Soft Ideal Spaces

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Abstract: As a result of evolution in science by researchers and scientists led to quantum leaps in several different fields of science, including medicine, engineering, and chemistry, physics and other Humanities and social as this evolution based on evolution in various branches of mathematics which is the infrastructure to develop the rest of science. Where definition of (Soft Set) came to solve many problems facing humans. This is because of researcher Molodtsov in 1999 [1]. Where the definition of soft sets was based universal set, $\mathcal{E}_X$ represent the all parameters of X , also the function from $\mathcal{E}_X$ to $\mathbb{P}(X)$. The motivation behind this paper is to present the thought of soft ideal in soft set theory. The idea of soft closure is likewise presented. These ideas are talked about with a view to discover unprecedented soft topologies from the first one The fundamental structure, particularly a reason for such produced soft topologies likewise considered here. At long last, the idea of soft closure compatibility with soft topologies is presented and some proportional conditions. As well as the concept of soft ψ_Γ –Operator and study of the most significant properties

Keywords: soft set, soft closure, soft ψ_Γ –Operator, soft closure compatibility, soft ideal space.

1. Introduction

Researchers who study to resolve complex problems in economics, engineering, environmental sciences, sociology and medicine handles science and many other areas with complex problems of uncertain data modeling that includes vulnerabilities. The Theory of Probability, Theory of fuzzy sets, Theory of Intuitionistic Fuzzy Sets, Theory of Vague Sets, Theory of Interval Mathematics and Theory of Rough Sets are considered as scientific devices for managing vulnerabilities. In 1999, Molodstov [1] called attention to that these speculations which are considered as numerical instruments for managing with vulnerabilities, have certain confinements. He further called attention to that the purpose behind these confinements is, perhaps, the deficiency of the parameterization device of the hypothesis. The soft



set Theory presented by Molodstov [1] is very not quite the same as these hypotheses in this unique circumstance. In 2003, Maji, Biswas and Roy [2] considered the theory of soft sets started by Molodstov and built up a few essential thoughts of soft set theory. At present, specialists are contributing on the augmentation of soft set theory. As of late, In 2011, Shabir and Naz [3] started the investigation of soft topological spaces. They characterized soft topology $\tilde{\mathcal{T}}$ on the gathering of soft sets over $\mathcal{X}$. Subsequently, they characterized essential ideas of soft topological spaces, for example, soft open and closed sets, soft subspace, and set up a considerable lot of their properties. Many researchers have also presented several concepts (see [4], [5],[6])

Hussain and Ahmad [7] examined the attribute of soft open (closed), soft nbd. They additionally characterized and talked about the attribute of soft interior, soft exterior also soft boundaries which are key for further research on soft topology and will reinforce the establishments of the hypothesis of soft topological spaces, many researchers have studied several properties on those sets see ([8],[9]). The motivation behind this paper is to present the thought of soft ideal in soft set theory. The idea of soft closure function is likewise presented. These ideas are talked about with a view to discover new soft topologies from the first one. The fundamental structure, particularly a reason for such produced soft topologies likewise considered here. At long last, the idea of soft closure compatibility with soft topologies is presented and some proportional conditions concerning this theme are set up here.

2. Preliminaries

Throughout this paper, the term $\mathcal{X}$ will refer to an underlying universe set and $\mathcal{E}_{\mathcal{X}}$ be a gathering of all conceivable parameter as for $\mathcal{X}$, where parameters are the qualities or properties of items in $\mathcal{X}$, a set $\mathcal{A} \subset \mathcal{E}_{\mathcal{X}}$ and $\mathbb{P}(\mathcal{X})$ signify the power set of $\mathcal{X}$. Furthermore, The family of all soft sets signified by $\mathcal{SS}(\mathcal{X})$.

Definition 2.1 [10]: The soft set $\mathcal{F}_{\mathcal{A}}$ is called

- 1- Null soft if $\mathcal{F}(e) = \varphi$, for all $e \in \mathcal{A}$, denoted by $\tilde{\varphi}$.
i.e. $\tilde{\varphi} = \{(e, \varphi) : e \in \mathcal{A}\}$.
- 2- Absolute soft if $\mathcal{F}(e) = \mathcal{X}$, for all $e \in \mathcal{A}$, denoted by $\tilde{\mathcal{X}}$.
i.e. $\tilde{\mathcal{X}} = \{(e, \mathcal{X}) : e \in \mathcal{A}\}$.

Definition 2.2 [10]: Let $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \in \mathcal{SS}(\mathcal{X})$.

- 1- if $\mathcal{F}(e) \subseteq \mathcal{G}(e)$ for every $e \in \mathcal{A}$. Then $\mathcal{F}_{\mathcal{A}}$ is called a soft subset of $\mathcal{G}_{\mathcal{A}}$, denoted by $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{G}_{\mathcal{A}}$.

Visibly, for all $\mathcal{F}_{\mathcal{A}} \in \mathcal{SS}(\mathcal{X})$ we get $\tilde{\varphi} \subseteq \mathcal{F}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$.

- 2- The soft sets $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{G}_{\mathcal{A}}$ are called a soft equal iff $\mathcal{F}(e) = \mathcal{G}(e)$ for all $e \in \mathcal{A}$. It is clear that, $\mathcal{F}_{\mathcal{A}} = \mathcal{G}_{\mathcal{A}}$ iff $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{G}_{\mathcal{A}}$ and $\mathcal{G}_{\mathcal{A}} \subseteq \mathcal{F}_{\mathcal{A}}$.

Definition 2.3 [11]: The soft complement of $\mathcal{F}_{\mathcal{A}}$ denoted by $\mathcal{F}_{\mathcal{A}}^c$ where $\mathcal{F}_{\mathcal{A}}^c: \mathcal{A} \rightarrow \mathcal{P}(\mathcal{X})$ is a function It was awarded by $\mathcal{F}_{\mathcal{A}}^c = \mathcal{X} \setminus \mathcal{F}(e), e \in \mathcal{A}$.

Definition 2.4 [10]: Let $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \in \mathcal{SS}(\mathcal{X})$. We define a soft (Union, Intersection, and Deference) of $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}}$ as follows

- 1- $\mathcal{F}_{\mathcal{A}} \cup \mathcal{G}_{\mathcal{A}} = \{(e, \mathcal{F}(e) \cup \mathcal{G}(e)) : e \in \mathcal{A}\}$.
- 2- $\mathcal{F}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}} = \{(e, \mathcal{F}(e) \cap \mathcal{G}(e)) : e \in \mathcal{A}\}$.
- 3- $\mathcal{F}_{\mathcal{A}} \setminus \mathcal{G}_{\mathcal{A}} = \{(e, \mathcal{F}(e) \setminus \mathcal{G}(e)) : e \in \mathcal{A}\}$.

Definition 2.5 [6]: The soft point of $\mathcal{F}_{\mathcal{A}}$ define as follow: $\mathcal{F}_e^x(n) = \begin{cases} \{x\} & \text{if } n = e \\ \tilde{\varphi} & \text{if } n \in \mathcal{A} \setminus \{e\} \end{cases}$, For all $e \in \mathcal{A}$.

Definition 2.6 [12]: For any collection of soft sets on the universal set $\mathcal{X}$ with the same parameters $\mathcal{A}$ is called soft topology $\tilde{\mathcal{T}}$, in the event that fulfil the following properties:

- 1- $\tilde{\varphi}, \tilde{\mathcal{X}} \in \tilde{\mathcal{T}}$.
- 2- $\forall \mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \in \tilde{\mathcal{T}} \Rightarrow \mathcal{F}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}} \in \tilde{\mathcal{T}}$.

$$3- \mathcal{F}_{\wedge \mathcal{A}} \tilde{\in} \tilde{\mathcal{T}} \forall \lambda \in \Lambda \Rightarrow \bigcup_{\lambda \in \Lambda} \mathcal{F}_{\wedge \mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}.$$

The triplet $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A})$ is called a soft topological space on $\mathcal{X}$.

Definition 2.7: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A})$ be soft topological space, $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$. Then

1- Soft interior of $\mathcal{F}_{\mathcal{A}}$ is symbolized by $\text{int}(\mathcal{F}_{\mathcal{A}})$ is defined as

$$(\text{int}(\mathcal{F}_{\mathcal{A}})) = \bigcup \{ \mathcal{H}_{\mathcal{A}} : \mathcal{H}_{\mathcal{A}} \subseteq \mathcal{F}_{\mathcal{A}}, \mathcal{H}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}} \}. [7]$$

2- Soft closure of $\mathcal{F}_{\mathcal{A}}$ is symbolized by $\text{cl}(\mathcal{F}_{\mathcal{A}})$ is defined as

$$(\text{cl}(\mathcal{F}_{\mathcal{A}})) = \bigcap \{ \mathcal{H}_{\mathcal{A}} : \mathcal{F}_{\mathcal{A}} \subseteq \mathcal{H}_{\mathcal{A}}, \mathcal{H}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}^c \}. [13]$$

Definition 2.8: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A})$ be soft topological space. Then

1- $\mathcal{F}_{\mathcal{A}}$ is soft open iff $\text{int}(\mathcal{F}_{\mathcal{A}}) = \mathcal{F}_{\mathcal{A}}$. [7]

2- $\mathcal{F}_{\mathcal{A}}$ is soft closed iff $\text{cl}(\mathcal{F}_{\mathcal{A}}) = \mathcal{F}_{\mathcal{A}}$. [13]

Definition 2.9 [14]: The collection $\tilde{\mathcal{I}}_{\mathcal{A}} \subseteq \mathcal{SS}(\mathcal{X})$ is called soft ideal that fulfil the following condition:

1- $\mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}} \Rightarrow \mathcal{F}_{\mathcal{A}} \cup \mathcal{G}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$.

2- $\mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \subseteq \mathcal{F}_{\mathcal{A}} \Rightarrow \mathcal{G}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$.

3. Soft closure function:

Definition 3.1: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space. $\mathcal{F}_{\mathcal{A}}$ a soft sub set of $\tilde{\mathcal{X}}$. Then we define $\Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{I}}_{\mathcal{A}}, \tilde{\mathcal{T}}) = \bigcup \{ \mathfrak{f}_e^x \tilde{\in} \tilde{\mathcal{X}} : \mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\notin} \tilde{\mathcal{I}}_{\mathcal{A}} \text{ for all } \mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x) \}$ Where $\tilde{\mathcal{T}}(\mathfrak{f}_e^x) = \{ \mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}} : \mathfrak{f}_e^x \tilde{\in} \mathcal{U}_{\mathcal{A}} \}$ is called the soft closure function of $\mathcal{F}_{\mathcal{A}}$ as for $\tilde{\mathcal{I}}_{\mathcal{A}}$ and $\tilde{\mathcal{T}}$.

Lemma 3.2: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space. Then $\mathcal{F}_{\mathcal{A}}^* \subseteq \Gamma \mathcal{F}_{\mathcal{A}}$ for all $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$.

Proof: Let $\mathfrak{f}_e^x \tilde{\in} \mathcal{F}_{\mathcal{A}}^*$. Then $\mathcal{F}_{\mathcal{A}} \cap \mathcal{U}_{\mathcal{A}} \tilde{\notin} \tilde{\mathcal{I}}_{\mathcal{A}}$ for all $\mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x)$ since

$\mathcal{F}_{\mathcal{A}} \cap \mathcal{U}_{\mathcal{A}} \subseteq \mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}})$, so we get $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\notin} \tilde{\mathcal{I}}_{\mathcal{A}}$. Therefore $\mathfrak{f}_e^x \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$.

Theorem 3.3: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A})$ be a soft ideal space. $\tilde{\mathcal{I}}_{\mathcal{A}}, \tilde{\mathcal{L}}_{\mathcal{A}}$ be two soft ideals of $\tilde{\mathcal{X}}$ and $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$. Then

1- If $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{G}_{\mathcal{A}}$, then $\Gamma \mathcal{F}_{\mathcal{A}} \subseteq \Gamma \mathcal{G}_{\mathcal{A}}$;

2- If $\tilde{\mathcal{I}}_{\mathcal{A}} \subseteq \tilde{\mathcal{L}}_{\mathcal{A}}$, then $\Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{I}}_{\mathcal{A}}) \supseteq \Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{L}}_{\mathcal{A}})$;

3- $\Gamma \mathcal{F}_{\mathcal{A}} = \text{cl}(\Gamma \mathcal{F}_{\mathcal{A}})$;

4- If $\mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\Gamma \mathcal{F}_{\mathcal{A}} = \emptyset$.

Proof:

1- Assume that $\mathfrak{f}_e^x \tilde{\notin} \Gamma \mathcal{G}_{\mathcal{A}}$. Then there exist $\mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x)$ such that $\mathcal{G}_{\mathcal{A}} \cap \mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$. Since $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \subseteq \mathcal{G}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}})$, so $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$. Hence $\mathfrak{f}_e^x \tilde{\notin} \Gamma \mathcal{F}_{\mathcal{A}}$. Thus $\Gamma \mathcal{F}_{\mathcal{A}} \subseteq \Gamma \mathcal{G}_{\mathcal{A}}$.

2- Assume that $\mathfrak{f}_e^x \tilde{\notin} \Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{I}}_{\mathcal{A}})$. Then there exist $\mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x)$ such that $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$. Since $\tilde{\mathcal{I}}_{\mathcal{A}} \subseteq \tilde{\mathcal{L}}_{\mathcal{A}}$, then $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\in} \tilde{\mathcal{L}}_{\mathcal{A}}$, $\mathfrak{f}_e^x \tilde{\notin} \Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{L}}_{\mathcal{A}})$. Therefore $\Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{L}}_{\mathcal{A}}) \subseteq \Gamma \mathcal{F}_{\mathcal{A}}(\tilde{\mathcal{I}}_{\mathcal{A}})$.

3- We have $\Gamma \mathcal{F}_{\mathcal{A}} \subseteq \text{cl}(\Gamma \mathcal{F}_{\mathcal{A}}) \dots \dots (1)$. Let $\mathfrak{f}_e^x \tilde{\in} \text{cl}(\Gamma \mathcal{F}_{\mathcal{A}})$. Then $\mathcal{U}_{\mathcal{A}} \cap \Gamma \mathcal{F}_{\mathcal{A}} \neq \emptyset$ for every $\mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x)$. Therefore, there exist $\mathfrak{f}_e^y \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}} \cap \mathcal{U}_{\mathcal{A}}$, $\mathcal{U}_{\mathcal{A}} \in \tilde{\mathcal{T}}(\mathfrak{f}_e^y)$. Since $\mathfrak{f}_e^y \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$, $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\notin} \tilde{\mathcal{I}}_{\mathcal{A}}$ hence $\mathfrak{f}_e^x \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$. So, we have $\text{cl}(\Gamma \mathcal{F}_{\mathcal{A}}) \subseteq \Gamma \mathcal{F}_{\mathcal{A}} \dots \dots (2)$. From (1) and (2) we get $\Gamma \mathcal{F}_{\mathcal{A}} = \text{cl}(\Gamma \mathcal{F}_{\mathcal{A}})$.

4- Assume that $\mathfrak{f}_e^x \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$, $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\notin} \tilde{\mathcal{I}}_{\mathcal{A}} \forall \mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{T}}(\mathfrak{f}_e^x)$. But since $\mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\mathcal{F}_{\mathcal{A}} \cap \text{cl}(\mathcal{U}_{\mathcal{A}}) \tilde{\in} \tilde{\mathcal{I}}_{\mathcal{A}}$. This cont. so $\Gamma \mathcal{F}_{\mathcal{A}} = \emptyset$.

Theorem 3.4: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space. $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$. Then

1- $\Gamma \emptyset = \emptyset$

2- $\Gamma \mathcal{F}_{\mathcal{A}} \cup \Gamma \mathcal{G}_{\mathcal{A}} = \Gamma(\mathcal{F}_{\mathcal{A}} \cup \mathcal{G}_{\mathcal{A}})$

Proof:

1- The proof is obvious

2- It follows from theorem 2.3 we get $\Gamma \mathcal{F}_{\mathcal{A}} \cup \Gamma \mathcal{G}_{\mathcal{A}} \subseteq \Gamma(\mathcal{F}_{\mathcal{A}} \cup \mathcal{G}_{\mathcal{A}})$.

To prove the invert incorporation. Let $f_e^x \notin \Gamma \mathcal{F}_A \cup \Gamma \mathcal{G}_A$. Then f_e^x belongs neither $\Gamma \mathcal{F}_A$ nor to $\Gamma \mathcal{G}_A$. So there exist $u_{f_e^x}, v_{f_e^x}$ such that $\mathcal{F}_A \tilde{\cap} cl(u_{f_e^x}) \in \mathcal{I}$ and $\mathcal{G}_A \tilde{\cap} cl(v_{f_e^x}) \in \tilde{\mathcal{I}}_A$ since $\tilde{\mathcal{I}}_A$ is additive. Then $(\mathcal{F}_A \tilde{\cap} cl(u_{f_e^x})) \cup (\mathcal{G}_A \tilde{\cap} cl(v_{f_e^x})) \in \tilde{\mathcal{I}}_A$, imply that $(\mathcal{F}_A \cup \mathcal{G}_A) \tilde{\cap} (cl(u_{f_e^x} \tilde{\cap} v_{f_e^x})) \in \tilde{\mathcal{I}}_A$

Since $u_{f_e^x} \tilde{\cap} v_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$, then $f_e^x \notin \Gamma(\mathcal{F}_A \cup \mathcal{G}_A)$. Hence, we get $\Gamma(\mathcal{F}_A \cup \mathcal{G}_A) \subseteq \Gamma \mathcal{F}_A \cup \Gamma \mathcal{G}_A$.

Therefore $\Gamma \mathcal{F}_A \cup \Gamma \mathcal{G}_A = \Gamma(\mathcal{F}_A \cup \mathcal{G}_A)$.

Lemma 3.5: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_A)$ be a soft ideal space, $\mathcal{F}_A, \mathcal{G}_A \subseteq \tilde{\mathcal{X}}$.

Then $\Gamma \mathcal{F}_A \setminus \Gamma \mathcal{G}_A = \Gamma(\mathcal{F}_A \setminus \mathcal{G}_A) \setminus \Gamma \mathcal{G}_A$.

Proof: We have by Theorem 2.4 $\Gamma \mathcal{F}_A = \Gamma[(\mathcal{F}_A \setminus \mathcal{G}_A) \cup (\mathcal{F}_A \tilde{\cap} \mathcal{G}_A)]$
 $= \Gamma[(\mathcal{F}_A \setminus \mathcal{G}_A) \cup \Gamma(\mathcal{F}_A \tilde{\cap} \mathcal{G}_A)] \subseteq \Gamma[(\mathcal{F}_A \setminus \mathcal{G}_A) \cup \Gamma(\mathcal{G}_A)]$. Thus $(\Gamma \mathcal{F}_A \setminus \Gamma \mathcal{G}_A) \subseteq \Gamma(\mathcal{F}_A \setminus \mathcal{G}_A) \setminus \Gamma(\mathcal{G}_A)$.
 By Theorem 2.3, we get $\Gamma(\mathcal{F}_A \setminus \mathcal{G}_A) \subseteq \Gamma(\mathcal{F}_A)$, $\Gamma(\mathcal{F}_A \setminus \mathcal{G}_A) \setminus \Gamma(\mathcal{G}_A) \subseteq \Gamma \mathcal{F}_A \setminus \Gamma \mathcal{G}_A$.

So we get $\Gamma \mathcal{F}_A \setminus \Gamma \mathcal{G}_A = \Gamma(\mathcal{F}_A \setminus \mathcal{G}_A) \setminus \Gamma \mathcal{G}_A$.

Corollary 3.6: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_A)$ be a soft ideal space, $\mathcal{F}_A, \mathcal{G}_A \subseteq \tilde{\mathcal{X}}$ with $\mathcal{G}_A \in \tilde{\mathcal{I}}_A$. Then $\Gamma(\mathcal{F}_A \cup \mathcal{G}_A) = \Gamma \mathcal{F}_A = \Gamma(\mathcal{F}_A \setminus \mathcal{G}_A)$.

Proof: Since $\mathcal{G}_A \in \tilde{\mathcal{I}}_A$, by Theorem 2.3, $\Gamma \mathcal{G}_A = \emptyset$. By Lemma 2.5

$\Gamma \mathcal{F}_A = \Gamma(\mathcal{F}_A \setminus \mathcal{G}_A)$. and by Theorem 2.4 $\Gamma(\mathcal{F}_A \cup \mathcal{G}_A) = \Gamma \mathcal{F}_A \cup \Gamma \mathcal{G}_A = \Gamma \mathcal{F}_A$

4. Soft closure compatibility with soft topology

Definition 4.1: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_A)$ be a soft ideal space. We say the $\tilde{\mathcal{T}}$ is closure soft compatible with the soft ideal $\tilde{\mathcal{I}}_A$, if for every $f_e^x \in \mathcal{F}_A$ there exist $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$ such that $\mathcal{F}_A \tilde{\cap} cl(u_{f_e^x}) \in \tilde{\mathcal{I}}_A$ then $F_A \in \tilde{\mathcal{I}}_A$ denoted $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{I}}_A$

Theorem 4.2: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_A)$ be a soft ideal space. The following properties are equivalent:

- 1- $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{I}}_A$
- 2- If $\mathcal{F}_A \subseteq \tilde{\mathcal{X}}$ has a open soft cover of sets each of whose soft closure intersection with $\mathcal{F}_A$ in $\tilde{\mathcal{I}}_A$, then $F_A \in \tilde{\mathcal{I}}_A$
- 3- For every $F_A \in \tilde{\mathcal{I}}_A$, $(\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A) = \emptyset$ implies that $F_A \in \tilde{\mathcal{I}}_A$;
- 4- For every $\mathcal{F}_A \subseteq \tilde{\mathcal{X}}$, $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \in \tilde{\mathcal{I}}_A$;
- 5- For every $\mathcal{F}_A \subseteq \tilde{\mathcal{X}}$, if $\mathcal{F}_A$ contains no non null soft subset $\mathcal{G}_A$ with $\mathcal{G}_A \subseteq \Gamma \mathcal{G}_A$ then $F_A \in \tilde{\mathcal{I}}_A$;

Proof: (1) $\Rightarrow$ (2): The proof evident.

(2) $\Rightarrow$ (3): Let $f_e^x \in \mathcal{F}_A \subseteq \tilde{\mathcal{X}}$, $f_e^x \notin \Gamma \mathcal{F}_A$ so there exist $v_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x) \ni \mathcal{F}_A \tilde{\cap} cl(v_{f_e^x}) \in \tilde{\mathcal{I}}_A$. Therefore, we have $\mathcal{F}_A \subseteq \cup \{v_{f_e^x}: f_e^x \in \mathcal{F}_A\}$, $v_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$ by (2) we get $F_A \in \tilde{\mathcal{I}}_A$.

(3) $\Rightarrow$ (4): For a set $\mathcal{F}_A$, $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \subseteq \mathcal{F}_A$ and $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \tilde{\cap} \Gamma(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \subseteq (\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \tilde{\cap} \mathcal{F}_A = \emptyset$. By (3) we get $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \in \tilde{\mathcal{I}}_A$.

(4) $\Rightarrow$ (5): By (4) for every soft set $\mathcal{F}_A \subseteq \tilde{\mathcal{X}}$, $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \in \tilde{\mathcal{I}}_A$

Let $(\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) = \mathcal{J}_A \in \tilde{\mathcal{I}}_A$, then $\mathcal{F}_A = \mathcal{J}_A \cup (\mathcal{F}_A \setminus \Gamma \mathcal{F}_A)$, by Theorem 2.4 (2) and Theorem 2.6 (5)

$$\Gamma \mathcal{F}_A = \Gamma \mathcal{J}_A \cup \Gamma(\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A) = \Gamma(\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A).$$

Therefore, we have $\Gamma \mathcal{F}_A \tilde{\cap} \mathcal{F}_A = \mathcal{F}_A \tilde{\cap} \Gamma(\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A) \subseteq \Gamma(\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A)$,

$\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A \subseteq \mathcal{F}_A$ By the assumption $\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A = \emptyset$.

Hence $\mathcal{F}_A = (\mathcal{F}_A \setminus \Gamma \mathcal{F}_A) \in \tilde{\mathcal{I}}_A$.

(5) $\Rightarrow$ (1): Let $\mathcal{F}_A \subseteq \tilde{\mathcal{X}}$, $f_e^x \in \mathcal{F}_A$. there exist $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$, $\mathcal{F}_A \tilde{\cap} cl(u_{f_e^x}) \in \tilde{\mathcal{I}}_A$. Then $\mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A = \emptyset$. Suppose that $\mathcal{F}_A$ contains $\mathcal{G}_A$ such that $\mathcal{G}_A \subseteq \Gamma \mathcal{G}_A$, then

$\mathcal{G}_A = \mathcal{G}_A \tilde{\cap} \Gamma \mathcal{G}_A \subseteq \mathcal{F}_A \tilde{\cap} \Gamma \mathcal{F}_A = \emptyset$. Therefore $\mathcal{F}_A$ contains no null soft sub set $\mathcal{G}_A$ with $\mathcal{G}_A \subseteq \Gamma \mathcal{G}_A$. Hence $\mathcal{F}_A \in \tilde{\mathcal{I}}_A$.

Theorem 4.3: Let $(\tilde{X}, \tilde{T}, \mathcal{A}, \tilde{J}_{\mathcal{A}})$ be a soft ideal space. If $\tilde{T}$ is closure soft compatible with $\tilde{J}_{\mathcal{A}}$, then the following equivalent properties are holds :

- 1- For every $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$ implies that $\Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$;
- 2- For every $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) = \check{\emptyset}$;
- 3- For every $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) = \Gamma \mathcal{F}_{\mathcal{A}}$.

Proof: First, we show that (1) holds if the soft topology $\tilde{T}$ is closure soft compatible with soft ideal $\tilde{J}_{\mathcal{A}}$.

Let $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$. By Theorem 3.2, $\mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{J}_{\mathcal{A}}$ and by Theorem 2.3 (5) $\Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$.

Now, (1) $\Rightarrow$ (2): Assume that for every $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$ implies that $\Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$.

$$\begin{aligned} \text{Let } \mathcal{G}_{\mathcal{A}} &= \mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}. \text{ Then } \mathcal{G}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{G}_{\mathcal{A}} = [(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) \tilde{\cap} \Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})] \\ &= (\mathcal{F}_{\mathcal{A}} \tilde{\cap} (\tilde{X} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})) \tilde{\cap} \Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} (\tilde{X} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})) \\ &\subseteq (\mathcal{F}_{\mathcal{A}} \tilde{\cap} (\tilde{X} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})) \tilde{\cap} (\Gamma \mathcal{F}_{\mathcal{A}} \tilde{\cap} (\Gamma(\tilde{X} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}))) = \check{\emptyset} \end{aligned}$$

By (1), we have $\Gamma \mathcal{G}_{\mathcal{A}} = \check{\emptyset}$. Hence $\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) = \check{\emptyset}$

(1) $\Rightarrow$ (3): Assume that for ever $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$,

$$\begin{aligned} \Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) &= \check{\emptyset}, \mathcal{F}_{\mathcal{A}} = (\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) \tilde{\cup} (\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) \Gamma \mathcal{F}_{\mathcal{A}} \\ &= \Gamma[(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) \tilde{\cup} (\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})] = (\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})) \tilde{\cup} (\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}})) = \Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) \end{aligned}$$

(3) $\Rightarrow$ (1): Assume that for ever $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}} = \check{\emptyset}$ and $\Gamma(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \Gamma \mathcal{F}_{\mathcal{A}}) = \Gamma \mathcal{F}_{\mathcal{A}}$.

This implies that $\check{\emptyset} = \Gamma \check{\emptyset} = \Gamma \mathcal{F}_{\mathcal{A}}$.

Theorem 4.4: Let $(\tilde{X}, \tilde{T}, \mathcal{A}, \tilde{J}_{\mathcal{A}})$ be a soft ideal space. Then the following properties are equivalent:

- 1- $\tilde{J}_{\mathcal{A}} \tilde{\cap} cl(\tilde{T}) = \check{\emptyset}$, where $cl(\tilde{T}) = \{cl(\mathcal{V}_{\mathcal{A}}) : \mathcal{V}_{\mathcal{A}} \tilde{\in} \tilde{T}\}$;
- 2- If $\mathcal{J}_{\mathcal{A}} \tilde{\in} \tilde{J}_{\mathcal{A}}$, then $int(\mathcal{J}_{\mathcal{A}}) = \check{\emptyset}$;
- 3- For every soft open $\mathcal{F}_{\mathcal{A}}$, $\mathcal{F}_{\mathcal{A}} \subseteq \Gamma \mathcal{F}_{\mathcal{A}}$;
- 4- $\tilde{X} = \Gamma \tilde{X}$.

Proof: (1) $\Rightarrow$ (2): if it was possible $int(\mathcal{J}_{\mathcal{A}}) = \check{\emptyset}$, so $f_e^x \tilde{\in} int(\mathcal{J}_{\mathcal{A}})$ this implies that

$\tilde{J}_{\mathcal{A}} \tilde{\cap} cl(\tilde{T}) \neq \check{\emptyset}$ which contradiction. Hence $int(\mathcal{J}_{\mathcal{A}}) = \check{\emptyset}$ for every $\mathcal{J}_{\mathcal{A}} \tilde{\in} \tilde{J}_{\mathcal{A}}$.

(2) $\Rightarrow$ (3): If possible $f_e^x \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$, then there exist $\mathcal{U}_{f_e^x} \tilde{\in} \tilde{T}(f_e^x)$ such that $\mathcal{F}_{\mathcal{A}} \tilde{\cap} cl(\mathcal{U}_{f_e^x}) \tilde{\notin} \tilde{J}_{\mathcal{A}}$, $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{U}_{f_e^x} \tilde{\in} \tilde{J}_{\mathcal{A}}$. Since $\mathcal{F}_{\mathcal{A}}$ is soft open by (2) $f_e^x \tilde{\in} \mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{U}_{f_e^x} = int(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{U}_{f_e^x}) \subseteq int(\mathcal{F}_{\mathcal{A}} \tilde{\cap} cl(\mathcal{U}_{f_e^x}))$. This is contradiction.

Hence $f_e^x \tilde{\in} \Gamma \mathcal{F}_{\mathcal{A}}$.

(3) $\Rightarrow$ (4): It is obtained directly

(4) $\Rightarrow$ (1): $\tilde{X} = \Gamma \tilde{X} = \tilde{\cup} \{f_e^x \tilde{\in} \tilde{X} : \tilde{X} \tilde{\cap} cl(\mathcal{U}_{\mathcal{A}}) = cl(\mathcal{U}_{\mathcal{A}}) \tilde{\in} \tilde{J}_{\mathcal{A}} \text{ for every soft open } \mathcal{U}_{\mathcal{A}} \text{ containing } f_e^x\}$. So $cl(\tilde{T}) \tilde{\cap} \tilde{J}_{\mathcal{A}} = \check{\emptyset}$.

5. Soft ψ_{Γ} -Operator

Definition 5.1: Let $(\tilde{X}, \tilde{T}, \mathcal{A}, \tilde{J}_{\mathcal{A}})$ be a soft ideal space. $\mathcal{A}$ a soft operator $\psi_{\Gamma}: \delta\delta(\tilde{X}) \rightarrow \tilde{T}$ is defined as follows :

For all $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \tilde{\cup} \{f_e^x \tilde{\in} \tilde{X} \exists \mathcal{U}_{\mathcal{A}} \tilde{\in} \tilde{T}(f_e^x) \text{ such that } cl(\mathcal{U}_{\mathcal{A}}) \tilde{\cap} \mathcal{F}_{\mathcal{A}} \tilde{\in} \tilde{J}_{\mathcal{A}}\}$

Observe that $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \tilde{X} \tilde{\cap} \Gamma(\tilde{X} \tilde{\cap} \mathcal{F}_{\mathcal{A}})$.

A few essential realities concerning of the soft ψ_{Γ} -Operator are included in the following Theorem.

Theorem 5.2: Let $(\tilde{X}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space. The following properties hold:

- 1- If $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})$ is soft open.
- 2- If $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{G}_{\mathcal{A}}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \subseteq \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}})$.
- 3- If $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \subseteq \tilde{X}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \cap \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}})$.
- 4- If $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \psi_{\Gamma}(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}))$ iff $\Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) = \Gamma(\Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}))$
- 5- If $\mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = (\tilde{X} \setminus \Gamma \tilde{X})$.
- 6- If $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{I}_{\mathcal{A}} \in \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{I}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})$.
- 7- If $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$, $\mathcal{I}_{\mathcal{A}} \in \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \cup \mathcal{I}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})$.
- 8- If $(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{G}_{\mathcal{A}}) \cup (\mathcal{G}_{\mathcal{A}} \setminus \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{I}}_{\mathcal{A}}$, then $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}})$.

Proof:

- 1- This follows from Theorem 2.3 (3)
- 2- This follows from Theorem 2.3 (1)
- 3-
$$\begin{aligned} \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}}) &= \tilde{X} \setminus \Gamma[\tilde{X} \setminus (\mathcal{F}_{\mathcal{A}} \cap \mathcal{G}_{\mathcal{A}})] \\ &= \tilde{X} \setminus \Gamma[(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) \cup (\tilde{X} \setminus \mathcal{G}_{\mathcal{A}})] \\ &= \tilde{X} \setminus \Gamma[\Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) \cup \Gamma(\tilde{X} \setminus \mathcal{G}_{\mathcal{A}})] \\ &= [\tilde{X} \setminus \Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}})] \cap [\tilde{X} \setminus \Gamma(\tilde{X} \setminus \mathcal{G}_{\mathcal{A}})] \\ &= \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \cap \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}}) \end{aligned}$$
- 4- This follows from the facts :
 - (i) $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \tilde{X} \setminus \Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}})$
 - (ii)
$$\begin{aligned} \psi_{\Gamma}(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})) &= \tilde{X} \setminus \Gamma[\Gamma(\tilde{X} \setminus \Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}))] \\ &= \tilde{X} \setminus \Gamma(\Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}})) \end{aligned}$$
- 5- : By Corollary 2.10 we get $\Gamma(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) = \Gamma(\tilde{X})$ if $\mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{I}}_{\mathcal{A}}$.
- 6- : This follows from Corollary 2.6 and

$$\begin{aligned} \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{I}_{\mathcal{A}}) &= \tilde{X} \setminus \Gamma[\tilde{X} \setminus (\mathcal{F}_{\mathcal{A}} \setminus \mathcal{I}_{\mathcal{A}})] = \tilde{X} \setminus \Gamma[(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) \cup \mathcal{I}_{\mathcal{A}}] \\ &= \tilde{X} \setminus \Gamma[(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}})] = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \end{aligned}$$
- 7- : This follows from Corollary 2.6 and

$$\begin{aligned} \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \cup \mathcal{I}_{\mathcal{A}}) &= \tilde{X} \setminus \Gamma[\tilde{X} \setminus (\mathcal{F}_{\mathcal{A}} \cup \mathcal{I}_{\mathcal{A}})] = \tilde{X} \setminus \Gamma[(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}}) \setminus \mathcal{I}_{\mathcal{A}}] \\ &= \tilde{X} \setminus \Gamma[(\tilde{X} \setminus \mathcal{F}_{\mathcal{A}})] = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \end{aligned}$$
- 8- : Assume $(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{G}_{\mathcal{A}}) \cup (\mathcal{G}_{\mathcal{A}} \setminus \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{I}}_{\mathcal{A}}$, Let $\mathcal{F}_{\mathcal{A}} \setminus \mathcal{G}_{\mathcal{A}} = \mathcal{I}_{\mathcal{A}}$ and $\mathcal{G}_{\mathcal{A}} \setminus \mathcal{F}_{\mathcal{A}} = \mathcal{L}_{\mathcal{A}}$
Observe that $\mathcal{I}_{\mathcal{A}}, \mathcal{L}_{\mathcal{A}} \in \tilde{\mathcal{I}}_{\mathcal{A}}$, $\mathcal{G}_{\mathcal{A}} = (\mathcal{F}_{\mathcal{A}} \setminus \tilde{\mathcal{I}}_{\mathcal{A}}) \cup \mathcal{I}_{\mathcal{A}}$
Thus $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{I}_{\mathcal{A}}) = \psi_{\Gamma}[(\mathcal{F}_{\mathcal{A}} \setminus \mathcal{I}_{\mathcal{A}}) \cup \mathcal{I}_{\mathcal{A}}] = \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}})$ by (6) and (7)

Theorem 5.3: Let $(\tilde{X}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space, $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{X}$. Then the following properties hold:

- 1- $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) = \bigcup \{ \mathcal{U}_{\mathcal{A}} \in \tilde{\mathcal{T}} : (cl(\mathcal{U}_{\mathcal{A}}) \setminus \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{I}}_{\mathcal{A}} \}$
- 2- $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \cong \bigcup \{ \mathcal{U}_{\mathcal{A}} \in \tilde{\mathcal{T}} : (cl(\mathcal{U}_{\mathcal{A}}) \setminus \mathcal{F}_{\mathcal{A}}) \cup (\mathcal{F}_{\mathcal{A}} \setminus cl(\mathcal{U}_{\mathcal{A}})) \in \tilde{\mathcal{I}}_{\mathcal{A}} \}$

Proof:

- 1- : This follows immediately from the definition soft ψ_{Γ} -Operator.

- 2- : since $\tilde{\mathcal{I}}_{\mathcal{A}}$ is heredity, it is obvious that

$$\begin{aligned} \bigcup \{ \mathcal{U}_{\mathcal{A}} \in \tilde{\mathcal{T}} : (cl(\mathcal{U}_{\mathcal{A}}) \setminus \mathcal{F}_{\mathcal{A}}) \cup (\mathcal{F}_{\mathcal{A}} \setminus cl(\mathcal{U}_{\mathcal{A}})) \in \tilde{\mathcal{I}}_{\mathcal{A}} \} \\ \subseteq \bigcup \{ \mathcal{U}_{\mathcal{A}} \in \tilde{\mathcal{T}} : (cl(\mathcal{U}_{\mathcal{A}}) \setminus \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{I}}_{\mathcal{A}} = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \}, \text{ for every soft set } \mathcal{F}_{\mathcal{A}} \subseteq \tilde{X} \end{aligned}$$

Theorem 5.4: Let $(\tilde{X}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{I}}_{\mathcal{A}})$ be a soft ideal space. If $\sigma = \{ \mathcal{F}_{\mathcal{A}} \subseteq \tilde{X} : \mathcal{F}_{\mathcal{A}} \subseteq \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \}$. Then σ is a soft topology for $\tilde{X}$.

Proof: Let $\sigma = \{ \mathcal{F}_{\mathcal{A}} \subseteq \tilde{X} : \mathcal{F}_{\mathcal{A}} \subseteq \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \}$, Since $\emptyset \in \tilde{\mathcal{I}}_{\mathcal{A}}$, by Theorem 2.3 (5) $\Gamma \emptyset = \emptyset$ and

$$\psi_{\Gamma}(\tilde{X}) = \tilde{X} \setminus \Gamma(\tilde{X} \setminus \tilde{X}) = (\tilde{X} \setminus \Gamma \emptyset) = \tilde{X}$$

Also, $\psi_{\Gamma}(\emptyset) = \tilde{X} \setminus \Gamma(\tilde{X} \setminus \emptyset) = (\tilde{X} \setminus \tilde{X}) = \emptyset$. Hence, we get $\emptyset, \tilde{X}$ belongs to σ

Now, if $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \in \sigma$, then by Theorem 4.2 $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}} \subseteq \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\cap} \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}}) = \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}})$. This implies that $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}} \in \sigma$. Finally, if $\{\mathcal{F}_{\Lambda\mathcal{A}} : \Lambda \in \Delta\} \subseteq \sigma$, then $\mathcal{F}_{\Lambda\mathcal{A}} \subseteq \psi_{\Gamma}(\mathcal{F}_{\Lambda\mathcal{A}}) \subseteq \psi_{\Gamma}(\bigcup \mathcal{F}_{\Lambda\mathcal{A}})$ for every Λ . So we get $\bigcup \mathcal{F}_{\Lambda\mathcal{A}} \subseteq \psi_{\Gamma}(\bigcup \mathcal{F}_{\Lambda\mathcal{A}})$.

This shows that σ is soft topology.

Theorem 5.5: Let $\sigma_0 = \{\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}} : \mathcal{F}_{\mathcal{A}} \subseteq \text{Int}(cl(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})))\}$, then σ_0 is a soft topology for $\tilde{\mathcal{X}}$.

Proof: By Theorem 4.2, for any soft sub set $\mathcal{F}_{\mathcal{A}}$ of $\tilde{\mathcal{X}}$ $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})$ is soft open and $\sigma \subset \sigma_0$.

Hence $\emptyset, \tilde{\mathcal{X}} \in \sigma_0$. Let $\mathcal{F}_{\mathcal{A}}, \mathcal{G}_{\mathcal{A}} \in \sigma_0$. By Theorem 4.2, we have $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}} \subseteq \text{int}[(cl(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}))) \tilde{\cap} \text{int}[(cl(\psi_{\Gamma}(\mathcal{G}_{\mathcal{A}})))] = \text{int}[cl(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\cap} \psi_{\Gamma}(\mathcal{G}_{\mathcal{A}}))] = \text{int}[cl(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}}))]$. Hence $\mathcal{F}_{\mathcal{A}} \tilde{\cap} \mathcal{G}_{\mathcal{A}} \in \sigma_0$. Now, let $\mathcal{F}_{\Lambda\mathcal{A}} \in \sigma_0$ for each $\Lambda \in \Delta$.

By Theorem 4.2, for each $\Lambda \in \Delta$, $\mathcal{F}_{\Lambda\mathcal{A}} \subseteq \text{int}[cl(\psi_{\Gamma}(\mathcal{F}_{\Lambda\mathcal{A}}))] \subseteq \text{int}[cl(\psi_{\Gamma}(\bigcup \mathcal{F}_{\Lambda\mathcal{A}}))]$.

Hence $\bigcup \mathcal{F}_{\Lambda\mathcal{A}} \subseteq \text{int}[cl(\psi_{\Gamma}(\bigcup \mathcal{F}_{\Lambda\mathcal{A}}))]$, so $\bigcup \mathcal{F}_{\Lambda\mathcal{A}} \in \sigma_0$. This shows that σ_0 is soft topology.

Theorem 5.6: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{J}}_{\mathcal{A}})$ be a soft ideal space. Then $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{J}}_{\mathcal{A}}$ iff $(\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$ for every soft set $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$.

Proof: Necessity, assume that $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{J}}_{\mathcal{A}}$ and let $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$. Observe that $f_e^x \in (\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}})$ iff $f_e^x \notin \mathcal{F}_{\mathcal{A}}$ and $f_e^x \in \Gamma(\tilde{\mathcal{X}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}})$ iff $f_e^x \notin \mathcal{F}_{\mathcal{A}}$ there exist $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$ such that $(cl(u_{f_e^x}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$ iff there exist $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$ such that $f_e^x \in (cl(u_{f_e^x}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$.

Now, for each $f_e^x \in (\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}})$ and $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x)$, $cl(u_{f_e^x}) \tilde{\cap} (\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$.

Hence $\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{J}}_{\mathcal{A}}$ by assumption that $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{J}}_{\mathcal{A}}$.

Sufficiency: Let $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$, $f_e^x \in \mathcal{F}_{\mathcal{A}}$, $u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x) : cl(u_{f_e^x}) \tilde{\cap} \mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{J}}_{\mathcal{A}}$. Observe that

$$\begin{aligned} (\psi_{\Gamma}(\tilde{\mathcal{X}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}})) \tilde{\setminus} (\tilde{\mathcal{X}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) &= (\mathcal{F}_{\mathcal{A}} \tilde{\setminus} \Gamma \mathcal{F}_{\mathcal{A}}) \\ &= \{f_e^x : \text{there exist } u_{f_e^x} \in \tilde{\mathcal{T}}(f_e^x) \text{ such that } f_e^x \in cl(u_{f_e^x}) \tilde{\cap} \mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{J}}_{\mathcal{A}}\}. \end{aligned}$$

Thus we have $\mathcal{F}_{\mathcal{A}} \subseteq [\psi_{\Gamma}(\tilde{\mathcal{X}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}})] \tilde{\setminus} (\tilde{\mathcal{X}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$. Hence $\mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{J}}_{\mathcal{A}}$ by heredity of $\tilde{\mathcal{J}}_{\mathcal{A}}$.

Proposition 5.7: Let $(\tilde{\mathcal{X}}, \tilde{\mathcal{T}}, \mathcal{A}, \tilde{\mathcal{J}}_{\mathcal{A}})$ be a soft ideal space. with $\tilde{\mathcal{T}} \approx_{\Gamma} \tilde{\mathcal{J}}_{\mathcal{A}}$, $\mathcal{F}_{\mathcal{A}} \subseteq \tilde{\mathcal{X}}$, if $\emptyset \neq \mathcal{G}_{\mathcal{A}}$ is soft open sub set of $(\Gamma \mathcal{F}_{\mathcal{A}} \tilde{\cap} \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}))$, then $\mathcal{G}_{\mathcal{A}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}} \in \tilde{\mathcal{J}}_{\mathcal{A}}$ and $cl(\mathcal{G}_{\mathcal{A}}) \tilde{\cap} \mathcal{F}_{\mathcal{A}} \notin \tilde{\mathcal{J}}_{\mathcal{A}}$.

Proof: If $\mathcal{G}_{\mathcal{A}} \subseteq \Gamma \mathcal{F}_{\mathcal{A}} \tilde{\cap} \psi_{\Gamma}(\mathcal{F}_{\mathcal{A}})$, then $\mathcal{G}_{\mathcal{A}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}} \subseteq (\psi_{\Gamma}(\mathcal{F}_{\mathcal{A}}) \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$ by Theorem 4.6, we get $(\mathcal{G}_{\mathcal{A}} \tilde{\setminus} \mathcal{F}_{\mathcal{A}}) \in \tilde{\mathcal{J}}_{\mathcal{A}}$. since $\emptyset \neq \mathcal{G}_{\mathcal{A}} \in \tilde{\mathcal{T}}$ and $\mathcal{G}_{\mathcal{A}} \subseteq \Gamma \mathcal{F}_{\mathcal{A}}$, we have $cl(\mathcal{G}_{\mathcal{A}}) \tilde{\cap} \mathcal{F}_{\mathcal{A}} \notin \tilde{\mathcal{J}}_{\mathcal{A}}$ by definition of $\Gamma \mathcal{F}_{\mathcal{A}}$.

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