Multi-body modelling of single-mast stacker cranes

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Abstract: In the frame structure of stacker cranes during non-stationary phases of movement due to inertial forces undesirable mast vibrations may occur. This effect can reduce the stability and positioning accuracy of these machines. The aim of this paper is to introduce an accurate and quite simple dynamical model of single-mast stacker cranes, which is suitable for investigating the mast vibrations of these machines. The multi-body modelling approach is selected to generate the differential equations of motion for this model. The solution of these equations is performed by means of the so-called modal coordinate transformation or modal superposition method. In this model structural damping is taken into consideration by means of the so-called proportional damping (Rayleigh damping) approach. The main advantage of the presented multi-body model is that with this model the mast-vibrations can be investigated in various positions of the mast. Dynamic models with varying lifted load positions can also be generated in simple way by using the introduced modelling technique. The main properties, i.e., the state space representation of our model as well as time domain simulation results, are also introduced.

Keywords: multi-body modelling; modal superposition; modal truncation; stacker cranes; proportional damping; dynamic modelling.

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1 Introduction

The performance of modern automated storage/retrieval systems (AS/RS) of warehouses depends on – besides many other parameters – the performance of material handling systems. The most important material handling machinery of AS/RS is the stacker crane which performs directly the storage/retrieval operation into/from rack position. The advanced stacker cranes therefore must meet the fast working cycle and reliable, economical operation. Today this material handling equipment often has more than one ton pay-load capacity with 50 m lifting height, 250 m/min velocity and 2 m/s^2 acceleration in the direction of the aisle. Consequently, these machines have very high dynamic load. Due to the requirements of economical manufacturing and low energy consumption operation the dead-weight of stacker crane frame structures is reduced. These requirements make the designer of stacker cranes attempt to increase operation velocities and accelerations as well as at the same time reduce energy consumption and dead-weight.

The reduction of dead-weight may result in decreasing the stiffness of frame structure. This structure is more responsive to dynamical loads. During operation undesirable vibrations, low frequency and high amplitude mast sways may occur in the frame structure due to the different inertial forces. The high amplitude mast sway may reduce the stability and positioning accuracy of the stacker crane and in an extreme case it may damage the structure.

Consequently, it is necessary to reduce undesirable mast-sway by controlling the travelling motion of the stacker crane (i.e., the motion towards the aisle of warehouse). The motion control of stacker cranes as well as the estimation of structural sway during the design period or dynamic investigation of an existing structure requires a dynamic model of the flexible structure. This model must be sufficiently accurate and at the same time simple to fulfil the requirements of control synthesis techniques.

In our work the multi-body modelling approach has been chosen to describe the dynamic behaviour of single-mast stacker cranes. The multi-body modelling technique [besides finite element modelling (FEM)] is a widely used modelling method of structural dynamics. It has a very extensive literature in the area of dynamic investigation of engineering structures (Jalón and Bayo, 1994; Angeles and Kecskeméthy, 1995; Keskinen et al., 2007; Ziaei-Rad et al., 2007) as well as stacker cranes (Arnold and Schumacher, 1993; Reisinger, 1998; Schumacher, 1994; Arnold and Dietzel, 2000; Dietzel, 1999; Köhne, 2003). In most cases for time-domain analysis of single-mast stacker crane structures relatively high order models (models with more than ten degrees of freedom) are applied (Arnold and Schumacher, 1993; Reisinger, 1998; Schumacher, 1994). However, in some references a few degree of freedom (DOF) models can be found for simulation and control synthesis (Schumacher, 1994; Arnold and Dietzel, 2000; Dietzel, 1999; Köhne, 2003).

In order to determine a model with sufficient approximation of actual stacker cranes the structural damping also must be taken into account. In this paper structural damping is taken into consideration by means of the so-called proportional damping (Rayleigh damping) approach. The determination of attributes of proportional damping is presented in detail by the following references: Spears and Jensen (2009), Chowdhury and Dasgupta (2003) and Pápai et al. (2012).

The relatively high order multi-body model is not suitable for control design methods, thus the investigated model can be reduced with a suitable model order reduction method. Using a smaller size model also speeds up the simulation process during the design validation phase. More detailed information about dynamic model order reduction methods is presented in the following references: Benner et al. (2003), Nowakowski et al. (2013), Dukic and Saric (2012) and Balas et al. (2007).

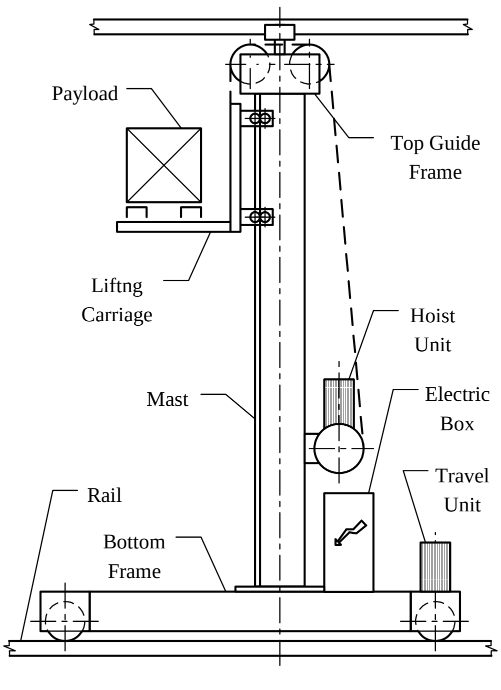
The aim of this work is to generate a dynamic model which is sufficiently simple to fulfill the requirements of control synthesis techniques and time domain simulations. The main advantage of our multi-body model to be introduced in the next section is that with this model mast vibrations can be investigated at various locations of the mast. Dynamic models with several lifted load positions can also be generated in a simple way by using this modelling technique.

The structure of this paper is as follows. Section 2 introduces a linear dynamic model of single-mast stacker cranes based on the multi-body modelling approach. In Section 3 the modal superposition method is presented, which is useful for taking structural damping into consideration. Section 4 presents the structural damping by means of the Rayleigh damping approach. In Section 5 the properties and time domain simulation results of our model are presented.

2 Multi-body model of single-mast stacker cranes

Practically the stacker cranes have two fundamental structural constructions: the so-called single-mast and twin-mast structures. In the work the single-mast stacker cranes is analysed since this construction has less stiffness. Thus considerable mast vibrations may occur in the frame structure of these machines. A line drawing of a single-mast stacker crane with its main components is shown in Figure 1.

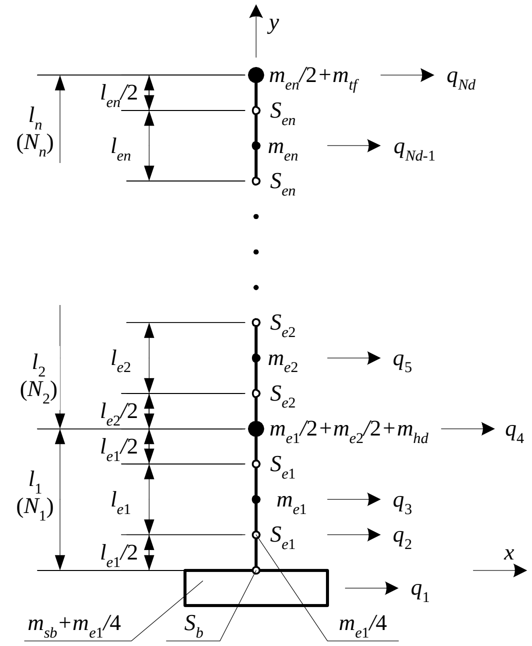
Figure 1 Single mast stacker crane



In this section a linear dynamic model of single-mast stacker cranes is introduced, which is simple, relatively low order and capable of investigating the dynamic behaviour of stacker cranes under varying excitation effects. The multi-body modelling technique, besides FEM, is one of the most widely used methods in structural analysis. Benefits of this method are the lower DOF (compared with FEM) and simpler equations of motion. With an adequate selection of generalised coordinates the matrices of motion equations can be generated in a diagonal form.

A linear multi-body model of a stacker crane structure is shown in Figure 2. The mast structure is divided into sections between the components of stacker crane, e.g., bottom frame, hoist unit, top guide frame, etc. The continuum sections are approximated by lumped mass elements. These rigid elements are generated by the division of sections (with length l_i) into N_i pieces.

Figure 2 Multi-body model of single-mast stacker crane



Lumped masses are located in the centre of elements (in the so called nodes). All inertia effects are concentrated at the nodes. The magnitudes of lumped masses are equal to the mass of corresponding beam pieces. The nodes in Figure 2 are denoted by solid dots. At either end of sections, because of nature of modelling, so called 'half elements' are located. In this way it is easy to generate connections between neighbouring sections by adding together the corresponding lumped masses.

The elements are interconnected by elastic but massless links (in Figure 2 denoted by hollow dots). This elasticity approximates the bending elasticity of original beam. Elasticity is provided by spiral springs (with spring stiffness S_{ei}) connected parallel to the ideal, frictionless hinges. The magnitude of this spring stiffness is calculated by means of strength of materials.

As Figure 2 shows further components of the stacker crane are modelled by lumped masses, i.e., the gross weight of the bottom frame (with idle and drive wheel blocks, electric box, etc.), the masses of the hoist unit and the top guide frame. The mass of the bottom frame is denoted by m_{sb}, the mass of the hoist unit by m_{hd} and the mass of the top guide frame by m_{tf}. The elasticity of the bottom frame beam is approximated by a spiral spring (S_b) between the lumped mass of the bottom frame and the lower end of the mast. The main parameters of the dynamic model are shown in Table 1.

In the generation of the motion equations of the multi-body model several equivalent choices of generalised coordinates are exist. With the adequate selection of generalised coordinates the mass and stiffness matrices of motion equations can be simplified and transformed into a diagonal form. One of these possible choices of generalised coordinates (i.e., the q_i vertical displacements of each lumped mass) is shown in Figure 2. The DOF of the model is denoted by N_d.

Table 1 Main parameters of dynamical model

Denomination	*Denotation*	*Value*
Payload	m_p	1,200 kg
Mass of lifting carriage	m_{lc}	410 kg
Mass of hoist unit	m_{hd}	470 kg
Mass of top guide frame	m_{tf}	70 kg
Mass of bottom frame	m_{sb}	2,418 kg
Lifted load position	h_h	1–44 m
Length of sections	l_1	3,5 m
	l_2	11,5 m
	l_3	30 m
Cross-sectional areas	A_1; A_2	0.02058 m^2
	A_3	0.01518 m^2
Second moments of areas	I_{z1}; I_{z2}	0.00177 m^4
	I_{z3}	0.00106 m^4

The generalised coordinate vector of the model is:

$$q = \begin{bmatrix} q_1 & q_2 & \dots & q_{N_d} \end{bmatrix}^T. \quad (1)$$

The differential equations of motion can be determined in several ways, e.g., by means of using the Euler-Lagrange equations. The detailed derivation of mass and stiffness matrices and dynamic equations for the before-mentioned multi-body model can be found in Hajdu and Gáspár (2013). The general form of matrix equation of motion in case of natural vibrations (i.e., without external excitation forces) is:

$$M\ddot{q} + Sq = 0, \quad (2)$$

where M is the mass matrix and S is the stiffness matrix of the system. For the investigation of excited vibrations of the flexible structure it is necessary to determine the matrix motion equation subject to external excitation forces, which is:

$$M\ddot{q} + Sq = F. \quad (3)$$

where F in general is the vector of external excitation forces. In this paper a single-input system is investigated, where the input signal of the model is the external force acting in the direction of q_1 generalised coordinate. Thus in vector F only this coordinate has a value other than zero.

3 The modal superposition approach

For the analysis of multiple DOF dynamic problems, in most cases, a matrix motion equation presented in (3) must be solved. Several kinds of (either analytical or numerical) methods exist for this purpose. In this section the so-called modal superposition approach is introduced which is useful for taking structural damping into consideration as well as for model order reduction in the following sections.

It is assumed that the solution of the homogenous part of (3) in the following form is:

$$q(t) = \psi e^{\lambda t}. \tag{4}$$

Substituting (4) into the homogenous part of equation (3) leads to the following general eigenvalue problem.

$$\left(\lambda^2 M + S\right)\psi = 0. \tag{5}$$

The number of eigenvalues of this problem equals to the N_d DOF of system (3). Since in most cases the mass matrix is symmetric positive definite and the stiffness matrix is symmetric positive semidefinite the eigenvalues of (5) are non-positive real and the elements of eigenvectors are also real. The λ_j^2 eigenvalues are the squares of natural frequencies of the dynamic system thus the complex natural frequencies are $\lambda_j = \pm\alpha_j$. Since the investigated model is a free model, i.e., it has rigid body motion facility (unconstrained DOF), thus the smallest eigenvalue equals to zero. The ψ_j eigenvector corresponding to the λ_j^2 eigenvalue is also known as the j^{th} mode shape of dynamic system.

It can be proved that eigenvectors have the following orthogonality properties:

$$\begin{aligned} \psi_j^T M \psi_k &= 0, \\ \psi_j^T S \psi_k &= 0, \end{aligned} \tag{6}$$

for all $j \neq k$. The presented properties in (6) are also known as the orthogonality properties of mode shapes. From eigenvectors the so-called modal matrix is generated:

$$\Psi = \begin{bmatrix} \psi_1 & \psi_2 & \cdots & \psi_{N_d} \end{bmatrix}. \tag{7}$$

Owning to the orthogonality properties presented in expressions (6) the original mass and stiffness matrices can be transformed into a diagonal form by means of the modal matrix:

$$\Psi^T M \Psi = diag\left\{m_j\right\}. \tag{8}$$

and

$$\Psi^T S \Psi = diag\left\{s_j\right\}. \tag{9}$$

These diagonal matrices are known as the modal mass and stiffness matrices containing modal mass and stiffness values for each one of the modes. Since the eigenvectors can be arbitrarily scaled, the mode shapes can be scaled so that the modal mass value for each mode is equal to one. The modal mass matrix is therefore an identity matrix. In the following derivations this kind of scaling method is applied. It can be proved that with this scaling method the modal stiffness matrix will be a diagonal matrix containing squares of natural frequencies for each one of the modes.

$$\Psi^T M \Psi = I, \tag{10}$$

and

$$\Psi^T S \Psi = diag\left\{\alpha_j^2\right\} = \Lambda. \tag{11}$$

The solution of (3) coupled matrix motion equation of multiple degrees of freedom dynamic systems can be significantly simplified. The following coordinate transformation (the so-called modal coordinate transformation) is introduced:

$$q = \Psi p. \tag{12}$$

The elements in p vector are referred as the modal coordinates or modal participation factors. Substituting the (12) coordinate transformation into equation (3) and pre-multiplying the resulting equation by transposing the modal matrix yields:

$$\Psi^T M \Psi \ddot{p} + \Psi^T S \Psi p = \Psi^T F, \tag{13}$$

i.e.,

$$I\ddot{p} + \Lambda p = \Phi, \tag{14}$$

where $\Psi^T F = \Phi$ is the so-called modal excitation force. Since the modal mass and stiffness matrices are diagonal matrices equation (3) has been decoupled. In other words, the original problem has been transformed from a large multiple DOF problem into a set of single DOF problems that can be solved using simple, well known methods. After solving each single DOF problems for p_j modal coordinates or modal participation factors the solution of the original problem can be generated by transforming (12). This coordinate transformation can be rewritten as:

$$q = \Psi p = \sum_{j=1}^{N_d} p_j \psi_j. \tag{15}$$

It can be seen that the q solution of (3) original problem is generated by means of linear combination of elementary motion components, each j^{th} one having shape of the j^{th} mode of vibration ψ_j and amplitude defined by the j^{th} modal participation factor p_j. This method is also known as the modal superposition approach.

The calculation of p_j modal coordinates can be achieved, e.g., by means of Laplace transformation or analytical solution of the j^{th} equation in the decoupled (14) system:

$$\ddot{p}_j + \alpha_j^2 p_j = \phi_j. \tag{16}$$

4 Structural damping

The investigation of damping plays a significant role in the dynamic analysis of the engineering structures. However, the available knowledge about damping in most cases is strongly limited. In addition, the method which takes the structural damping into consideration must be simple and numerically applicable in the dynamic analysis of the structure. In the dynamic analysis of damped systems the damping matrix D appears. In general it is not possible to state that the damping matrix can be diagonalised by the modal coordinate transformation. However, there are some cases of damping where this useful property exists.

$$M\ddot{q} + D\dot{q} + Sq = F. \tag{17}$$

The most effective way to introduce structural damping into dynamic analysis is the approximation of real damping value with equivalent Rayleigh damping in form of:

$$D = aM + bS, \tag{18}$$

where D is the damping matrix of (17) damped dynamic system as well as a and b are predefined constants. As (17) shows the damping matrix is a linear combination of mass and stiffness matrices. This kind of damping is usually referred as proportional damping. The Rayleigh damping is particularly relevant because the damping matrix of this approach can be diagonalised by the (12) modal coordinate transformation, and thus the (17) system of N_d equations of motion can be decoupled into N_d single DOF equations. It can be proved that the j^{th} equation of this decoupled system in general is:

$$\ddot{p}_j + \left(a + b\alpha_j^2\right)\dot{p}_j + \alpha_j^2 p_j = \phi_j. \tag{19}$$

Unfortunately a general theoretical method for the determination of a and b unknown parameters does not exist. However, from the structure of equation (19) some important observations can be made. More detailed information about experimental determination of the parameters of proportional damping can be found in Spears and Jensen (2009), Chowdhury and Dasgupta (2003) and Pápai et al. (2012).

The general form of the (19) motion equation of single DOF system is:

$$\ddot{p}_j + 2\zeta_j\alpha_j\dot{p}_j + \alpha_j^2 p_j = \phi_j, \tag{20}$$

where ζ_j is the damping ratio corresponding to the j^{th} modal coordinate. From the comparison of equations (19) and (20) the relation between modal damping ratio and natural frequency is:

$$2\zeta_j\alpha_j = a + b\alpha_j^2, \tag{21}$$

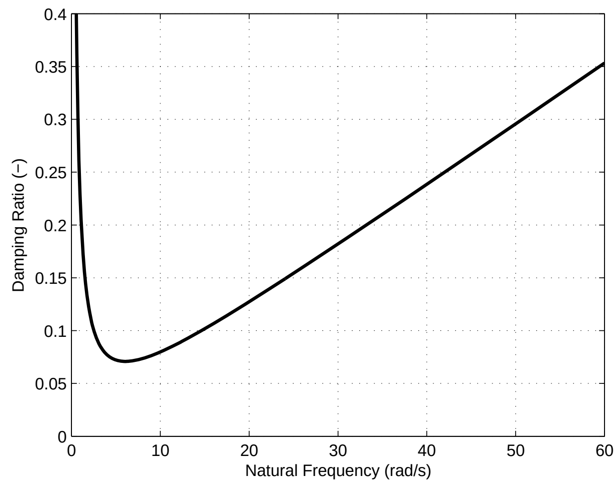
thus:

$$\zeta_j = \frac{a}{2\alpha_j} + \frac{b\alpha_j}{2}. \tag{22}$$

A typical diagram of equation (22) is shown in Figure 3. From the diagram it can be concluded that in the lower range of the natural frequency the first (nonlinear) term of (22) dominates and as the natural frequency increases the effect of the first term diminishes and the second (linear) term begins dominating. This property helps determine the unknown parameters of damping.

For determination of the two unknown parameters of proportional damping two pairs of corresponding damping ratio and natural frequency values are necessary. With this data and by means of (21) a system of linear equations can be constructed and the unknown parameters calculated. In the design period of stacker cranes the engineer unfortunately must rely on previous experience in determining this dataset. However, when an actual stacker crane is available the necessary dataset can be measured by the help of experimental modal analysis. With this method more than the necessary two data pairs can be measured, thus the (18) proportionality assumption can be verified.

Figure 3 Relation between damping ratio and natural frequency



In the damped case (similarly to the undamped case presented in the last section) the analytical solution of (17) matrix equation of motion is determined by the Laplace transformation of (20) and using the (15) modal superposition equation. The Laplace transformation of q solution of (17) equation of motion and the transfer function matrix are presented in the following equation:

$$q(s) = \sum_{j=1}^{N_d} \frac{\psi_j^T \psi_j}{s^2 + 2\zeta_j\alpha_j s + \alpha_j^2} F(s) = H(s)F(s), \tag{23}$$

where $H(s)$ is the transfer function matrix form the input vector F to the generalised coordinate vector q. With the help of this transfer function matrix the time functions of generalised coordinates can be determined by means of transformation (23) back into time domain.

As mentioned before the aim of this work is to generate a dynamic model which is suitable for not only the investigation of mast-vibrations but also control synthesis purposes. Since most of control design methods use the state space representation of the model, the matrix equation of motion (17) must be transformed into the state space form. The input signal of our model is the external force acting in the direction of q_1 generalised coordinate. In the further steps of research the dynamic model is applied in the synthesis of the control which realises the mast-vibration free positioning control of single-mast stacker cranes. Therefore the outputs of the investigated state space representation can be classified into two groups. The first one is used for analysing the mast-vibrations in arbitrary mast locations. These performance outputs are the inclinations of the mast, i.e., the horizontal position difference between the lowest point of the mast and an arbitrary location of the mast. The second one is the measurement output, which is the horizontal position or velocity of the stacker crane.

The state space realisation of a linear time invariant system in general is described by the following equations:

$$\begin{aligned} \dot{x} &= Ax + B_1 d + B_2 u, \\ z &= C_1 x + D_{11} d + D_{12} u, \\ y &= C_2 x + D_{21} d, \end{aligned} \tag{24}$$

where x, d, u are the state vector, disturbance and control input respectively and $x_0 \in R^n$ is the initial state of the system. Here n is known as the order of the system and m and p are the number of input and output variables of the system respectively. Let us define the state vector with the generalised coordinate vector of (17) as follows:

$$x = \begin{bmatrix} \dot{q} & q \end{bmatrix}^T. \tag{25}$$

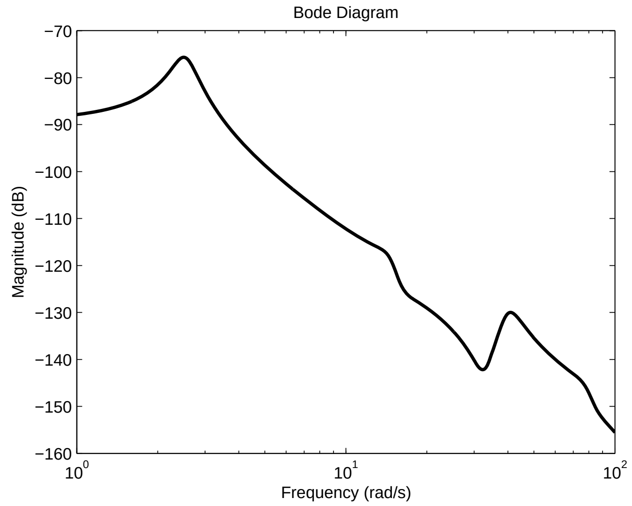
With this definition the state space representation of our investigated multi-body model can be generated taking notice of the above mentioned definition of input and output signals. A detailed derivation of the state space representation of a similar multi-body model is presented in Hajdu and Gáspár (2013).

5 Simulation results

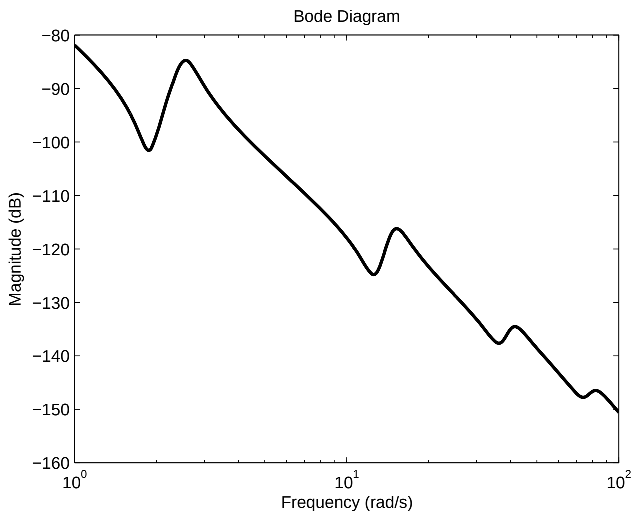
In this section the analysis of the system both in time and frequency domain is presented. In Figure 4(a) the Bode diagram of the transfer function from tractive force F to mast inclination at mast tip is presented, and in Figure 4(b) the Bode diagram of transfer function from tractive force to stacker crane position is shown.

The time domain simulations are carried out by using two kinds of tractive force functions. The first one is a simple step function while the second one is a more complicated, piecewise linear function (see in the first diagrams of Figures 5 to 7). In the first two simulation cases mast vibration is investigated at three locations of the mast near the fixed (highest) lifted load position. Location L_1 is placed at the mast tip, location L_2 is at the height of approximately 30 m while location L_3 is at the height of approximately 15 m. In the third simulation case the vibrations of the mast tip are investigated near varying lifted load position. The load positions h_1-h_5 are evenly placed in the admissible position range. The time functions of excitation (tractive) forces, stacker crane positions and velocities as well as mast vibrations are presented in Figures 5 to 7.

Figure 4 Bode diagrams of transfer functions, (a) from tractive force to mast inclination at mast tip (b) from tractive force to stacker crane position



(a)



(b)

Figure 5 Simulation results, case #1

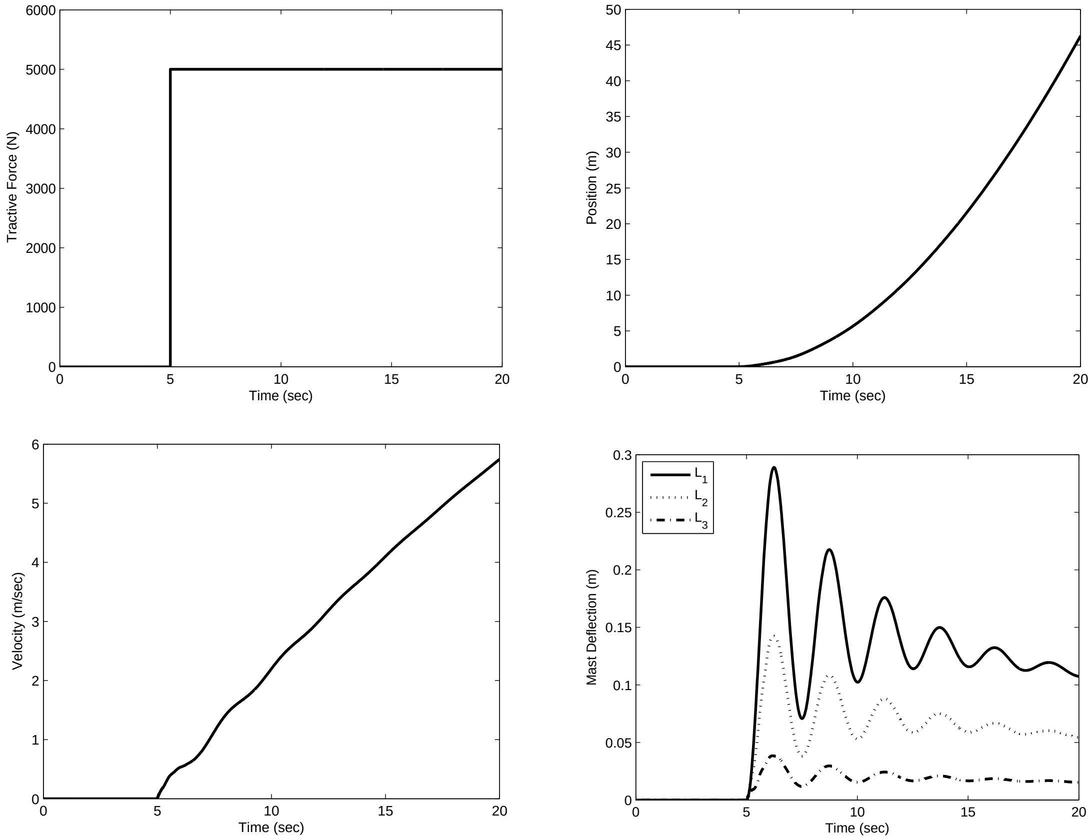


Figure 6 Simulation results, case #2

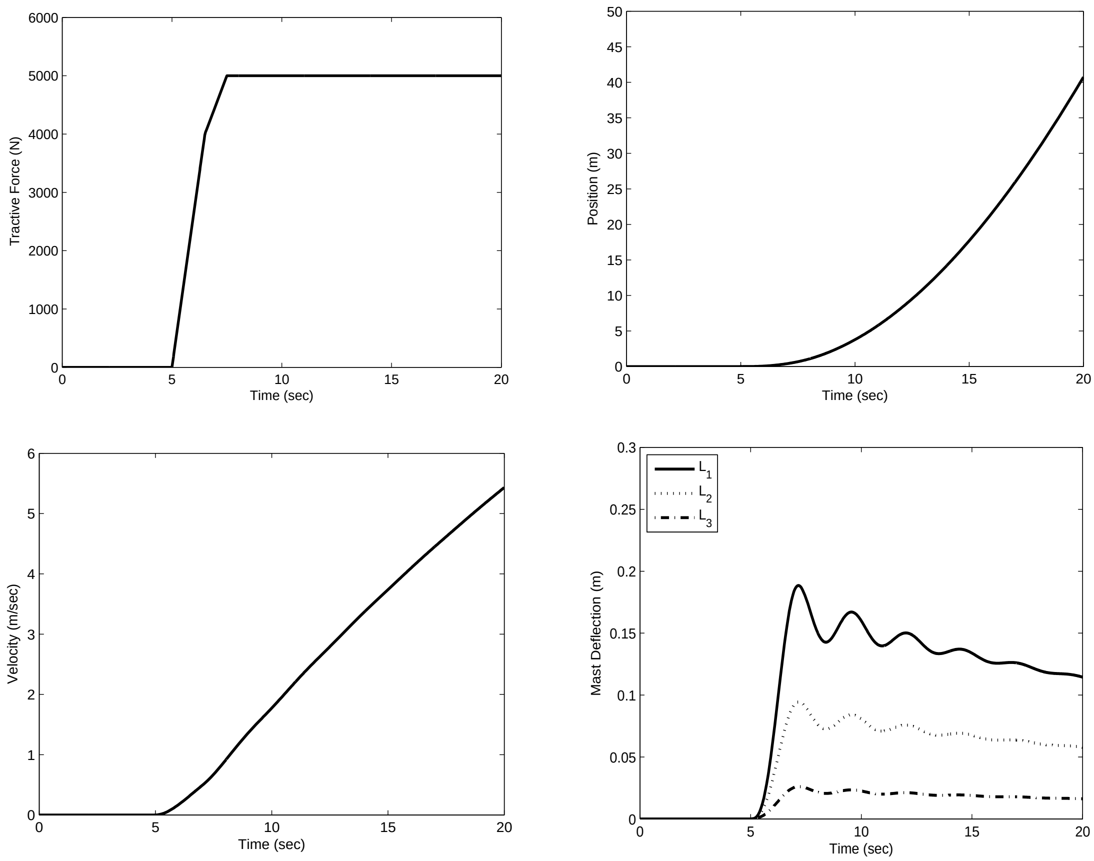
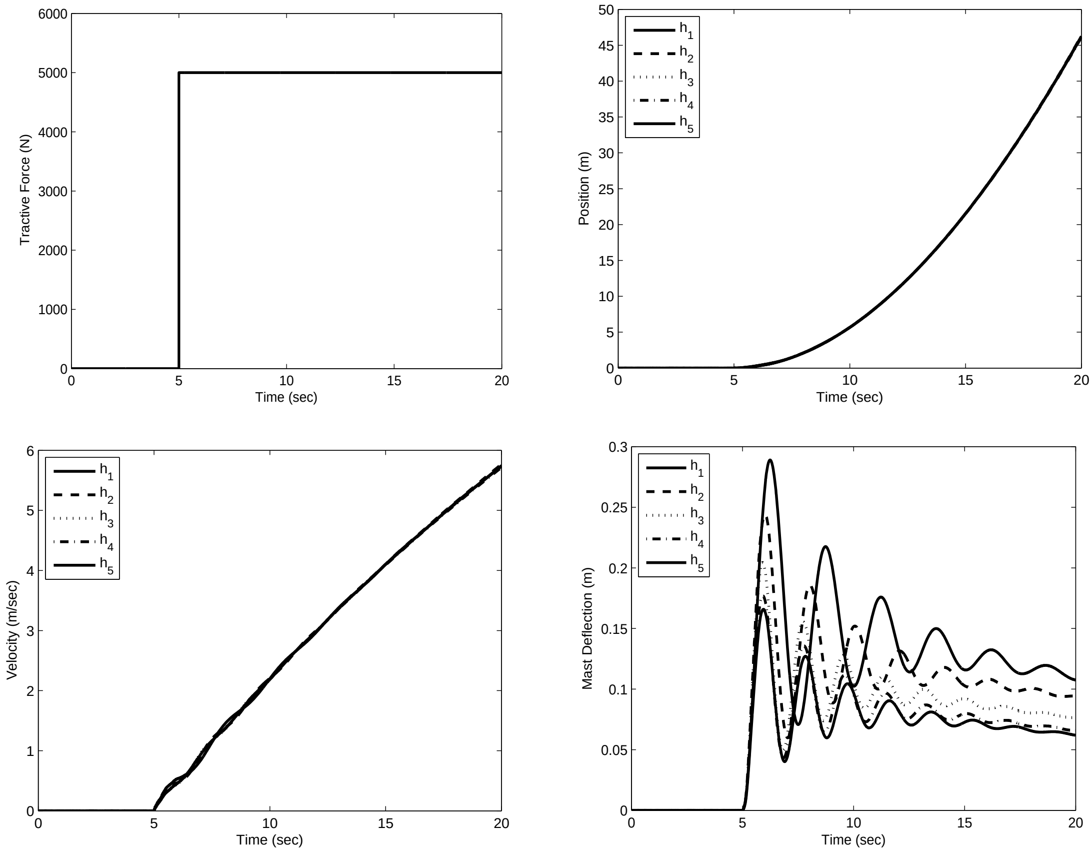


Figure 7 Simulation results, case #3



6 Conclusions

In the paper dynamic modelling considerations of single-mast stacker crane structures has been presented. The aim of this work is to generate an appropriate dynamic model of single-mast stacker cranes for investigating mast vibrations and designing the position controllers. To generate the motion equations of single-mast stacker cranes the multi-body modelling approach is selected. To solve the motion equations the modal superposition approach is introduced, which is also useful in taking structural damping into consideration. Structural damping was approximated by the Rayleigh damping assumption, which makes it possible to calculate the modal damping values based on engineers' experience or results of experimental modal analyses. The dynamic model is useful in simulating the mast vibrations of single-mast stacker cranes as well as in designing the position control of these machines. The main advantage of this modelling approach is that with this model mast vibrations can be investigated simply at various locations of mast and various lifted load positions.

The results of this work can be implemented in the design of single-mast stacker cranes to predict the dynamical behaviour of the machine. Another implementation area is the control design of these machines. The introduced model (after applying the necessary model-order reduction methods) is suitable for designing robust controllers to control the position of stacker cranes with reduced mast vibrations. This model can also be useful after the control design phase in the validation of results.

References

Angeles, J. and Kecskeméthy, A. (1995) *Kinematics and Dynamics of Multi-body Systems*, Springer-Verlag, New York.

Arnold, D. and Dietzel, M. (2000) 'Aktive Schwingungsdämpfung von Regalbediengeräten', *F+H Fördern und Heben*, Vol. 50, Nos. 1–2, pp.50–52.

Arnold, D. and Schumacher, M. (1993) 'Optimierung fördertechnischer Komponenten unter dynamischen Belastungen', *F+H Fördern und Heben*, Vol. 43, Nos. 1–2, pp.18–25.

Balas, G., Chiang, R., Packard, A. and Safonov, M. (2007) *Robust Control Toolbox 3 User's Guide*, The MathWorks Inc., Natick, MA.

Benner, P., Quintana-Ortí, E.S. and Quintana-Ortí, G. (2003) 'State-space truncation methods for parallel model reduction of large-scale systems', *Parallel Computing – Special Issue: Parallel and Distributed Scientific and Engineering Computing*, Vol. 29, Nos. 11–12, pp.1701–1722.

Chowdhury, I. and Dasgupta, S. (2003) 'Computation of Rayleigh damping coefficients for large systems', *The Electronic Journal of Geotechnical Engineering*, Vol. 8, No. 3, pp.1–11.

Dietzel, M. (1999) *Beeinflussung des Schwingungsverhaltens von Regalbediengeräten durch Regelung des Fahrantriebs*, Dissertation, Institut für Fördertechnik Karlsruhe.

Dukic, S.D. and Saric, A.T. (2012) 'Dynamic model reduction: an overview of available techniques with application to power systems', *Serbian Journal of Electrical Engineering*, Vol. 9, No. 2, pp.131–169.

Hajdu, S. and Gáspár, P. (2013) 'Investigation of the influence of lifted load on dynamical behavior of stacker cranes through unstructured uncertainties', *CINTI: Proceeding of the 14th IEEE International Symposium on Computational Intelligence and Informatics*, pp.179–184.

Jalón, J.G. and Bayo, E. (1994) *Kinematic and Dynamic Simulation of Multibody Systems. The Real-Time Challenge*, Springer-Verlag, New-York.

Keskinen, E., Kuokkala, V-T., Vuoristo, T. and Martikainen, M. (2007) 'Multi-body wave analysis of axially elastic rod systems', *Proc. IMechE, Part K: J. Multi-body Dynamics*, Vol. 221.

Köhne, M. (2003) 'Analyse und Modellierung des dynamischen Verhaltens von Regalbediengeräten', *VDI-Berichte*, No. 1756, pp.1039–1048.

Nowakowski, C., Kürschner, P., Eberhard, P. and Benner, P. (2013) 'Model reduction of an elastic crankshaft for elastic multibody simulations', *ZAMM – Journal of Applied Mathematics and Mechanics*, Vol. 93, No. 4, pp.198–216.

Pápai, F., Adhikari, S. and Wang, B. (2012) 'Estimation of modal dampings for unmeasured modes', *Slovak Journal of Civil Engineering*, Vol. 20, No. 4, pp.1–11.

Reisinger, K.H. (1998) *Schwingungssimulation von Regalförderzeugen*, Dissertation, Technische Universität Graz.

Schumacher, M. (1994) *Untersuchung des Schwingungsverhaltens von Einmast-Regalbediengeräten*, Dissertation, Institut für Fördertechnik Karlsruhe, pp.11–24.

Spears, R.E. and Jensen, S.R. (2009) 'Approach for selection of Rayleigh damping parameters used for time history analysis', *ASME Pressure Vessels and Piping Division Conference*, Prague.

Ziaei-Rad, S., Ariaei, A. and Imregun, M. (2007) 'Vibration analysis of Timoshenko beams under uniform partially distributed moving masses', *Proc. IMechE*, *Part K: J. Multi-body Dynamics*, Vol. 221.