

Chantler *et al.* Reply: We find that Epp’s [1] hypothesis—namely, that the current experimental data set is well represented by a small constant or zero offset from the predictions of Artemyev *et al.* [2]—is not supported by standard statistical analysis. We find that adding the new datum recommended by Epp makes the modeled discrepancy that we reported earlier grow with Z even more strongly (rather than more weakly, as Epp suggests). Our primary conclusion [3] of an apparent Z -dependent discrepancy between experiment and theory is unchanged. While Epp suggests that our results extrapolate to problems at very high Z , our present and previous data sets both include data up to and including $Z = 92$, and the reduced chi-squared χ_r^2 values in our original work [3] showed a negligible discrepancy of the fit over this range of Z .

Upon replacing the value for $Z = 18$ from our data set with Epp’s recommended weighted average of 3139.5805(49) eV, we obtain the “expanded” data set. Following the analysis that produced our original Fig. 2 [3], we present in Fig. 1 the χ_r^2 of the monomial function $y = aZ^n$, where y is the discrepancy between experiment and the theoretical calculations of Artemyev *et al.* [2] under the assumption of various orders, n . Epp’s hypothesis ($n = 0$) yields the high $\chi_r^2 = 3.7$; however, the optimum exponent is not $n = 0$ but $n = 5$, suggesting a stronger Z dependence than the original data set. Applying the standard chi-squared statistical hypothesis test to the case $n = 0$, i.e., Epp’s null hypothesis (constant offset hypothesis), we find the probability of observing residuals at least as large as observed to be extremely low: $p = 0.00003$, providing strong evidence that the null hypothesis is false. It is not correct to claim, as Epp suggests, that the data are “fairly fitted” by a constant offset.

The original data set gave similar values of χ_r^2 for $n = 3$ and $n = 4$. If we allow the exponent to take on nonintegral values, we find that the minimum χ_r^2 is an excellent 1.1 for $n = 3.5$. A similar analysis for the expanded data set gives $n = 5.0$, but with χ_r^2 being high: 2.4.

Although the inclusion of the new datum [4] does not change the main conclusions of our earlier paper [3], and the exponent is in reasonable agreement with our previous result, the minimum χ_r^2 more than doubles, suggesting that the expanded data set includes a data point with greatly underestimated uncertainty or that the assumed form of the deviation should be revised.

Epp’s statement that any data set “could be easily fitted more accurately by a full polynomial of third order” is not relevant to our original analysis since we have only presented results in terms of a monomial. The number of fit parameters is the same for our hypothesis and his. The agreement in our original fit remains remarkable.

We take exception to Epp’s implication that our model function is not physically justified and that our “fit does not provide any meaningful insight.” Our physical motivation

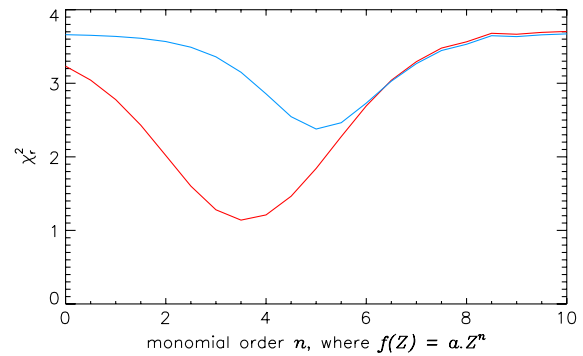


FIG. 1 (color online). Reduced chi square for the original (red, bottom curve) and expanded (blue, top curve) data set as a function of monomial order n , for $N = 12$ values of Z , with one fit parameter.

for assuming a monomial form of the divergence is that this is the underlying *ab initio* analytic form taken by each term in the QED calculation; see e.g., Refs. [5,6]. This provides guidance as to which QED expansion terms might be inadequately computed, although we repeat our caution that “the actual divergence could be the result of a variety of orders and Z dependencies” [3].

As in our original publication, we conclude that to present a Z -dependent divergence between experiment and theory is observed for $Z > 20$. We encourage more work in both theory and experiment that will lead to better characterization, and ultimately better physical understanding, of any disagreement.

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