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One-loop radiative corrections to the helicity amplitudes of QCD processes involving four quarks and one gluon ⋆

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Abstract

We present analytic results for the one-loop corrections of the helicity amplitudes of the QCD five-parton subprocesses involving four quarks and one gluon obtained with a standard Feynman diagram calculation using dimensional reduction.

The technical developments achieved recently in perturbative QCD calculations (helicity method [1–4], string theory based derivations [5,6]) made calculations of one-loop corrections to helicity amplitudes up to five-parton and perhaps also to six-parton processes feasible. As a first result, Bern, Dixon and Kosower published recently the one-loop corrections to the five-gluon amplitude in QCD and in $N = 1,2,4$ supersymmetric Yang-Mills theories. They used a string based technique and helicity method and obtained a remarkably short analytic answer. In a previous paper [7] we presented a simple analytic result for the singular parts of all helicity amplitudes of all five-parton processes (in full agreement with the results of [6] in the case of the five gluon amplitudes). In this letter we present the complete one-loop corrections for processes involving four quarks (equal or unequal flavors) and one gluon. We used the conventional Feynman-diagram method, however, the use of the helicity technique was decisively important. Our method was tested previously by the diagrammatic evaluation of the one-loop corrections of the helicity amplitudes of all $2 \to 2$ parton processes [8]. A systematic description together with the non-trivial details of our calculation will be presented in a later publication. Here we mention only that the tensor integrals have been reduced to scalar integrals with a reduction method similar to [9]. Calculating the integrals this way and the use of the helicity method allow us to eliminate all integrals which are more complicated than pentagon (box) tensor-integrals with one (two) integration momenta in the numerator. As a result, the Gram determinants in the denominator which blow up the size of intermediate expressions were eliminated at the very beginning of the calculation. For the sake of simplicity we performed the calculation in the dimensional reduction scheme (in $D = 4 - 2\varepsilon$ dimensions). Transition rules to different schemes - such as conventional dimensional regularization, 't Hooft–Veltman - have been derived in [8].

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It is convenient to give our result in a crossing symmetric form for the unphysical channel where all particles are outgoing $0 \to \bar{q}\bar{Q}Qqg$. The momenta of the partons are labeled as

$$0 \to \text{antiquark}_1(\bar{q}) + \text{antiquark}_2(\bar{Q}) + \text{quark}_2(Q) + \text{quark}_1(q) + \text{gluon}(g)\ . \tag{1}$$

The color structure of the amplitudes is the same at one loop as at tree level:

$$\begin{aligned}
&\mathcal{A}^{(i)}(\bar{q},h_{\bar{q}};\bar{Q},h_{\bar{Q}};Q,h_Q;q,h_q;g,h_g)\\
&= g^3\left(\frac{g}{4\pi}\right)^{2i}\Bigg[\sum_{(q_1\neq q_2)\in\{q,Q\}}(T^g)_{q_1\bar{q}_2}\delta_{q_2\bar{q}_1}a^{(i)}_{q_1\bar{q}_2}(\bar{q},h_{\bar{q}};\bar{Q},h_{\bar{Q}};Q,h_Q;q,h_q;g,h_g)\\
&\quad-\sum_{(q_1\neq q_2)\in\{q,Q\}}\frac{1}{N_c}(T^g)_{q_1\bar{q}_1}\delta_{q_2\bar{q}_2}a^{(i)}_{q_1\bar{q}_1}(\bar{q},h_{\bar{q}};\bar{Q},h_{\bar{Q}};Q,h_Q;q,h_q;g,h_g)\Bigg]\ ,
\end{aligned} \tag{2}$$

where $i=0$ means tree level and $i=1$ means one-loop approximation.

At a given order in perturbation theory, there are only two independent color subamplitudes because we have the symmetry properties

$$a^{(i)}_{q\bar{Q}}(\bar{q},h_{\bar{q}};\bar{Q},h_{\bar{Q}};Q,h_Q;q,h_q;g,h_g) = a^{(i)}_{Q\bar{q}}(\bar{Q},h_{\bar{Q}};\bar{q},h_{\bar{q}};q,h_q;Q,h_Q;g,h_g)\ , \tag{3}$$

$$a^{(i)}_{q\bar{q}}(\bar{q},h_{\bar{q}};\bar{Q},h_{\bar{Q}};Q,h_Q;q,h_q;g,h_g) = a^{(i)}_{Q\bar{Q}}(\bar{Q},h_{\bar{Q}};\bar{q},h_{\bar{q}};q,h_q;Q,h_Q;g,h_g)\ . \tag{4}$$

Furthermore, we should consider only four helicity configurations. If we change the sign of all helicities, we obtain the corresponding amplitudes simply by replacing the spinor products $\langle\ldots\rangle \to -[\ldots]$, where the angle bracket and squared bracket denote spinor products with minus-plus and plus-minus helicities,

$$\langle pq\rangle = \overline{\psi_-(p)}\psi_+(q) \quad \text{and} \quad [pq] = \overline{\psi_+(p)}\psi_-(q)\ .$$

Due to helicity conservation along a fermion line, we have $h_q = -h_{\bar{q}}$ and $h_Q = -h_{\bar{Q}}$. We present our result for positive gluon helicity and label the remaining helicities with h_q and h_Q.

In order to be able to write down the result for arbitrary values of h_q and h_Q, we introduce the helicity dependent momenta

$$\begin{aligned}
&r(h_q) = q \quad \text{if} \quad h_q = - \qquad \text{and} \quad r(h_q) = \bar{q} \quad \text{if} \quad h_q = +,\\
&R(h_Q) = Q \quad \text{if} \quad h_Q = - \qquad \text{and} \quad R(h_Q) = \bar{Q} \quad \text{if} \quad h_Q = +.
\end{aligned} \tag{5}$$

We shall suppress the helicity dependence of r and R.

For the sake of completeness, we recall the tree-level amplitudes $a^{(0)}_{ij}$

$$a^{(0)}_{ij}(h_q,h_Q,+) = \mathrm{i}p_a(h_q,h_Q,+)\,\frac{\langle ij\rangle}{\langle ig\rangle\langle gj\rangle}\ . \tag{6}$$

We note that the helicity dependence can be absorbed completely in the factor p_a:

$$p_a(h_q,h_Q,+) = (-1)^{\delta_{h_q h_Q}}\frac{\langle rR\rangle^2}{\langle q\bar{q}\rangle\langle Q\bar{Q}\rangle}\ , \tag{7}$$

where δ is the usual Kronecker δ.

At one loop, the result can naturally be decomposed into soft contributions $S_{ij}(h_q,h_Q,+)$ given in [7], ultraviolet renormalization terms $\mathcal{R}_{ij}(h_q,h_Q,+)$, terms coming from the expansion of collinear $1/\epsilon$ singularities

$\mathcal{C}_{ij}(h_q, h_Q, +)$ given by vertex and self energy integrals, finite terms composed from spinor products, dot products and single logarithms $\mathcal{D}_{ij}(h_q, h_Q, +)$, and finite terms $\mathcal{E}_{ij}(h_q, h_Q, +)$ containing factors of Li_2, $\ln^2$ and π^2 coming from pentagon and box integrals. The labels i, j run over q, Q, $\bar{q}$, $\bar{Q}$ similarly to the corresponding labels in the color subamplitudes $a_{ij}^{(1)}$. The color subamplitude $a_{ij}^{(1)}$ has then the decomposition

$$\begin{aligned} & a_{ij}^{(1)}(\bar{q}, -h_q; \bar{Q}, -h_Q; Q, h_Q; q, h_q; g, +) \\ & = \mathcal{S}_{ij}(h_q, h_Q, +) + \mathcal{R}_{ij}(h_q, h_Q, +) + \mathcal{C}_{ij}(h_q, h_Q, +) + \mathcal{D}_{ij}(h_q, h_Q, +) + \mathcal{E}_{ij}(h_q, h_Q, +), \end{aligned} \tag{8}$$

where we suppressed the dependence on the momenta $\bar{q}, q, \bar{Q}, Q, g$. For completeness we recall the soft contributions

$$\begin{aligned} S_{Q\bar{q}}(h_q, h_Q, +) = & -\frac{c_\Gamma}{\varepsilon^2} \Big\{ N_c a_{Q\bar{q}}^{(0)}(h_q, h_Q, +) \left[\mathcal{P}_{\bar{q}g} + \mathcal{P}_{\bar{Q}q} + \mathcal{P}_{Qg} \right] \\ & - \frac{1}{N_c} a_{Q\bar{q}}^{(0)}(h_q, h_Q, +) \left[-\mathcal{P}_{\bar{q}\bar{Q}} + \mathcal{P}_{\bar{q}Q} + \mathcal{P}_{\bar{q}q} + \mathcal{P}_{\bar{Q}Q} + \mathcal{P}_{\bar{Q}q} - \mathcal{P}_{Qq} \right] \\ & - \frac{1}{N_c} a_{Q\bar{Q}}^{(0)}(h_q, h_Q, +) \left[-\mathcal{P}_{\bar{q}\bar{Q}} + \mathcal{P}_{\bar{q}g} + \mathcal{P}_{\bar{Q}q} - \mathcal{P}_{qg} \right] \\ & - \frac{1}{N_c} a_{q\bar{q}}^{(0)}(h_q, h_Q, +) \left[\mathcal{P}_{\bar{Q}q} - \mathcal{P}_{\bar{Q}g} - \mathcal{P}_{Qq} + \mathcal{P}_{Qg} \right] \Big\} \end{aligned} \tag{9}$$

and

$$\begin{aligned} S_{Q\bar{Q}}(h_q, h_Q, +) = & -\frac{c_\Gamma}{\varepsilon^2} \Big\{ N_c a_{Q\bar{Q}}^{(0)}(h_q, h_Q, +) \left[\mathcal{P}_{\bar{q}q} + \mathcal{P}_{\bar{Q}g} + \mathcal{P}_{Qg} \right] \\ & - \frac{1}{N_c} a_{Q\bar{Q}}^{(0)}(h_q, h_Q, +) \left[-\mathcal{P}_{\bar{q}\bar{Q}} + \mathcal{P}_{\bar{q}Q} + \mathcal{P}_{\bar{q}q} + \mathcal{P}_{\bar{Q}Q} + \mathcal{P}_{\bar{Q}q} - \mathcal{P}_{Qq} \right] \\ & + N_c a_{Q\bar{q}}^{(0)}(h_q, h_Q, +) \left[\mathcal{P}_{\bar{q}\bar{Q}} - \mathcal{P}_{\bar{q}q} - \mathcal{P}_{\bar{Q}g} + \mathcal{P}_{qg} \right] \\ & + N_c a_{q\bar{Q}}^{(0)}(h_q, h_Q, +) \left[-\mathcal{P}_{\bar{q}q} + \mathcal{P}_{\bar{q}g} + \mathcal{P}_{Qq} - \mathcal{P}_{Qg} \right] \Big\}, \end{aligned} \tag{10}$$

where we introduced

$$\mathcal{P}_{ij} = \left(-\frac{\mu^2}{s_{ij}} \right)^{\varepsilon}, \quad c_\Gamma = (4\pi)^{\varepsilon} \frac{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)} .$$

We note that the helicity dependence of the soft contribution is given by the Born factors $a_{ij}^{(0)}$. This holds for the ultraviolet renormalization contributions and the collinear $(1/\varepsilon)$ singularity as well. We record the result in such a form that the helicity dependence of the $\mathcal{C}$ terms is again completely factored into the Born terms [1].

$$\mathcal{R}_{ij}(h_q, h_Q, +) = -\frac{(4\pi)^{\varepsilon}}{2\,\varepsilon\Gamma(1-\varepsilon)} (11 N_c - 2 N_f)\ a_{ij}^{(0)}(h_q, h_Q, +), \tag{11}$$

$$\begin{aligned} \mathcal{C}_{Q\bar{Q}}(h_q, h_Q, +) = & \frac{c_\Gamma}{\varepsilon} a_{Q\bar{Q}}^{(0)}(h_q, h_Q, +) \left(\frac{2}{3}(N_c - N_f)\mathcal{P}_{\bar{q}q} + \frac{3}{2}\frac{1}{N_c}(\mathcal{P}_{\bar{q}q} + \mathcal{P}_{\bar{Q}Q}) \right) \\ & + a_{Q\bar{Q}}^{(0)}(h_q, h_Q, +) \left(\frac{29}{18} N_c + \frac{13}{2}\frac{1}{N_c} - \frac{10}{9} N_f \right), \end{aligned} \tag{12}$$

[1] Note that the $\mathcal{C}$ terms are not uniquely determined since we can always shift finite contributions between $\mathcal{D}$ and $\mathcal{C}$ terms.

$$\mathcal{C}_{Q\bar{q}}(h_q,h_Q,+) = \frac{c_\Gamma}{\varepsilon} a^{(0)}_{Q\bar{q}}(h_q,h_Q,+)\left(\frac{2}{3}(N_c-N_f)\mathcal{P}_{\bar{Q}Q} + \frac{3}{2}\frac{1}{N_c}(\mathcal{P}_{\bar{q}q}+\mathcal{P}_{\bar{Q}Q})\right)$$
$$+a^{(0)}_{Q\bar{q}}(h_q,h_Q,+)\left(\frac{29}{18}N_c + \frac{13}{2}\frac{1}{N_c} - \frac{10}{9}N_f\right). \tag{13}$$

As expected, the next-to-leading order corrections destroy some of the symmetries of the Born terms (see (6),(7)). However, there are several symmetry relations which remain valid even at one loop:

$$a^{(1)}_{Q\bar{Q}}(-,-,+) = -a^{(1)}_{Q\bar{Q}}(+,+,+)|_{\bar{q}\leftrightarrow q,\bar{Q}\leftrightarrow Q}, \tag{14}$$
$$a^{(1)}_{Q\bar{Q}}(+,-,+) = -a^{(1)}_{Q\bar{Q}}(-,+,+)|_{\bar{q}\leftrightarrow q,\bar{Q}\leftrightarrow Q}, \tag{15}$$
$$a^{(1)}_{Q\bar{q}}(-,-,+) = -a^{(1)}_{Q\bar{q}}(+,+,+)|_{\bar{q}\leftrightarrow Q,q\leftrightarrow \bar{Q}}. \tag{16}$$

We find that $\mathcal{S}_{ij}$, $\mathcal{R}_{ij}$, $\mathcal{E}_{ij}$ and $\mathcal{C}_{Q\bar{Q}}$ respect these symmetries separately. As a result the $\mathcal{D}_{Q\bar{Q}}$ terms must also satisfy the relations (14) and (15). In the case of $\mathcal{C}_{Q\bar{q}}$, if we factor the helicity dependence into the Born terms as in Eq. (13), then the symmetry relation (16) is slightly violated which is compensated by a corresponding change in $\mathcal{D}_{Q\bar{q}}$:

$$\mathcal{D}_{Q\bar{q}}(-,-,+) = -\mathcal{D}_{Q\bar{q}}(+,+,+)|_{\bar{q}\leftrightarrow Q,q\leftrightarrow\bar{Q}} + \frac{2}{3}(N_c-N_f)a^{(0)}_{Q\bar{q}}(-,-,+)\ln\left(\frac{s_{\bar{Q}Q}}{s_{\bar{q}q}}\right). \tag{17}$$

In order to give the results in a more compact form, we introduce the following notation[2]:

$$\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_0 = \ln\left(\frac{-s_{ij}}{\mu^2}\right) - \ln\left(\frac{-s_{kl}}{\mu^2}\right), \qquad \left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_1 = \frac{1}{s_{kl}-s_{ij}}\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_0,$$
$$\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_2 = \frac{1}{(s_{kl}-s_{ij})^2}\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_0 + \frac{1}{s_{kl}(s_{kl}-s_{ij})}, \qquad \left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_3 = \frac{1}{(s_{kl}-s_{ij})^3}\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_0 + \frac{(s_{ij}+s_{kl})}{2\, s_{ij}s_{kl}(s_{kl}-s_{ij})^2}, \tag{18}$$

$$\langle ijkl\rangle = \frac{\langle ik\rangle\langle jl\rangle}{\langle il\rangle\langle jk\rangle}. \tag{19}$$

For the $a_{Q\bar{Q}}$ color structure the $\mathcal{D}$ terms take the form

$$\mathcal{D}_{Q\bar{Q}}(+,+,+) = a^{(0)}_{Q\bar{Q}}(+,+,+)$$
$$\times\Bigg\{\frac{N_c^2+1}{N_c}\Bigg[\langle\bar{q}gq\bar{Q}\rangle\langle\bar{q}Qq\bar{Q}\rangle\frac{s_{qQ}s_{qg}}{s_{\bar{q}q}s_{\bar{Q}Q}} + \langle\bar{q}gQ\bar{Q}\rangle^2 s_{Qg}\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}q \end{smallmatrix}\right\}_1 - \langle\bar{q}gQ\bar{Q}\rangle\langle\bar{q}Qg\bar{Q}\rangle s_{Qg}\left\{\begin{smallmatrix} \bar{Q}Q \\ \bar{q}q \end{smallmatrix}\right\}_1$$
$$+\langle\bar{q}Qq\bar{Q}\rangle\frac{s_{qQ}}{2}\left(s_{\bar{Q}g}\left\{\begin{smallmatrix} \bar{q}q \\ \bar{Q}g \end{smallmatrix}\right\}_2 - \langle\bar{q}gq\bar{Q}\rangle s_{qg}\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}q \end{smallmatrix}\right\}_2\right) - \langle\bar{q}gq\bar{Q}\rangle\langle\bar{q}Qg\bar{Q}\rangle s_{Qg}s_{qg}\left\{\begin{smallmatrix} \bar{q}q \\ \bar{Q}Q \end{smallmatrix}\right\}_2\Bigg]$$
$$+N_c\left[\frac{3}{2}\langle\bar{q}gQ\bar{Q}\rangle s_{Qg}\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}q \end{smallmatrix}\right\}_1\right]$$
$$+\frac{1}{N_c}\Bigg[\frac{3}{2}\left\{\begin{smallmatrix} \bar{Q}Q \\ \bar{Q}g \end{smallmatrix}\right\}_0 + \langle\bar{q}gq\bar{Q}\rangle^2 s_{qg}\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}Q \end{smallmatrix}\right\}_1 - \langle\bar{q}gq\bar{Q}\rangle\langle\bar{q}qg\bar{Q}\rangle s_{qg}\left\{\begin{smallmatrix} \bar{Q}q \\ \bar{q}Q \end{smallmatrix}\right\}_1$$
$$-\langle\bar{q}qQ\bar{Q}\rangle\langle\bar{q}Qq\bar{Q}\rangle s_{qQ}\left(\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}q \end{smallmatrix}\right\}_1 + \left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}Q \end{smallmatrix}\right\}_1\right) - \frac{1}{2}\langle\bar{q}Qq\bar{Q}\rangle s_{qQ}\left\{\begin{smallmatrix} \bar{Q}g \\ \bar{q}q \end{smallmatrix}\right\}_1\Bigg]\Bigg\} \tag{20}$$

and

[2] Note that the function $\left\{\begin{smallmatrix} ij \\ kl \end{smallmatrix}\right\}_n$ has $-2n$ mass dimensions. These functions are related to similar ones introduced in Ref. [6].

$$
\begin{aligned}
\mathcal{D}_{Q\bar Q}(-,+,+) &= a^{(0)}_{Q\bar Q}(-,+,+) \\
&\times \Bigg\{ \frac{N_c^2+1}{N_c} \Bigg[\langle \bar q \bar Q q g\rangle \langle \bar q \bar Q q Q\rangle \frac{s_{\bar q g} s_{\bar q Q}}{s_{\bar q q} s_{\bar Q Q}} + \langle \bar Q Q g q\rangle^2 s_{Qg} \left\{\begin{smallmatrix} \bar Q g \\ \bar q q \end{smallmatrix}\right\}_1 + \langle \bar q q \bar Q Q\rangle \langle \bar q \bar Q q Q\rangle s_{\bar q Q} \left\{\begin{smallmatrix} \bar Q g \\ q Q \end{smallmatrix}\right\}_1 \\
&- \langle \bar q \bar Q q g\rangle s_{\bar q g} \left(\langle \bar q \bar Q q g\rangle \left\{\begin{smallmatrix} \bar Q g \\ q Q \end{smallmatrix}\right\}_1 - \langle \bar q q \bar Q g\rangle \left\{\begin{smallmatrix} q Q \\ \bar q \bar Q \end{smallmatrix}\right\}_1 \right) - \langle \bar q \bar Q q Q\rangle \left(1 + \langle \bar q \bar Q q Q\rangle\right) s_{\bar q Q} \left\{\begin{smallmatrix} \bar Q g \\ \bar q q \end{smallmatrix}\right\}_1 \\
&- \langle \bar Q g Q q\rangle s_{Qg} \left(\langle \bar Q Q g q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 + \langle \bar q \bar Q q g\rangle s_{\bar q g} \left\{\begin{smallmatrix} \bar q q \\ \bar Q Q \end{smallmatrix}\right\}_2 \right) + \frac{1}{2} \langle \bar q \bar Q q Q\rangle^2 s^2_{\bar q Q} \left\{\begin{smallmatrix} \bar Q g \\ \bar q q \end{smallmatrix}\right\}_2 \Bigg] \\
&+ N_c \left[\frac{3}{2} \left\{\begin{smallmatrix} \bar Q g \\ \bar q q \end{smallmatrix}\right\}_0 + 3 \langle \bar q \bar Q q Q\rangle s_{\bar q Q} \left\{\begin{smallmatrix} \bar Q g \\ \bar q q \end{smallmatrix}\right\}_1 \right] + \frac{3}{2N_c} \left\{\begin{smallmatrix} \bar Q Q \\ \bar Q g \end{smallmatrix}\right\}_0 \Bigg\}. \qquad (21)
\end{aligned}
$$

Since the results for the $a_{Q\bar q}$ color structure are somewhat more complicated we make the decomposition

$$
\begin{aligned}
\mathcal{D}_{Q\bar q}(h_q, h_Q, +) &= a^{(0)}_{Q\bar q}(h_q, h_Q, +) \\
&\times \left[\frac{2}{3}(N_c - N_f) \mathcal{D}^f_{Q\bar q}(h_q, h_Q) + \frac{N_c^2+1}{N_c} \mathcal{D}^m_{Q\bar q}(h_q, h_Q) + N_c \mathcal{D}^l_{Q\bar q}(h_q, h_Q) + \frac{1}{N_c} \mathcal{D}^s_{Q\bar q}(h_q, h_Q) \right]. \qquad (22)
\end{aligned}
$$

We have to record the $\mathcal{D}_{Q\bar q}(h_q, h_Q)$ terms for three different helicity combinations. In the case of $h_q = h_Q = +$, for the flavor dependent part we get

$$
\begin{aligned}
\mathcal{D}^f_{Q\bar q}(+,+) &= \langle \bar q \bar Q g Q\rangle s_{\bar Q g} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 \\
&- \frac{1}{2} \langle \bar q \bar Q q Q\rangle \langle \bar q q g \bar Q\rangle s_{qg} s_{Qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_2 + \langle \bar q g q Q\rangle \langle \bar q Q g \bar Q\rangle^2 s_{qg} s^2_{Qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_3, \qquad (23)
\end{aligned}
$$

while the term proportional to $(N_c^2+1)/N_c$ reads

$$
\begin{aligned}
\mathcal{D}^m_{Q\bar q}(+,+) &= \frac{1}{2} \langle \bar q q g Q\rangle + \langle \bar q q g \bar Q\rangle s_{qg} \left(\langle q Q \bar q \bar Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 - \langle \bar q g q Q\rangle \langle \bar q g q \bar Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q g \end{smallmatrix}\right\}_1 \right) \\
&+ \frac{1}{2} \langle \bar q g q Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q g \end{smallmatrix}\right\}_0 - \frac{1}{2} \langle \bar q q g Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_0 - 2 \langle \bar q q Q \bar Q\rangle \langle \bar q Q q \bar Q\rangle s_{qQ} \left\{\begin{smallmatrix} q \bar Q \\ \bar q g \end{smallmatrix}\right\}_1 \\
&+ \langle \bar q g q Q\rangle \langle \bar q q g \bar Q\rangle^2 s_{qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 + \frac{1}{2} \langle \bar q g q Q\rangle \langle \bar q Q q \bar Q\rangle^2 s^2_{qQ} \left\{\begin{smallmatrix} \bar q g \\ \bar Q Q \end{smallmatrix}\right\}_2 \\
&+ \frac{1}{2} \langle q Q \bar q \bar Q\rangle \langle \bar q q g \bar Q\rangle \left(\langle \bar q Q g \bar Q\rangle s^2_{Qg} \left\{\begin{smallmatrix} \bar q q \\ \bar Q Q \end{smallmatrix}\right\}_2 - \langle \bar q g q \bar Q\rangle s^2_{qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_2 \right) \qquad (24)
\end{aligned}
$$

and the leading and subleading color terms are

$$
\begin{aligned}
\mathcal{D}^l_{Q\bar q}(+,+) &= \langle \bar q Q q \bar Q\rangle \left(2 \left\{\begin{smallmatrix} q \bar Q \\ \bar q g \end{smallmatrix}\right\}_0 - \left\{\begin{smallmatrix} \bar Q Q \\ q \bar Q \end{smallmatrix}\right\}_0 \right) + \langle \bar q Q g \bar Q\rangle s_{Qg} \left(2 \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 + 3 \langle \bar q Q q \bar Q\rangle \left\{\begin{smallmatrix} q \bar Q \\ \bar q g \end{smallmatrix}\right\}_1 \right) \\
&+ \langle \bar q q g \bar Q\rangle \langle \bar q g q Q\rangle s_{qg} \left(2 + \langle \bar q q Q \bar Q\rangle\right) \left\{\begin{smallmatrix} \bar Q Q \\ \bar q g \end{smallmatrix}\right\}_1 - \langle q Q \bar q \bar Q\rangle \langle \bar q q g \bar Q\rangle^2 s_{qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1, \qquad (25) \\
\mathcal{D}^s_{Q\bar q}(+,+) &= -\frac{1}{2} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_0 + \langle \bar q g q Q\rangle s_{qg} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 \left(1 - 3 \langle \bar q q g \bar Q\rangle\right) + 2 \langle \bar q Q q \bar Q\rangle^2 s_{qQ} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q g \end{smallmatrix}\right\}_1 \\
&+ 2 \langle q Q \bar q \bar Q\rangle \langle \bar q q g \bar Q\rangle s_{qg} \left(\langle \bar q q g \bar Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q g \end{smallmatrix}\right\}_1 - \langle \bar q g q \bar Q\rangle \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 \right) + \langle \bar q \bar Q g Q\rangle s_{\bar Q g} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1. \qquad (26)
\end{aligned}
$$

The results for the helicity configuration $h_q = -, h_Q = +$ are

$$
D^f_{Q\bar q}(-,+) = \langle \bar q \bar Q g Q\rangle s_{\bar Q g} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_1 + \langle \bar q \bar Q q Q\rangle s_{Qg} \left(\frac{1}{2} s_{\bar q g} \left\{\begin{smallmatrix} \bar q q \\ \bar Q Q \end{smallmatrix}\right\}_2 - \langle \bar q \bar Q q g\rangle s^2_{\bar q g} \left\{\begin{smallmatrix} \bar Q Q \\ \bar q q \end{smallmatrix}\right\}_3 \right), \qquad (27)
$$

$$D^m_{Q\bar q}(-,+) = \frac{1}{2} + \frac{1}{2}\langle \bar q\bar Q qQ\rangle^2 \frac{s_{\bar q g}s_{\bar q Q}}{s_{\bar q q}s_{\bar Q Q}} - \langle \bar q\bar Q qQ\rangle s_{Qg}\left\{{\bar Q Q \atop \bar q q}\right\}_1 - \frac{3}{2}\langle \bar q\bar Q qQ\rangle s_{\bar q g}\left\{{\bar Q Q \atop \bar q q}\right\}_1$$
$$-\frac{1}{2}\langle \bar q\bar Q qQ\rangle\langle \bar Q g Q q\rangle s_{Qg}(s_{\bar q g} - s_{Qg})\left\{{\bar q q \atop \bar Q Q}\right\}_2, \tag{28}$$

$$D^l_{Q\bar q}(-,+) = \frac{3}{2}\langle \bar Q g Q q\rangle s_{Qg}\left\{{\bar Q Q \atop \bar q q}\right\}_1 + \frac{3}{2}\langle \bar q\bar Q q g\rangle s_{\bar q g}\left\{{\bar Q Q \atop \bar q q}\right\}_1, \tag{29}$$

$$D^s_{Q\bar q}(-,+) = 2\langle \bar q\bar Q qQ\rangle\left(\left\{{\bar q\bar Q \atop \bar Q Q}\right\}_0 + \langle \bar Q Q g q\rangle s_{Qg}\left\{{qQ \atop \bar q\bar Q}\right\}_1 - \langle \bar q\bar Q q g\rangle s_{\bar q g}\left\{{\bar Q Q \atop \bar q q}\right\}_1\right). \tag{30}$$

Finally we give the results for $h_q = +, h_Q = -$. The flavor dependent part is

$$\mathcal{D}^f_{Q\bar q}(+,-) = \langle \bar q\bar Q g Q\rangle s_{\bar Q g}\left(\left\{{\bar Q Q \atop \bar q q}\right\}_1 - \frac{1}{2}\langle \bar q g q Q\rangle s_{qg}\left\{{\bar Q Q \atop \bar q q}\right\}_2 + \langle \bar q g q Q\rangle\langle \bar q\bar Q g Q\rangle s_{\bar Q g}s_{qg}\left\{{\bar Q Q \atop \bar q q}\right\}_3\right). \tag{31}$$

For the term proportional to $(N_c^2+1)/N_c$ we have

$$\mathcal{D}^m_{Q\bar q}(+,-) = \frac{1}{2} + \frac{1}{2}\langle \bar q\bar Q g Q\rangle^2 \frac{s_{\bar Q g}}{s_{\bar Q Q}}\left(1 - \langle \bar q g q Q\rangle\frac{s_{\bar Q g}}{s_{\bar q g}}\right) - \frac{1}{2}\langle \bar q\bar Q g Q\rangle\langle \bar q\bar Q qQ\rangle^2 \frac{s^2_{q\bar Q}}{s_{\bar q q}s_{\bar Q Q}}$$
$$+\langle \bar q g q Q\rangle\langle \bar q q g Q\rangle^2 s_{qg}\left(\left\{{\bar Q Q \atop \bar q g}\right\}_1 + \left\{{\bar Q Q \atop \bar q q}\right\}_1\right) - \frac{1}{2}\langle \bar q g q Q\rangle s_{\bar q g}\left\{{\bar Q Q \atop \bar q q}\right\}_1$$
$$+\langle \bar q g\bar Q Q\rangle\langle \bar q\bar Q g Q\rangle s_{\bar Q g}\left(\frac{1}{2} + \langle \bar q g\bar Q Q\rangle\right)\left\{{\bar Q Q \atop \bar q q}\right\}_1 + \langle \bar q\bar Q g Q\rangle\langle \bar q g\bar Q Q\rangle^2 s_{\bar Q g}\left\{{Qg \atop \bar q q}\right\}_1$$
$$+\frac{1}{2}\langle \bar q g q Q\rangle\langle \bar q\bar Q g Q\rangle s_{qg}\left(\langle \bar q q g Q\rangle s_{\bar Q g}\left\{{\bar Q Q \atop \bar q g}\right\}_2 - \langle \bar q\bar Q qQ\rangle s_{\bar Q q}\left\{{Qg \atop \bar q q}\right\}_2\right)$$
$$+\frac{1}{2}\langle \bar q\bar Q g Q\rangle\langle \bar q\bar Q qQ\rangle s_{Qg}s_{q\bar Q}\left\{{\bar q q \atop Qg}\right\}_2$$
$$+\frac{1}{2}\langle \bar q g q Q\rangle\langle \bar q\bar Q g Q\rangle^2 s_{\bar Q g}\left(s_{\bar Q g} - s_{qg}\right)\left\{{\bar q q \atop \bar Q Q}\right\}_2 - \frac{1}{2}\langle \bar q g q Q\rangle\langle \bar q q g Q\rangle s_{qg}s_{\bar Q Q}\left\{{\bar q g \atop \bar Q Q}\right\}_2, \tag{32}$$

the leading color term reads

$$\mathcal{D}^l_{Q\bar q}(+,-) = \frac{3}{2}\langle \bar q g q Q\rangle\langle \bar q q g Q\rangle s_{qg}\left\{{\bar Q Q \atop \bar q g}\right\}_1 + \frac{3}{2}\langle \bar q g\bar Q Q\rangle\langle \bar q\bar Q g Q\rangle s_{\bar Q g}\left\{{\bar q q \atop Qg}\right\}_1, \tag{33}$$

and the subleading color term is

$$\mathcal{D}^s_{Q\bar q}(+,-) = -\langle \bar q g q Q\rangle\left(\frac{1}{2}\left\{{\bar q g \atop \bar q q}\right\}_0 + \left\{{\bar Q Q \atop \bar q q}\right\}_0 - 2\langle \bar q\bar Q qQ\rangle\left\{{qQ \atop \bar q g}\right\}_0\right)$$
$$+\langle \bar q q g Q\rangle\left(\frac{3}{2}\left\{{\bar Q Q \atop \bar q q}\right\}_0 + 2\langle \bar q\bar Q qQ\rangle\left\{{\bar q\bar Q \atop \bar q q}\right\}_0\right) - \langle \bar q g\bar Q Q\rangle\langle \bar q\bar Q g Q\rangle s_{\bar Q g}\left(\frac{3}{2} + 2\langle \bar q\bar Q qQ\rangle\right)\left\{{Qg \atop \bar q q}\right\}_1$$
$$+\langle \bar q g\bar Q Q\rangle\langle \bar q\bar Q g Q\rangle s_{\bar Q g}\left(2\langle \bar q\bar Q qQ\rangle\left\{{qQ \atop \bar q g}\right\}_1 + 2\langle \bar q g q Q\rangle\left\{{qQ \atop \bar q\bar Q}\right\}_1\right)$$
$$+\langle \bar q\bar Q g Q\rangle^2 s_{\bar Q g}\left(2\langle \bar q g q Q\rangle + 3\langle \bar Q g q Q\rangle\right)\left\{{\bar Q Q \atop \bar q q}\right\}_1 - \langle \bar q g q Q\rangle\langle \bar q q g Q\rangle s_{qg}\left\{{\bar Q Q \atop \bar q g}\right\}_1$$
$$-\langle \bar q g q Q\rangle\langle \bar q\bar Q qQ\rangle s_{q\bar Q}\left(\frac{1}{2} + 2\langle \bar q\bar Q qQ\rangle\right)\left\{{\bar Q Q \atop \bar q g}\right\}_1 + 2\langle \bar q\bar Q qQ\rangle\langle \bar q g q Q\rangle\langle \bar q q g Q\rangle s_{qg}\left\{{Qg \atop \bar q\bar Q}\right\}_1. \tag{34}$$

The $\mathcal{E}$ contributions can conveniently be given in terms of an auxiliary function

$$(ijk)_n = \langle r(h_q)ikR(h_Q)\rangle^n\mathcal{F}(i,j,k), \tag{35}$$

where $\langle \ldots \rangle$ has been defined in (19) and

$$\mathcal{F}(i,j,k,) = -\mathrm{Li}_2\left(1-\frac{s_{mn}}{s_{ij}}\right) - \mathrm{Li}_2\left(1-\frac{s_{mn}}{s_{jk}}\right) - \frac{1}{2}\ln^2\left(\frac{s_{ij}}{s_{jk}}\right) - \frac{\pi^2}{6}. \quad (36)$$

Li_2 is the dilogarithm and m,n denote the complementary labels to (i,j,k) in the set of labels $(\bar{q},\bar{Q},Q,q,g)$

$$\{m,n\} = \{\bar{q},\bar{Q},Q,q,g\}\backslash\{i,j,k\}. \quad (37)$$

We use the convention that if both $\langle rikR\rangle = 0$ and $n=0$ then $(ijk)_n = \mathcal{F}(i,j,k)$. Then the $\mathcal{E}_{Q\bar{q}}(h_q,h_Q)$ term for arbitrary h_q, h_Q quark helicity configuration is given as

$$\begin{aligned}\mathcal{E}_{Q\bar{q}}(h_q,h_Q,+) &= -a^{(0)}_{Q\bar{q}}(h_q,h_Q,+)\,\mathcal{G}^{h_qh_Q}_{Q\bar{q}}(\bar{q},\bar{Q},Q,q,g)\\ &+a^{(0)}_{Q\bar{Q}}(h_q,h_Q,+)\,\mathcal{G}^{h_qh_Q}_{Q\bar{Q}}(\bar{q},\bar{Q},Q,q,g) + a^{(0)}_{q\bar{q}}(h_q,h_Q,+)\,\mathcal{G}^{h_qh_Q}_{q\bar{q}}(\bar{q},\bar{Q},Q,q,g),\end{aligned} \quad (38)$$

where

$$\begin{aligned}\mathcal{G}^{h_qh_Q}_{Q\bar{q}}(\bar{q},\bar{Q},Q,q,g) &= N_c\left[(\bar{q}gQ)_0 + (\bar{q}q\bar{Q})_2 + (q\bar{q}g)_2 + (gQ\bar{Q})_2 + (q\bar{Q}Q)_2\right]\\ &-\frac{1}{N_c}\left[(qg\bar{Q})_0 + (\bar{q}gQ)_0 - (\bar{q}g\bar{Q})_0 - (qgQ)_0 + (\bar{q}Q\bar{Q})_2 + (\bar{q}q\bar{Q})_2\right.\\ &\left.+(q\bar{q}Q)_2 + (q\bar{Q}Q)_2 - (qQ\bar{Q})_2 - (q\bar{q}\bar{Q})_2 - (\bar{q}\bar{Q}Q)_2 - (\bar{q}qQ)_2\right],\end{aligned} \quad (39)$$

$$\begin{aligned}\mathcal{G}^{h_qh_Q}_{Q\bar{Q}}(\bar{q},\bar{Q},Q,q,g) &= N_c(gQ\bar{Q})_2\\ &-\frac{1}{N_c}\left[-(\bar{q}g\bar{Q})_0 + (\bar{Q}gq)_0 + (\bar{q}qg)_2 - (q\bar{q}g)_2 + (\bar{q}Q\bar{Q})_2 - (\bar{q}q\bar{Q})_2 - (qQ\bar{Q})_2 + (q\bar{q}\bar{Q})_2\right.\\ &\left.-(\bar{Q}Qg)_{2\delta_{QR}} + (\bar{q}\bar{Q}g)_{2(1-\delta_{QR})} - (\bar{q}Qg)_{2\delta_{QR}} + (qQg)_{2\delta_{QR}} - (q\bar{Q}g)_{2(1-\delta_{QR})}\right],\end{aligned} \quad (40)$$

$$\begin{aligned}\mathcal{G}^{h_qh_Q}_{q\bar{q}}(\bar{q},\bar{Q},Q,q,g) &= N_c(q\bar{q}g)_2\\ &-\frac{1}{N_c}\left[(\bar{Q}gq)_0 - (Qgq)_0 - (gQ\bar{Q})_2 + (g\bar{Q}Q)_2 + (q\bar{q}Q)_2 - (q\bar{Q}Q)_2 - (q\bar{q}\bar{Q})_2 + (qQ\bar{Q})_2\right.\\ &\left.-(g\bar{q}q)_{2(1-\delta_{rq})} + (g\bar{q}\bar{Q})_{2(1-\delta_{rq})} - (gq\bar{Q})_{2\delta_{rq}} + (gqQ)_{2\delta_{rq}} - (g\bar{q}Q)_{2(1-\delta_{rq})}\right].\end{aligned} \quad (41)$$

As noted before, the $\mathcal{E}_{Q\bar{Q}}$ terms satisfy the symmetry relations (14) and (15), therefore it is sufficient to present the $\mathcal{E}_{Q\bar{Q}}$ type contributions for $h_q = +$ and arbitrary h_Q:

$$\begin{aligned}\mathcal{E}_{Q\bar{Q}}(+,h_Q,+) &= a^{(0)}_{Q\bar{Q}}(+,h_Q,+)\,\mathcal{H}^{h_Q}_{Q\bar{Q}}(\bar{q},\bar{Q},Q,q,g)\\ &+a^{(0)}_{Q\bar{q}}(+,h_Q,+)\,\mathcal{H}^{h_Q}_{Q\bar{q}}(\bar{q},\bar{Q},Q,q,g) + a^{(0)}_{q\bar{Q}}(+,h_Q,+)\,\mathcal{H}^{h_Q}_{q\bar{Q}}(\bar{q},\bar{Q},Q,q,g),\end{aligned} \quad (42)$$

where the auxiliary functions, $\mathcal{H}^{h_Q}_{ij}$ have the form

$$\begin{aligned}\mathcal{H}^{h_Q}_{Q\bar{Q}}(\bar{q},\bar{Q},Q,q,g) &= N_c\left[(\bar{q}gq)_0 + (Qqg)_0 - (\bar{Q}g\bar{q})_0 - (Qgq)_0 - (Qq\bar{q})_0 - (gq\bar{q})_0\right.\\ &\left.+ (g\bar{q}\bar{Q})_2 - (q\bar{q}\bar{Q})_2 - (g\bar{q}q)_2\right]\\ &-\frac{1}{N_c}\left[(\bar{q}qQ)_0 - (\bar{q}q\bar{Q})_0 + (\bar{q}\bar{Q}g)_0 - (\bar{q}Qg)_0 + (q\bar{q}\bar{Q})_2 - (q\bar{q}Q)_2\right.\\ &\left.- (Q\bar{Q}g)_{2(1-\delta_{RQ})} - (q\bar{Q}g)_{2(1-\delta_{RQ})} + (qQg)_{2\delta_{RQ}} - (\bar{Q}Qg)_{2\delta_{RQ}}\right],\end{aligned} \quad (43)$$

$$\mathcal{H}^{h_Q}_{Q\bar{q}}(\bar{q},\bar{Q},Q,q,g) = N_c\,\big[(\bar{q}g\bar{Q})_0 - (\bar{q}gq)_0 + (Qq\bar{q})_0 - (Qqg)_0 - (Q\bar{Q}\bar{q})_0 - (g\bar{q}\bar{Q})_2 + (g\bar{q}q)_2 + (g\bar{Q}Q)_2\big]\,, \tag{44}$$

$$\mathcal{H}^{h_Q}_{q\bar{Q}}(\bar{q},\bar{Q},Q,q,g) = N_c\,\big[(qgQ)_0 - (qg\bar{q})_0 - (Qqg)_0 + (\bar{q}qg)_0 + (q\bar{q}\bar{Q})_2 - (g\bar{q}\bar{Q})_2 + (gQ\bar{Q})_2 - (qQ\bar{Q})_2 + (g\bar{q}q)_2 - (q\bar{q}g)_2\big]\,. \tag{45}$$

The above results are valid in the unphysical region, where the dot products are negative, therefore the arguments of the logarithms and dilogarithms are away from the branch cuts. To obtain the amplitudes in any physical channel, one has to continue analytically to the corresponding physical region and make the usual substitution

$$s_{ij} \longrightarrow s_{ij} + i\eta. \tag{46}$$

This defines all functions in a unique way.

As a consistency check we investigated the limiting values of our results for configurations when two of the external momenta, say a and b, become collinear. As expected, at most single pole terms $1/\langle ab\rangle$ or $1/[ab]$ have been found[3]. Due to color ordering some color amplitudes must remain finite for certain collinear configurations. Our amplitudes fulfill also these requirements.

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